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SPOC

Simulated Processing of Ore and Coal

Chapter 3.1 BILMAT Computer Program

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The **SPOC** Manual

Chapter 3.1 BILMAT Computer Program **BILMAT — Computer Program for Material Balance Data Adjustment**

D. Hodouin and F. Flament

Editor: D. Laguitton

Contributors: R. Poirier, J. Wilson and M.D. Evereli

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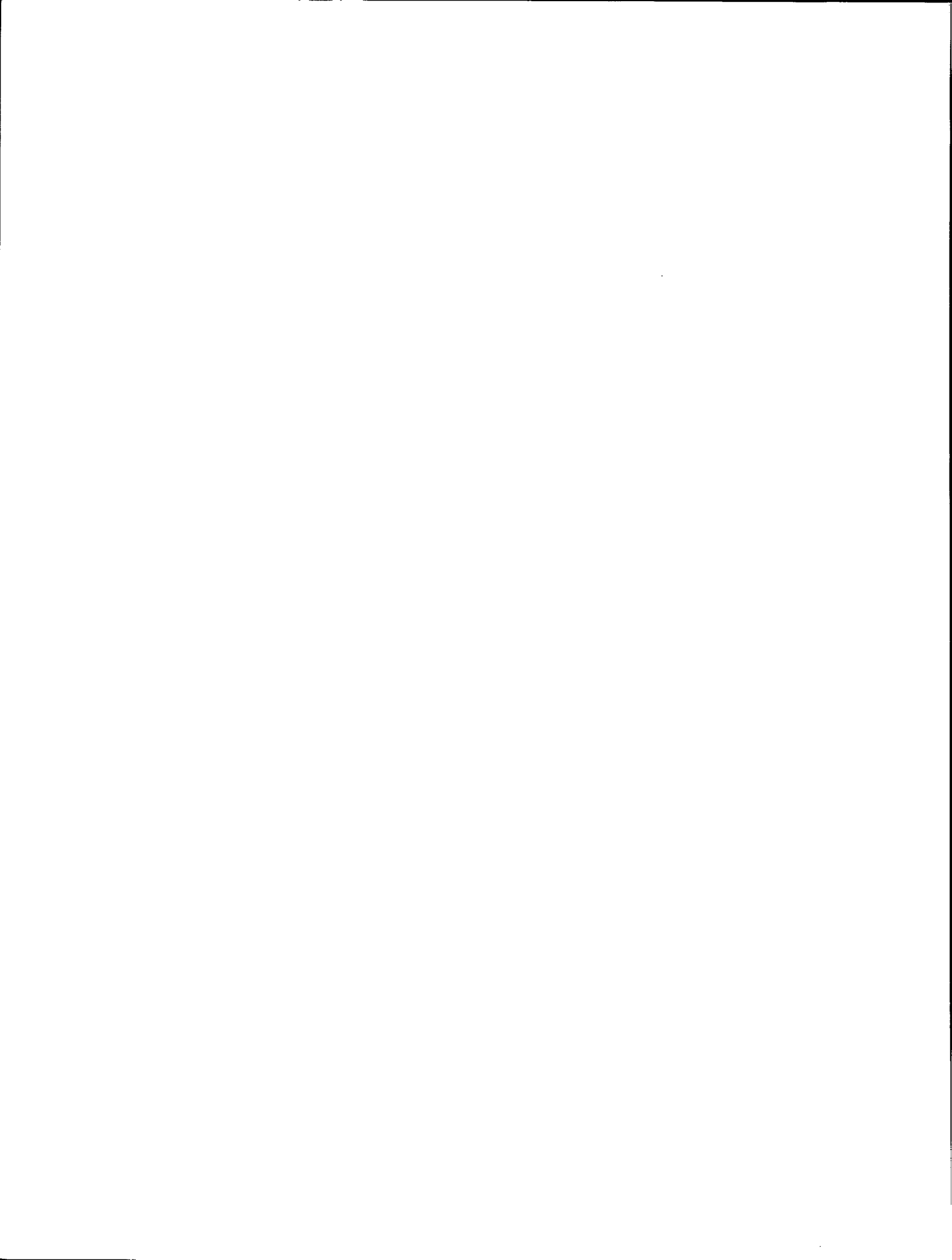
The **SPOC*** manual consists of eighteen chapters, published separately. Their numbers and short titles are as follows:

- | | |
|------------------------------------|--------------------------------------|
| 1. Summary | 5. Unit Models: Part A |
| 2. Sampling Methodology | 5.1 Unit Models: Part B |
| 2.1 SAMBA Computer Program | 5.2 Unit Models: Part C |
| 2.2 Grinding Circuit Sampling | 6. Flowsheet Simulators |
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*Simulated Processing of Ore and Coal



FOREWORD

High energy costs and depleting ore reserves combine to make process evaluation and optimization a challenging goal in the 80's. The spectacular growth of computer technology in the same period has resulted in widely available computing power that can be distributed to the most remote mineral processing operations. The SPOC project, initiated at CANMET in 1980, has undertaken to provide Canadian industry with a coherent methodology for process evaluation and optimization assisted by computers. The SPOC Manual constitutes the written base of this methodology and covers most aspects of steady-state process evaluation and simulation. It is expected to facilitate industrial initiatives in data collection and model upgrading.

Creating a manual covering multidisciplinary topics and involving contributions from groups in universities, industry and government is a complex endeavour. The reader will undoubtedly notice some heterogeneities resulting from the necessary compromise between ideals and realistic objectives or, more simply, from oversight. Critiques to improve future editions are welcomed.

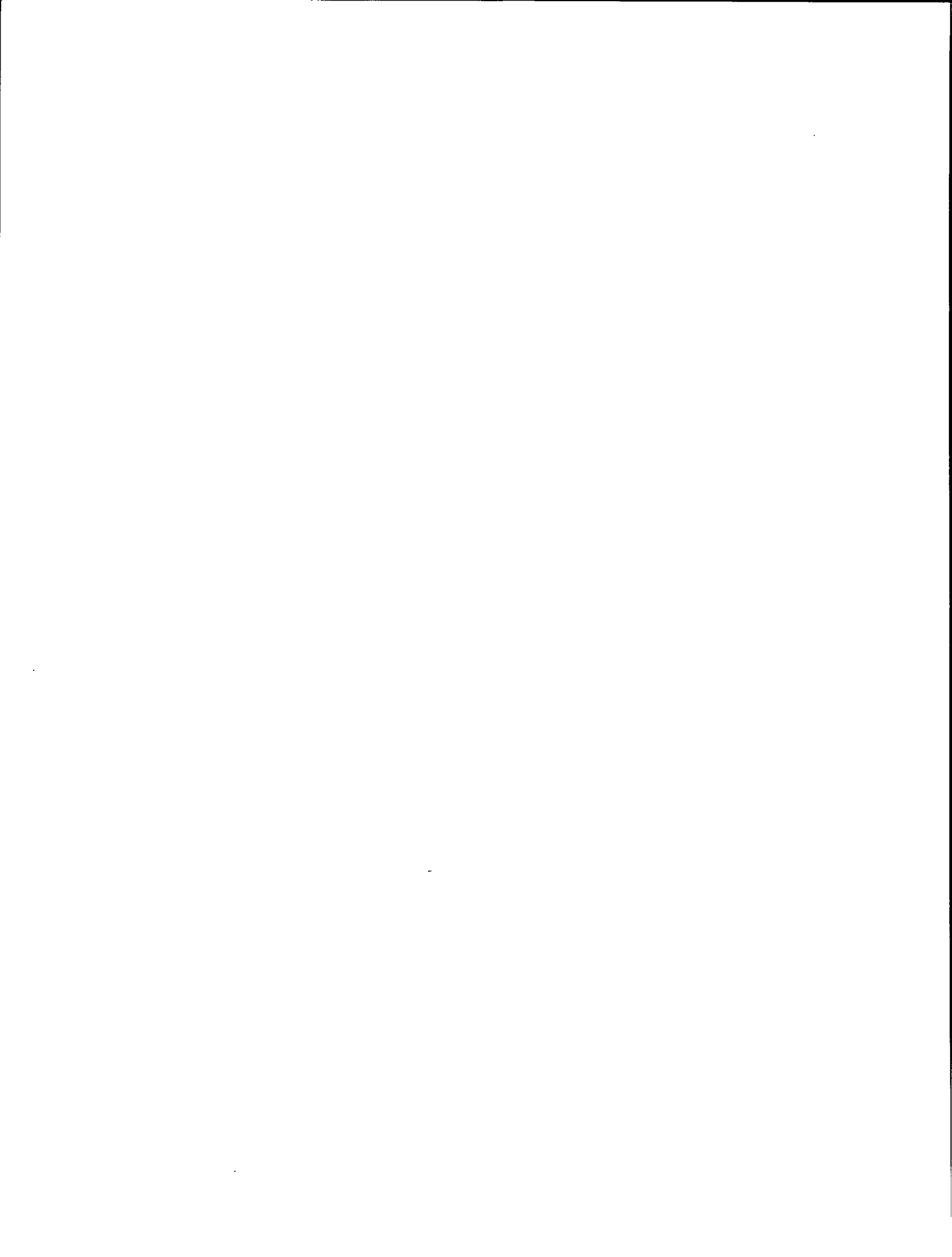
D. Laguitton
SPOC Project Leader
Canada Centre for Mineral and Energy Technology

AVANT-PROPOS

La croissance des coûts de l'énergie et l'appauvrissement des gisements ont fait de l'évaluation et de l'optimisation des procédés un défi des années 80 au moment même où s'effectuait la dissémination de l'informatique jusqu'aux concentrateurs les plus isolés. Le projet SPOC, a été lancé en 1980 au CANMET, en vue de développer pour l'industrie canadienne, une méthodologie d'application de l'informatique à l'évaluation et à l'optimisation des procédés minéralurgiques. Le Manuel SPOC constitue la documentation écrite de cette méthodologie et en couvre les différents éléments. Les retombées devraient en être une vague nouvelle d'échantillonnages et d'amélioration de modèles.

La rédaction d'un ouvrage couvrant différentes disciplines et rassemblant des contributions de groupes aussi divers que les universités, l'industrie et le gouvernement est une tâche complexe. Le lecteur notera sans aucun doute des ambiguïtés ou contradictions qui ont pu résulter de la diversité des sources, de la traduction ou tout simplement d'erreurs. La critique constructive est encouragée afin de parvenir au format et au contenu de la meilleure qualité possible.

D. Laguitton
Chef du projet SPOC,
Centre canadien de la technologie des minéraux et de l'énergie



ABSTRACT

Material balance computations for complex ore and coal flowsheets are based on large experimental data sets and require a computer. The BILMAT program has been developed to solve such complex problems and compute a sensitivity analysis of the estimated process variables.

This user's guide has been written for two main types of users: the process engineer who wants to access it as a black-box to compute his material balances and the research engineer interested in the method and the program itself. The FORTRAN routines are fully documented and two sample runs are presented.

The source programs are available on tape or diskettes from CANMET.

RÉSUMÉ

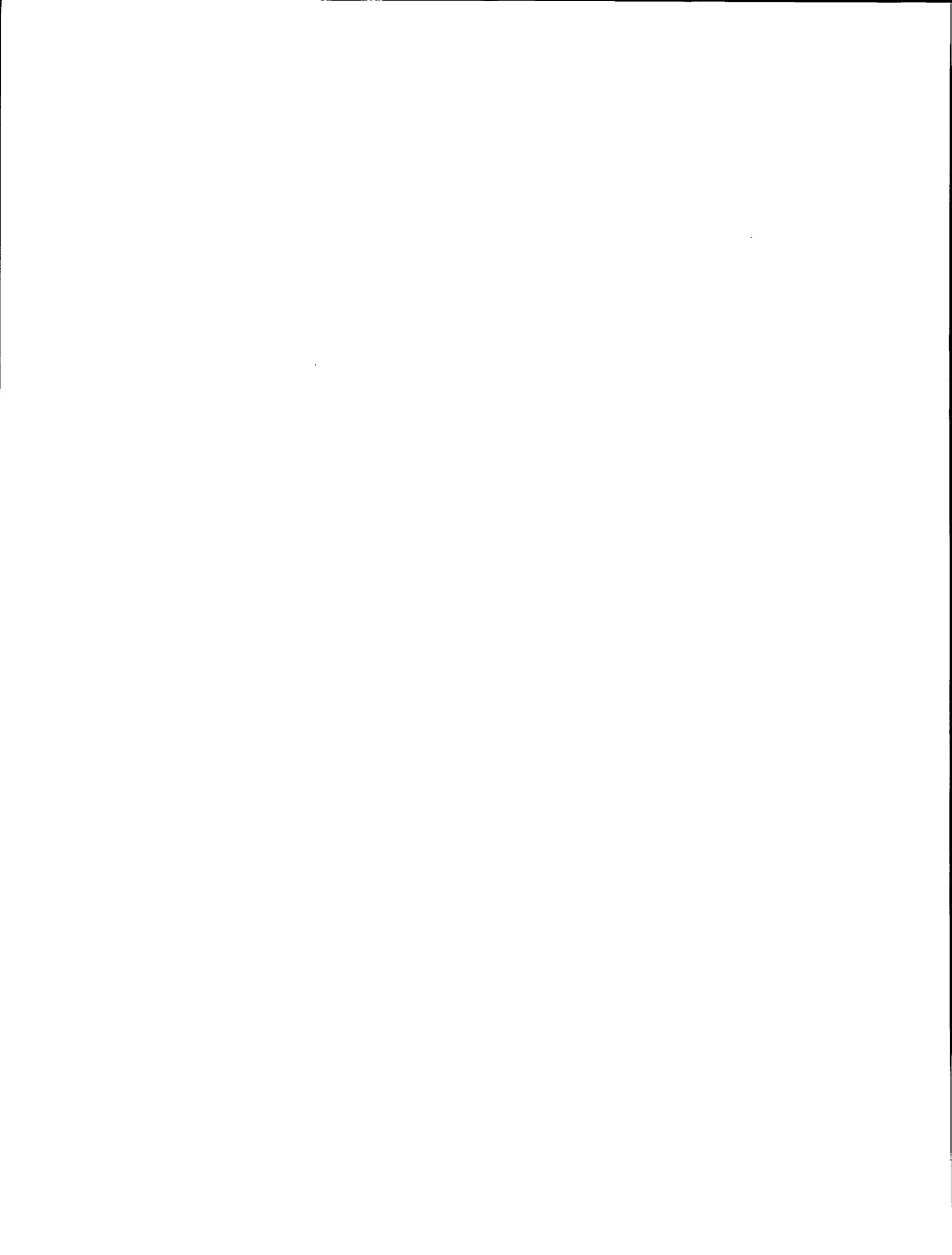
Les calculs de bilans matière pour des circuits complexes de traitement des minerais et du charbon utilisent un grand nombre de mesures et requièrent l'accès à un ordinateur. Le programme BILMAT permet de résoudre de tels problèmes et effectue un calcul de sensibilité des variables calculées.

Ce manuel s'adresse à deux types principaux d'utilisateurs: l'ingénieur de procédé qui veut l'utiliser comme boîte noire pour calculer ses bilans matière et le chercheur intéressé au détail de la méthode et du programme. Les programmes en FORTRAN sont pleinement documentés et deux problèmes types sont présentés et résolus à l'aide du programme.

Les programmes sont disponibles sur ruban magnétique ou sur disquettes en communiquant avec CANMET.

ACKNOWLEDGEMENTS

The SPOC project has benefited from such a wide range of contributions throughout the industry, the university, and the government sectors that a nominal acknowledgement would be bound to make unfair omissions. The main groups that contributed are: the various contractors who completed project elements; the Industrial Steering Committee members who met seven times to provide advice to the project leader; the various users of project documents and software who provided feedback on their experience; the CANMET Mineral Sciences Laboratories staff members who handled the considerable in-house task of software development, maintenance, and documentation; the EMR Computer Science Centre staff who were instrumental in some software development; and the CANMET Publications Section. Inasmuch as in a snow storm, every flake is responsible, their contributions are acknowledged.



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1. INTRODUCTION

1.1 IDENTIFICATION OF THE PROBLEM

The computation of a material balance (that is, the determination of stream flow rates and compositions) around any metallurgical or mineral processing unit is a basic step in process evaluation. In spite of the simplicity of the physical laws of mass conservation and of the very fundamental need to know the flow of components around a circuit, it appears, especially in mineral processing, that material balancing is quite often a difficult task and often leads to unreliable results.

The reasons for this are the problems associated with collecting data in ore and coal processing plants. The plants are generally poorly instrumented and data must often be collected through hand-sampling. Due to the heterogeneous nature of the streams, segregation occurs and, therefore, samples may not be representative — a problem made worse by the nonexistence of circuit-sampling facilities which are often completely ignored by plant designers. Furthermore, the nature of measurements to be performed (for instance, particle size distributions in the fine range) frequently produces inaccurate results.

In most cases, main variables, such as the flow rates of various slurry streams, are not measured and must be obtained by computation — a procedure which is very sensitive to measurement errors in stream composition. This is particularly true when the process flowsheet is complex and contains many recycling streams. The various formulae available to calculate the flow rates often lead to different results. Furthermore, the results vary with the manner of selecting the experimental data which are generally more abundant than necessary to calculate flow rates.

1.2 STATE OF THE ART

During the 70's several significant R&D projects were performed to find systematic methods of simultaneously balancing all the components in all the streams of a process flowsheet. The proposed methods are generally based on least-squares procedures: they correct the raw data for measurement errors in order to make them consistent from a mass conservation point of view and simultaneously calculate the unknown variables, such as the streams flow rates.

Publications in this area indicate that there is great interest in such data-adjustment techniques for various purposes (1 to 13). First of all they can be used for global inventory of the inputs and outputs of a plant, or for evaluation of material and energy exchanges between plants. More frequently, they are used to obtain the best *image* of the state of a process. They are also powerful tools to evaluate the performance of processes since they can give accurate values of the various component

flows. They are also a preliminary and necessary method to upgrade raw data obtained after sampling around various units of a process and to give a reliable base of data for the building of unit models which are the basic elements of a process or plant computer simulator (14,15).

The programs developed in the 70's are generally based on the same fundamental principles which take into consideration the statistical properties of the data errors and all the mass conservation equations implied in a flowsheet. They differ by the mathematics used to solve the problem, by their versatility, by the variety of data which can be handled, and, finally, by their availability to outside users (which is usually limited).

1.3 PROJECT OBJECTIVES

The purpose of the present project was to develop a FORTRAN mass balance program offering more capabilities, versatility, and user-friendliness than other existing packages. The following requirements for the package were especially emphasized:

- to facilitate improvement of the package using modular programming style and extensive documentation;
- to facilitate entry, retrieval, and modification of data;
- to provide full sample runs;
- to optimize execution and to minimize memory requirements.

1.4 REPORT CONTENTS

This report is an attempt to clearly define the principles of the BILMAT method, the program structure and capabilities, and the technique to use the program. This report is directed towards various users:

1. Users who want to use the program as a *black-box* to analyze their industrial raw data.
2. Users who want to optimize their material balances, who understand well the principles and wish to rethink their sampling strategy or to compare various methods.
3. Users who require a tool for very accurate analysis of complex data for process modelling and simulation.
4. Users involved in mineral processing R & D who wish to modify or improve BILMAT with new features.

This report is divided into five parts. The first is the Introduction. Part 2 contains the basic principles, the complete mathematics of the problem and the basic structure and capabilities of BILMAT. Part 3 is devoted to the software. Part 4 is very short and deals with the operation of the BILMAT system. Part 5 contains two samples. The inputs and outputs of the examples are well detailed and discussed. Finally, Appendices A-J give mathematical details of the method, while Appendix K presents a procedure to run BILMAT.

There are various ways to read this report depending upon the user's needs. To the first type of BILMAT user, we recommend the reading of:

Part 1: all sections;

Part 2: sections 2.1, 2.2.1, 2.2.2, 2.2.4.1, 2.6, 2.7, 2.8, 2.9;

Part 3: sections 3.1, 3.2, 3.3, 3.5, 3.6, 3.8;

Part 4: all sections;

Part 5: sections 5.1, 5.2;

Appendices: G,H,K.

To the second type of user, we recommend, in addition, the reading of:

Part 2: sections 2.2.3, 2.2.4, 2.3, 2.5, 2.10;

Part 5: section 5.3;

Appendix: I.

To the third type of user, we recommend, in addition to all the previous sections, the reading of:

Part 2: section 2.4;

Appendices: B,C,E,J;

Reference: 16.

Quite obviously, the fourth type of user must read this manual *completely* and eventually request more information from the authors or CANMET.

2. ENGINEERING DOCUMENTATION

2.1 NARRATIVE DESCRIPTION

2.1.1 History

The BILMAT algorithm was developed originally at Laval University for process modelling. Several years ago it became quite obvious to Laval researchers and students that the use of industrial data to build models requires first of all a method to properly evaluate the process variables from the raw data which are generally not suitable for process modelling, being incomplete, inaccurate and inconsistent from the mass conservation point of view. The basic mathematical principles were developed in the course of the study of a three-stage grinding circuit for a New Brunswick complex sulphide ore (14,15). Data were complex (ore and water flow rates, particle size distributions, subsieve analyses of particle sizes, chemical assays,...) and had to be simultaneously adjusted from the rod mill up to the flotation head.

None of the published methods were able to meet our requirements and it was thought preferable to develop a method adapted to the problem. The technique used consists in an original decomposition of the calculation process into a so-called hierarchical minimization.

This technique was found powerful and promising by CANMET and it was decided that the hierarchical procedure should be extended in order to process data of any kind of flowsheet or process. It was also decided that this would be the best approach to upgrade collected plant data prior to using them for unit and process simulator building. This led to the BILMAT FORTRAN version which gives the users much freedom to study their mass balance problems. However, the first version still proved to be unwieldy for the novice and wasteful of computer storage. So a second version of the program was developed to overcome these shortcomings.

Finally, the third version, which is presented here, includes significant improvements to the data entry and to the sensitivity analysis capabilities.

2.1.2 Problems Associated with Material Balance Calculations

The main purpose of BILMAT is to evaluate the flow rates of each stream of an ore or coal processing circuit: flow rates of ore, water, copper, iron, particles finer than 10 μm ,... These flow rates are *balanced* if they verify laws of material conservation. Consider, for example, the simple case of a three-branch unit (e.g., a cyclone, or a distributor, or a flotation cell,...) as represented in Figure 1, D_1, D_2, D_3 are solid (ore) flow rates; x_1, x_2, x_3 are concentrations (of zinc, of particles of size between 20 and 28 mesh,...); and S_1, S_2, S_3 are mass fractions of solids in pulp. Typical equations of material conservation are:

$$\text{for ore: } D_1 - D_2 - D_3 = 0 \quad \text{Eq 1}$$

for each individual species:

$$D_1 x_1 - D_2 x_2 - D_3 x_3 = 0 \quad \text{Eq 2}$$

$$\text{for water: } D_1 \frac{1 - S_1}{S_1} - D_2 \frac{1 - S_2}{S_2} - D_3 \frac{1 - S_3}{S_3} = 0 \quad \text{Eq 3}$$

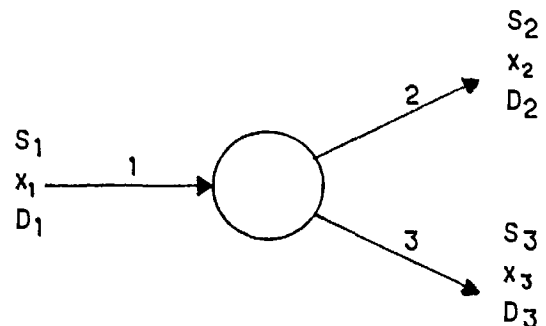


Fig. 1 - A three-stream unit

A typical material balance problem could be the following: D_1 is known; x_1, x_2, x_3 are measured for various individual species; and S_1, S_2, S_3 are measured. What are the values of D_2 and D_3 ?

If measurements are taken only for one individual species, the solution of the system of Equation 1 and Equation 2 leads to values of D_2 and D_3 and is known as the two-product formula (17). However, if more than one element is analyzed there are more than two equations of material conservation and, in general, there are no existing values of D_2 and D_3 which satisfy simultaneously all the equations. In this case the measurements are said to be redundant; that is, there is more information than needed for flow rate computation. The impossibility of solving exactly the system of Equation 1 and Equation 2 for D_2 and D_3 is a consequence of the errors which arise in the measurements due to sampling, sample splitting, handling, and analysis. In summary, the problem of mass balance computations comes from the fact that the information is redundant and erroneous.

A simple example taken from Smith (18) will illustrate this point. Consider the zinc flotation circuit of Figure 2 and the assays around the cleaner cell. The material conservation equation can be written:

$$\frac{D_3}{D_1} = \frac{x_1 - x_2}{x_3 - x_2} \quad \text{Eq 4}$$

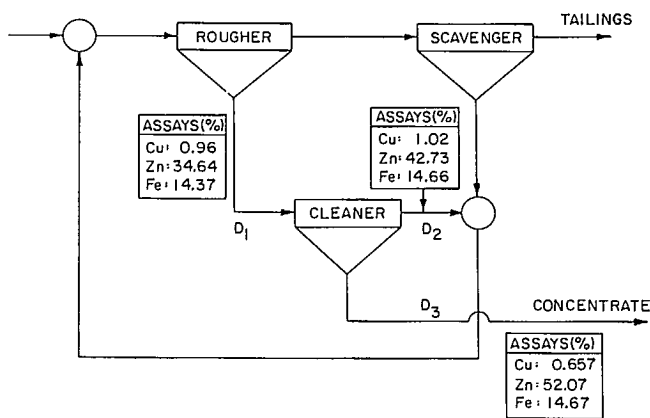


Fig. 2 – Zinc flotation circuit [after Smith (18)]

If the copper assays are used, a value of 0.165 is found for D_3/D_1 , which can be considered inaccurate since Smith (18) gives 0.125 as being the most probable value. If the zinc assays are used, a negative value is found, which is absurd; and for the iron, the material balance is impossible since $x_3 - x_2$ is very close to zero.

Another illustration of the data inconsistency is given by the hydrocyclone example of Figure 3 and Table 1. The feed flow rate is assumed to be one, the flow rate K is measured and is equal to 0.85. The size distributions x_1, x_2, x_3 are measured. Size distribution x_1 can be calculated using size i particles conservation equation:

$$\hat{x}_{1i} = (1 - K)x_{2i} + K x_{3i} \quad \text{Eq 5}$$

Table 1 shows x_{1i} compared to \hat{x}_{1i} . The problem here is to decide how the error must be shared on the three size distributions which are certainly all inaccurate, not only the one of stream 1.

Table 1 – Comparison between calculated and observed feed size distributions for the hydrocyclone of Figure 3

| Particle size (μm) | Observed x_1 (% passing) | Calculated \hat{x}_1 (% passing) |
|------------------------------------|----------------------------------|--|
| 841 | 100 | 100 |
| 595 | 99.6 | 99.8 |
| 420 | 99.2 | 99.5 |
| 297 | 96.9 | 97.7 |
| 210 | 89.0 | 91.4 |
| 149 | 74.6 | 78.4 |
| 105 | 60.3 | 64.1 |
| 74 | 47.4 | 50.0 |
| 53 | 41.0 | 41.1 |
| 37 | 32.7 | 32.4 |
| 26 | 27.9 | 24.3 |
| 18 | 23.0 | 18.6 |
| 13 | 18.8 | 14.3 |
| 9 | 15.6 | 11.5 |
| 6.6 | 13.0 | 9.7 |
| 4.6 | 10.7 | 7.9 |
| 3.3 | 9.0 | 6.7 |

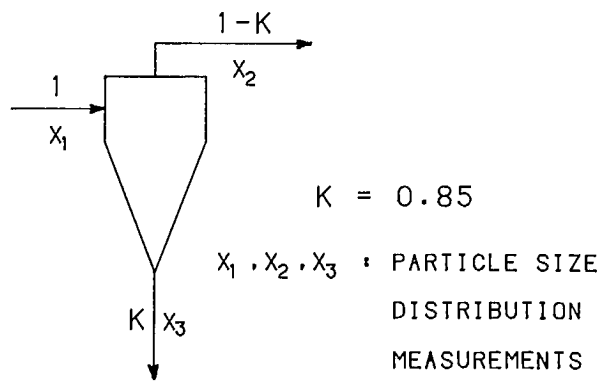


Fig. 3 – Desliming hydrocyclone

A final illustration of the problem of material balance calculations is given by the calculation of the circulating load ratio (CLR) of the grinding circuit for a New Brunswick complex sulphide ore (Fig. 4) using size distributions and pulp per cent solids measurements. Each material conservation equation written allows the calculation of a CLR and we know from the first example given that the result depends upon the individual species considered; furthermore, it depends upon the branches and nodes considered (see Section 2.1.4 for definition of branches and nodes). As illustrated in Table 2, it is possible to consider material conservation around either the sump, the cyclone, or a node which embodies the sump and the cyclone.

Table 2 lists the values obtained for the various methods of calculating CLR. It also gives the results obtained by three methods which use simultaneously all the information on size distributions. Considering, for example, the hydrocyclone as represented in Figure 5, the conservation of particles of size i gives:

$$\text{CLR} = \frac{x_{2i} - x_{1i}}{x_{1i} - x_{3i}} \quad \text{Eq 6}$$

where it appears that CLR is the slope of a straight line passing through the origin in the coordinate system $(x_1 - x_3), (x_2 - x_1)$. As a consequence, the slope of the straight line, such that the sum of squared distances

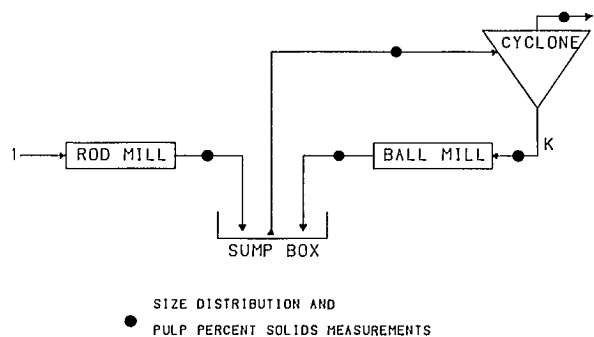


Fig. 4 – Heath Steele Mines primary grinding circuit

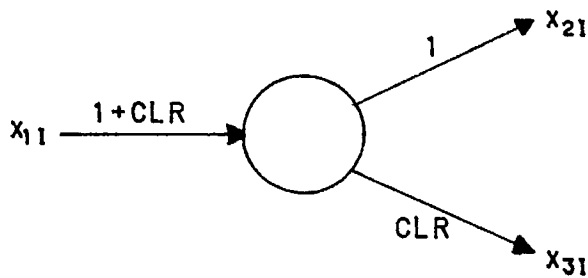


Fig. 5 – Schematic representation of the hydrocyclone of Fig. 4 (x_{1i} , x_{2i} , x_{3i} : mass fractions of particle of size i)

from experimental points to the line is minimum, is an estimation of the CLR. This simple method, called *orthogonal regression*, has been detailed and is available in BASIC (19).

Table 2 gives the results of this procedure applied to the non-cumulative mass fractions in column 5, and to cumulative fractions of particles finer than size i in column 6. Using the same basic data presented in two different manners (on a cumulative or a non-cumulative basis) leads to different results which is due to the different implicitly-assumed error models (5). The last column corresponds to the procedure used to calculate the starting values of the iterative routine used by BILMAT. It is referred to in a previous paper (2) as the *node imbalance technique* and consists in minimizing the sum of the squares of the deviations from zero of all

the material conservation Equations 1, 2 and 3, written with the measured x_i values. This simple method has been generalized to complex flowsheets and is available in BASIC (20) and in FORTRAN (28) but it presents important limitations when the flowsheet contains many streams (16).

As it is clear that the results of material balance computation depend strongly upon the method and the information used, there is a need for a technique which optimally uses the available information and which is capable of evaluating how good the results obtained are. Furthermore, this method should help in the evaluation of the quality of the experimental information available, with respect to the objectives of the mass balance calculations. In other words, it should evaluate the data-acquisition procedure and give guidance in the correct selection of the measurements to be performed. A mass balance program should provide guidelines to answer questions such as: Is the experimental set too redundant, or hardly sufficient, to define the mass balance calculation? What are the critical data which contain a large amount of information? What are the superfluous data? Which measurements have to be improved since the results are sensitive to them? Which components and streams should be sampled? What flow rates should be measured? Which are the very poor measurements and the data outliers?

In view of all the problems associated with mass balance calculations, there is a need for a program which:

Table 2 – Circulating load ratio values (%) calculated by various methods for the comminution circuit of Figure 4

| | | Experimental information considered | | | | | | |
|---------------------------------------|--|-------------------------------------|-----------|------------|----------|-----------------------|------------------|-------------------|
| | | -400 mesh | +100 mesh | 20/28 mesh | % solids | All size intervals | | |
| | | | | | | Least-squares I | Least-squares II | Least-squares III |
| Mass Conservation Equation Considered | | 1740 | <0 | <0 | <0 | 350 | 584 | 166 |
| | | 202 | 234 | 295 | 243 | 226 | 229 | 221 |
| | | 333 | 312 | 643 | 99 | 349 | 318 | 323 |
| | | single information | | | | redundant information | | |

*RMD = Rod Mill Discharge; BMD = Ball Mill Discharge; CF = Cyclone Feed; COF = Cyclone Overflow; CUF = Cyclone Underflow.

- computes unmeasured variables, such as flow rates;
- uses simultaneously all experimental data on the whole flowsheet;
- satisfies simultaneously all mass-balance constraints;
- corrects all measured data according to an assumed error model;
- evaluates the reliability of its calculation results;
- allows an evaluation of alternate data-acquisition strategies.

The BILMAT method is a response to this need.

2.1.3 Proposed Solution

The purpose of the BILMAT algorithm is to solve the system of mass balance constraint equations (Eq 1 to 3) in the case where the measurements are redundant — that is, in a number greater than needed to calculate the missing data. Since the material conservation equations are not verified due to errors in measurement, one possible way to solve exactly the system of material conservation equations is to correct the measurements. These corrections must be as small as possible and in agreement with the reliability of the measurements. This means that a measurement obtained by poor sampling or poor analysis is allowed to be corrected more than a measurement which is expected to be very accurate.

The weighted least-squares procedure is used to meet these requirements. It consists in minimizing the sum of the squared residuals (difference between the measured value of a variable and its corrected value) simultaneously for all the available data. Each residual is weighted by the inverse of the standard deviation (*) of the corresponding measurement. Simultaneous to the minimization process all the corrected data are forced to verify the mass conservation equations.

The program is user-oriented and consists of inputs and outputs. **Inputs** are:

- a coded description of the flowsheet;
- a list of the values of the measurements on the various streams: chemical assays, particle size distributions, flow rates, per cent solids...;
- a list of standard deviations associated with the data;
- a list of coded numbers representing the program options selected by the user.

The program **outputs** consist of:

- a table of the estimated values of all the variables measured as well as unmeasured;
- a table of the standard deviations of the corrected measured values, or of the calculated values for the unmeasured variables, and a table of the outlier data.

The second part of the output can be obtained outside the mass balance computation when it is required to study the effect of changing the data structure and to evaluate alternative data collection strategies.

2.1.4 Glossary of Terms

Adjustment of data

This is the calculation process by which the measured values of the variables are adjusted (or corrected) to produce estimates of the measured variables.

Centered reduced value

This is the difference between a variable value and its mean value, divided by its standard deviation. It has a standard deviation of one.

Component

A component of a stream is any class of material within a stream: it can be a chemical element, a compound, a material identified by its physical properties (size of particles, specific gravity, colour ...), a mineral, a gangue, ash,....

Complete (incomplete) analysis

This is an analysis of a given material with respect to all its components, such that the sum of the component mass fractions is equal to one. An incomplete analysis does not verify this property.

Convergence check

This is the procedure by which the minimization process of a function is tested to evaluate if the solution is close to the true mathematical value within a required accuracy.

Estimate of a variable

This is the calculated value of a variable which is considered as being the best evaluation of the true value from a statistical point of view. An estimate of a variable can be obtained by the least-squares calculation process. The variable may be measured or not. If the estimate corresponds to a measured variable it can be called the *adjusted* or *corrected* value of the variable.

Independent set of variables

This is a subset of variables from which the values of all the other variables pertinent to a particular problem may be calculated.

**In practice these values are not always accurately available. However, in Section 5.1 some hints are given to evaluate these data.*

Least-squares method

This is a process which consists of minimizing the sum of weighted squared residuals to produce estimates of the mass balance variables.

Measurement error

This is the difference between the true value and the measured value of a variable. The true value can never be known, only approximated.

Minimization algorithm

This is a scheme of calculation which finds the minimum value of a function. The practical realization of an algorithm on a computer is a *program*.

Minimization levels

This is a way to classify the various minimization algorithms depending on their position in the hierarchy of the whole computation.

Network nodes, branches

Components of a graphical representation of a flow-sheet, from the point of view of material conservation laws. Branches are analogous to streams, and nodes to units (or to assemblies of units and streams).

Outlier

This is an experimental value which does not belong to the assumed distribution of the variable, because an unusual or systematic error or bias has affected the measurement.

Process flowsheet (or flowsheet, flow-diagram)

This is a chart which represents the inter-connection between streams and units.

Redundant set of data

This is a set of data containing more than the minimum information required to calculate a solution of the material balance problem.

Residual

This is the difference between the measured value and the estimated value of a variable. It is relative when it is divided by the measured value and can be expressed in per cent.

Standard deviation

This is the square root of the variance of a variable. If divided by the mean value of the variable, it is called the *relative* standard deviation (or coefficient of variation) and can be expressed in per cent.

True value of a variable

This is an unattainable value which is approximated by the estimate of the variable.

Unit

This is an elementary processing device of a circuit (sump, classifier, flotation cell,...).

Variable

This is the general word given to a *quantifiable* property of a stream, or, of a component of a stream.

Variance

This characterizes the distribution of the values of a variable. In particular cases, it can be:

- the measured value variance;
- the variance of the error of the measured value (assumed identical to the previous one);
- the estimate variance;
- the residual variance.

Variable levels

This is a way of classifying the variables into subsets related to various subdivisions of a stream. Associated with each level of variable are:

- a network level;
- a data level;
- a component level.

Variance matrix

Sometimes identified as the *covariance* matrix, this is the variance and covariance values of a vector of variables. If the variables are assumed to be not correlated, the matrix is diagonal (no covariance terms).

Weighting factor

This is a coefficient which multiplies each squared residual in the least-squares method. For the present application, it is the inverse of the variance of a measured variable.

2.2 MASS CONSERVATION LAWS**2.2.1 Nature of Components and Variables**

The components (or physical properties) which constitute a stream of a given ore or coal processing circuit can be divided into four levels as schematically shown in Figure 6. From the first to the fourth level, the stream is more and more finely analyzed with respect to its constituents or properties. The pulp (first level) is divided into solids and liquid (second level), then the solids into components (third level) (particles, chemical elements ...), then each of these components into subcomponents (fourth level), which can be, for instance, the chemical elements within each particle size interval of the third level, or the particles size distribution within a specific gravity interval of the solids at the third level, and so on.

To each of these levels of the stream, variables correspond which express the stream state. For instance, at the first level, the stream could be characterized by the pulp specific gravity or by its volume flow rate. However, at this level, BILMAT can process only pulp mass flow rate variables. At the second level, only water and ore

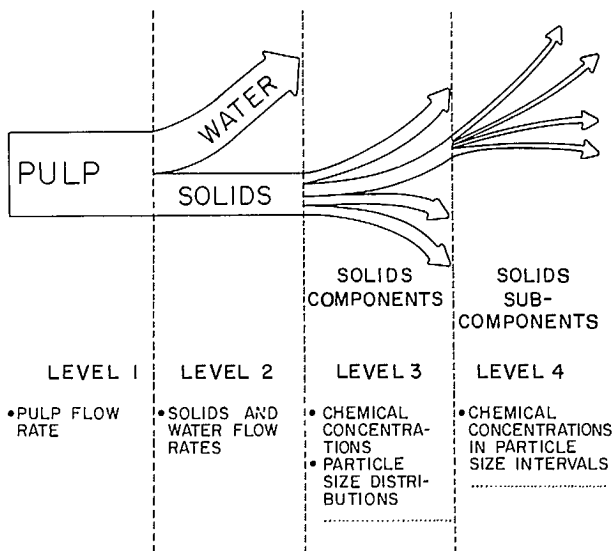


Fig. 6 – Various levels of mass balance data

mass flow rate are permissible variables and, finally, at the third and fourth levels, BILMAT uses variables which pertain to the mass fractions of the solids. For instance, at the third level a variable could be the mass fraction of zinc in the ore, and at the fourth level it could be the mass fraction of copper in the particles of the 100/150 mesh size interval.

One of the basic assumptions of the method is that the measured value of a variable is a random variable distributed according to a normal statistical distribution, the mean of which is the true value (unbiased data). The error is the difference between the measured value and the true value. Neither the error nor the true value can ever be exactly known.

A further assumption is that the errors of the various measurements are independent (or uncorrelated). This is a simplification since it may not always be true. For instance, the sieving technique produces data where errors are known to be correlated: particles missing in a given size interval are redistributed to the other size intervals.

2.2.2 Flow Networks

The mass conservation laws are handled graphically using a flow network. A network is a flowsheet where the nodes correspond generally to process units and the streams (or branches) correspond to the pipes which interconnect the various units of the circuit flowsheet. However, it may happen that nodes or streams do not represent physical parts of the circuit but are a way of expressing relationships of mass conservation. The best way to understand the mechanism of flow network construction is with examples.

The first one concerns the Heath Steele Mines grinding circuit represented in Figure 7, with the available data (14). Theoretically there is one flow network for each component of the streams; however, practically, many components have the same flow network. The various networks which must be defined in this case are given in Figure 8. Overall is the pulp network (Fig. 8a) (pure water being considered as pulp with 0% solids). The ore network (Fig. 8b) is obtained by removing pure water streams and nodes from the pulp network. The network for particle sizes corresponds to the ore network in which mill nodes are removed (Fig. 8c) since there is no conservation of particle size in a grinding mill. The last two networks for subsieve analyses and lead assays (see respectively Fig. 8d and 8e) are obtained by removing the streams where these analyses were not performed.

The second example was obtained from Laguitton (21) for a flotation circuit as represented in Figure 9, with experimental data as shown. In this case two networks must be defined: one for silver, and one for Cu, Pb, and Zn conservation (Fig. 10). Branch 10 does not appear in the network because this stream is embedded in node III and the analyses are thus useless for mass balance computation.

BILMAT can deal with a maximum of eight different networks (Table 3). The first network corresponds to first-level component conservation laws (generally pulp including pure water and dry or wet ore). The second one is used to describe second-level component conservation laws (generally the ore or the coal). A third network is defined for third-level complete analyses; i.e. for components such that the sum of their mass fractions is equal to one. Generally, this third network corresponds to particle size distributions which verify this property. With regard to third-level components which do not satisfy this property (incomplete analyses), three networks are available. These networks correspond generally to chemical assays data. Two networks are available for the fourth level components: one for data such that the sum of mass fractions is equal to one (generally sub-sieve analyses), and one for other data (generally chemical assays data).

Table 3 – Available networks for BILMAT application

| Level | Component | Network number |
|-------|---------------------|-----------------------|
| 1 | Pulp | 1 |
| 2 | Ore/Coal | 2 |
| 3 | { Complete analysis | 3 |
| | | { Incomplete analysis |
| 4 | { Complete analysis | 4 |
| | | { Incomplete analysis |

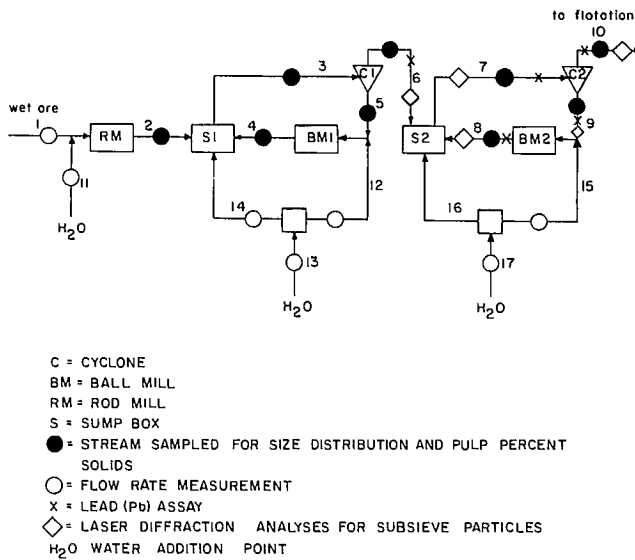


Fig. 7 - Heath Steele Mine grinding circuit

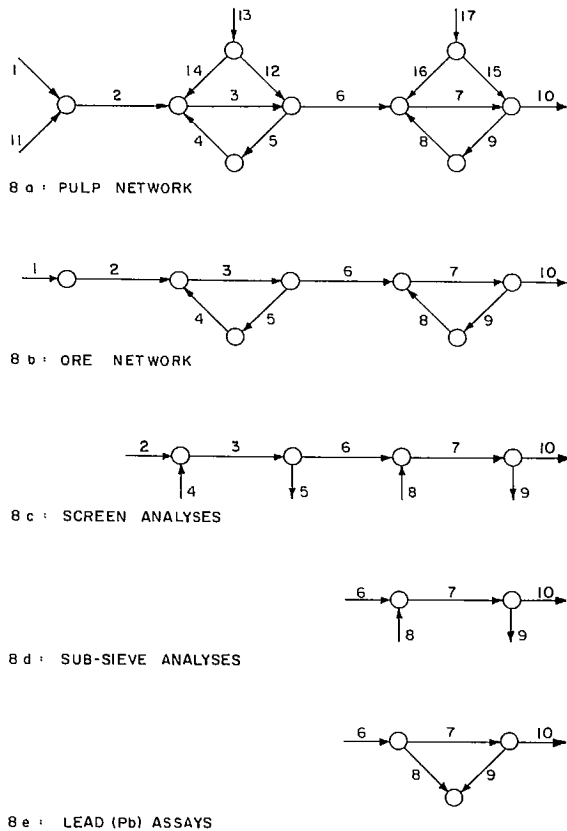


Fig. 8 - Various networks corresponding to the grinding circuit of Fig. 7

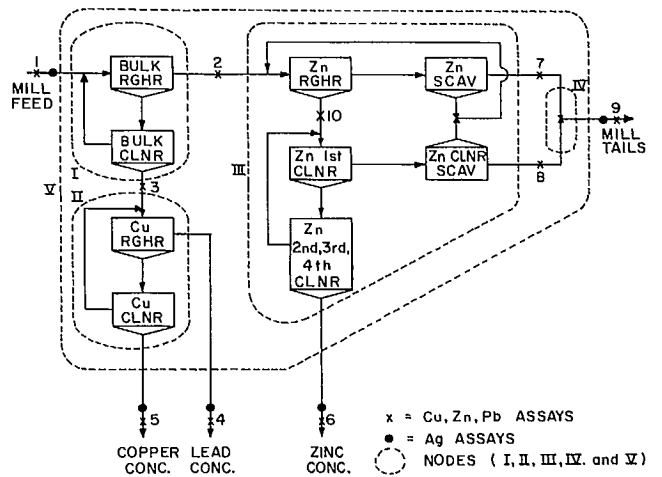


Fig. 9 - Pb/Zn/Cu flotation circuit [after Laguitton (21)]

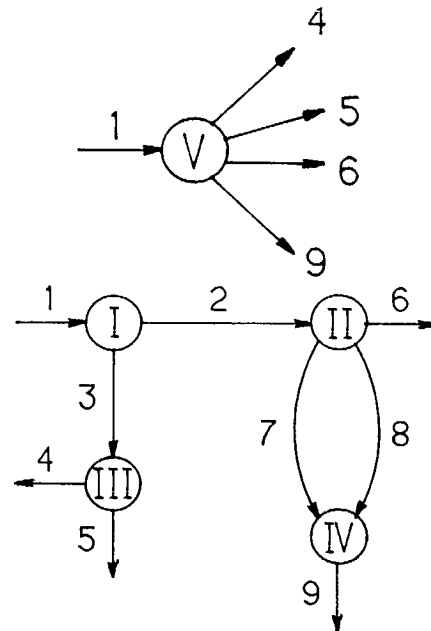


Fig. 10 - Various networks corresponding to the flotation circuit of Fig. 9

For the preceding example of the grinding circuit, the following networks are used:

- Network 1: pulp and water (Fig. 8a);
- Network 2: ore (Fig. 8b);
- Network 3: particle size distributions (Fig. 8c);
- Network 4: subsieve analyses (Fig. 8d);
- Network 5: lead assays (Fig. 8e).

For the flotation example, the networks are:

- Network 1: pulp, identical to network 2;
- Network 2: ore (Fig. 10a);
- Network 5: Cu, Pb, Zn (identical to network 2);
- Network 6: Ag (Fig. 10b).

Some general rules can be given to correctly construct the networks required by BILMAT:

- Network 1 must be the largest network; i.e., it must include all the seven sub-networks corresponding to the levels 2, 3, and 4 components.
- Each network must be as small as possible and should not contain un-sampled streams.
- Nodes connected by unmeasured branches must be condensed into a single node.
- If a pipe is sampled at two different locations, a supplementary node can be created if both data points are to be used.

2.2.3 Symbols

Before giving an alphabetical list of the symbols, we prefer to explain the logical basis of our notation system. Some systematic rules are followed for the writing of this text and knowledge of these rules often makes the symbols self-explanatory. Furthermore, each symbol is defined after its first appearance in the text.

Index rules

k is the index of a network.

j is the index of a stream.

i is the index of a level-3 component.

ℓ is the index of a level-4 component.

Dimension rules

q is a number of nodes.

p is a number of streams.

n is a number of components.

Variables

D is a flow rate; **d** a relative flow rate, **θ** an independent relative flow rate.

S is a mass fraction of solids in the pulp.

δ is a pulp dilution (mass of water divided by mass of solids).

x is a level-3 mass fraction.

y is a level-4 mass fraction.

Estimates

The estimate of a variable — that is, its value calculated by BILMAT — is indicated by **a** on top of the variable.

Measured values

Subscript **o** indicates the measured value of a variable (“observed value”). In the case of mass fractions, to alleviate the notation, we have selected **a** and **b** respectively instead of x_o and y_o .

Standard deviation and variance

Standard deviations are noted σ , and variances σ^2 .

Nature of component

The nature of the component considered is indicated by superscripts (or subscripts when preferable). **p** is for pulp, **w** for water, **s** for mass fraction of solids in the pulp, **δ** for dilution, **x** or **a** for third level mass fractions and **y** or **b** for fourth level mass fractions. The absence of sub or superscript corresponds to total solids.

Examples of scalar notation

Dp_j is the flow rate of pulp observed on stream **j**.

σ_j is the standard deviation of the total solids flow rate measured on stream **j**.

$\sigma_{kji\ell}$ is the standard deviation of subcomponent **ℓ**, in component **i**, on stream **j**, in network **k**.

n_k is the number of components pertinent to network **k**.

Matrix notation

Components of an array may be arranged into a column matrix. In this case, the index disappears. For example x_{ki} is the column matrix of the x_{kji} 's for all **j** values.

Covariance matrices are represented by **V** and the network incidence matrices are denoted **m**.

Diagonal matrix

When a column matrix must be organized into a diagonal form, the corresponding variable is underlined; example: \underline{x}_{ki} .

There are other notations used in this text, and in some cases they overlap the ones defined above. However, they are defined and used only within one section and the context excludes misinterpretation.

The following is an alphabetical list of the main notations used in the text.

2.2.3.1 Main Notations

a_{kji} : measured value of the mass fraction of component **i** in stream **j** of level 3 network **k**.

a_{ki} : measured values of the mass fraction of component **i** of level 3 network **k** for all the streams.

a_k : measured values of the mass fractions of all the components of network **k**.

B, B_i , **b** : matrices used for expressing mass conservation constraints for complete analyses.

$b_{kji\ell}$: measured value of the mass fraction of subcomponent **ℓ** in component **i** in stream **j** of level 4 network **k**.

| | | | |
|--------------------------------|--|--|--|
| $b_{k\ell}$ | : measured values of the mass fractions of subcomponent ℓ in component i , for all the streams of network k . | m_k | : incidence matrix of network k . |
| b_{ki} | : measured values of the mass fractions of all the subcomponents of network k in component i . | M_1, M_2 | : block matrices of the solids network for identification of a set of independent relative flow rates. |
| d_j | : relative ore flow rate in stream j . | n_k | : number of components analyzed in network k . |
| d, \underline{d} | : relative ore flow rate in column and diagonal matrices respectively. | p^k | : number of branches in network k . |
| $\hat{d}, \underline{\hat{d}}$ | : relative ore flow rate estimates. | q_k | : number of nodes in network k . |
| \underline{d}_k | : diagonal of relative solids flow rates corresponding to network k . | r_{ji} | : residual value for mass fraction of component i on stream j . |
| d_o | : measured values of relative solids flow rates. | \bar{r}_j | : average value of r_{ji} for all components i . |
| D_j, D_j^w, D_j^p | : flow rates of solids, water, and pulp in stream j . | S_j | : pulp solid fraction in stream j . |
| D, D^w, D^p | : column matrices of D_j, D_j^w, D_j^p . | S_{oj} | : measurement value of S_j . |
| $D_{oj}, D_{oj}^w, D_{oj}^p$ | : measured values of solids, water, and pulp flow rates in stream j . | T | : J_{12} term not function of water flow rates. |
| D_o, D_o^w, D_o^p | : column matrices of $D_{oj}, D_{oj}^w, D_{oj}^p$. | u_j | : coefficients of $(D_j^w)^2$ in J_{12} . |
| \hat{D}_1 | : estimate of solids feed rate. | u | : matrix of u_j . |
| \hat{D}^w | : estimate value of D^w . | V_{ki}^x | : variance matrix of mass fractions of component i on network k . |
| $H_3=0$ | : mass conservation constraints for level 3 components. | V_k^x | : variance matrix of mass fractions for all components of network k . |
| $H_3^k=0$ | : mass conservation constraints for level 3 components on network k . | V_{ki}^{x*} | : variance matrix of mass fractions of subcomponent ℓ of network k in level 3 component i . |
| $H_3^{ki}=0$ | : mass conservation constraints for level 4 components on network k , within level 3 component i . | V_{ki}^y | : variance matrix of mass fractions for all subcomponents of network k in level 3 component i . |
| i, j, k, k', ℓ | : indices. | w_j | : coefficient of D_j^w in J_{12} . |
| I | : number of independent solid flow rates. | w | : matrix of w_j . |
| J | : least-squares function. | x_{kji} | : mass fraction of component i in stream j of network k . |
| J_{12} | : part of J corresponding to levels 1 and 2 data. | $x_{ki}, \underline{x}_{ki}, \hat{x}_{ki}$ | : mass fractions of component i in all streams of network k (column matrix, diagonal matrix, estimates). |
| J_3 | : part of J corresponding to level 3 data. | x_k, \hat{x}_k | : mass fractions on network k and their estimates. |
| J_4 | : part of J corresponding to level 4 data. | $y_{kji\ell}$ | : mass fraction of subcomponent ℓ of network k , in level 3 component i in stream j . |
| J_3^k | : part of J corresponding to level 3 data on network k . | $y_{ki\ell}, \hat{y}_{ki\ell}$ | : mass fraction of subcomponent ℓ of network k , in level 3 component i in all streams and their estimates. |
| J_3^{ki} | : part of J corresponding to level 3 data for component i on network k . | y_{ki}, \hat{y}_{ki} | : mass fractions of subcomponents in component i on network k and their estimates. |
| J^* | : sum of the minimum values of J_{12}, J_3 and J_4 . | α | : calculation auxiliary matrix. |
| L, ℓ | : matrices for calculation of solids flow rates from an independent set of values. | δ_j | : dilution on stream j . |
| | | δ_{oj} | : measured value of δ_j . |
| | | δa | : error on \hat{x} . |

- δd : small variation of estimate d .
 δd_o : error on d_o .
 δx : small variation of estimate x .
 $\delta \theta$: small variation of estimate θ .
 ϵ_{ij} : 'imbalance' value of node j for component i .
 ϵ_i : 'node imbalances' for component i .
 γ : Lagrange multiplier.
 σ_{kji}^x or σ_{ji}^x : standard deviation of component i mass fraction estimate in stream j of network k .
 σ_{kji}^a or σ_{ji}^a : standard deviation of component i mass fraction in stream j of network k .
 $\sigma_{kji\ell}^b$: standard deviation of sub-component ℓ mass fraction in component i in stream j of network k .
 σ_j : standard deviation of solids flow rate in stream j .
 σ_j^w : standard deviation of water flow rate in stream j .
 σ_j^p : standard deviation of pulp flow rate in stream j .
 σ_j^s : standard deviation of pulp mass fraction of solids in stream j .
 σ_j^d : standard deviation of pulp dilution in stream j .
 $\theta, \hat{\theta}$: independent relative solids flow rates and their estimates.
 T : matrix transposition (used as a superscript).

$$\left(\begin{array}{l} \min \\ x, y \\ g=0 \end{array} \right) f : \text{minimum of the function } f \text{ with respect to } x \text{ and } y \text{ and under constraint } g(x, y) = 0.$$

2.2.4 Mass Conservation Matrix Notation

Using the network graphical representation of mass conservation of each stream component, it is easy to write the corresponding mathematical equations using matrix algebra.

2.2.4.1 Network incidence matrix

Each line of this matrix corresponds to a node of the network and each column to a stream. If a stream converges to — or diverges from — a node, a value + 1 or - 1 respectively is entered at the corresponding position in the matrix. If a stream is not connected to a node a zero value is entered. For instance, the reader will easily verify that the following matrix is the incidence matrix of the network of Figure 10a:

| | | Streams | | | | | | | | |
|---|-----|---------|----|----|----|----|----|----|----|----|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| N | | | | | | | | | | |
| o | I | 1 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 |
| d | II | 0 | 1 | 0 | 0 | 0 | -1 | -1 | -1 | 0 |
| e | III | 0 | 0 | 1 | -1 | -1 | 0 | 0 | 0 | 0 |
| s | IV | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | -1 |

2.2.4.2 Ore and water conservation

Defining D as the column matrix of the ore flow rate D_j on each stream j of the ore network, it can be easily shown that the ore conservation equation for the whole network (generalization of Eq 1) is:

$$m_2 D = 0 \quad \text{Eq 7}$$

where m_2 is the incidence matrix of the ore network.

In a similar way, defining the column matrix of water flow rates by D^w , the water conservation equation is:

$$m_1 D^w = 0 \quad \text{Eq 8}$$

where m_1 is the incidence matrix of the pulp network.

2.2.4.3 Independent relative ore flow rates

The system of Equation 7 is homogeneous, that is, the D_j values can be defined only on a relative basis since the multiplication of all the D_j values by the same constant factor does not change the system. The ore feed rate to the circuit is selected as 1, consequently relative ore flow rates d can be defined by:

$$d = \frac{D}{D_1} \quad \text{Eq 9}$$

and from Equation 7 we have also:

$$m_2 d = 0 \quad \text{Eq 10}$$

The independent relative ore flow rates are a subset of flow rates from which the flow rates of all the other streams of the network can be calculated using Equation 10. For a complex network there are many subsets of such flow rates. Selecting one of these, set θ , it is possible to calculate all the values d_j by:

$$d = L\theta + \ell \quad \text{Eq 11}$$

where L and ℓ are matrices which can be deduced from m_2 as detailed in Appendix A.

2.2.4.4 Level 3 components conservation

Let us define m_k as the network matrix corresponding to the conservation of components of level 3 on the k^{th} network. For BILMAT, k is 3, 5, 6 or 7 (see Table 3). The

generalization of the conservation Equation 2 for a complex network can be written in a matrix form as:

$$m_k d_k x_{ki} = 0 \quad \text{for } i = 1 \dots n_k \quad \text{Eq 12}$$

$$k = 3, 5, 6, 7$$

where d_k is the subset of relative ore flow rates pertinent to network k (diagonal matrix), and n_k the number of components pertinent to the k^{th} network. x_{ki} is the column matrix of the mass fractions of the i^{th} component on the streams of network k .

As indicated in Section 2.2.2, network 3 on one hand, and networks 5, 6 and 7, on the other hand, differ by the possibility of having in network 3 a complete analysis such that the sum of the components mass fractions is equal to unity (the case of particles size distributions, for example). In this case, the following mass conservation equations must be added:

$$\sum_{j=1}^{p_3} x_{kji} = 1 \quad i = 1 \dots n_3 \quad \text{Eq 13}$$

However, to guarantee the independence of the mass conservation equations, Equation 12 must be written for $n_3 - 1 - 1$ components only:

$$m_k d_k x_{ki} = 0 \quad i = 1 \dots, n_3 - 1 \quad \text{Eq 14}$$

2.2.4.5 Level 4 components conservation

Let us consider a single node network as the one in Figure 1. The mass conservation equation of level 4 component ℓ , in level 3 component i , is:

$$D_1 x_{1i} y_{1i\ell} - D_2 x_{2i} y_{2i\ell} - D_3 x_{3i} y_{3i\ell} = 0 \quad \text{Eq 15}$$

where $y_{j\ell}$ is the mass fraction of level 4 component ℓ in level 3 component i in stream j .

This equation can be generalized to a complex flow-sheet by the following matrix formulation:

$$m_k d_k x_{k'k} y_{k\ell} = 0 \quad \begin{array}{l} \ell = 1 \dots n_k \\ i = 1 \dots n_{k'} \end{array} \quad \text{Eq 16}$$

where k is the number of fourth level networks and k' the number of third level networks. In practice, for BILMAT, the validity of this equation is limited to subcomponents analyses in the components of network 3 ($k' = 3$) in the two following cases:

| | | | |
|---|---|----|---|
| I | $k = 4$ $\ell = 1 \dots n_4$ $i = n_3$ complete level 4 analysis | II | $k = 8$ $\ell = 1 \dots n_8$ $i = 1 \dots n_3$ incomplete level 4 analysis |
|---|---|----|---|

The first case corresponds generally to a subsieve analysis and the second one to chemical assays in particle size intervals. This will be detailed some more in section 2.6 (program capabilities).

Furthermore BILMAT in its present version does not allow the same components in level 3 and level 4. For example, it may arise that copper is analyzed both in the ore as a whole and in each particle size interval. In this case, there are supplementary mass conservation relationships which interconnect level 3 and 4 for the copper. BILMAT cannot presently handle these relationships.

2.3 THE LEAST-SQUARES SOLUTION TO THE MASS BALANCE PROBLEM

The basic problem of a mass balance calculation is to solve the system of mass conservation equations written in the previous section. These are summarized below:

$$m_2 D = 0 \quad \text{ore conservation} \quad \text{Eq 17a}$$

$$m_1 D^w = 0 \quad \text{water conservation} \quad \text{Eq 17b}$$

$$H_3 = 0 \left\{ \begin{array}{l} m_k d_k x_{ki} = 0 \\ \sum_{j=1}^{pk} x_{kji} = 1 \end{array} \right\} \quad \begin{array}{l} \text{level 3} \\ \text{conservation} \end{array} \quad \text{Eq 17c}$$

$$H_4 = 0 \left\{ \begin{array}{l} m_k d_k x_{k'k} y_{k\ell} = 0 \\ \sum_{j=1}^{pk} y_{kji\ell} = a \text{ constant} \end{array} \right\} \quad \begin{array}{l} \text{level 4} \\ \text{components} \\ \text{conservation} \end{array} \quad \text{Eq 17e}$$

For the version of BILMAT presented here, these equations are valid for the following indices:

$$\text{Eq 17c: } \begin{array}{l} k = 3 \quad i = 1 \dots (n_3 - 1); \\ k = 5, 6, 7; i = 1 \dots n_k \end{array}$$

$$\text{Eq 17d: } k = 3 \quad i = 1 \dots n_3$$

$$\text{Eq 17e: } k' = 3 \left\{ \begin{array}{l} k = 4 \quad i = n_3; \ell = 1 \dots n_4 - 1 \\ k = 8 \quad i = 1 \dots n_3; \ell = 1 \dots n_8 \end{array} \right.$$

$$\text{Eq 17f: } k' = 3 \quad k = 4 \quad i = n_3; \ell = 1 \dots n_4$$

2.3.1 Redundant System of Mass Balance Equations

To solve the system of Equation 17, experimental data need to be collected. Let us assume that some variables of the system are measured: for instance, flow rates such as D_j^w or D_j for some j values and mass fractions x_{kji} or $y_{kji\ell}$. In some cases, indirect measurements of the system variables can be performed such as those of the pulp flow rates and the pulp solid mass fractions. These measurements are related to the variables of the system and the system must be augmented by the relationship between these indirect variables:

$$S_j = \frac{D_j}{D_j + D_j^w} \quad \text{Eq 18a}$$

$$D_j^p = D_j + D_y \quad \text{Eq 18b}$$

If in the system of Equation 17 (eventually augmented by Equation 18), we replace the measurable variables by their measured values, the system of Equations 17 and 18 must be solved with respect to the unmeasured variables. Three situations can occur:

1. The system has more unknowns than equations and consequently does not have a unique solution. This is the case when insufficient data have been collected to calculate all the unmeasured variables.
2. The system is exactly defined (i.e., has as many equations as unknowns) and consequently has a unique solution. This is the case when just sufficient data have been collected to calculate all the unmeasured variables.
3. The system has more equations than unknowns. This is the case when more than sufficient data have been collected to calculate all the unmeasured variables. In this case, the set of data is said to be redundant. If, as mentioned previously, the measured values are randomly distributed, it is very improbable that they will be coherent from the point of view of mass conservation. In other words, it will be impossible to find values of the unmeasured variables such that all the mass balance equations of the system of Equations 17 and 18 are simultaneously verified.

The BILMAT program is designed to solve the mass balance problem in this latter case when data are redundant and erroneous. However, it works also for case 2 and in a hybrid situation when the system of Equation 17 and Equation 18 is exactly defined for some variables and overdefined for others.

2.3.2 Principles of the Least-Squares Solution

Since it is known that measurements deviate from true values, one of the possible ways of solving the system of Equations 17 and 18 is to correct the measured values in order to make them coherent while calculating the unmeasured variables. It can be shown (10) that the weighted least-squares method allows all the data to be corrected and produces best estimates of all the variables (measured and unmeasured). When the weighting factors are selected as the reciprocals of the measure-

ment variances, these estimates are of minimum variance if the measurements are unbiased and gaussian. Since in practice these two conditions are never exactly true, BILMAT results are not always, strictly speaking, the statistically most probable values, they are however generally much better than the ones obtained by conventional methods.

The variables of the mass balance system of Equations 17 and 18 can be separated into two sets: those which are measured and those which are not measured. Let us generate these two sets, respectively, by variables (*) z_i^m and z_i^u , and note z_i^m the measured value of z_i^m which is assumed to belong to a normal population of variances σ_i^2 and to be independent of the other measured variables.

The least-squares solution of the mass balance problem can be formulated as:

$$\text{Minimize: } J = \sum_i (z_i^m - z_i^m)^2 \frac{1}{\sigma_i^2} \quad \text{Eq 19}$$

with respect to z_i^m and z_i^u and under mass balance constraints of the system of Equations 17 and 18.

2.4 THE MINIMIZATION PROCEDURE

The minimization of J with respect to measured and unmeasured variables is performed iteratively using a hierarchical procedure based on a decomposition into smaller parts of the function to be minimized. To facilitate understanding of the concept of hierarchical minimization of a function, let us consider a function f of two variables x and y such that we have:

$$f(x, y) = g(x) + h(y) \quad \text{Eq 20}$$

The minimum of f with respect to x and y can be obtained by minimization of g and h respectively with respect to x and y, according to the following rule:

$$\min_{x,y} f = \min_x g + \min_y h \quad \text{Eq 21}$$

Let us now consider a function f of three variables x, y, z such that:

$$f(x, y, z) = g(x, z) + h(y, z) \quad \text{Eq 22}$$

we have:

$$\min_{x,y,z} f = \min_z (\min_x g + \min_y h) \quad \text{Eq 23}$$

z is said to be the common variable of the minimization problem and the minimization is performed at the lower level with respect to x for g and to y for h and, at the upper level (supervisory level), with respect to z. These

(*) Here the i subscript refers to the variable considered and must not be confused with a component index.

principles are applied to the minimization of the least-squares criterion J , to perform minimization on smaller parts of J containing fewer variables than the whole function J .

2.4.1 Decomposition Principles

2.4.1.1 Partition of the least-squares criterion

The variables can be split into three sets: the first one contains levels 1 and 2 variables; the second one, level 3 variables; and the third one level 4 variables. This allows the partition of the J function into three parts corresponding to the three sets of variables defined:

$$J = J_{12} + J_3 + J_4 \quad \text{Eq 24}$$

The part J_{12} is explicitly the following function:

$$J_{12} = \sum_j (D_j^P - D_{oj}^P)^2 \frac{1}{\sigma_j^{P2}} + \sum_j (D_j^W - D_{oj}^W)^2 \frac{1}{\sigma_j^{W2}} + \sum_j (D_j - D_{oj})^2 \frac{1}{\sigma_j^2} + \sum_j (\delta_j - \delta_{oj})^2 \frac{1}{\sigma_j^{\delta 2}} \quad \text{Eq 25}$$

where j can take values from 1 to p_1 (*), depending on the measurements.

In this equation for the simplification of later calculations, the pulp mass fractions of solids S_j have been replaced by pulp dilutions δ_j whose measured values δ_{oj} are related to S_{oj} by:

$$\delta_{oj} = \frac{1 - S_{oj}}{S_{oj}} \quad \text{Eq 26}$$

For consistency, Equation 18a should be also replaced by:

$$\delta_j = \frac{D_j^W}{D_j} \quad \text{Eq 27}$$

In order to solve exactly the initial problem using pulp mass fractions of solids (which is the usual way of presenting the data), the variances should be correspondingly replaced by:

$$(\sigma_j^{\delta})^2 = \frac{1}{(S_{oj})^4} (\sigma_j^S)^2 \quad \text{Eq 28}$$

In Equation 25, D_j^P and δ_j can be replaced by their definition Equations 18b and 27, and D_j by $D_1 d_j$.

J_{12} becomes a quadratic function of D_j^W :

$$J_{12} = \sum_j u_j (D_j^W)^2 - 2 \sum_j w_j D_j^W + T \quad \text{Eq 29}$$

where:

$$u_j = \frac{1}{(\sigma_j^S)^2 d_j^2 D_1^2} + \frac{1}{(\sigma_j^P)^2} + \frac{1}{(\sigma_j^W)^2} \quad \text{Eq 30}$$

$$w_j = \frac{\delta_{oj}}{(\sigma_j^S)^2 d_j D_1} + \frac{D_{oj}^P}{(\sigma_j^P)^2} - \frac{d_j D_1}{(\sigma_j^P)^2} + \frac{D_{oj}^W}{(\sigma_j^W)^2} \quad \text{Eq 31}$$

and where T contains all the remaining terms which are not a function of D_j^W .

In Equation 24, J_3 and J_4 are:

$$J_3 = \sum_k \sum_j \sum_i (X_{kji} - a_{kji})^2 \frac{1}{(\sigma_{kji}^a)^2} \quad \text{Eq 32}$$

$$J_4 = \sum_k \sum_j \sum_i \sum_\ell (Y_{kjie} - b_{kjie})^2 \frac{1}{(\sigma_{kjie}^b)^2} \quad \text{Eq 33}$$

To perform a hierarchical minimization of J , it is necessary to clearly understand which variables are common to each part of J and to its corresponding constraint. From the previous evaluation of J_{12} , it appears that it is a function of D^W , D_1 and d .

The variable d is also present in the constraints $H_3 = 0$ and $H_4 = 0$ associated with J_3 and J_4 and, consequently, the relative ore flow rates d are common to the three minimization sub-problems. The level 3 mass fractions x are common to J_3 and to the constraint $H_4 = 0$ associated with J_4 (Eq 17e and f).

The partition of minimization problems becomes very powerful when the parts of the whole problem are easy to minimize and especially when the minimum can be found analytically without an iterative computation. When the function to be minimized is quadratic with respect to variables and the associated constraints linear with respect to the same variables, the minimum can be explicitly found by a set of linear equations whose solution can be found without iterative calculation.

Looking at the three parts of the minimization problem it appears that this situation is met in the following cases:

- minimization of J_{12} under $M_1 D^W = 0$ if variables d and D_1 are maintained at constant values;
- minimization of J_3 under $H_3 = 0$ (Eq 17) with variables d maintained at constant values;
- minimization of J_4 under $H_4 = 0$ (Eq 17) with variables d and x maintained at constant values.

In all these cases the minimum of the constrained function can be found by a direct solution of a linear system.

(*) p_1 is the number of branches of the largest network (network 1).

2.4.1.2 Horizontal and vertical decompositions of the minimization procedure

To take advantage of the previous observations the minimization of J under the system of constraints is decomposed vertically and horizontally according to the scheme of Figure 11.

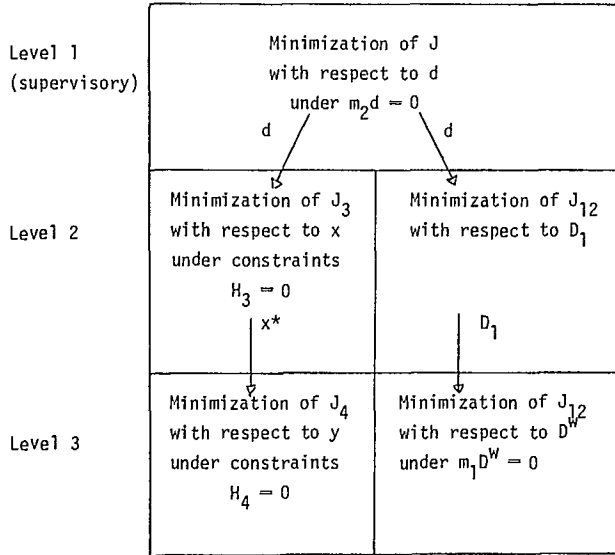


Fig. 11 – Hierarchical decomposition of the minimization procedure

At the supervisory level, the minimization is performed with respect to the relative ore flow rates d under the corresponding constraint $m_2d = 0$ (d are common variables similar to z in Eq 22). At the second level, the d values being defined by the supervisory level J_3 can be minimized with respect to x and J_{12} with respect to D_1 .

At the third level, J_4 can be minimized with respect to y since x and d are defined by the upper levels, and J_{12} can be minimized with respect to water flow rates D^w . In mathematical formulation, the minimization procedure can be summarized by:

$$\left(\min_{D_1, d, D^w, x, y} J \right) = \min_{m_2 d = 0} \left\{ \left(\min_{\substack{x \\ H_3 = 0}} J_3 \right) + \left(\min_{\substack{y \\ H_4(x^*) = 0}} J_4 \right) \right. \\ \left. + \min_{D_1} \left(\min_{m_1 D^w = 0} J_{12} \right) \right\} \quad \text{Eq 34a}$$

The constraints $H_4 = 0$ are written giving to x the value x^* obtained by the minimization of J_3 at the upper level. Due to this modification of the system of equations, the solution (when level 4 data are present) is not exactly the true mathematical minimum since J_3 and J_4 are not simultaneously minimized with respect to x . However, this sub-optimal solution, which gives priority to third level in comparison to fourth level data, is quite acceptable in the majority of cases.

The true mathematical minimum is:

$$\left(\min_{D_1, d, D^w, x, y} J \right) = \min_{m_2 d = 0} \left\{ \min_{\substack{x \\ H_3 = 0}} \left(J_3 + \min_{\substack{y \\ H_4 = 0}} J_4 \right) \right. \\ \left. + \min_{D_1} \left(\min_{m_1 D^w = 0} J_{12} \right) \right\} \quad \text{Eq 34b}$$

However, this solution is much more difficult to handle due to the large number of common x 's in J_3 and H_4 .

Another sub-optimal solution for the estimation of level 4 variables is available in the BILMAT program. It is less optimal than the previous one, but recommended for a first look at a mass balance computation. It consists in a first step to omit level 4 data and to solve the following problem:

$$\left(\min_{D_1, d, D^w, x} J \right) = \min_{m_2 d = 0} \left\{ \left(\min_{\substack{x \\ H_3 = 0}} J_3 \right) \right. \\ \left. + \min_{D_1} \left(\min_{m_1 D^w = 0} J_{12} \right) \right\} \quad \text{Eq 34c}$$

and then to minimize J_4 with respect to y under:

$$H_4(\hat{d}, \hat{x}, y) = 0 \quad \text{Eq 35}$$

where \hat{d} and \hat{x} are solutions of Equation 34c. In section 2.4.2.4, solving Equations 34c and 35 is identified as sub-optimal solution No. 1, and solving Equation 34a as sub-optimal solution No. 2.

2.4.2 Minimization Algorithms

For each of the minimization sub-problems defined in Figure 11, it is possible to use the best optimization algorithm to take advantage of the hierarchical decomposition which has been performed.

2.4.2.1 Calculation of water flow rates

The water flow rates are calculated at the third minimization level (Fig. 11). To simplify, Equation 25 can be rewritten using matrix notation:

$$J_{12}(D^w) = (D^w)^T U D^w - 2W D^w + T \quad \text{Eq 36}$$

Coefficient U is the diagonal matrix of terms u_i of Equation 30, and coefficient W is the column matrix of terms w_j of Equation 31. U , W , and T are only functions of d and D_1 and in the present case are considered as constant coefficients.

The minimization of $J_{12}(D^w)$ with respect to D^w and under the constraint $m_1 D^w = 0$ yields the following values of D^w :

$$\hat{D}^w = U^{-1}W - U^{-1}m\bar{I} (m_1U^{-1}m\bar{I})^{-1} m_1U^{-1}W \quad \text{Eq 37}$$

The details of the general solution of the minimization of a quadratic function under linear constraints are given in Appendix B.

2.4.2.2 Calculation of circuit feed rate

At the third vertical level, J_{12} is minimized with respect to D^w , as explained in the previous section, and, at the second level, this minimum has to be minimized with respect to D_1 (Fig. 11):

$$\min_D \left(\min_{D^w} J_{12} \right) = D_1 J_{12}(\hat{D}^w) \quad \text{Eq 38}$$

$J_{12}(\hat{D}^w)$ is generally a high degree function of D_1 , and its minimization cannot be performed analytically. However, when only pulp per cent solids and solids flow rates have been measured, the minimum can be found analytically. Generally, $J_{12}(\hat{D}^w)$ is minimized with respect to D_1 using a one-dimension optimization algorithm. We have selected the COGGIN algorithm which is based on a method of quadratic adjustment of the function to be minimized (Appendix C). However, we have not tried other one-dimension algorithms which eventually could work better.

In the case where only S_j and D_j variables have been measured, a careful examination of the terms u_i and w_j (Eq 30 and 31) shows that the solution \hat{D}^w of Equation 37 can be written:

$$\hat{D}^w = D_1 \hat{d}^w \quad \text{Eq 39}$$

where the relative water flow rates \hat{d}^w are not a function of D_1 .

In this particular case J_{12} is:

$$J_{12} = \sum_j (D_j - D_{oj})^2 \frac{1}{\sigma_j^2} + \sum_j (\delta_j - \delta_{oj})^2 \frac{1}{\sigma_j^{s2}} \quad \text{Eq 40}$$

and consequently $J_{12}(\hat{D}^w)$ can be written as the following function of D_1 :

$$J_{12}(\hat{D}^w) = J_{12}(D_1) = \left(\sum_j d_j^2 / \sigma_j^2 \right) D_1^2 - 2 \left(\sum_j D_{oj} d_j / \sigma_j^2 \right) D_1 + \sum_j D_{oj}^2 / \sigma_j^2 + \sum_j \left(\frac{\hat{d}_j^w}{d_j} - \delta_{oj} \right)^2 / (\sigma_j^s)^2 \quad \text{Eq 41}$$

$J_{12}(D_1)$ is a second degree function of D_1 and can be easily minimized by zeroing its derivative. The minimum is found for the following value of D_1 :

$$\hat{D}_1 = \left(\sum_j D_{oj} d_j / \sigma_j^2 \right) / \left(\sum_j d_j^2 / \sigma_j^2 \right) \quad \text{Eq 42}$$

2.4.2.3 Calculation of third-level mass fractions

At this second minimization level (Fig. 11), the relative solid flow rates d are defined by the first minimization level (supervisory) and the constraints $H_3 = 0$ (Eq 17c and d) become linear with respect to x . As a consequence the minimum can be found explicitly using the general method given in Appendix B. Two cases must be distinguished: one when the mass fractions do not verify Equation 17d (incomplete analyses), and the other one when Equation 17d is verified (complete analyses). The first corresponds to networks 5, 6 and 7, and the second one to network 3.

Therefore, first of all, J_3 can be horizontally partitioned into several functions, each one corresponding to a given network k :

$$J_3 = \sum_k J_3^k \quad k = 3, 5, 6, 7 \quad \text{Eq 43}$$

The corresponding constraints $H_3 = 0$ can also be partitioned according to the networks:

$$H_3 = 0 \quad H_3^k = 0 \quad k = 3, 5, 6, 7 \quad \text{Eq 44}$$

Since the variables in the various level 3 networks are assumed to be independent, we have:

$$\left(\begin{array}{c} \min \\ x \\ H = 0 \end{array} \right) J_3 = \sum_k \left(\begin{array}{c} \min \\ x_k \\ H_3^k = 0 \end{array} \right) J_3^k \quad \text{Eq 45}$$

Incomplete analyses

In this case, there is no relationship between the mass fractions of the various elements i in the same stream j . The function J_3^k can be minimized independently for each element according to:

$$\left(\begin{array}{c} \min \\ x_k \\ H_3^k = 0 \end{array} \right) J_3^k = \sum_{i=1}^{n_k} \left(\begin{array}{c} \min \\ x_{ki} \\ H_3^{ki} = 0 \end{array} \right) J_3^{ki} \quad \text{Eq 46}$$

Finally, the minimization to be performed in matrix notation is:

$$\text{minimize } J_3^k = (x_{ki} - a_{ki})^T (V_{ki}^k)^{-1} (x_{ki} - a_{ki}) \quad \text{Eq 47}$$

$$\text{under } m_{k\underline{d}} x_{ki} = 0 \quad i = 1 \dots n_k \quad \text{Eq 48}$$

This problem is highly non-quadratic and the solution is only tractable by a direct optimization method. The algorithm selected here, the BOTM algorithm of conjugate gradients without derivatives evaluation (Appendix E), is recognized as being powerful and safe with respect to the convergence criterion. However, more recent methods available from the Harwell library (22) might be more efficient and could be substituted for the present one.

Due to the solids conservation equation $m_2 d = 0$, the variables d are not independent and the minimization is performed only on a set θ of independent d values. Then for the lower minimization levels the d values are generated by Equation 11.

The algorithm must be initialized by a set of θ values. This can be done automatically or by the program user. The automatic method used is identified as the "nodes imbalance technique" and has already been described in detail and programmed in BASIC (20). The method consists in selecting a network k (3 or 5) and in minimizing with respect to θ the sum of squared "nodes imbalances" ϵ_i defined by:

$$m_k d_k a_{ki} = \epsilon_i, \quad i = 1 \dots n_k \quad \text{Eq 62}$$

which is the mass conservation constraint written for the experimental values a_{ki} of mass fractions x_{ki} . The function to be minimized is:

$$J = \sum_j \sum_i \epsilon_{ij}^2 \quad \text{Eq 63}$$

Substituting Equation 62 into Equation 63 and expressing d as a function of θ , J becomes:

$$J(\theta) = \sum_i a_{ki}^T d_k m_k^T m_k d_k a_{ki} \quad \text{Eq 64}$$

$$J(\theta) = \sum_i a_{ki}^T (\underline{L}\theta + \underline{\ell}) m_k^T m_k (\underline{L}\theta + \underline{\ell}) a_{ki} \quad \text{Eq 65}$$

whose derivatives with respect to θ must vanish.

The solution is finally:

$$\theta = -(\underline{L}^T \alpha \underline{L})^{-1} \underline{L}^T \alpha \underline{\ell} \quad \text{Eq 66}$$

$$\text{with } \alpha = \sum_{i=1}^{n_k} a_{ki}^T m_k^T m_k a_{ki} \quad \text{Eq 67}$$

The automatic initialization option can be activated only when the set of independent streams selected by BILMAT is contained in the selected network k (3 or 5). As the user never knows which streams will be selected as independent streams by the program, a sufficient (but not necessary) condition to meet this requirement is that networks 2 and k (3 or 5) are identical. If not, it is still possible to use this option, but the program will stop execution and print an error message each time this requirement is not satisfied. In this case, the users

should deactivate option IZ (12) and introduce their own initial values in the data file and restart the program.

Figure 12 summarizes the principles of the hierarchical minimization of J . Algorithms 1, 2, 3 and 4 correspond to the analytical solution of the minimum of a linearly-constrained quadratic function (Appendix B).

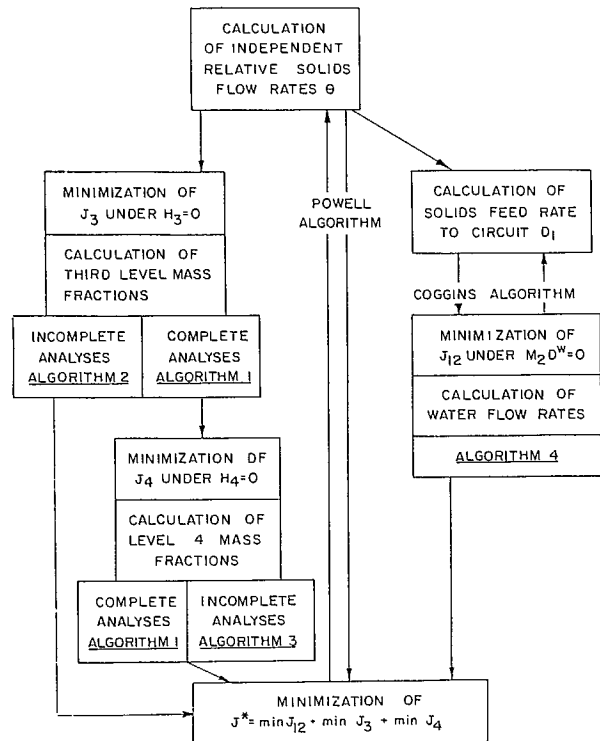


Fig. 12 - Flowsheet of the minimization algorithm

2.5 SENSITIVITY ANALYSIS

2.5.1 Introduction

The problem faced here is to find what kind of errors are induced on the estimates of the variables of the mass balance problem resulting from the BILMAT computation. The question is: How sensitive are the results of a mass balance computation to errors in measured variables? Or alternatively expressed: what is the reliability of the results, or what are their variances? These questions are pertinent to the variables which are not measured but derived from the mass balance computation, as well as to the variables which have been measured and then adjusted.

Besides providing the evaluation of the reliability of the results, the sensitivity analysis allows a classification of the data with respect to the amount of information they contain. This aspect of the analysis is very important since it helps to define a more appropriate sampling strategy. The measurement corresponding to very informative data should be very carefully performed, at the same time variables which contain very little information can be eventually ignored in the data collection step.

Finally, sensitivity analysis aims at detecting outliers by comparison of the adjustment calculated by BILMAT to the error model assumptions. However, it must be pointed out that the concept of outliers is relative to the assumed error model and a particular value may behave as an outlier only because the error model is inadequate due to a bias or an underevaluated variance.

2.5.2 Principles of Calculation

The sensitivity analysis is limited to solid flow rates and level-3 data. Whenever possible, pulp flow rates and pulp per cent solids are combined to compute solid flow rates. Other sources of data are not considered. Finally, the extra constraint for complete analysis (sum of mass fractions equal to 1 on each stream) is not taken into account.

The problem of sensitivity is schematically represented in Figure 13. As can be seen, the problem is to find the variances of the estimates knowing the measured values and the error models. It must be pointed out that the variances of the estimates are valid only within the assumptions of the selected error models. If the models are incorrect (wrong selection of variances), or the implicit assumptions (unbiased and uncorrelated normal distributions) not verified, the estimates variances could also be incorrect. The implicit assumptions are generally not exactly true but can often be considered acceptable approximations. Consequently, the estimates of the variances are acceptable and give valuable information on the sensitivity of the method.

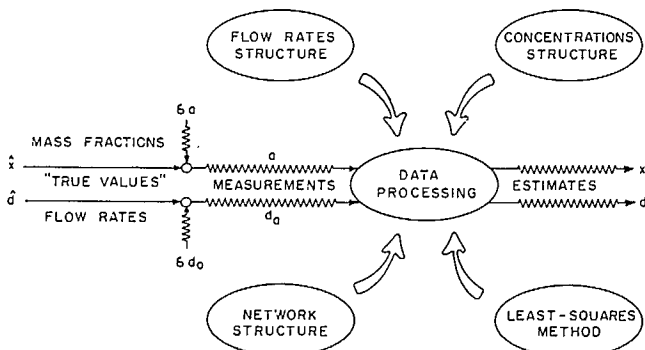


Fig. 13—Propagation of errors through mass balance computation

The measurement errors are used by the program to produce variance estimates. One of the factors which affect the calculation is the structure of the data (values of flow rates and mass fractions): it is well known, for example, that it is difficult to calculate mass flow rates when very small concentration variations take place in various units of a flowsheet. Another factor is the structure of the network: it is also intuitive that, when the network is complex (containing a large number of recycling streams, for example), the estimates of flow rates are not accurate if they have not been measured. Lastly, the statistical method used to calculate the estimates also has an influence.

The BILMAT method, based on the maximum likelihood principle, leads to the minimum variance estimates, so it is to be expected that simplified approaches (e.g. the **nodes imbalance** method) would lead to higher variances of the estimates (16).

The procedure used here is to generate disturbances on the BILMAT estimates \hat{x} and \hat{d} , obtained for a given set of data considered as *true values*, and to study their propagation through the material balance calculations.

Starting from the values \hat{x} and \hat{d} , which necessarily verify mass conservations laws, data are created by:

$$\begin{aligned} a &= \hat{x} + \delta a \\ d_o &= \hat{d} + \delta d_o \end{aligned} \quad \text{Eq 68}$$

where d_o is the measured value of relative ore flow rates.

These data are processed by the selected least-squares technique and produce estimates which are:

$$\begin{aligned} x &= \hat{x} + \delta x \\ d &= \hat{d} + \delta d \\ \text{or } \theta &= \hat{\theta} + \delta \theta \end{aligned} \quad \text{Eq 69}$$

The study of the distribution of δx and δd gives the variances of the estimates. The Monte Carlo simulation technique can be used to solve this problem: it creates synthetic sets of data, using the error models, and afterwards studies the statistical properties of the estimates. This method has been proposed by Laguitton (6,7) in the MATBAL program.

The method used here consists in linearizing equations and in analytically solving the data adjustment problem to directly find distribution laws of the estimates. It is only valid for small deviations around the true values, but requires considerably less computation time than the Monte Carlo simulation technique.

2.5.3 Results Reliability

Appendix F gives the relationships developed for the variances of the solids flow rates and the mass fractions at the third level. They allow evaluation of the reliability of the results. For variables which have been measured, the variances of the residuals are also calculated by BILMAT using:

$$\begin{aligned} \text{Variance of residual} &= \text{variance of measurement} \\ &\quad - \text{variance of estimate} \end{aligned} \quad \text{Eq 70}$$

From this formula it appears that the variances of the estimates, as well as the variances of the residuals, have values less than the variances of the measurements (for unmeasured variables the variances of the estimates can take any value between zero and infinity). It must be pointed out that the variances of the estimates (or of the residuals) cannot be used directly to evaluate the measurement variances.

2.5.4 Evaluation of Data Collection Strategies

The sensitivity analysis can also be used to classify variables with respect to the amount of information they contain. An element of data contains more information than another one if removing it from (or adding it to) the data set increases (or decreases) all the estimate variances more than another one would.

A variable can contain a large amount of information because it is in itself more accurately measured or because it is critical with respect to the mass balance problem structure. BILMAT allows this evaluation since it is possible to repeat the sensitivity calculation removing or adding some data without going through all the mass balance calculations.

Some calculation tests made on a two-product separator (16) show interesting results concerning the amount of information contained in mass fraction data. It appears that the mass fraction data of the components which concentrate in the low flow rates provide the greater information for the flow rates evaluation. It has been demonstrated that the more selective a flowsheet is with respect to a given component, the more informative the data on this component.

The same study (16), shows (using a cement clinker grinding circuit as an example) how to use this sensitivity analysis technique to determine the best location for measuring solids flow rates, i.e., the flow measurements which significantly increase the overall reliability of the results.

Examination of the estimates and residuals variances also provides indications on the role of a variable in the mass balance calculation. Some data contain a large amount of information, that is, their values almost fix the values of flow rates. The mass balance computation is said to be sensitive to them. After adjustment, they have low residuals and low residual variances (since the estimated values are supposed to be unbiased). As a consequence (Eq 70), the estimated value is not significantly more reliable than the measured value. Such variables have to be carefully identified and measured since they have an important weight in the calculations and since they are not significantly better after adjustment.

On the other hand, some data do not contain a large amount of information. After completion of calculations, the corresponding estimated values are more reliable and may result from strong adjustments.

2.5.5 Evaluation of Error Models and Detection of Outliers

Finally, sensitivity results are used in BILMAT to identify outliers by the integer part of the ratio of the residuals and their calculated standard deviations. A value of n indicates that the true ratio is between n and $n + 1$.

Generally, all variables whose residual values are greater than twice the residuals standard deviation must be carefully examined. Two cases are possible:

- The data point is a true outlier, i.e. large errors occurred in the process of collection of the raw data: mixing of samples, of tags, incorrect analysis or sample preparation . . . In this case the datum must be corrected or rejected from the data set and BILMAT analysis restarted.
- The error model variance is underestimated: this may appear if systematically a given component or a given stream gives high residuals compared to their standard deviations. In this case, the error model must be reevaluated and BILMAT restarted. To help in this analysis, the program provides mean values of the reduced residuals r_{ji} of mass fractions:

$$r_{ji} = \frac{x_{ji} - a_{ji}}{[(\sigma_{\beta})^2 - (\sigma_{\gamma})^2]^{1/2}} \quad \text{Eq 71}$$

Theoretically, from a statistical point of view, it is impossible to derive properties of the measurement errors if there is no repetition in the data collection and mass balance calculations on a given ore or coal processing circuit. However, sources of error can occur many times during the same data collection process. For example, the same component can be analyzed at various points in the flowsheet. Consequently, if the method used for the analysis of this component constitutes the main source of error in the final measurement value, the mean of the reduced residuals for this component contains information regarding this source of error.

On the other hand, it may be assumed that on each stream there is a typical sampling error which is transmitted on all analyzed components. Consequently, a mean value of the reduced residuals on the same stream can eventually give information about the relative quality of the sampling from one stream to another.

BILMAT calculates such means per stream and per component using algebraic or absolute value of the residuals:

$$\bar{r}_j = \frac{1}{n} \sum_{i=1}^n r_{ij}$$

$$\bar{r}_i = \frac{1}{p} \sum_{j=1}^p r_{ij}$$

$$|\bar{r}_j| = \frac{1}{n} \sum_{i=1}^n |r_{ij}|$$

$$|\bar{r}_i| = \frac{1}{p} \sum_{j=1}^p |r_{ij}|$$

Theoretically, the mean value of the absolute values of a centered and reduced normal variable is equal to ≈ 0.8 . Therefore, if the means deviate strongly from this value, an inadequate selection of the error model may be suspected. Furthermore, the comparison of the mean values (for absolute values) from stream to stream and from component to component may help in classifying the sampling or analysis procedures and give guidelines on methods to improve data collection. The mean values of the algebraic residuals may eventually give hints on the existence of biases, if abnormal deviations from null values are observed systematically.

To summarize, sensitivity analysis is an essential part of a mass balance program because it gives the reliability of the unmeasured variables as well as of the measured ones. It is also a way to improve data collection and error modelling assumptions. It is recommended that the printout given by BILMAT (giving assumed error standard deviations, residuals, calculated residuals and estimates standard deviations) be carefully examined by the user. It is not possible to give all the possibilities which are provided by the sensitivity analysis but examples are given in Section 5.

2.6 BILMAT PROGRAM AND CAPABILITIES

Mass balance calculations and sensitivity analysis calculations are two somewhat independent steps. Therefore, it is convenient to perform them in two different programs (Section 3).

Due to the large variety of data that can be handled by the programs and the numerous combinations of options, no guarantee of proper program execution can be given for all cases. Based on the authors' experience, the most probable sources of temporary failures are bad data entry, ill-conditioned data, or inadequate values given to the iterative algorithm parameters.

2.6.1 Mass Balance Program Capabilities

The BILMAT program is used to calculate mineral and coal processing flowsheet data which satisfy given mass balance constraints. The results are obtained by adjusting given experimental data in order to minimize the weighted sum of squares of residuals.

As described previously in Sections 2.1 and 2.2, four levels of data and eight networks can be handled by BILMAT. However, some limitations are imposed on the program and these are now described.

Limitation 1: When network 1 is drawn and numbers are given to its streams, the main feed stream to the circuit must be given the number 1. All other streams containing ore or coal must be numbered sequentially (but

the order in which the streams are picked up does not matter) and the water-only streams must have the highest numbers in the series.

Limitation 2: The measured values must be the measurement of mass flow rates, per cent solids, and mass fractions.

Limitation 3: The number of available networks is limited. However, when required, this problem can be overcome by grouping the extra data on the already-used networks. For instance, if networks 5, 6 and 7 are not sufficient to accommodate all the components analyzed on various level-3 networks, it is possible to select one of the networks 5, 6 or 7 which can include the supplementary network needed, and distribute the data on the extra components on the selected network. This is done by creating arbitrary data on the unanalyzed streams with corresponding very high variances (Section 2.8.3).

Limitation 4: Network 4 can accommodate only a complete analysis in the last component of network 3. This situation has been programmed to process subsieve analyses. It is not a theoretical limitation, so the program can be easily modified to generalize the calculation of complete analyses in all the components of network 3 (section 2.4.5).

Limitation 5: Network 8 requires incomplete analyses in all the components of network 3, but not in components of networks 5, 6 and 7 (Section 2.4.5).

Limitation 6: The same components cannot theoretically be processed simultaneously at variable levels 3 and 4 because the supplementary mass conservation constraints are not taken into account by the algorithm. However, BILMAT can be executed with such data but the results may contain some minor inconsistencies between level 3 and level 4.

2.6.2 Sensitivity Analysis Program Capabilities

As described in Section 2.5, the reliability of the results can be evaluated from the variances of the estimated values produced by the mass balance program. This analysis of the variances is done through the sensitivity analysis program.

The program requires that relative ore flow rates be known on all the streams of network 2. It is therefore necessary to perform a mass balance calculation prior to a sensitivity analysis, but it is not necessary to test the sensitivity on all the data used in the mass balance part. This feature is very convenient in evaluating a sampling strategy.

With the present version of the program, only data on networks 3, 5, 6 and 7 and measured solid flow rates can be processed. If, on a given stream, the solid flow rate is unknown but both pulp flow rate and pulp per cent solid are measured, a solid flow rate value can be calculated and may be used for the purpose of the sensitivity analysis.

2.7 DATA INPUT

In order to facilitate data entry for the mass balance program on one hand, and the sensitivity analysis program on the other hand, two data file editors are proposed. They are both FORTRAN programs.

2.7.1 Data File Editor for the Mass Balance Program

EDIMAT is an interactive program dedicated to the editing of the BILMAT system main data file.

EDIMAT can be used to create a new data file, to modify an existing data file, or to produce a commented listing of a data file. All entries performed by the user are in free-field format. Some tests are also done to check options and data compatibility.

The following is a description of the sequence of data-entry steps:

1. Title.
2. Options selected.
3. Network 1 description and stream names.
4. Network 2 description.
5. Network 3 description and data (measured values, associated relative accuracies and component names) if corresponding option is selected.
6. Same as step 5 for network 4.
7. Same as step 5 for network 5.
8. Same as step 5 for network 6.
9. Same as step 5 for network 7.
10. Same as step 5 for network 8.
11. Flow rate measurement data if option is selected.
12. Pulp per cent solid measurement data if option is selected.
13. **BOTM** algorithm parameters (Section 5.1.6).
14. **COGGIN** algorithm parameters (Section 5.1.6) if the combination of selected options requests it.

When utilized in the modification mode, EDIMAT accepts modifications if they are presented in the same order. To facilitate the operation, a menu of modification commands is displayed.

A detailed sample run of the EDIMAT program and the resulting data file are presented in Appendix G.

2.7.2 Data File Editor for the Sensitivity Analysis Program

The sensitivity analysis program uses the same basic data as the mass balance program, plus the results of the mass balance calculations (i.e., relative ore flow rates and adjusted values of all the measurements). Therefore, to reduce the number of files involved, both programs use the same data file. As a consequence one can understand that the mass balance program extends the data file for the sensitivity analysis program by adding the results of the mass balance problem to its own input data file. This operation is user-transparent.

As discussed in Section 2.5, the sensitivity analysis may be used to evaluate different sampling designs. This can be done by comparing the results obtained with a subset of the data and those obtained by using all the data. It is also possible to modify the relative standard deviations of some of the measurements to test their effect on the calculations. To perform all the data manipulations, an editor is proposed — EDISIV. Sample runs of this program are presented in Appendix G.

2.8 PROGRAM OPTIONS

2.8.1 Mass Balance Program Options

Data input and calculation path options are controlled by setting the code variable IZ, an integer array of twelve elements which define the structure of data and the calculations to be performed. The IZ elements take the values listed below or the zero value if not specified:

- IZ (1):** It must be set to 1 if some flow rates have been measured.
- IZ (2):** It must be set to 1 if some pulp per cent solids have been measured.
- IZ (3):** It must be set to 1 if data on network 3 are available (e.g., particle size analyses, or any level 3 complete analysis).
- IZ (4):** It must be set to 1 if data on network 4 are available (e.g., subsieve analyses).
- IZ (5):** It must be set to 1 if the first branch does not include water.
- IZ (6):** It must be set to 1 if data on network 5 are available (e.g., chemical assays, or any level 3 incomplete analyses).
- IZ (7):** It must be set to 1 if data on network 6 are available (e.g., chemical assays, or any level 3 incomplete analyses).
- IZ (8):** It must be set to 1 if data on network 7 are available (e.g., chemical assays, or any level 3 incomplete analyses).

- IZ (9):** It must be set to 1 if data on network 8 are available (e.g., chemical assays on each component of network 3).
- IZ (10):** It must be set to 0 if sub-optimal solution No. 1 is selected for networks 4 or 8 data processing, and set to 1 if sub-optimal solution No. 2 is selected (Section 2.4.1.2). If there is no network 4 or 8 data, IZ (10) must be set to zero.
- IZ (11):** It must be set to 1 if the BILMAT user wants to assign positive values to relative ore flow rates during mass balance calculations.
- IZ (12):** It must be set to 0 to read the user's own starting values for the BOTM algorithm. It must be set to 1, to activate calculation of these starting values using network 3 data. It must be set to 2, to activate calculation of these starting values using network 5 data.

When using the EDIMAT program, the IZ array is filled in by the program as the user answers to prompts.

2.8.2 Sensitivity Analysis Program Options

The code variable IX has for the sensitivity analysis program the same role as IZ for the mass balance program. Five options are proposed to permit the selection of measurements to be used by this particular program. IX array has eight elements, three of which are reserved for future expansion. Others correspond to types of measurement as follows:

- IX (1):** solid flow rate measurements;
- IX (3):** network 3 data;
- IX (6):** network 5 data;
- IX (7):** network 6 data;
- IX (8):** network 7 data.

EDISIV does the setting of the IX array as the user answers to prompts.

2.8.3 Measurement Variances

To construct the least-squares criterion, BILMAT needs the values of the variances associated with the measurements (Section 5.1). They are entered in the program as relative standard deviations expressed in per cent. However, for some special cases the following rules must be adopted:

- If a measured value is zero, BILMAT reads an absolute standard deviation in per cent. This rule was used in Sample Run 2 (refer to Section 5.3 and Appendix H).
- If a measured value is considered very accurate, the user may want to constrain it to its value. This occurs, for instance, when a particle size distribution contains zeros in the top size intervals, which are **true values** considering the nature of the process. To constrain a variable to its measured value, the user must enter a zero value for the corresponding standard deviation in the input file. This rule is also illustrated in Sample Run 2.
- If a measurement is not performed on a given stream and if the corresponding estimated value must be known, the user must enter an arbitrary *measured value* and a 999.999 relative standard deviation (refer to Sample Run 2 for an illustration of the use of this rule).

2.9 PRINTED OUTPUT

In order to fit most of the printers, all output tables, comments, and warning messages are less than 80 characters in width. It is possible, however, to change the format of printing for special cases.

2.9.1 Optional Printings

The mass balance program can produce, upon request, partial results during the iterative calculations. This feature could be useful in the resolution of ill-conditioned data sets. Two printing parameters are available, one for each of the two iterative algorithms.

IPRIN1 is the printing parameter of the BOTM algorithm. It allows the printing of the values of the criterion to be minimized and of the independent relative ore flow rates:

- If IPRIN1 = 1, after completion of the minimization process along one direction within each iteration;
- If IPRIN1 = 2, after completion of each iteration (this is the preferable value in many cases);
- If IPRIN1 = 3, after completion of the overall minimization process (Appendix E).

IPRIN2 is the printing parameter of the COGGIN algorithm (Appendix C). It allows the printing of the criterion to be minimized and the solid flow rate value on stream #1. Four options are proposed here. The first three have a similar effect to IPRIN1 for the BOTM algorithm, and the fourth one, IPRIN2 = 4, inhibits printing of the COGGIN algorithm. This last option is usually recommended in most applications.

Warning messages may also be printed when the data or data file is ill-conditioned or when the BOTM algorithm finds a negative ore flow rate. The former produces fatal error messages and leads to an abort of the program (Section 4.2).

2.9.2 Mass Balance Results

Mass balance results are printed according to IZ values. These results are listed network by network in tabular form. For the corresponding network data, each table gives:

- identification name of the data;
- the measured values (mass fractions are expressed as per cent);
- the estimate values (mass fractions are expressed as per cent);
- the relative residuals (%);
- the relative standard deviations of the measurement error (%).

Note that for network 4 data (which may be any level 4 complete analysis data) two tables are provided. Since BILMAT considers these data as subsieve particle size analyses, the results are presented as complete mass fractions at level 4 and are also linked to level 3 complete mass fractions to produce a particle size analysis as a whole (Sample Run 2).

2.9.3 Sensitivity Analysis Results

The sensitivity analysis results are printed out in three parts. A first table gives the standard deviations of the relative ore flow rates. Then a series of tables, one for the measured solid flow rates and one for each component of networks 3, 5, 6 and 7 taken into consideration during the sensitivity calculation, gives for each stream the measured value, the adjusted value, the residual value, and their standard deviations. The integer part of the ratios of residual values over their standard deviations are also given since they represent the magnitude of the bias detected by the computation. A value greater than two should be particularly investigated since it means the adjusted value differs from the measured value by more than two times the standard deviation of the residual. All standard deviations are absolute, compared to the relative standard deviations printed out by the mass balance program.

The third part of the printout consists of two tables resulting from a statistical analysis of the residuals. For each stream and each component, the mean value of the reduced residuals is given in absolute and algebraic forms (Section 2.5.4).

2.10 FLOWSHEET

Figure 14 is a general overview of the BILMAT system. It shows the interactions between the various components of the system; i.e., data files and programs.

Section 3 gives a technical description of the system while Appendix K presents the listing of a deck of control cards used to run the BILMAT system under the NOS/BE operating system on a Cyber.

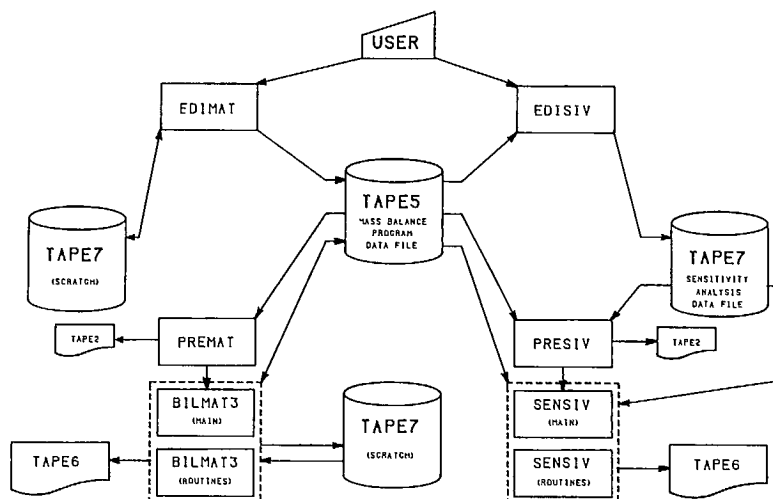


Fig. 14 - General Overview of the BILMAT system

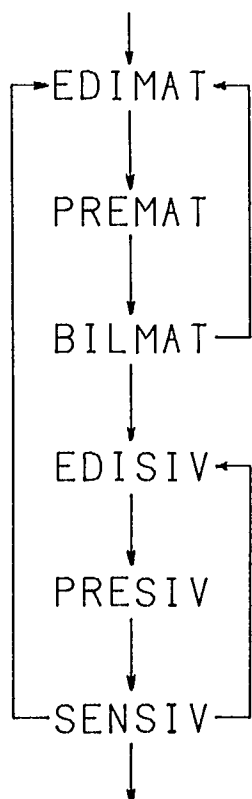
3. SYSTEM DOCUMENTATION

3.1 COMPUTER EQUIPMENT

BILMAT runs on a CDC Cyber 730 computer, operating under NOS/BE, with disk drive, card reader, and line printer or interactive terminal.

3.2 GENERAL OVERVIEW

The BILMAT system utilizes six different programs which are run independently. The various data files are the only links between them. Since these files are created or modified by the programs, the order of execution should be as follows:



The first three programs form the mass balance part, while the last three form the sensitivity analysis part. Both parts are *independent* and can be executed several times successively. A data file which contains the input data to the mass balance program and its results constitutes the input file to the sensitivity analysis programs. Appendix K presents a sequence of control cards to run the whole system and a detailed flowsheet.

3.3 EDIMAT PROGRAM

3.3.1 Purpose

EDIMAT is the interactive editor of the input data file of the mass balance program. It uses a free-field question-

and-answer format. This is intended to relieve the user of the tedious task of typing a lengthy formatted data file. It is a stand-alone program which can be used to create a new data file, to produce a documented listing of an existing data file, or to modify it. Interaction between files and main subroutines is shown in Figure 15.

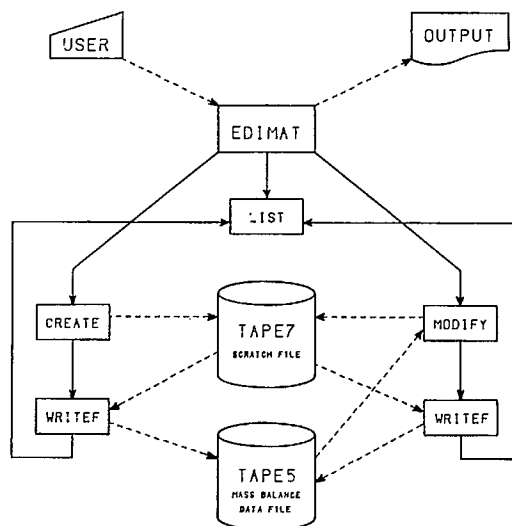


Fig. 15 - General flowsheet of data-input editor EDIMAT

3.3.2 Sequence of Steps

The user can choose to run EDIMAT in three different modes: **list**, **create**, or **modify**.

Under the **list** mode, the sequence of steps is:

- Step 1: to rewind data file;
- Step 2: to read file sequentially and to transfer data with interlaced comments to output file (TAPE3);
- Step 3: to transfer the user to modification mode, if so desired.

Under the **create** mode, the sequence of steps is:

- Step 1: to obtain data from the user (Section 2.7.1), to perform some compatibility checks, and to record options in variable IZ, dimensions in variable PAR, and data in file TAPE7;
- Step 2: to build data file TAPE5 from IZ, PAR, and TAPE7;
- Step 3: to transfer the user to list mode, if so desired.

Under the **modify** mode, the sequence of steps is:

- Step 1: to display the menu of possible modifications;
- Step 2: to get a modification command;
- Step 3: to execute commands by reading data on TAPE5, and writing modified data on TAPE7, or to go to step 5;
- Step 4: to go back to step 2;
- Step 5: to transfer the content of scratch file TAPE7 to data file TAPE5;
- Step 6: to transfer the user to list mode, if so desired.

3.3.3 Variables and Dimensions

In order to minimize memory requirements, EDIMAT makes extensive use of scratch variables. In fact, only four variables have a constant meaning throughout the program.

IZ is the array of options (Section 2.8.1) and PAR contains all the integer variables used for dimensioning. In modification mode, IZ and PAR contain old values, while NIZ and NPAR contain the new (modified) values. Other variables are shared, as needed, by the various parts of the program.

Table 5 is a reference map of the three labelled COMMON areas used by EDIMAT.

Some arrays have to be dimensioned. This might cause trouble for the user who would like to overflow the limits. For simplification purposes, all these variables are gathered into one labelled COMMON statement. Furthermore, all the dimensions are functions of only one variable: the number of streams in network 1. Refer to Table 4 to change the dimensions which are set in this version to a maximum of $N = 30$ streams.

3.3.4 Data Structure

Besides the interactive input and output files, EDIMAT uses two data files. The file called TAPE7 is a scratch file used to store records temporarily. The file called TAPE5 is the final data file produced by the EDIMAT program. This data file is the major data file for the system: it is used by BILMAT, PREMAT, EDISIV and PRESIV as an input file.

During a creation run, data entered by a user are stored temporarily on TAPE7. When all entries are completed, TAPE5 is created from TAPE7. During a list run, only TAPE5 is used. During a modification run, TAPE5 content is transferred sequentially to TAPE7 and modifications are done during the transfer. When modifications are completed, a new TAPE5 is created from TAPE7.

TAPE5 format is presented in Table 6. Each record may use several lines or cards of 80 characters each, some of which are optional.

Table 4 – Main arrays used by EDIMAT

| Array name | Dimension | |
|------------|-----------|----------|
| | Value | Meaning |
| IVEC | 60 | 2.N |
| RVEC | 30 | N |
| IMAT | 30,30 | N,N |
| RMAT | 30,30 | N,N |
| RMEM | 30 | N |
| OPTFOR | 51 | CONSTANT |
| FLAG | SCALAR | CONSTANT |
| RVEC1 | 30,3 | N,3 |

Table 5 – Labelled COMMON areas used by EDIMAT

| Common block | Used by |
|--------------|--|
| PARAM | EDIMAT, LISTE, NETWRK, AVANCE, TRANSF, CALPLA, CALOPT, MATBIN, CHECK |
| FICH | EDIMAT, LISTE, NETWRK, AVANCE, MODIF, MATBIN, READF, WRITEF, PASSIZ |
| WORK | EDIMAT, LISTE, NETWRK, CALOPT, MODIF, READF, PASSIZ |

Table 6 – TAPE5 structure in EDIMAT program

| Record No. | Record description | Format | Option |
|------------|--|----------|-----------------|
| 1 | IZ options | 12I2 | None |
| 2 | Number of nodes and streams in networks 1 and 2 | 4I2 | None |
| 3 | Number of nodes, streams and components in network 3 | 3 | IZ(3) |
| 4 | Same as record 3 but for network 4 | 3I2 | IZ(4) |
| 5 | Same as record 3 but for network 5 | 3I2 | IZ(6) |
| 6 | Same as record 3 but for network 6 | 3I2 | IZ(7) |
| 7 | Same as record 3 but for network 7 | 3I2 | IZ(8) |
| 8 | Same as record 3 but for network 8 | 3I2 | IZ(9) |
| 9 | Number of pulp, solid, and water flow rate measurements | 3I2 | IZ(1) |
| 10 | Number of pulp per cent solid measurements | I2 | IZ(2) |
| 11 | Title | 18A4 | None |
| 12 | Labels for network 1 streams | 6(3A4) | None |
| 13 | Network 1 incidence matrix | 35I2 | None |
| 14 | Numbers of network 1 streams used to describe network 2 | 35I2 | None |
| 15 | Network 2 incidence matrix | 35I2 | None |
| 16-17 | Same as 14-15 but for network 3 | | IZ(3) |
| 18 | Network 3 data | 10F7.3 | IZ(3) |
| 19 | Relative standard deviation associated with network 3 data | 10F7.3 | IZ(3) |
| 20 | Network 3 component labels | 9(2A4) | IZ(3) |
| 21-25 | Same as 16-20 but for network 4 | | IZ(4) |
| 26-30 | Same as 16-20 but for network 5 | | IZ(6) |
| 31-35 | Same as 16-20 but for network 6 | | IZ(7) |
| 36-40 | Same as 16-20 but for network 7 | | IZ(8) |
| 41-42 | Same as 16-17 but for network 8 | | IZ(9) |
| 43-44 | Same as 18-19 but for network 8 data related to the first component of network 3 | | IZ(9) |
| 45 | Network 8 component labels or blank | 9(2A4) | IZ(9) |
| NOTE | Use 43-45 record structure for all other components of network 3 | | |
| 46 | Stream number, pulp flow rate value, relative standard deviation | I2,2F7.3 | IZ(1) |
| NOTE | Use as many records as pulp flow rate measurements | | |
| 47 | Same as 46 but for solid flow rate measurements | | IZ(1) |
| 48 | Same as 46 but for water flow rate measurements | | IZ(1) |
| 49 | Same as 46 but for pulp per cent solid measurements | | IZ(2) |
| 50 | One record per stream beginning with stream 2 and containing the relative ore flow rate estimate and a tolerance in per cent | 2F7.3 | IZ(12) |
| 51 | BOTM algorithm parameters: escale value, maximum number of iterations, printing option | | |
| 52 | COGGIN algorithm parameters: a solid flow rate estimate for stream 1, a tolerance in per cent, a step size and a printing option | 3F7.3,I2 | IZ(1) and IZ(2) |

Interactive input and output files are identified as TAPE2 and TAPE3 respectively.

3.3.5 Subroutines

EDIMAT uses 14 subroutines. A reference map is given in Table 7. The following is a short description of the main purpose of each.

| | |
|----------------|--|
| LISTE | to produce on the output file the documented listing of the data file TAPE5. |
| DUMPIZ | to produce a documented listing of the mass balance options as selected through the IZ array. |
| IZQUERY | to organize the query of the mass balance options and to store them in the IZ array. |
| MATBIN | to organize the description of a network and to build its incidence matrix. |
| NETWRK | to organize the modifications of the data related to a given network. |
| MODIF | to organize the modifications of the incidence matrix of a given network. |
| CALOPT | to perform several tests to prevent conflicts between options during a modification run. |
| CHECK | to check if the modification command entered by the user is a valid one. |
| READF | to read the first records of TAPE5, to interpret them, and to store the values in IZ and PAR arrays. |
| PASSIZ | to perform a dummy read of the first records of TAPE5. |
| TRANSF | to transfer records from one file to the other. Files are not rewound before or after execution. |

| | |
|---------------|--|
| CALPLA | to compute the number of the record of TAPE5 where the stream numbers used to describe a given network reside. |
| AVANCE | to transfer a given number of records from TAPE5 to TAPE7. |
| REPOND | to validate the user's answer to a YES/NO question. |

Table 7 – Reference map for EDIMAT subroutines

| Subroutine | Called by |
|------------|----------------|
| LISTE | EDIMAT |
| NETWRK | EDIMAT |
| AVANCE | EDIMAT, NETWRK |
| DUMPIZ | EDIMAT, LISTE |
| CALPLA | EDIMAT, NETWRK |
| TRANSF | EDIMAT, WRITEF |
| CALOPT | EDIMAT |
| MODIF | EDIMAT, NETWRK |
| READF | EDIMAT, LISTE |
| WRITEF | EDIMAT |
| PASSIZ | EDIMAT |
| MATBIN | EDIMAT, MODIF |
| CHECK | EDIMAT |
| REPOND | IZQUERY |
| IZQUERY | EDIMAT |

3.3.6 Flowsheet

A general overview of the three modes of operation of EDIMAT and the interactions of the various files is presented in Figure 15, (Section 3.3.1).

3.4 PREMAT PROGRAM

The PREMAT program is a stand-alone program used as a preprocessor to the BILMAT subroutines. Its function is to write a FORTRAN main program to call the subroutines used by BILMAT.

The purpose of this is to adjust array dimensions to the requirements of each specific data set. This has two advantages: it saves memory space in all applications, and relieves the user of the tedious and time-consuming task of changing dimensions of arrays when necessary.

PREMAT reads the actual dimensions of the problem from the first records of TAPE5 (the mass balance data file) and writes on TAPE3 a coded main program with dimension statements filled in with values that fit the data set requirements. The main program, called BILMAT, that is written in FORTRAN, includes several DIMENSION, COMMON, and DATA statements, plus three CALL statements: one for the data input subroutine, another for the subroutine which executes mass balance computation, and a last one for the subroutine which prints out the results.

Due to the selection of options, all arrays are not necessarily used; in which case their dimensions are set to 1, since FORTRAN does not accept null dimensions.

3.5 BILMAT PROGRAM

3.5.1 Purpose

The purpose of the BILMAT program is to perform the mass balance calculations. It is composed of a main program and several subroutines. As explained in Section 3.4, the main program is different for each application since it is written by a preprocessor to reflect the actual size of the problem. It also must be compiled each time it is run. It is used essentially to allocate memory to variables and to call the three major subroutines, INTIAL, MASCAL, and IMPRES. The subroutines are classified under four headings according to their purpose: read the input data file, perform the computation, display the results, and assist other subroutines.

3.5.2 Sequence of Steps

The three main steps are performed by INTIAL, MASCAL, and IMPRES subroutines:

- Step 1: to read input data file;
- Step 2: to perform computation;
- Step 3: to display results and extend data file.

More details are given in Figure 16.

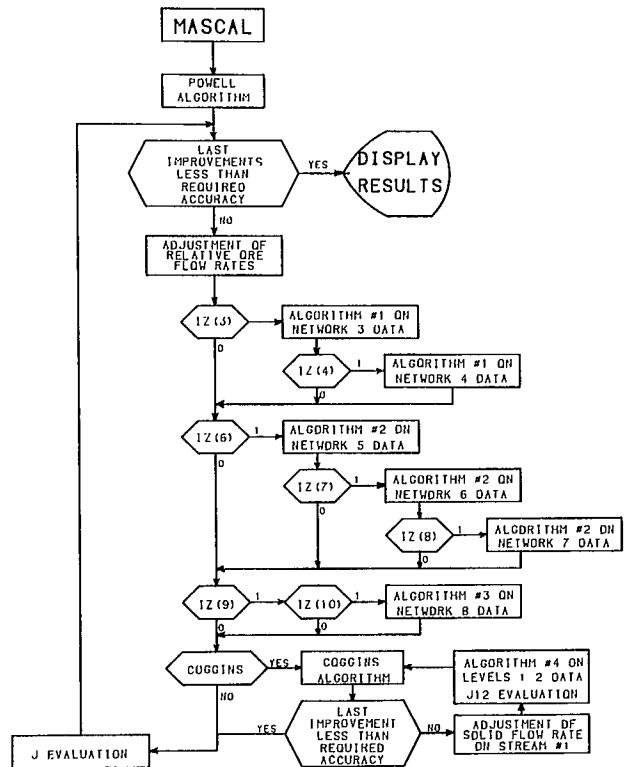


Fig. 16 - General flowsheet of the mass balance calculation subroutine MASCAL

3.5.3 Variables and Dimensions

The main arrays and their dimensions are listed in Table 8.

Table 8 – Arrays dimensioned by the preprocessor

| Array name | Dimension | Array name | Dimension |
|------------|---------------|------------|---|
| MATG1 | (3,PG1,MG1) | MVDSR | MIC |
| MVG1 | MG1 | DSR | MIC |
| NRG1 | (NG1,MG1) | DSA | MIC |
| G1U | PG1 | VMACU | MAC |
| MATG2 | (3,PG2,MG2) | X1 | MIC-NIC-1 |
| MVG2 | MG2 | E1 | MIC-NIC-1 |
| NRG2 | (NG2,MG2) | GL | MIC.(MIC-NIC-1) |
| G2U | PG2 | WA5 | (PG1-1).MIC.(MIC + 1)/2 if PG1 > PG2 else (PG2-1).MIC.(MIC + 1)/2 |
| MATC1 | (3,PC1,MC1) | MATCGX | (3,PCG,MCG) |
| MVC1 | MC1 | NRAUX | (NIC,MIC) |
| NRC1 | (NC1,MC1) | ANR1 | (NIC,MIC-NIC) |
| C1U | PC1 | ANR2 | (NIC,NIC) |
| MATC2 | (3,PC2,MC2) | W1 | MIC-NIC-1 |
| MVC2 | MC2 | MATAUX | (3,MG2,PG2) |
| NRC2 | (NC2,MC2) | DILU | (3,MDILU) |
| C2U | PC2 | DS | (3,MDS) |
| MATC3 | (3,PC3,MC3) | DP | (3,MDP) |
| MVC3 | MC3 | DE | (3,MDE) |
| NRC3 | NC3,MC3 | SL | MIC |
| C3U | PC3 | BØ | NIC.MIC |
| VARCG | (PG1,PCG,MCG) | V11 | MAC |
| CG | (PG1,PCG,MCG) | V12 | MAC |
| CGA | (PG1,PCG,MCG) | V21 | MAC |
| MVCG | MCG | V22 | MAC |
| NRCG | (NCG,MCG) | V23 | MAC |
| CDGU | PCG | B1 | (PCG,MCG) |
| NRAC | (NAC,MAC) | WERK | See Section 3.5.1 |
| NRIC | (NIC,MIC) | | |
| DEA | MAC | | |
| LV | NIC | | |
| MVIC | MIC | | |

Other variables are explained through comment cards in the source. BILMAT uses several COMMON areas; these are listed in Table 9.

Table 9 – Labelled COMMON areas used by BILMAT

| Common block | Used by |
|--------------|---|
| BOTSU | MAIN, INTIAL, ESTIME, CONTRL, MASCAL, BOTSUP |
| COGGINS | MAIN, INTIAL, MASCAL, COGGIN, CALMIA |
| IO | MAIN, INTIAL, DATAIN, DEBITS, ANALYS, ESTIME, CONTRL, MASCAL, BOTSUP, COGGIN, CALMIA, IMPRES, RESMAT, RESMIC, ERROR, PASS |
| UTILITY | MAIN, INTIAL, DATAIN, MASCAL, BOTSUP, CALMIA, IMPRES, RESMAT |

The array named WERK is a variable used internally to store results temporarily. Its dimension depends on the dimension of several other variables. For a given application, PREMAT gives to WERK a dimension equal to the maximum value of one of the three following expressions:

$$\begin{aligned} & \text{MAC} \cdot (\text{MAC} + \text{NAC} + 3) + (\text{NAC} \cdot \text{NAC}) \\ & \text{MIC} \cdot (2 \cdot \text{MIC} - 1/2 \cdot \text{NIC} + 3/2) \\ & \text{MIC} \cdot (\text{MIC} + 1) \cdot 5/2 \end{aligned}$$

The meanings of MAC, NAC, MIC, NIC and all other integer variables used for dimensioning purposes are given in Table 10.

Table 10 – Integer variables used for dimensioning purposes

| Integer variables | Meaning |
|--|---|
| NAC, NIC, NG1, NG2, NC1, NC2, NC3, NCG | Numbers of nodes in networks 1 to 8 respectively |
| MAC, MIC, MG1, MG2, MC1, MC2, MC3, MCG | Numbers of streams in networks 1 to 8 respectively |
| PG1, PG2, PCI, PC2, PC3, PCG | Numbers of analyzed components in networks 3 to 8 respectively |
| MDP, MDS, MDE | Numbers of pulp, solid, and water flow rate measurements respectively |
| MDILU | Number of pulp per cent solid measurements |

3.5.4 Data Structure

As an input file, BILMAT uses the formatted data file created by EDIMAT and called TAPE5 (see Table 6). This file is extended at the end of the execution to record the results of the computation. The structure of the new records is given in Table 11.

Table 11 – Data added to file TAPE5 after execution of BILMAT

| Variable | Format |
|--|--------|
| Matrix L (see Appendix A) | 20F3.0 |
| Water flow rate estimates | 7F10.6 |
| Solid flow rate estimates | 7F10.6 |
| Relative ore flow rate estimates | 7F10.6 |
| Estimates of network 3 mass fractions (if IZ(3) = 1) | 7F10.6 |
| Estimates of network 5 mass fractions (if IZ(6) = 1) | 7F10.6 |
| Estimates of network 6 mass fractions (if IZ(7) = 1) | 7F10.6 |
| Estimates of network 7 mass fractions (if IZ(8) = 1) | 7F10.6 |

3.5.5 Subroutines

BILMAT uses 29 subroutines. The following briefly describes their main tasks. They are classified under four groups: the subroutines related to the reading of the input data file; the subroutines which perform the mass balance computation; the subroutines which print out results; and the subroutines (here called *utilities*) which assist other subroutines. A reference map of all the subroutines used by BILMAT is presented in Table 12.

3.5.5.1 Input file-reading subroutines

- INTIAL** This is the main subroutine in this category. From the array of options, it organizes the reading of the input data file, the initialization of various arrays and the search for a set of independent streams.
- DATAIN** Called by INTIAL, it organizes the reading of networks 3 to 8 data and of flow rate measurements.
- ANALYS** Called by DATAIN, it reads the data related to a given network: measured values, relative standard deviations, and labels.
- DEBITS** Called by DATAIN, it reads flow rate or pulp per cent solid measurements.

Table 12 – Reference map for BILMAT subroutines

| Subroutine | Called by |
|------------|---|
| INTIAL | MAIN |
| DATAIN | INTIAL |
| DEBITS | DATAIN |
| ANALYS | DATAIN |
| ESTIME | INTIAL |
| GREATL | INTIAL |
| CONTRL | INTIAL |
| DEBDEP | INTIAL |
| MASCAL | MAIN |
| BOTSUP | MASCAL |
| COGGIN | CALMIA |
| JMICO | CALMIA |
| JMICFL | CALMIA, CONGRA |
| CALMIA | BOTSUP, MASCAL |
| CONGRA | CALMIA, MASCAL |
| CALCV | CALMIA |
| CALCFX | CALMIA, COGGIN |
| IMPRES | MAIN |
| RESMAT | IMPRES |
| RESMIC | RESMAT |
| PRINT7 | IMPRES |
| ERROR | ESTIME, GREATL, DEBDEP, JMICO, JMICFL, CALMIA, CALCFX |
| PASS | INTIAL |
| PMSYM | ESTIME, JMICO |
| SINV | ESTIME, JMICO |
| MFSO | SINV |
| GMPRD | ESTIME, GREATL, JMICO, JMICFL, CALCFX |
| MINV | GREATL |
| SIMQ | JMICO, JMICFL, CALMIA, CALCFX |

DEBDEP Called by INTIAL, it searches for a valid set of independent streams in network 2.

GREATL Called by INTIAL, it computes matrix L as described in Appendix A.

ESTIME Called by INTIAL, it computes the values of a set of first estimates for the independent relative ore flow rates [see option IZ(12) in Section 2.8.1] and reads BOTM algorithm parameters.

CONTRL Called by INTIAL, it reads the independent relative ore flow rate estimates provided by the user [see option IZ(12) in Section 2.8.1] and BOTM algorithm parameters.

3.5.5.2 Mass balance computation subroutines

MASCAL This is the main subroutine in this section. It reads the COGGIN algorithm parameters if necessary, and then organizes the mass balance computation. It also prints out the values of the first estimates of the independent relative ore flow rates.

BOTSUP This is the BOTM algorithm subroutine (see Appendix E). Called by MASCAL, it searches for the values of the relative ore flow rates which minimize the J criterion of Equation 24.

CALMIA Called by BOTSUP or MASCAL, it organizes the computation of the value of the J criterion from a set of relative ore flow rate values.

JMICO Called by CALMIA, it computes the value of the criterion part pertaining to network 3 or 4 using algorithm #1 (see Section 2.4.2.3).

JMICFL Called by CALMIA or CONGRA, it computes the value of the criterion part pertaining to networks 5, 6, 7 or 8 using algorithm #2 (see Section 2.4.2.3).

CONGRA Called by CALMIA, it organizes the computation of the criterion part pertaining to network 8.

COGGIN This is the COGGIN algorithm (see Appendix C). It is called only when the data structure requires it. Its purpose is to find the value of the solid flow rate of stream #1 which minimizes the criterion part related to levels 1 and 2 data.

CALCFX Called by CALMIA or COGGIN, it computes the criterion part pertaining to levels 1 and 2 data using algorithm #4 (see Section 2.4.2.2).

CALCV Called by CALCFX, it computes U and W values of algorithm #4 (see Section 2.4.2.1).

3.5.5.3 Printout subroutines

- IMPRES** This is the main subroutine in this section. It organizes the printout of the mass balance results. It also controls the extension of the input data file.
- RESMAT** Called by IMPRES, it controls the printout of the results related to levels 1 and 2 data.
- RESMIC** Called by RESMAT, it controls the printout of the results related to levels 3 and 4 data.
- PRINT7** Called by IMPRES, it performs the extension of the input data file. It temporarily uses another file during the process.

3.5.5.4 Utilities subroutines

- ERROR** This displays the error messages (Section 4.2).
- PASS** Called by INTIAL, it is used to skip the first records of the input data file.
- PMSYM** This performs one of the following matrix products.
$$A*B*AT \text{ or } AT*B*A$$
where B is a symmetric matrix and AT stands for the transposed matrix of A. The resultant matrix is symmetric.
- SINV** This computes the inverse of a symmetric matrix. Matrix origin is lost after execution.
- MFSD** This is used by SINV.
- GMPRD** This computes the product of two matrices.
- MINV** This computes the invert of a general matrix.
- SIMQ** This solves a linear system of equations.

3.5.6 Flowsheet

A general flowsheet of the mass balance computation is outlined in Figure 16, (Section 3.5.2).

3.6 EDISIV PROGRAM

3.6.1 Purpose

EDISIV is an interactive editor to the sensitivity analysis data files. It allows the user to modify the structure of the data set used for the mass balance computation and to alter some of the standard deviations of the measured values. This permits both the evaluation and the design of a sampling campaign (Section 2.5).

3.6.2 Sequence of Steps

The data file extended by BILMAT is used as an input file by EDISIV. The program generates two data files called TAPE15 and TAPE7. TAPE15 emulates the input data file but reflects the modifications brought by the user. TAPE7 contains the new data set structure.

EDISIV goes through the following steps during execution.

- Step 1: To interpret, from input data file, the actual data structure and to transfer it to TAPE15.
- Step 2: To read network 3 data (if any), to ask for the user's modifications, and to transfer modified data to TAPE15.
- Step 3: To transfer network 4 data (if any), to TAPE15. These data are not processable by the sensitivity analysis program but, nevertheless, stay in data files.
- Step 4: Same as step 2 but for network 5 data.
- Step 5: Same as step 2 but for network 6 data.
- Step 6: Same as step 2 but for network 7 data.
- Step 7: Same as step 3 but for network 8 data.
- Step 8: To read flow rate measurements and pulp per cent solid measurements; to display processable data and ask for the user's selection; data are then transferred to TAPE15.
- Step 9: To transfer estimated values (all the input data file extension) to TAPE15 and record the new data set structure on TAPE7 (Section 3.6.4).

3.6.3 Variables and Dimensions

The only purpose of EDISIV is data manipulation. Almost no calculations are performed. As a consequence, dummy variables can be used here as they are in EDIMAT. The only explicit variables are those which deal with dimensions, data structure, and the references to the user's data selection. These variables are arrays with fixed dimensions. Table 13 lists them and should be used if the DIMENSION statements have to be modified by a user whose data set does not fit with the prescribed dimensions. In the proposed version, EDISIV assumes a maximum of $N = 30$ streams in network 2, and a maximum of $P = 20$ analyzed components per network.

Table 13 – Main arrays used by EDISIV

| Array name | Dimension | |
|------------|-----------|---------|
| | Value | Meaning |
| VMACU | 30,3 | N,3 |
| PVG1S | 20 | P |
| PVC1S | 20 | P |
| PVC2S | 20 | P |
| PVC3S | 20 | P |
| GCU | 20,2 | P,2 |
| V | 600 | N,P |
| DP | 3,30 | 3,N |
| DS | 3,30 | 3,N |
| DE | 3,30 | 3,N |
| DILU | 3,30 | 3,N |
| MVDSS | 30 | N |
| MVDPS | 30 | N |

Besides these arrays with modifiable dimensions, EDISIV uses two arrays with fixed dimensions: the array of options (IZ, Section 2.8.1) of the mass balance program and the array of options (IX, Section 2.8.2) of the sensitivity analysis program. Most of the data are stored in labelled COMMON areas (Table 14).

Table 14 – Labelled COMMON areas used by EDISIV

| Common block | Used by |
|--------------|--|
| IO | EDISIV, ECRIRE, READF5, RESEAU, AVANCE, WRIT15, MODIFI |
| UTILITY | EDISIV, READF5, WRIT15 |

Table 15 – TAPE7 data file structure

| Record No. | Record description | Format | Option |
|------------|--|--------|-----------|
| 1 | IX: Sensitivity analysis options | 35I2 | None |
| 2 | Number of processable solid flow rates followed by the number and stream numbers of solid flow rate measurements to be processed | 35I2 | IX(1) |
| 3 | Number and component numbers of assays of network 3 to be processed | 35I2 | IX(3) = 1 |
| 4 | Number and component numbers of assays of network 5 to be processed | 35I2 | IX(6) = 1 |
| 5 | Number and component numbers of assays of network 6 to be processed | 35I2 | IX(7) = 1 |
| 6 | Number and component numbers of assays of network 7 to be processed | 35I2 | IX(8) = 1 |

3.6.4 Data Structure

Besides the interactive input and output files, EDISIV uses three data files; one as an input data file, and two as output data files.

The input data file, called TAPE5, is the file extended by BILMAT after completion of its execution.

The output data file, called TAPE15, is essentially an emulator of TAPE5. It has the same structure but reflects the user's modifications to the standard deviation of measured values. Tables 6 and 11 describe its structure.

The output file, called TAPE7, reflects the data set structure the user wishes to use during the sensitivity analysis calculations. This data set structure may differ from the one used during the mass balance calculations (see Section 3.6.1). Table 15 gives a description of each record in that particular data file.

The interactive input and output files are called TAPE8 and TAPE6 respectively.

3.6.5 Subroutines

EDISIV uses two COMMON areas (Table 14) and eight subroutines. A reference map is given in Table 16.

The following is a list of the subroutines and their main purposes.

| | |
|---------------|--|
| ECRIRE | To write the data file called TAPE7. |
| READF5 | To read, on TAPE5, the IZ vector and the data structure used in the mass balance computation. |
| REPEND | See EDIMAT subroutines, Section 3.3.5. |
| RESEAU | To ask the user for the selection of data to be processed by the sensitivity analysis program. |

| | |
|---------------|--|
| WRIT15 | To create the TAPE15 data file. |
| AVANCE | To transfer from TAPE5 to TAPE15 a given number of records. |
| MODIFI | To alter standard deviation values as requested by the user. |
| DPDILU | To compute, when possible, solid flow rate values from pulp flow rate and per cent solid measurements and to ask the user for the selection. |

Table 16 – Reference map for EDISIV subroutines

| Subroutine | Called by |
|------------|------------------------|
| ECRIRE | EDISIV |
| READF5 | EDISIV |
| REPOND | EDISIV, RESEAU, MODIFI |
| RESEAU | EDISIV |
| WRIT15 | EDISIV |
| AVANCE | EDISIV |
| MODIFI | RESEAU |
| DPDILU | EDISIV |

3.7 PRESIV PROGRAM

PRESIV is a stand-alone program very similar to PREMAT. Its function consists in writing a main program (i.e. a driver) for the sensitivity analysis subroutines. The driver reflects the actual dimensions of the arrays required to process data. The optimization of the memory space requirements is significant in this case since SENSIV does many matrix manipulations which could use a large amount of memory space if matrices were overdimensioned.

PRESIV reads the data structure and problem dimensions from the data files produced by EDISIV on TAPE15 and TAPE7, and writes a main program in FORTRAN on TAPE3. The main program has then to be compiled to be executed.

3.8 SENSIV PROGRAM

3.8.1 Purpose

The purpose of SENSIV is to compute the reliability of all the estimates produced by the mass balance program and to display the results.

3.8.2 Sequence of Steps

The sequence of steps is as follows:

- Step 1: To read data from TAPE15 and TAPE7 data files and to initialize arrays.

- Step 2: To compute the standard deviation of the relative ore flow rate estimates and to print results.

- Step 3: To compute the standard deviation of all the mass fraction estimates used for processing and to print results.

- Step 4: to perform a statistical analysis on the residuals and to print results.

3.8.3 Variables and Dimensions

SENSIV uses the same variables as BILMAT (Table 8) plus the arrays listed in Table 17. The WORK array is a workspace shared by the subroutines. It is used for temporary storage of partial results. Its dimension is the maximum of 7.MIC.NIC and 6.MAC.NAC.

SENSIV also uses two labelled COMMON areas, as shown in Table 18.

Table 17 – Main arrays used by SENSIV

| Array name | Dimension |
|------------|-----------------------|
| PVG1S | PG1 |
| PVC1S | PC1 |
| PVC2S | PC2 |
| PVC3S | PC3 |
| GA1S | MG1.PG1 |
| CA1S | MG1.PC1 |
| CA2S | MC2.PC2 |
| CA3S | MC3.PC3 |
| VG1S | MG1.PG1 |
| VC1S | MC1.PC1 |
| VC2S | MC2.PC2 |
| VC3S | MC3.PC3 |
| VDS | MIC.MIC |
| VDILS | MAC |
| DILAS | MAC |
| MVDSS | MDS |
| VID | MAC.MIC |
| VDEB | MIC.MIC |
| ABER1 | MIC |
| ABER2 | MIC |
| ABER3 | PG1 + PC1 + PC2 + PC3 |
| ABER4 | PG1 + PC1 + PC2 + PC3 |
| ABER5 | MIC |
| VD | MIC.MIC |
| WORK | see Section 3.8.3 |

Table 18 – Labelled COMMON areas used by SENSIV

| Common block | Used by |
|--------------|--|
| IO | SENSIV, PASS, DATSIV, DEBITS, ANALYS, INITIA, RESSIV, CVDEB, CALELE, CAPREC, CALVA, ERRSIV |
| UTILITY | SENSIV, DATSIV, INITIA, CVDEB, CAPREC |

3.8.4 Data Structure

SENSIV uses three files. TAPE7 and TAPE15 are the two data files created by EDISIV, and TAPE6 is the output file which contains all the results of the sensitivity analysis.

3.8.5 Subroutines

SENSIV uses 21 subroutines. A reference map is given in Table 19. Among them, seven are also used by BILMAT: DEBITS, ANALYS, PASS, PMSYM, SINV, MFSD, and GMPRD (Section 3.5.5).

The main functions of the other subroutines are given here.

| | |
|---------------|---|
| DATSIV | To organize the reading of the input data file called TAPE15. |
| INITIA | To perform arrays initialization and to read IX array; to organize the selection of data to be processed. |
| RESSIV | To perform the selection of data to be processed from networks 3, 5, 6 and 7. |
| READ15 | To read TAPE15 extension. |
| DILUDP | To compute all possible solid flow rates from pulp flow rate and per cent solid measurements and to select those to be processed. |
| CVDEB | To organize the computation of the standard deviations of relative ore flow rates and to print results. |
| CALELE | To compute the contribution of networks 3, 5, 6 and 7 data to the standard deviation values of relative ore flow rates. |
| CAPREC | To organize the computation of the standard deviations of the estimated values of networks 3, 5, 6 and 7 data; to perform a statistical analysis on the residuals and to print results. |
| CALVA | To compute the standard deviations of the estimated values of the data of a given network. |

| | |
|---------------|---|
| ERRSIV | To print error messages. |
| DIAG | To transform a single dimension array into a diagonal matrix. |
| SYMET | To transform the storage mode of a symmetric matrix from normal mode to symmetric mode (upper triangular part stored columnwise). |
| NORMAL | To transform the storage mode of a symmetric matrix from symmetric mode to normal mode. |
| GMTRA | To transpose a general matrix. |

Table 19 – Reference map for SENSIV subroutines

| Subroutine | Called by |
|------------|------------------------------|
| PASS | SENSIV |
| DATSIV | SENSIV |
| ANALYS | DATSIV |
| DEBITS | DATSIV |
| READ15 | SENSIV |
| INITIA | SENSIV |
| RESSIV | INITIA |
| CVDEB | SENSIV |
| CALELE | CVDEB |
| CAPREC | SENSIV |
| CALVA | CAPREC |
| NORMAL | CVDEB, CAPREC, CALVA |
| DIAG | CVDEB, CALELE, CAPREC, CALVA |
| SYMET | CVDEB, CALELE, CAPREC, CALVA |
| GMTRA | CAPREC |
| PMSYM | CVDEB, CALELE, CAPREC, CALVA |
| ERRSIV | CVDEB, CALELE, CAPREC, CALVA |
| SINV | CVDEB, CALELE, CAPREC, CALVA |
| MFSD | SINV |
| GMPRD | CAPREC, CALVA |
| DILUDP | DATSIV |

3.9 STORAGE REQUIREMENTS

The minimization of memory requirements was an objective during the coding of BILMAT: the six programs involved are run independently; preprocessors are implemented to use the minimum core amount to process the data of each particular application. All this leads to a system which could use overlays if necessary. The proposed version runs easily under a space of 70K octal words (29K decimal words of 32 bytes) for all data sets that have been tested.

3.10 MAINTENANCE AND UPDATES

The authors cannot provide any formal maintenance of the program. Several updates are expected to upgrade BILMAT capabilities. The main improvements might be

the ability to process together the same component analyses of third and fourth levels, to perform sensitivity analysis on all the data, and to perform material balance calculation on components in more than one phase (liquid and solid, for instance).

4. OPERATING DOCUMENTATION

4.1 OPERATOR INSTRUCTIONS

BILMAT has been developed and tested on the Cyber 730 computer of the Computer Science Centre, Energy, Mines and Resources Canada. The source language is FORTRAN extended 4,8, complying with the American National Standards Institute FORTRAN language.

The following documentation, which is almost always specific to the NOS/BE system, is provided as a guideline for adaptation of other systems.

4.2 OPERATING MESSAGES

Normal system messages are produced by the NOS/BE system. In addition, special diagnostics have been included in BILMAT to aid in trouble-shooting when aborts occur during execution. These messages, issued on the output file, are listed in Tables 20 and 21.

General meanings and suggested actions are provided, but no guarantee can be given since it is impossible to

anticipate all possible errors. Note that errors may be of three kinds:

- bad input file (see 2.7);
- ill-conditioned data;
- no data redundancy (see Appendix J).

4.3 CONTROL RECORDS

A listing of a deck of control records is given in Appendix K. They use the NOS/BE operating system. However, the flowsheet of Figure K1 and the comments interlaced with the control records should be sufficient for users who have to write a deck of control records using another operating system.

4.4 ERROR RECOVERY

The program must be restarted on error.

4.5 RUN TIME

Run time depends on the problem size, the chosen options, the convergence accuracies required, the initial values and, finally, on the structure of the data.

Table 20 – Error messages issued by SENSIV

| Message | Meaning | Action |
|---|---|---|
| ERROR IN SUBROUTINE CVDEB IN VDEB CALC. | A matrix is not invertible when CVDEB computes the relative ore flow rate standard deviations. | Check the redundancy of data selected through EDISIV. |
| ERROR IN SUBROUTINE CALELE NETWORK x DATA | A matrix is not invertible when CALELE computes the contribution of network x data to the standard deviation values of relative ore flow rates. | Check the redundancy of network x data selected through EDISIV. |
| ERROR IN SUBROUTINE CAPREC (2) | A matrix is not invertible when CAPREC computes the standard deviations of flow rates. | Check the redundancy of flow rate data. |
| ERROR IN SUBROUTINE CALVA NETWORK x DATA | A matrix is not invertible when CAPREC computes the standard deviations of network x data. | Check the redundancy of network x data and the values of the standard deviations of the measurements. |

NOTE: The user must be aware that these errors do not necessarily cause an abort of the program.

Table 21 – Error messages issued by BILMAT

| Message | Meaning | Action |
|--|--|---|
| SINGULAR MATRIX IN CALMIA-SIMQ | SIMQ, called by CALMIA, found a singular set of equations. | Check the network 2 matrix in the input file. |
| SINGULAR MATRIX IN JMICO-SINV | SINV, called by JMICO, cannot process its matrix inversion. | Check networks 3 and 4 matrices, or the relative standard deviation values associated with the complete analysis. |
| SINGULAR MATRIX IN JMICO-SIMQ | SIMQ, called by JMICO, found a singular set of equations. | Check complete analyses data. |
| SINGULAR MATRIX IN JMICFL-SIMQ | SIMQ, called by JMICFL, found a singular set of equations. | Check network 5, 6, 7 and 8 matrices, or the relative standard deviation values associated with the measurements. |
| SINGULAR MATRIX IN CALCFX-SIMQ | SIMQ, called by CALCFX, found a singular set of equations. | Check network 1 matrix, or flow rates, or pulp per cent solid measurements, or check data redundancy. |
| SINGULAR MATRIX IN GREATL-MINV | MINV, called by GREATL, cannot process its matrix inversion. | Check network 2 matrix, or modify the numbers given to its nodes. |
| SINGULAR MATRIX IN ESTIME-SINV | SINV, called by ESTIME, cannot process its matrix inversion. | Check networks 1 and 2 matrices and data required by ESTIME and specified in option IZ (12). |
| SINGULAR MATRIX IN DEBDEP | DEBDEP cannot process its independent branches search. | Check network 2 matrix. |
| NETWORK CHOSEN TO ESTIMATE RELATIVE ORE FLOW RATES IS UNSUITABLE | One of the independent streams selected by BILMAT does not appear in the list of streams used to describe the network selected through option IZ (12). | See Section 2.4.2.5. |
| A RELATIVE ORE FLOW RATE IS FOUND NEGATIVE | During the mass balance calculations BILMAT found a negative ore flow rate; according to option IZ (11), this value is deleted or kept and run is restarted. | When this message is printed out at each iteration of the BOTM algorithm, check the input data. |
| MAXIMUM CHANGE DOES NOT ALTER FUNCTION | See BOTM algorithm (Appendix E). | Check the input data. |
| ACCURACY LIMITED BY ERROR ON F | See BOTM algorithm (Appendix E). | Check the input data. |

Table 21 – Error messages issued by BILMAT (continued)

| Message | Meaning | Action |
|---|--|--|
| x ITERATIONS COMPLETED BY BOTSUP | See BOTM algorithm (Appendix E). | Change initial values or give a larger maximum number of iterations. |
| ACCURACY ON THE SEARCH VARIABLE IS LIMITED BY THE ACCURACY ON CRITERION | COGGIN algorithm cannot improve its search variable to reach the desired accuracy. | Change the initial value or its required accuracy. |

5. SAMPLE RUNS

5.1 GENERAL PROCEDURE TO BE FOLLOWED FOR BILMAT EXECUTION

Various steps to be followed by the user of the BILMAT program **before** the execution of a mass balance calculation are recommended in this section. It is also highly recommended that BILMAT users try the sample runs described in Sections 5.2 and 5.3 in various situations before processing their own problem.

5.1.1 Redundancy Analysis

The structure of the data must be analyzed with respect to the BILMAT description: nature of the measurements and level of the variables (see Sections 2.2.1 and 2.2.2). This corresponds to the coded values given to the first nine elements of the IZ vector (Section 2.8.1).

Special attention must be devoted to the identification of the degree of redundancy of the problem. This can be done by counting the number of mass conservation equations and the number of unmeasured variables. If the first one is greater than the second one, the problem as a whole is overdefined. However, it may arise that for some variables the problem is overdefined, whereas for others, it is underdefined or exactly defined. An example of this is given in Appendix J.

It may also arise that the system is theoretically overdefined but that, due to the numerical structure of the problem, some equations are almost linearly dependent leading to exactly defined or even numerically-underdefined systems. These ill-defined mass balance systems will be detected through error messages on singular matrices or through the sensitivity analysis if the computation does not fail during program execution.

5.1.2 Selection of Networks

With each data level, one or several networks (Section 2.2) which have to be correctly drawn must be associated. The branches and nodes must be numbered correctly (Section 2.6.1). As mentioned in Section 2.6, there is no unique way of describing the structure of a given mass balance problem, since arbitrary data can be created on some streams allowing a gathering of some networks. The result is theoretically independent of the way the problem is described. However, the number of iterations required could change as well as the final values depending on the convergence criterion.

5.1.3 Measured Values

The values of the data must be arranged in vectors and tables, in forms compatible with the numbering system adopted for the network branches. All mass fraction values must be expressed in percentage.

5.1.4 Standard Deviation Values

The standard deviations of the measured values must be arranged in the same way and expressed as a per cent (standard deviation relative to the measured value).

Input standard deviation values are one of the keys of the material balance calculation. However, it is also that part of the problem which is the least known and the most dependent on the problem studied. Thus, it is impossible to define a method to evaluate error models. However, our experience of mass balance allows us to give some guidelines.

For each measured value, which is assumed to belong to a normal unbiased population, a standard deviation σ must be selected, i.e., the value such that the measured value has a 68% probability of being within the bracket of the true value plus or minus σ . This value can be evaluated by a careful examination of the errors which are produced at each step of the method leading to the measured value.

For an on-line measurement (such as a pulp flow rate measurement with a flow meter, a pulp density measurement with a gauge, or an ore feed rate measurement by a weightometer) the accuracy of the device can be established by using the vendor figures and eventually by adding some other factors (such as accuracy variation within the variable ranges involved) and certain monitoring inaccuracies.

The problem of calibration is frequently present for on-line measurement devices. The best procedure is to calibrate these instruments periodically. However, if a bias is suspected or deduced from BILMAT results, the data can be corrected or the standard deviation eventually increased to accommodate an eventual bias, (this procedure is not strictly correct from a statistical point of view, but it avoids the introduction of large errors in the mass balance calculations).

For data collected through hand sampling, the errors come from the following steps.

Primary sampling: This is often the main source of errors, in particular for heterogeneous material flowing in pipes where segregation occurs. Good sampling is the first requirement in obtaining a good material balance. It is not always recognized that this is a very complex engineering problem on which much effort must be expended in order to do a good job and to be aware of the inevitable errors. The most extensive work available on this subject is by P. Gy (23). This aspect is also covered in Chapter 2 of the SPOC Manual (24). Some simplified recommendations can also be found in other publications (25 to 27).

It is essential that sampling and material balance computation be performed by the same engineer or the

same team of engineers, as this is the only way sampling errors can be correctly evaluated and the sensitivity analysis of BILMAT can be correctly interpreted to redefine error models or improve sampling practices. A practical way to define sampling error is to classify the samples into three or four classes depending upon a subjective evaluation of the relative quality of the samples taken at various locations in the circuit.

Secondary sampling: This is the step which, starting from a large composite sample obtained by primary sampling, leads to a small sample suitable for analysis. This step normally introduces smaller errors than the previous one if it is performed very carefully. However, it inevitably introduces errors which have to be taken into consideration.

Sample analysis: This is the last step and the error depends on the method used and on the component analyzed. However, it is the step for which, by repetition of analysis, it is generally possible to derive error models. For chemical assays obtained by physical (X-ray fluorescence) or chemical techniques this is quite possible; however, with the sieving technique it is more difficult because the error varies strongly from one ore to another and from one particle size distribution to another (and also with the sample size used and the ambient humidity of the laboratory!). Furthermore, the assumption of uncorrelated errors from one size interval to the others is not verified.

From repetitive sieving analyses of the same batch of particles, we have observed that the standard deviation qualitatively increases with the mass fraction retained in a definite size interval. We have found it suitable, in a first approximation, to use the following type of relationship:

$$\sigma = \epsilon + km$$

where **100 k** is the relative mean accuracy in per cent, **m** the mass fraction of particles, and **ε** a threshold which corresponds to the sensitivity of the method.

If 10 g of material are retained on a sieve, it is possible that the accuracy is ± 1 g (10% precision). However, a sieve retaining 0.1 g will certainly not have the same relative precision. In this hypothetical case, an acceptable error model could be $\sigma = 0.001 + 0.1 m$.

We have frequently observed that the material contained in the last particle size interval (the pan of the Tyler sieving column) has a significantly higher inaccuracy and that the σ value could be increased. For subsieve analyses (by cyclosizer, sedimentation, laser diffraction,...) we have also observed higher inaccuracies.

The final model of errors could be obtained by adding the contributions of the various error sources.

Again it must be emphasized that it is very important to select an adequate error model, even if not exact. It is particularly dangerous to identify as good, data which are wrong. Such an error could strongly bias the final

results, because of the very high contribution that poor data, (not recognized as poor by the error model), have to the least-squares criterion.

Finally, it must be remembered that the evaluation of an error model is an iterative process because the sensitivity analysis of BILMAT allows reevaluation of error models and because, if the same circuit is periodically sampled with the same procedures, the comparison of the adjustments applied to the data from data set to data set is a tool which helps to reveal the error structure.

5.1.5 Selection of Options

The BILMAT options to be used must be selected through code variables IZ and IX (Section 2.8):

- flow rate constraints; see option IZ (11);
- initialization of the relative independent flow rates; see option IZ (12);
- selection of the sub-optimal method for level 4 data; see option IZ (10);
- sensitivity analysis (vector IX).

5.1.6 Control Parameters Values

The control parameters of the BOTM (22) and COGGIN algorithms must be selected.

For the **BOTM** algorithm, we recommend the following values:

- Required accuracy: for each independent relative ore flow rate, a value in per cent must be given: 1 to 10% is generally sufficient, 10% being recommended for a first execution of BILMAT. If 1% is requested for the flow rates, it is recommended that the user request only 10% on the very low flow rates to decrease computation time.
- ESCALE factor: this factor is usually selected as 1 for a 10% accuracy; 10 for 1% accuracy; and 100 for a 0.1% accuracy.
- The maximum number of iterations can generally be selected as twice the number of independent flow rates, but not smaller than 5.

For the **COGGIN** algorithm, we recommend the following values:

- X: an estimated value of the solid flow rate of the feed stream as close as possible to the true value (the mea-

sured value, if available, is a good starting point);

- XLIM: a required accuracy between 0.1 and 1% of the X value, depending on the reliability of the data as a whole.

Note that the **COGGIN** algorithm is not required in the following cases:

- There are no level 1 and 2 measurements.
- Only solid flow rates and/or pulp per cent solids measurements are available at levels 1 and 2.

5.1.7 Running BILMAT

When the problem has been carefully analyzed, the data file can be created using EDIMAT and then program execution can be started. The execution may fail because of an incorrectly formulated mass balance problem containing some *islands* of underdefined mass conservation equations, errors in the data file, or misuses of BILMAT options. Error messages occurring mainly on matrix inversions allow the localization of the problem (see Section 4.2).

The computer printout must be carefully examined to detect anomalies in the calculations. The answers must be meaningful in a physical sense; negative values must not be present in the results. The residuals and the standard deviations of the measurements must be carefully examined, remembering that the standard deviations of the residuals are lower than the standard deviations of the measured values. If a sensitivity analysis has been performed, the reliability of the relative flow rates must be studied. It gives an indication of the amount of information which is contained in the data, and of the flow rates which are difficult to estimate. The residuals analysis must be carefully investigated to detect outliers and to readjust error models.

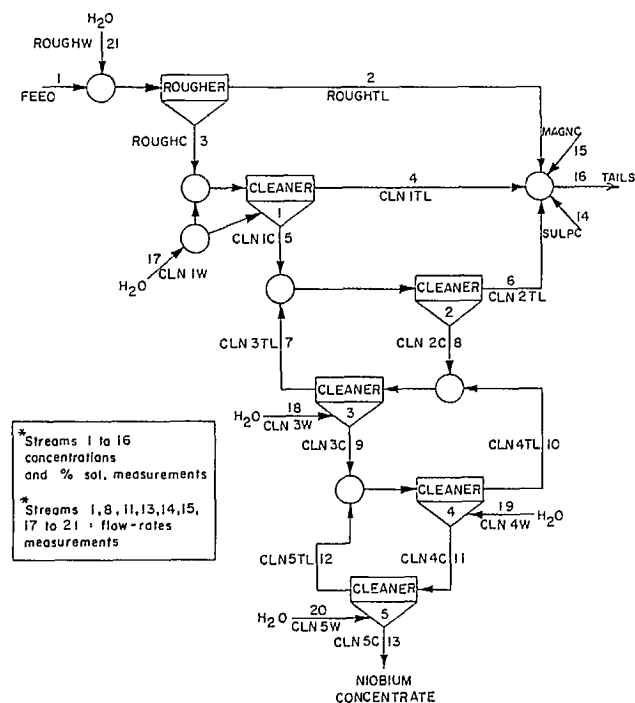
The user should remember that a BILMAT printout which contains ratios of residual-over-residual standard deviation greater than three are not acceptable (unless this situation occurs for variables or residual variances very close to zero).

5.2 PYROCHLORE FLOTATION CIRCUIT/ SAMPLE RUN 1

5.2.1 Problem Presentation

This first application of the BILMAT program presented in this manual is performed on the flotation circuit used by Les Services TMG Inc. (NIOBEC) to concentrate a niobium ore. In a first step, the gangue flotation unit produces a feed to the niobium concentration stage. Five cleaning stages with three tailings recycles are needed to produce the niobium concentrate. The gen-

eral tailings contain the flotation tailings, a sulphides concentrate stream, and a magnetic concentrate stream (Fig. 17).



*Streams 1 to 16 concentrations and % sol. measurements
*Streams 1, 8, 11, 13, 14, 15, 17 to 21 flow-rates measurements

Fig. 17—Les Services TMG Inc., Pyrochlore flotation circuit flowsheet

Data were collected by a team of researchers from Laval University and the TMG Services metallurgist. As shown in Figure 17, various streams were sampled for pulp per cent solids determination and chemical assays of Nb, Mg, Si, Ca, P, Fe and S. Composite samples were collected by repetitive stream cutting during one hour. Then, immediately after, some water or pulp flow rates were measured with the stop-watch method. The objective of the study was to determine the performance of the various stages by an evaluation of the flow rates and concentration values. Due to the various circulating load ratios and the number of variables to be considered, the problem is complex when processed by conventional methods and BILMAT proved helpful in this situation.

5.2.2 Data Presentation

Looking at the flowsheet (Fig. 17), the first network is quite easily drawn (Fig. 18). Note that branches without any information are deleted by the compression of corresponding nodes. The second network is deduced from the first one by removing water branches and nodes. There is no water node to be removed in this case. Note that, as required by the BILMAT program, the higher sequence numbers (17 to 21) are given to the water branches. Network 5, which is used to describe the conservation of chemical components, is identical to the third one since all chemical assays have been performed where samples were taken (Fig. 17).

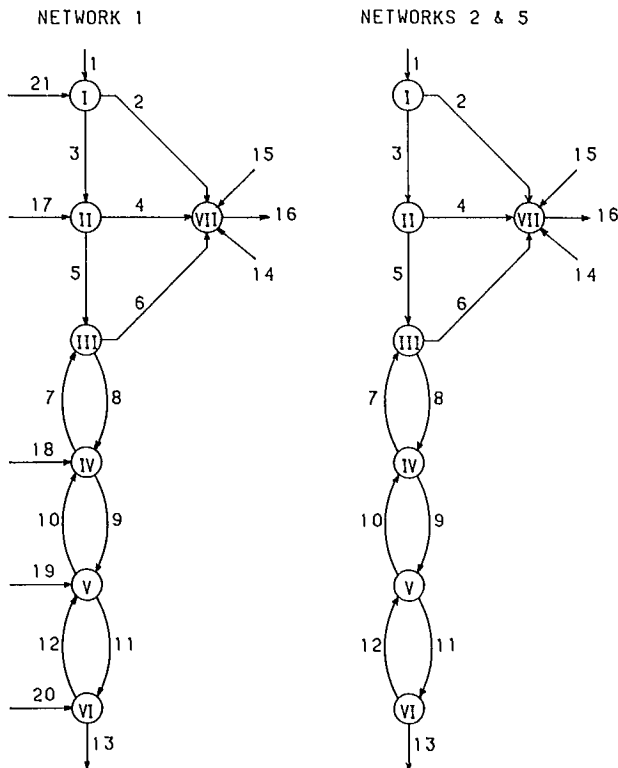


Fig. 18 – Networks corresponding to the circuit of Fig. 17

The chemical assays and the relative standard deviation evaluations were provided by Les Services TMG Inc. Flow rates and per cent solids were measured with different accuracies depending on the accessibility of the stream: relative standard deviations vary from 5 to 50%. A first run of BILMAT using option IZ (12) = 2 (internal estimation of relative ore flow rates using chemical assays data) gives a good approximation of the solution. It was used to start the final run of BILMAT for this problem.

5.2.3 Data File

A documented listing of the data file is presented in Appendix H.

5.2.4 Output File

BILMAT results appear in Appendix H. As specified through the printing parameters, only the final results of the minimization of the function are printed. One can see that eight independent relative ore flow rates were handled by the BOTM algorithm. Four iterations were needed and the function values are detailed for each minimization algorithm, i.e., for each part of the least-squares function.

The first results are relative to flow rates and per cent solids (level 1 and 2 variables); each table gives for each

branch, the measured value (0.00 if not measured, and in percentage for pulp per cent solids), the estimated value (in percentage for per cent solids), the relative residual (residual value/ measured value) in percentage, and the associated relative standard deviation (%) of the measured value.

Here, the level 3 results are relative to chemical assays. For each branch, a table gives, for each chemical component, the measured value (%), the estimate (%), the relative residual (as for level 1 and 2 results) and the associated relative standard deviation values (%) of the measured value.

A sensitivity analysis was performed. The first table gives for each branch the relative ore flow rate value as computed by BILMAT, and the values of the absolute and relative standard deviations.

The next table gives the measured values of solid flow rates, their estimated values and associated residuals and the corresponding **absolute** standard deviations.

Then for each chemical assay, a table is printed which gives the same kind of results: measured, estimated, and residual values and all their **absolute** standard deviations. An outlier flag is also printed (see Section 2.5.5).

The sensitivity analysis program ends with the printing of the results of the statistical analysis on the residuals (see Section 2.5.5).

5.2.5 Discussion of Results

The sensitivity analysis results may be read for three purposes:

1. To evaluate the accuracy of all the estimated values.
2. To review the error models of all the measured values and detect possible bias or outliers.
3. To help the user in the design of a new sampling campaign in order to improve the reliability of the estimated values.

Purpose 1 is straightforward and will not be discussed here.

Purpose 2 should always be followed mainly because a wrong interpretation of the mass balance results could be stated without it. Such a study is based on the review of all the outlier flags and on the statistical analysis of the residuals. In the present sample run, it can be seen that no strong outlier is detected. But the last two tables of the sensitivity analysis results reveal problems. The first column should contain values around 0% and the sec-

ond column values around 80%. The main items to point out are:

- The average of reduced residuals on each stream is not statistically significant since only seven values are used to compute the average. However, strong deviations can be observed around the rougher, the 3rd and 5th cleaners, and the tailings.
- There is no data up-grading on sulphides and magnetic concentrate streams. Because of the structure of the problem, these data are almost not redundant relative to the others. This is confirmed by the residual variances which have almost null values. In line with these values, the residuals are zero. Therefore, the variance analysis shows that the correct interpretation of

these low residuals is not that the measured values are very accurate, but that the corresponding variables cannot be structurally up-graded.

- Niobium assay, which is supposed to be known at 20%, is certainly more accurate than that.

Purpose 3 is probably the most interesting application of the sensitivity analysis program. In this case the statistical analysis results are not taken into account since the program is used to produce a virtual image of what would be the accuracy of the estimated values if the design of the sampling campaign were different. We cannot add more data than processed by the mass balance program, but we can at least modify the error models and mask some assays or solid flow rate measurements. For instance, Table 22 shows the relative standard deviations of the relative ore flow rates for various designs.

Table 22 – Relative standard deviations (%) of relative ore flow rates for Sample Run 1: A comparison

| 1 All | 2 -Nb | 3 Nb 5% | 4 -Mg | 5 -Si | 6 -P | 7 -Ca | 8 -Fe | 9 -S | 10 -DS | 11 Nb 5% DS,Fe,S,Mg |
|----------|----------|------------|----------|----------|---------|----------|----------|---------|-----------|---------------------------|
| 1.9 | 2.0 | 1.3 | 2.0 | 2.0 | 2.0 | 2.1 | 2.6 | 2.1 | 2.1 | 1.5 |
| 8.5 | 8.9 | 5.9 | 8.6 | 8.9 | 8.7 | 9.0 | 11.6 | 9.3 | 9.1 | 6.4 |
| 15.3 | 15.7 | 12.1 | 15.5 | 16.3 | 16.0 | 19.6 | 18.0 | 16.6 | 15.7 | 15.6 |
| 8.5 | 8.9 | 5.8 | 8.7 | 8.9 | 8.6 | 8.9 | 11.8 | 9.2 | 9.1 | 6.1 |
| 9.7 | 10.4 | 6.4 | 10.1 | 10.3 | 9.8 | 10.2 | 13.3 | 10.5 | 10.1 | 6.9 |
| 16.7 | 18.9 | 8.6 | 19.8 | 16.8 | 17.1 | 16.8 | 19.6 | 16.9 | 18.8 | 8.7 |
| 13.6 | 15.3 | 7.3 | 15.7 | 13.7 | 13.9 | 13.7 | 16.3 | 13.8 | 15.6 | 7.4 |
| 12.1 | 13.1 | 7.2 | 17.7 | 12.3 | 12.4 | 12.3 | 14.4 | 12.4 | 13.5 | 7.4 |
| 16.7 | 18.0 | 10.2 | 28.3 | 16.9 | 17.3 | 17.1 | 18.4 | 17.0 | 17.7 | 10.5 |
| 9.6 | 10.2 | 6.1 | 12.1 | 9.7 | 9.6 | 9.6 | 12.3 | 9.8 | 11.3 | 6.3 |
| 15.4 | 16.0 | 12.6 | 41.9 | 15.6 | 15.5 | 15.6 | 17.1 | 15.7 | 16.8 | 12.9 |
| 9.3 | 10.0 | 6.0 | 9.8 | 9.5 | 9.4 | 9.4 | 12.2 | 9.6 | 11.1 | 6.1 |
| 13.0 | 13.1 | 12.8 | 13.1 | 13.1 | 13.1 | 13.1 | 20.9 | 13.1 | 16.7 | 12.9 |
| 18.5 | 18.5 | 18.4 | 18.6 | 18.5 | 18.5 | 18.6 | 18.8 | 26.4 | 25.7 | 18.5 |
| 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.8 | 0.8 | 0.6 |

Column 1 is the reference column.

In **column 2**, the niobium assay has been removed. One can see that it does not greatly change the accuracy of the results. This is amazing since niobium is the selective component of the plant and, therefore, the results should be sensitive to that variable. The explanation is obvious: niobium is assayed with an accuracy of only 20%, and, as a consequence, its contribution to the computation is low.

In **column 3**, the niobium assay is given an accuracy of 5% instead of 20%. This significantly increases the reliability of the results. The component for which the plant is built should always be as accurate as possible since it is the most sensitive variable in the computation.

In **columns 4 to 9**, Mg, Si, P, Ca, Fe, and S assays respectively have been removed from the data set, one at a time. This does not greatly change the results, except locally around the 5th cleaner when Mg is removed, and on all the results when Fe or S are removed. These assays make a significant contribution to the computation.

In **column 10**, the solid flow rates have been removed from the data set. It can be seen that they have a high contribution to the calculation, particularly around the tailings.

Following these investigations, it was decided to run SENSIV another time and the results are presented in **column 11**. Niobium accuracy is set at 5% and the other measured values used are Fe, S, Mg, and the solid flow rates. The results show that a good accuracy can still be obtained and at a *lower cost*.

5.3 HYDROCYCLONE CLASSIFICATION OF MINERALS WITH VARIOUS SPECIFIC GRAVITIES/SAMPLE RUN 2

5.3.1 Problem Presentation

This example deals with a cyclone classifier fed with particles of galena (PbS), pyrite (FeS₂), and quartz (SiO₂). Due to the different specific gravities of the minerals, the cyclone, which classifies particles according to their weight, creates differences in the particle size distributions of the products as well as in their chemical compositions. Therefore in this case, it is extremely useful to have the chemical analysis within each particle size interval in order to determine the classification behaviour of each of the minerals.

The data used here have been created by simulation assuming a partition curve for the classifier, such that the cut size depends on the specific gravity. Furthermore, the material in the last sieve size interval is assumed to be analyzed for particle size using another method (laser diffraction, for instance). Pulp flow rates and pulp per cent solids are supposed to be measured on each stream. Some data are considered missing: i.e., the mass fractions of particles in the first and second size intervals of the feed stream, and the mass fraction of SiO₂ in the third sieve size interval of the underflow.

Error models have been arbitrarily selected and simulated data randomly disturbed to create inconsistent data from the mass conservation point of view. Finally, the null value in the first sieve size interval of the underflow is considered exactly true and constrained at zero using a zero variance.

5.3.2 Data Presentation

The first network, which was used to describe the conservation of pulp flow rates, is identical to the flowsheet (Fig. 19). Five other networks are necessary to express the conservation of all the measured components. Network 2 is used to express the solids conservation; network 3 the particles conservation in sieve size intervals; network 4 the particles conservation in the subsieve size intervals; network 5 the chemical components conservation; and network 8 the level 4 chemical components conservation. All these networks are identical (Fig.20).

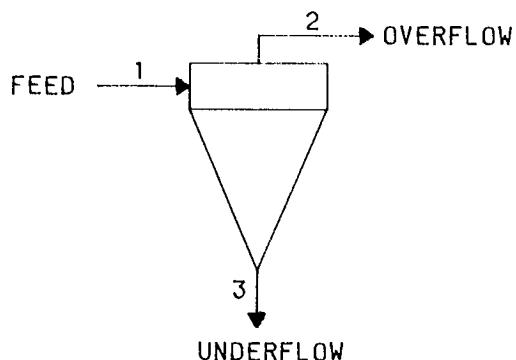


Fig. 19 – Hydrocyclone classifier circuit flowsheet

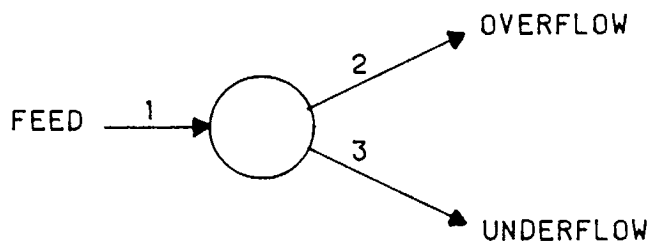


Fig. 20 – Networks corresponding to the circuit of Fig. 19

Among the data, those estimated are given arbitrary values and a relative standard deviation of 999.999%, which allows BILMAT to adjust them freely. The relative standard deviation of the *true* value 0 is set to 0.0.

5.3.3 Data File

A documented listing of the data file is given in Appendix I.

5.3.4 Output File

The output file is reported in Appendix I. After a recall of the BOTM algorithm, the results of the minimization are printed: i.e., values of each part of the function and relative ore flow rates on each branch.

Then, for each branch, the first two tables give the measured value, the estimate, the adjustment value (100 x residual/measurement value), and the relative standard deviation (%) of the flow rates and per cent solids.

The level 3 and 4 tables give:

- the sieve particle size analysis results, including for each branch the measured value (%), the estimate (%), the adjustment value (as above), and the relative standard deviation value (%) of each particle size interval;
- the subsieve particle size analysis results linked to the first one;

- the subsieve particle size analysis results;
- the level 3 chemical assays;
- the chemical assays for each size interval of the sieve particle size analysis.

This ends the printout of the mass balance results.

When the sensitivity analysis is completed, new tables are printed. The first one gives, for each branch, the relative ore flow rate value as computed by BILMAT, and its absolute and relative standard deviations.

The second table presents the results related to solid flow rates. None is measured, but since pulp flow rate and pulp per cent solids are known, solid flow rates can be computed from them and used in the sensitivity analysis.

For each branch and each size interval of the network 3 data set the third table gives the measured mass fraction (% retained), the adjusted mass fraction (% retained), the residual value (%) and the three respective standard deviation values. An outlier flag is also printed (see Section 2.9.3).

The fourth table is for the network 5 data set what the third table is for the network 3 data set.

The last two tables present the results of the statistical analysis on the residuals: it is, in this case, meaningless, due to the lack of data per stream and streams per measurement.

5.3.5 Discussion of Results

Let us see what happened to the null value of the sieve particle size distribution. The results show that it is not adjusted nor are the corresponding level 4 chemical assays, whose values become meaningless while the chemical assays of both other branches are made equal.

The unmeasured variables of the sieve particle size analysis have been freely estimated: here the adjustments are meaningless since the *measured* values are arbitrary values. As for the missing values of the chemical assay of the third size interval of the underflow, it can be seen that it was the only one to be *adjusted*: concentrations of SiO₂ are not adjusted on the other two branches.

The first table, which gives the reliability of the relative ore flow rates, shows that both overflow and underflow have the same absolute standard deviation. This is obvious since an error on the overflow must be compensated exactly by the same error (with an opposite sign) on the underflow.

The next two tables give the reliability of the mass fractions. It appears the estimates have almost the same reliability on the three branches. However, the underflow and overflow data are almost not up-graded while the feed is up-graded (reduction of estimate variance by comparison to measure variance).

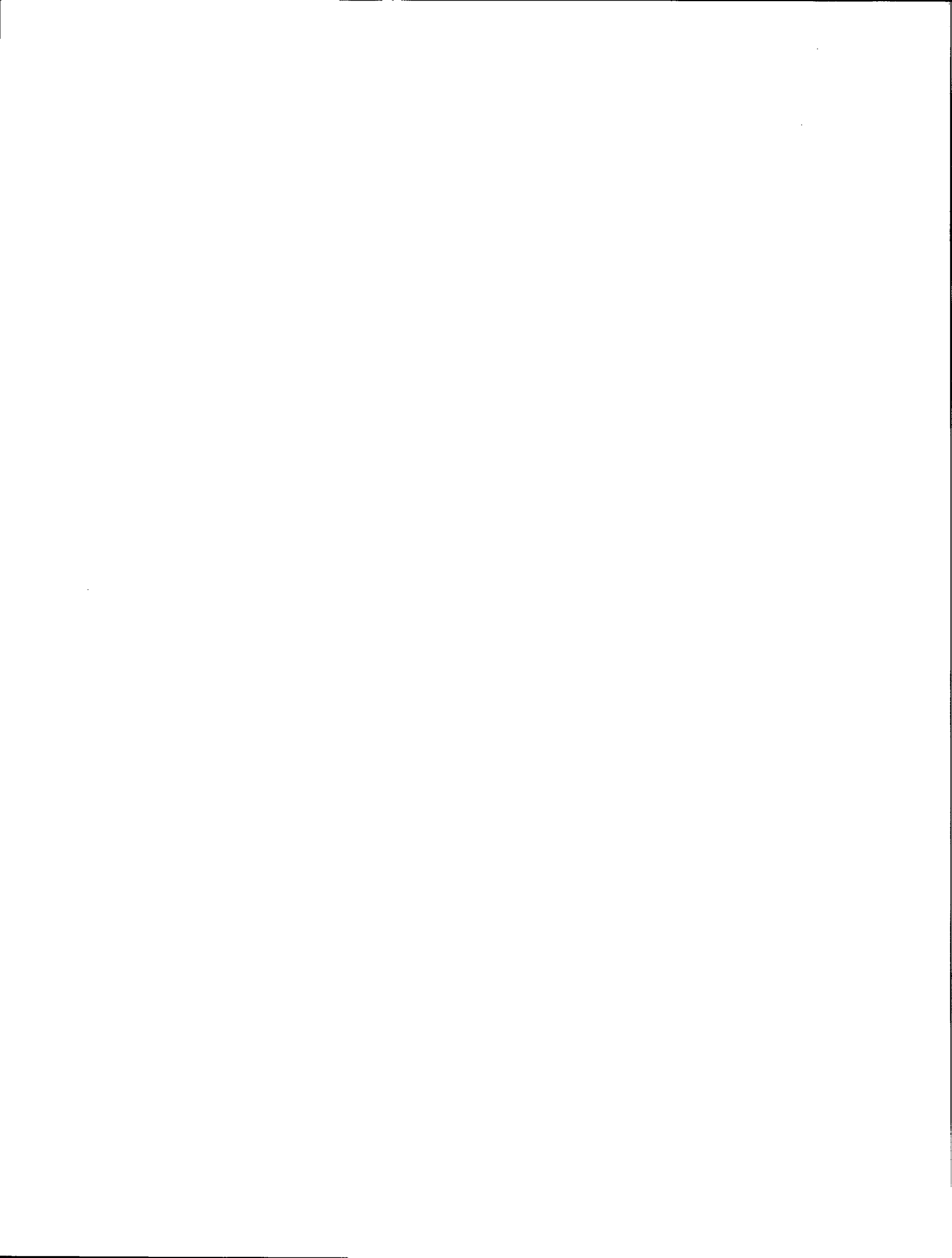
The two missing data in the feed size distributions have very high residual variances which allow them to be freely adjusted and the corresponding estimates have standard deviations comparable to the ones of the measured variables.

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APPENDIX A
INDEPENDENT RELATIVE ORE FLOW RATES



INDEPENDENT RELATIVE ORE FLOW RATES

To find one of the possible sets of independent ore flow rates (IOFR), BILMAT permutes rows and columns of the ore network incidence matrix m_2 , in order to decompose it into the following form:

$$\text{NODES} \begin{pmatrix} \text{STREAMS} \\ \begin{matrix} a & b & c & d & e & f & g & h & i \\ x & x & x & x & x & \otimes & 0 & 0 & 0 \\ x & x & x & x & x & x & \otimes & 0 & 0 \\ x & x & x & x & x & x & x & \otimes & 0 \\ x & x & x & x & x & x & x & x & \otimes \end{matrix} \end{pmatrix} = (M_1, M_2) \text{ Eq A.1}$$

where x stands for -1, 0, or 1; and \otimes for -1 or 1.

The permutation of the rows and the columns does not change the mass conservation equation which becomes:

$$(M_1, M_2)D = 0 \quad \text{Eq A.2}$$

where components D_j are ordered in the stream sequence a, b ... i.

In this representation it appears that a, b, c, d and e stream flow rates are independent and f, g, h and i are dependent. From the first row, f can be calculated if a, b, c, d, e are given; g can be calculated from the second row, and so on.

Due to the simple structure of the flow diagram matrices (which contain only 1, -1, or 0 terms, and a maximum number of two elements differing from zero in each column), one of the various row-echelon forms can be obtained simply by modifying the initial ordering sequence of nodes and branches. By this procedure, one obtains a new matrix which represents the same initial flow network and consists of the juxtaposition of two block matrices, M1 being rectangular and M2 lower triangular.

For the flotation example in Figure 9 of the main text, the network matrix is written as:

$$\text{NODES} \begin{pmatrix} \text{STREAMS} \\ \begin{matrix} I & 1 & 2 & 4 & 6 & 8 & 3 & 5 & 7 & 9 \\ II & 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ III & 0 & 0 & -1 & 0 & 0 & 1 & -1 & 0 & 0 \\ IV & 0 & 1 & 0 & -1 & -1 & 0 & 0 & -1 & 0 \\ & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 \end{matrix} \end{pmatrix} \begin{matrix} \hline \\ \\ \\ \\ \hline \end{matrix} \begin{matrix} \\ \\ \\ \\ \\ \hline \end{matrix} \begin{matrix} \\ \\ \\ \\ \\ \hline \end{matrix}$$

and the independent relative ore flow rates are those of streams 2, 4, 6 and 8. By convention, BILMAT always keeps the first branch (feed stream) in the IOFR set.

Considering that there are r (number of streams — numbers of nodes) independent ore streams, d can be written as:

$$d^T = (1, d_2 \dots d_r, d_{r+1} \dots d_p) \quad \text{Eq A.3}$$

where p is the number of solids branches.

Equation A.2 can be written:

$$(M_1, M_2)d = 0 \quad \text{Eq A.4}$$

which gives:

$$\begin{pmatrix} d_{r+1} \\ \vdots \\ d_p \end{pmatrix} = -M_2^{-1}M_1 \begin{pmatrix} 1 \\ d_2 \\ \vdots \\ d_r \end{pmatrix} \quad \text{Eq A.5}$$

and when θ , the set of variable IOFR is known, the whole set of relative ore flow rates can be obtained using:

$$d = \begin{pmatrix} 1 \\ -M_2^{-1}M_1 \end{pmatrix} \begin{pmatrix} 1 \\ \theta \end{pmatrix} \quad \text{Eq A.6}$$

which can be written:

$$d = L\theta + \ell$$

where L is the first term matrix of Equation A.6 in which the first column ℓ has been deleted.

For the flotation example in Figure 9, the reader can easily verify that:

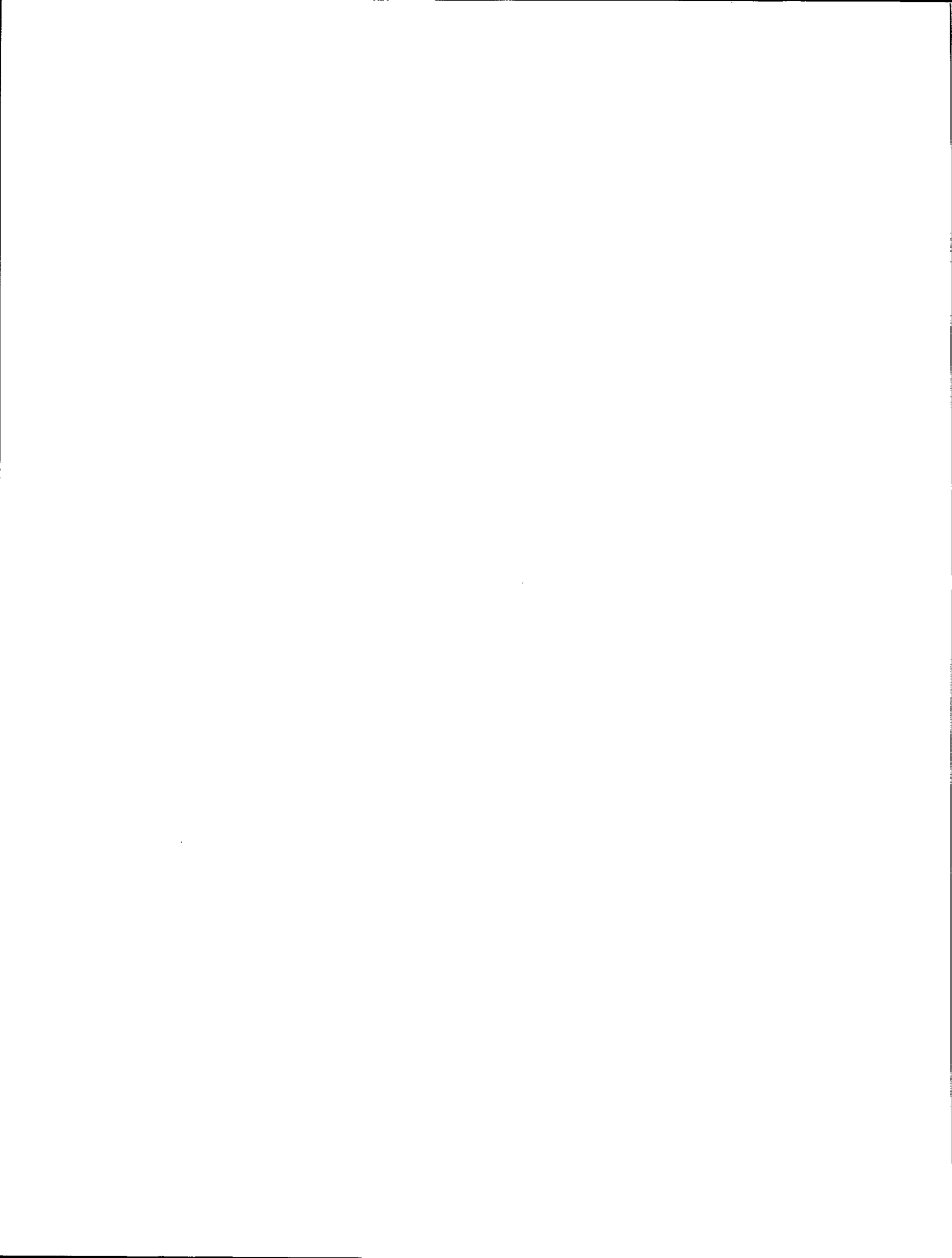
$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 1 & 0 & -1 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

$$\ell^T = (1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0)$$



APPENDIX B

**MINIMIZATION OF A QUADRATIC FUNCTION UNDER
LINEAR CONSTRAINTS**



MINIMIZATION OF A QUADRATIC FUNCTION UNDER LINEAR CONSTRAINTS

Let us assume that the function to be minimized is of second degree with respect to variable x and the constraints linear in x . We can write:

$$J(x) = x^T A x - 2Bx + D \quad \text{Eq B.1}$$

and the linear constraint to be verified is:

$$Cx = c \quad \text{Eq B.2}$$

The solution of this problem is obtained by forming a new function, named the Lagrangian, whose derivatives must vanish with respect to x and λ ; the new unknowns are called the Lagrange multipliers.

The Lagrangian is:

$$\ell(x, \lambda) = x^T A x - 2Bx + D + 2\lambda^T(Cx - c) \quad \text{Eq B.3}$$

The derivatives are set to zero:

$$\frac{1}{2} \frac{\partial \ell}{\partial x} = Ax - B^T + C^T \lambda = 0 \quad \text{Eq B.4}$$

$$\frac{\partial \ell}{\partial \lambda} = Cx - c = 0 \quad \text{Eq B.5}$$

Multiplying Equation B.4 by CA^{-1} , one obtains:

$$Cx - CA^{-1}B^T + CA^{-1}C^T \lambda = 0 \quad \text{Eq B.6}$$

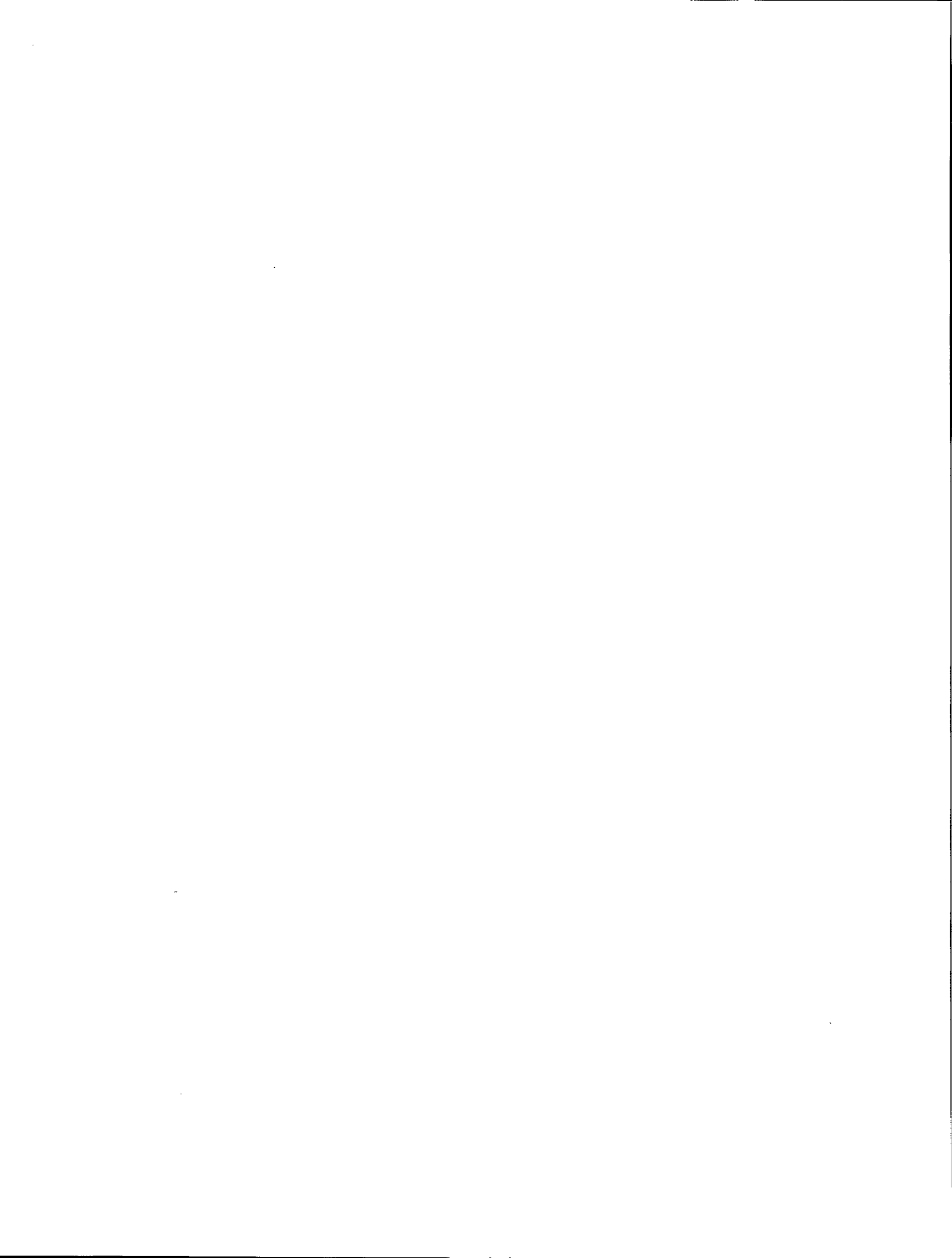
from which λ can be extracted:

$$\lambda = (CA^{-1}C^T)^{-1} (CA^{-1}B^T - c) \quad \text{Eq B.7}$$

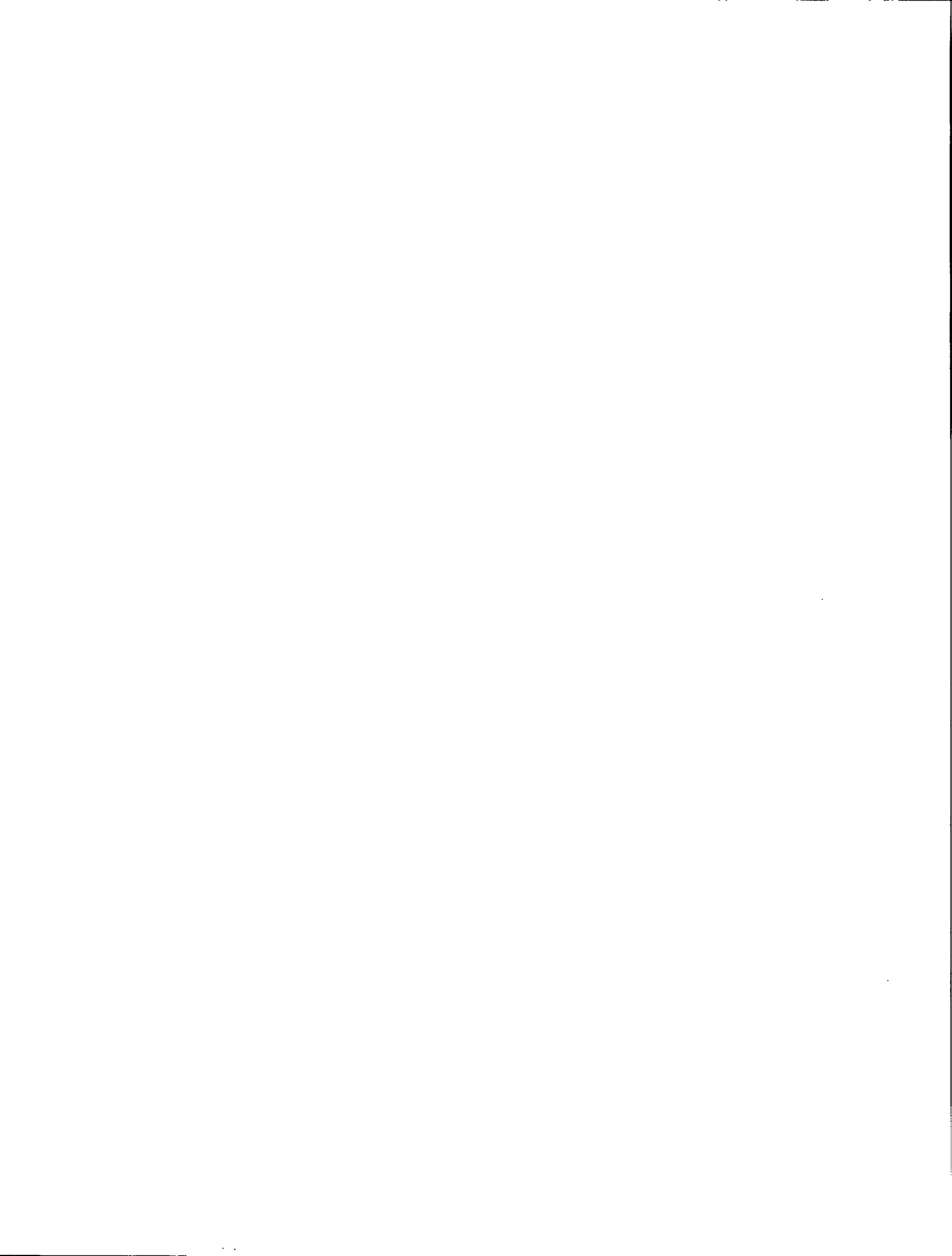
Substituting this value in Equation B.4 leads to:

$$x = A^{-1}B^T - A^{-1}C^T (CA^{-1}C^T)^{-1} (CA^{-1}B^T - c) \quad \text{Eq B.8}$$

which is the x value which minimizes J under the constraint.



APPENDIX C
COGGIN ALGORITHM



COGGIN ALGORITHM

This program finds the unconstrained minimum of a single variable nonlinear function. The procedure is based on the method of Coggins and on the COGGIN algorithm proposed by Box, Davies, and Swann (C.1). Unimodality is assumed and thus the use of a good starting point is recommended.

The algorithm proceeds as follows.

Step 0: The evaluation of the function Y_1 for the starting value of the search variable: point X_1 .

Step 1: The search variable is incremented by a step DELX to give point X_2 . Function Y_2 is evaluated:

- If an improvement is obtained, the step size is doubled and the program transfers to step 2.
- If an improvement is not obtained, the step size is reversed, points X_1 and X_2 are permuted and the program transfers to step 2.

At the end of the first step, two functions have been evaluated and points X_1 and X_2 are such that Y_1 is greater than Y_2 .

Step 2: Using the step size value defined in step 1, function Y_3 is evaluated at point X_3 .

- If an improvement is obtained, the step size is doubled, X_1 and Y_1 are replaced by X_2 and Y_2 , which are replaced by X_3 and Y_3 . Then step 2 is started.

- If an improvement is not obtained, the optimal point is straddled by points X_1 , X_2 and X_3 , so that Y_1 and Y_3 are greater than Y_2 . The program transfers to step 3.

Step 3: A quadratic equation, f , is curve fitted to the three points obtained at the end of step 2. The minimum location X_m , obtained by setting $df/dx = 0$, is:

$$X_m = \frac{1}{2} \frac{(X_2^2 - X_3^2)Y_1 + (X_3^2 - X_1^2)Y_2 + (X_1^2 - X_2^2)Y_3}{(X_2 - X_3)Y_1 + (X_3 - X_1)Y_2 + (X_1 - X_2)Y_3}$$

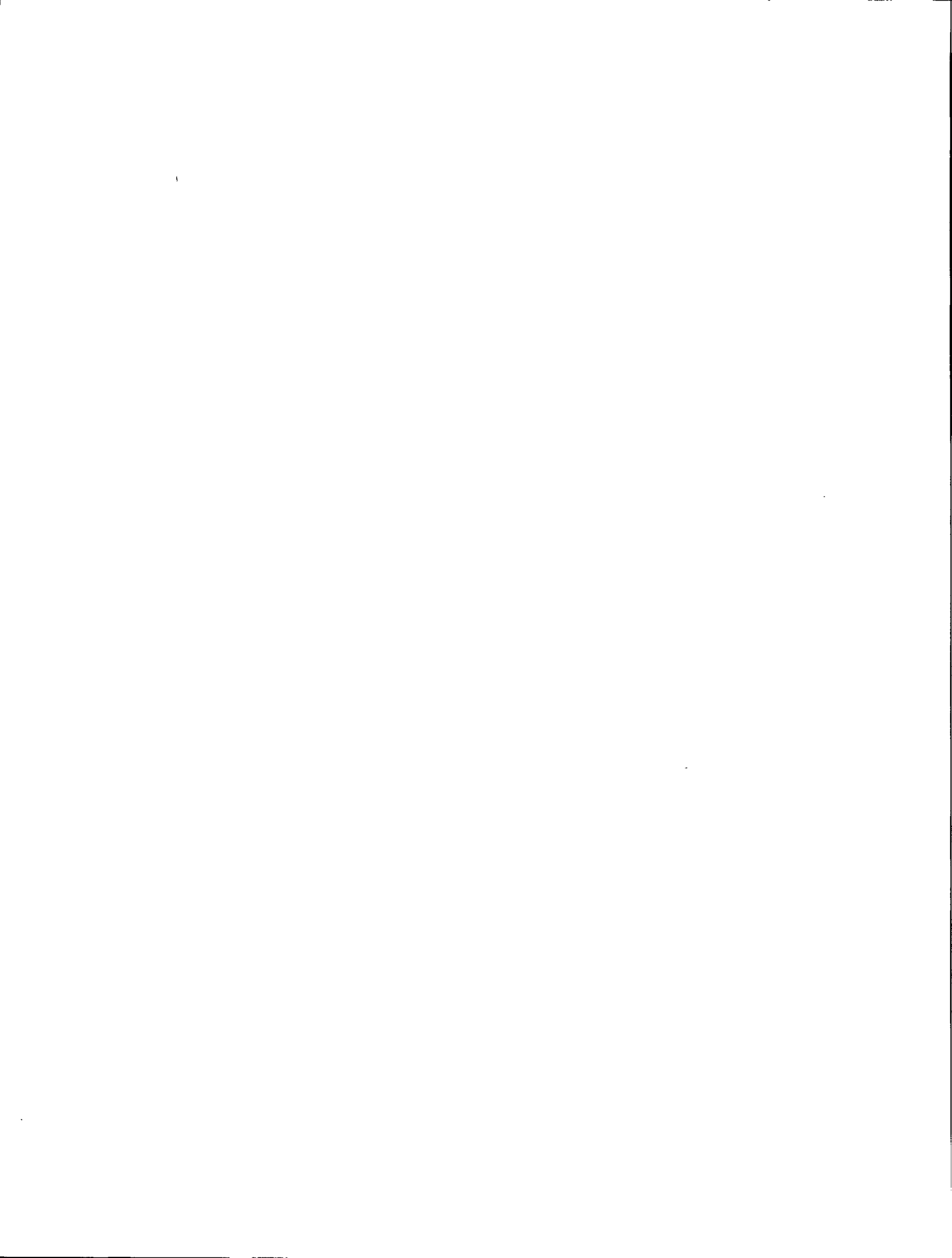
The function is evaluated at point X_m . If any of:

$$|X - X_i| < XLIM; i = 1, 2 \text{ or } 3$$

is satisfied, the minimization process is terminated. If not, the three points among X_1 , X_2 , X_3 and X_m which straddled the minimum are kept, reorganized, and step 3 is then restarted.

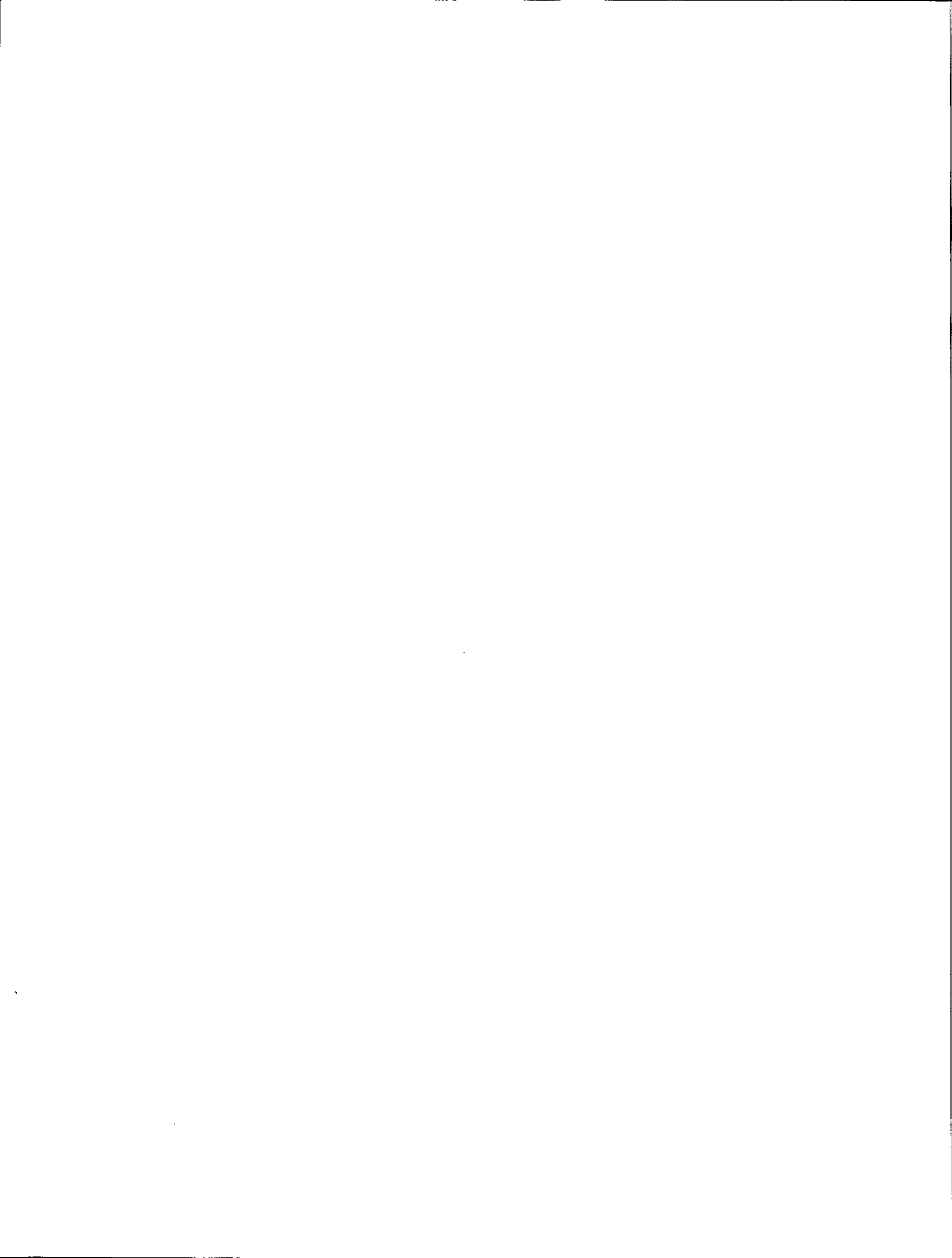
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APPENDIX D

**CALCULATION OF CORRECTED LEVEL 3
COMPLETE ANALYSES**



CALCULATION OF CORRECTED LEVEL 3 COMPLETE ANALYSES

Solution of Equation 54 in the main text, deleting index k and superscript x to simplify the notation, gives:

$$x = a - VB^T (BVB^T)^{-1} (Ba - b) \quad \text{Eq D.1}$$

Using the partitioned form of B (Eq 52) and of V , (if the errors of the various elements are assumed independent), one can write:

$$BVB^T = \begin{bmatrix} m V_1 \underline{d} m^T & & & & m \underline{d} V_1 \\ & m \underline{d} V_1 \underline{d} m^T & & & \\ 0 & & 0 & & m \underline{d} V_i \\ & & & & \vdots \\ V_1 \underline{d} m^T & \cdot & \cdot & \cdot & \cdot \\ & & & & \sum_{i=1}^n V_i \end{bmatrix} \quad \text{Eq D.2}$$

Equation D.1 can be written as:

$$x = a - VB^T Z \quad \text{Eq D.3}$$

with $Z = (BVB^T)^{-1} (Ba - b)$

Therefore, to obtain the value of Z , it is preferable, on a calculation cost basis, not to invert BVB^T but to solve for Z the linear system:

$$BVB^T Z = Ba - b \quad \text{Eq D.4}$$

Z can be partitioned as follows:

$$Z = \begin{bmatrix} Z_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Z_{n-1} \\ Z_n \end{bmatrix} \quad \text{Eq D.5}$$

where Z_1 to Z_{n-1} have dimensions $(n, 1)$ and Z_n $(p, 1)$.

Consequently, Equation D.4 can be written as:

$$m \underline{d} V_1 \underline{d} m^T Z_1 + m \underline{d} V_i Z_n = m \underline{d} a_i \quad i = 1, \dots, n - 1$$

$$\sum_{i=1}^{n-1} V_i \underline{d} m^T Z_i + \sum_{i=1}^n V_i Z_n = 0 \quad \text{Eq D.6}$$

which is equivalent to:

$$Z_i = \gamma_i^{-1} (m \underline{d} a_i - m \underline{d} V_i Z_n); \quad i = 1, \dots, n - 1 \quad \text{Eq D.7a}$$

$$\begin{bmatrix} \sum_{i=1}^{n-1} V_i \alpha_i V_i & - \sum_{i=1}^n V_i \end{bmatrix} Z_n = \sum_{i=1}^{n-1} \beta_i \quad \text{Eq D.7b}$$

$$\gamma_i = m \underline{d} V_i \underline{d} m^T; \quad i = 1, \dots, n - 1 \quad \text{Eq D.7c}$$

$$\alpha = \underline{d} m^T \gamma_i^{-1} m \underline{d}; \quad i = 1, \dots, n - 1 \quad \text{Eq D.7d}$$

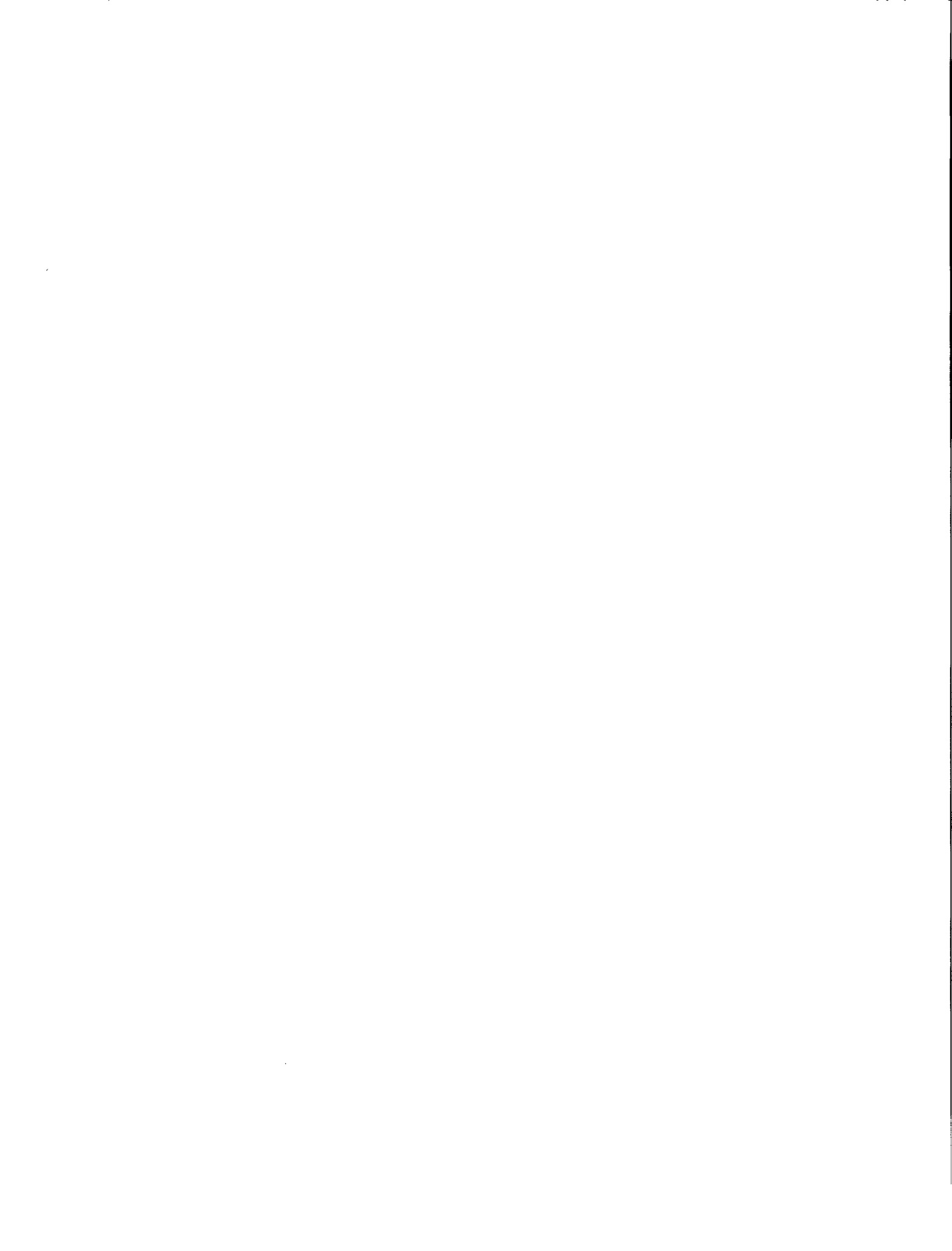
$$\beta_i = V_i m^T \underline{d} \gamma_i^{-1} m \underline{d} a_i; \quad i = 1, \dots, n - 1 \quad \text{Eq D.7e}$$

Finally, one can obtain the x solution by:

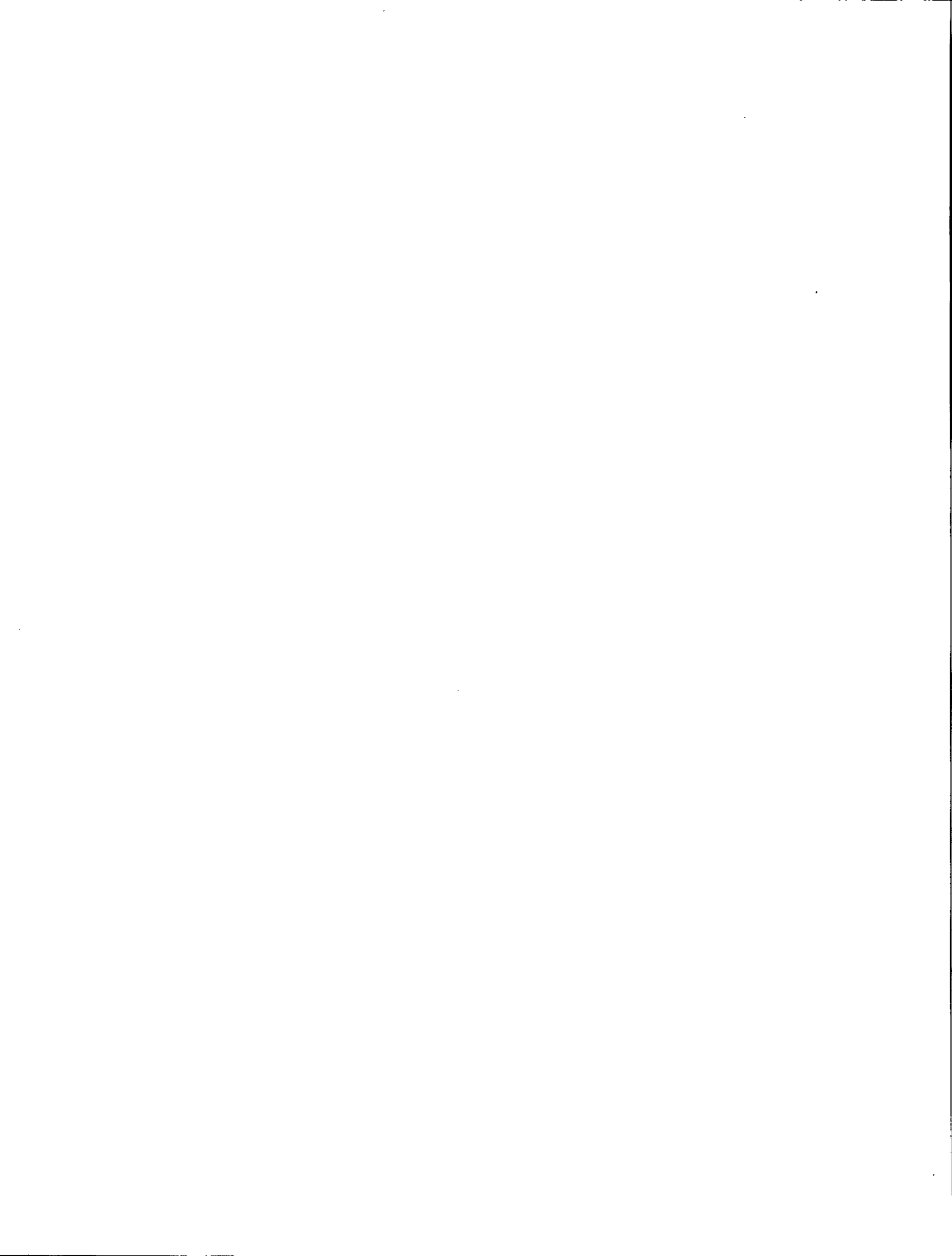
$$x_i = a_i - V_i \underline{d} m^T Z_i - V_i Z_n; \quad i = 1, \dots, n - 1$$

$$x_n = a_n - V_n Z_n \quad \text{Eq D.8}$$

where Z_n is obtained by resolving the linear system (Eq D.7b) and Z_i is obtained by Equation D.7a.



APPENDIX E
BOTM ALGORITHM



BOTM ALGORITHM

The BOTM algorithm is based on search directions, called *conjugate directions*. It is detailed in a paper by Botm(E.1) and the Fortran version used is published by Kuester and Mize(E.2). To simply illustrate the method, a two-variable example will be considered graphically.

Figure E.1 represents the lines of constant values of the least-squares function J as a function of two variables; θ_1 and θ_2 (relative independent ore flow rates). The starting point is A and the first direction is θ_1 ; J is minimized in this direction until point B is found. Then, starting from this point, J is minimized in the direction θ_2 until point C is reached. This process defines a new direction, AC , a conjugate direction, on which a new minimum, D , is found. This constitutes the first iteration.

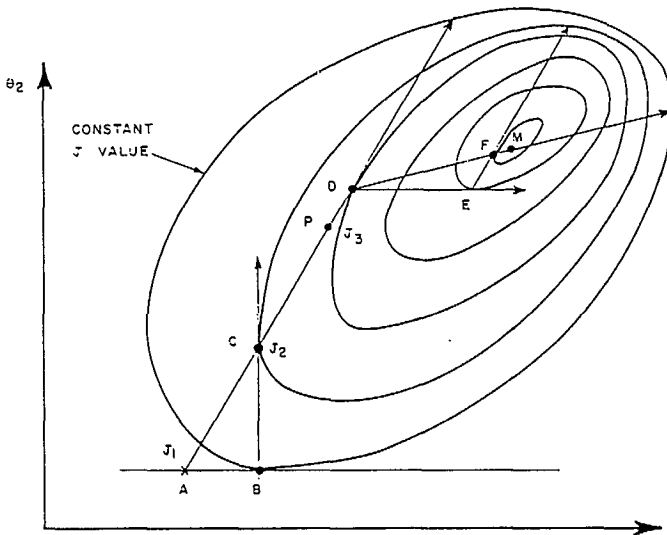


Fig. E.1 – Schematic illustration of the BOTM algorithm for a two-variable function $J(\theta_1, \theta_2)$

The search procedure is restarted from point D , replacing one of the two initial directions by the new conjugate directions. Two new minima, E and F , are located and a second conjugate direction is defined by DF , on which, for the case considered in Figure E.1, the minimum can be found. For a quadratic function, Botm has shown that after n iterations (for an n -variable minimization problem), all the directions are mutually conjugate and the exact minimum is found.

Modifications to the basic procedure

- At each iteration, define the direction in which the decrease of the function is maximum. Calculate the decrease Δ of the function in this direction.
- Calculate the value J_3 of the function at a point P at a distance $2 AC$ from A and the value J_1 at point A and J_2 at point C .

$$\text{If } J_3 > J_1 \text{ or if } (f_1 - 2f_2 + f_3)(f_1 - f_2 - \Delta)^2 > \frac{\Delta}{2}(f_1 - f_3)^2$$

- Use the old directions for the next iteration and point C as starting point.
- Otherwise, start the next iteration from point D and replace the direction of maximum decrease of J (Δ value) by the conjugate direction AC .

The disadvantage of this technique is that, for a high number of variables, there is a tendency for new directions to be chosen less often, or for conjugate directions to be discarded. However, it avoids the procedure of selecting nearly-dependent directions. The convergence criterion is satisfied when the values of all the independent variables between two successive iterations are less than 10% of the preset accuracies ϵ_i (one value per variable i).

Determination of the minimum along a direction

Let us suppose that function J is evaluated on three points on the direction ξ , defined by $p + a\xi$, $p + b\xi$, $p + c\xi$ where p is a reference point and ξ is defined by its components on directions $\theta_1, \theta_2, \dots$. The method consists of fitting a parabola passing through the three points $[a, J(p + a\xi)]$, $[b, J(p + b\xi)]$ and $[c, J(p + c\xi)]$ and finding its minimum which is located at $p + d\xi$ with:

$$d = \frac{1}{2} \frac{(b^2 - c^2)J_a + (c^2 - a^2)J_b + (a^2 - b^2)J_c}{(b - c)J_a + (c - a)J_b + (a - b)J_c}$$

The procedure is iterated and stops when the accuracy on the position of the minimum in direction ξ is such that, when compared with a, b and c , it is within the required accuracy which is $0.05\xi_i$. In the first iteration of the BOTM algorithm, the directions are those of the variables $\theta_1, \dots, \theta_n$, and a step q is selected as $0.1E\xi_i$ where E is named the ESCALE factor and the function J is evaluated for:

- p which are the initial values of the θ components
- $p + q\xi$
- $p - q\xi$ or
- $p + 2q\xi$, depending on whether $J(p)$ is less than, or greater than, $J(p + q\xi)$.

In the general situation (a, b, c), if the position of the minimum of the parabola is such that $p + d\xi$ have values greater than $E\epsilon_1, E\epsilon_2 \dots E\epsilon_n$, the maximum allowed step is taken in the direction of decreasing J and the function

*This test determines whether the function in this region is a valley, i.e., f_3 is an improvement but the surface is rising.

value at the point which is farthest from $p + d_\epsilon$ is discarded. In this way, the process may be repeated.

At the end of each search on a direction, the second derivative is calculated:

$$D = -2 \frac{(b-c)J_a + (c-a)J_b + (a-b)J_c}{(a-b)(b-c)(c-a)}$$

If in the next algorithm iteration the same ξ direction is used, its components are scaled by $1/\sqrt{D}$ and the new minimum is predicted directly using $J(p)$ and $J(p + q\xi)$ by:

$$d = \frac{1}{2}q \frac{J(p + q\xi) - J(p)}{q}$$

This procedure decreases the number of function values to be calculated. If a new direction is defined, three function values on this direction will have already been calculated, (J_a , J_b , J_c). Thus, a prediction of the minimum may be made immediately. Therefore, it is only during the first n linear searches of the first iteration that it is necessary to calculate two function values in addition to $J(p)$ for the initial prediction of the minimum in the direction ξ .

Except for the first iteration, the value of q is set to $0.4 \sqrt{\Delta J}$ where ΔJ is the total decrease of the function in the previous iteration.

The control parameters of this procedure are:

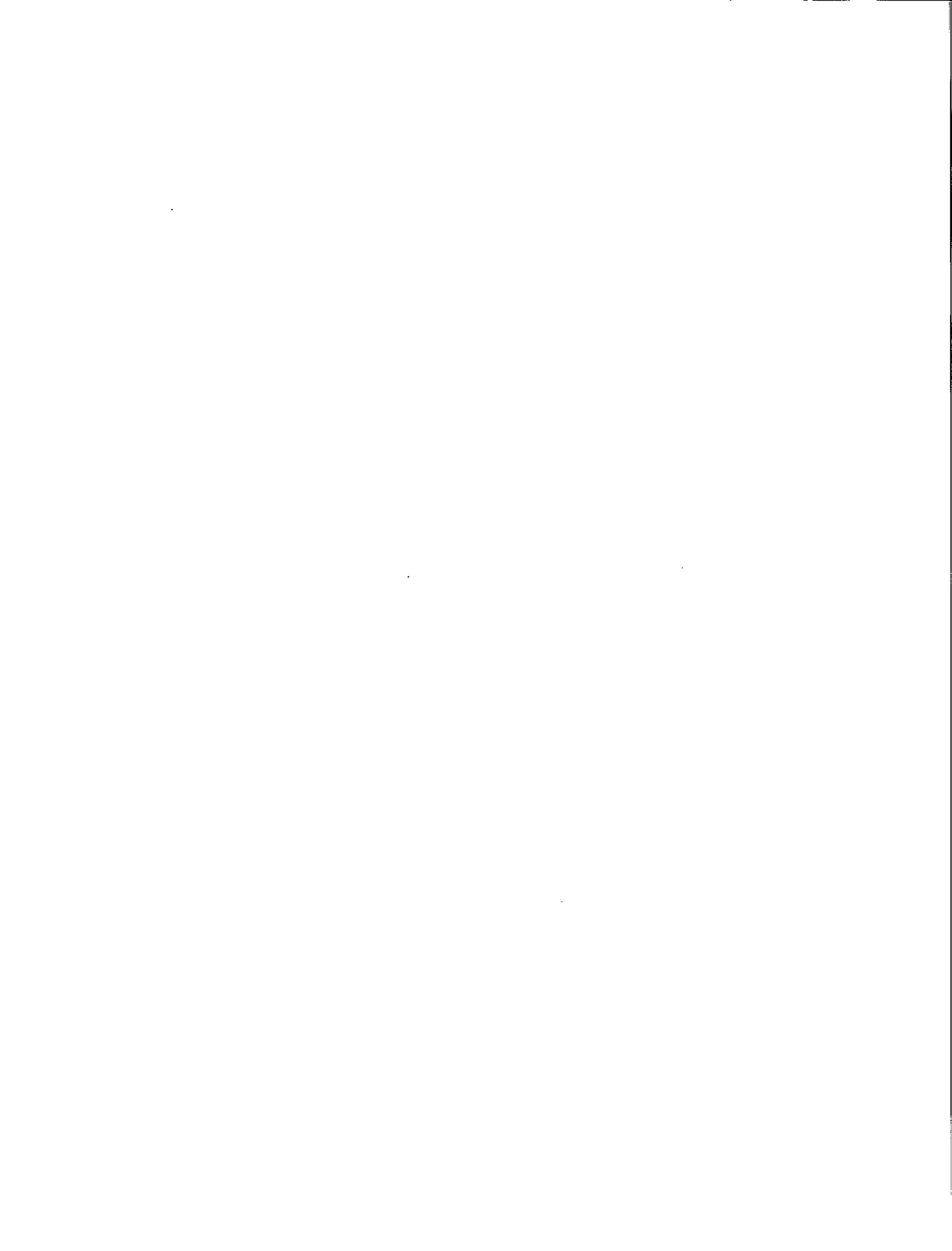
1. the initial values of the variables $\theta_1, \theta_2 \dots \theta_n$ (the relative independent ore flow rates);
2. the requested accuracy on $\theta_1, \dots \theta_n$ for the minimum position $\epsilon_1 \dots \epsilon_n$;
3. the ESCALE factor E ;
4. a printing control code variable (see 2.9);
5. the maximum number of iterations required.

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APPENDIX F

VARIANCES OF THE ESTIMATES OBTAINED BY THE BILMAT ALGORITHM



VARIANCES OF THE ESTIMATES OBTAINED BY THE BILMAT ALGORITHM

In the present version of the sensitivity analysis program, only the solid flow rate measurements and the mass fraction measurements related to networks 3, 5, 6 and 7 can be taken into consideration. Therefore, the derivative of the least-squares criterion J (see Section 2.4.1) is reduced to the following sum of derivatives:

$$J = J_S + J_{33} + J_{35} + J_{36} + J_{37} \quad \text{Eq F.1}$$

where

$$J_S = (dD - dD_0)^T V_D^{-1} (dD - dD_0) \quad \text{Eq F.2}$$

$$J_{33} = (dx_3 - da_3)^T V_3^{-1} (dx_3 - da_3) \quad \text{Eq F.3}$$

$$J_{35} = (dx_5 - da_5)^T V_5^{-1} (dx_5 - da_5) \quad \text{Eq F.4}$$

$$J_{36} = (dx_6 - da_6)^T V_6^{-1} (dx_6 - da_6) \quad \text{Eq F.5}$$

$$J_{37} = (dx_7 - da_7)^T V_7^{-1} (dx_7 - da_7) \quad \text{Eq F.6}$$

The value of J must be minimum and the mass balance constraints still verified.

The Constraints

The first constraint to be considered is the conservation of solid flow rates:

$$MD = 0 \quad \text{Eq F.7}$$

As explained in Appendix A, Equation F.7 can be transformed into Equation F.8 to put forward the IOFR and, therefore, to reduce the number of unknowns:

$$d = L\theta + \ell \quad \text{Eq F.8}$$

Since $d = D/D_1$, it is easy to express D as a function of θ :

$$D = D_1 (L\theta + \ell) \quad \text{Eq F.9}$$

The derivative of Equation F.9 is:

$$dD = D_1 dd + d dD_1 = D_1 L d\theta + dD_1 (L\theta + \ell) \quad \text{Eq F.10}$$

Equation F.10 can now be used to express the derivatives H'_i of the other mass balance constraints H_i :

$$H_i = M_i D_i x_i = 0 \quad i = 3, 5, 6 \text{ or } 7 \quad \text{Eq F.11}$$

$$H'_i = M_i dD_1 x_i + M_i D_1 dx_i = 0$$

$$H'_i = M_i D_1 L_i d\theta_i x_i + M_i dD_1 d_i x_i + M_i D_1 d_i dx_i = 0$$

$$H'_i = D_1 M_i (L_i d\theta_i x_i + d_i dx_i) + dD_1 M_i d_i x_i = 0$$

Since $M_i d_i x_i = 0$ and D_1 is a scalar, it follows:

$$H'_i = M_i (L_i d\theta_i x_i + d_i dx_i) = 0 \quad i = 3, 5, 6 \text{ or } 7 \quad \text{Eq F.12}$$

where

M_i is the incidence matrix of network i

L_i is the matrix L (see Appendix A) reduced to network i

x_i are the adjusted values of the component mass fractions of network i

d_i are the relative ore flow rates of network i

θ_i are the IOFR reduced to network i

Minimization of J

The solution in dx_i , dD_1 , $d\theta$ of the system of Equations F.2 to F.6, F.10 and F.12, is obtained by writing the Lagrangian ξ (Appendix B):

$$\xi = J_S + \sum_i (J_{3i} + \lambda_i H'_i) \quad i = 3, 5, 6 \text{ or } 7 \quad \text{Eq F.13}$$

The derivatives of ξ with respect to the unknowns must vanish so that $d\theta$, dx_i , dD_1 and λ_i are solutions of the following system of equations:

$$D_1 L^T V_D^{-1} dD - D_1^2 L^T V^{-1} D L d\theta - D_1 L^T V_D^{-1} d dD_1 = 0 \quad \text{Eq F.14a}$$

$$d^T V_D^{-1} dD - D_1 d^T V_D^{-1} L d\theta - d^T V_D^{-1} d dD_1 = 0 \quad \text{Eq F.14b}$$

$$V_i^{-1} da_i - V_i^{-1} dx_i - d_i M_i^T \lambda_i = 0 \quad \text{Eq F.14c}$$

$$M_i d_i dx_i + M_i L_i d\theta_i x_i = 0 \quad \text{Eq F.14d}$$

$$L_i^T x_i M_i \lambda_i = 0 \quad \text{Eq F.14e}$$

where $i = 3, 5, 6$ and 7 .

The solution in $d\theta$ of the system of Equations F.14 is first obtained, and the variance V_θ values are calculated using:

$$V_\theta = E (d\theta d\theta^T)$$

where E stands for the mathematical expectancy.

The V_0 values are:

$$V_0 = L^T v_D L + \sum_i L_i^T v_i L_i \quad i = 3, 5, 6 \text{ and } 7 \quad \text{Eq F.15}$$

where

$$v_D = D_D^2 V_D^{-1} - V_D^{-1} d (d^T V_D^{-1} d)^{-1} d^T V_D^{-1}$$

$$v_i = \sum_k v_{ik} \quad k = 1 \text{ to } p \text{ if } p \text{ is the number of components in network } i$$

$$v_{ik} = (\text{diag } x_{ik}) M_i^T \alpha^{-1}_{ik} M_i (\text{diag } x_{ik})$$

$$\alpha_{ik} = M_i d_i V_i^{-1} d_i M_i^T$$

From Equation F.15, the variances of all the relative ore flow rates are obtained by:

$$V_d = L^T V_0 L \quad \text{Eq F.16}$$

The solution in dx_i of the system of Equations F.14 gives access to the variances of the adjusted values of the component mass fractions of each network i .

For the component k , for example:

$$V_{ik} = \underline{V} + \gamma - (\beta \underline{D} \underline{V}) \quad \text{Eq F.17}$$

where:

$$\gamma = \beta \underline{x} V_d \underline{x} \beta^T$$

$$\beta = V \underline{D} M_i^T \alpha^{-1} M_i$$

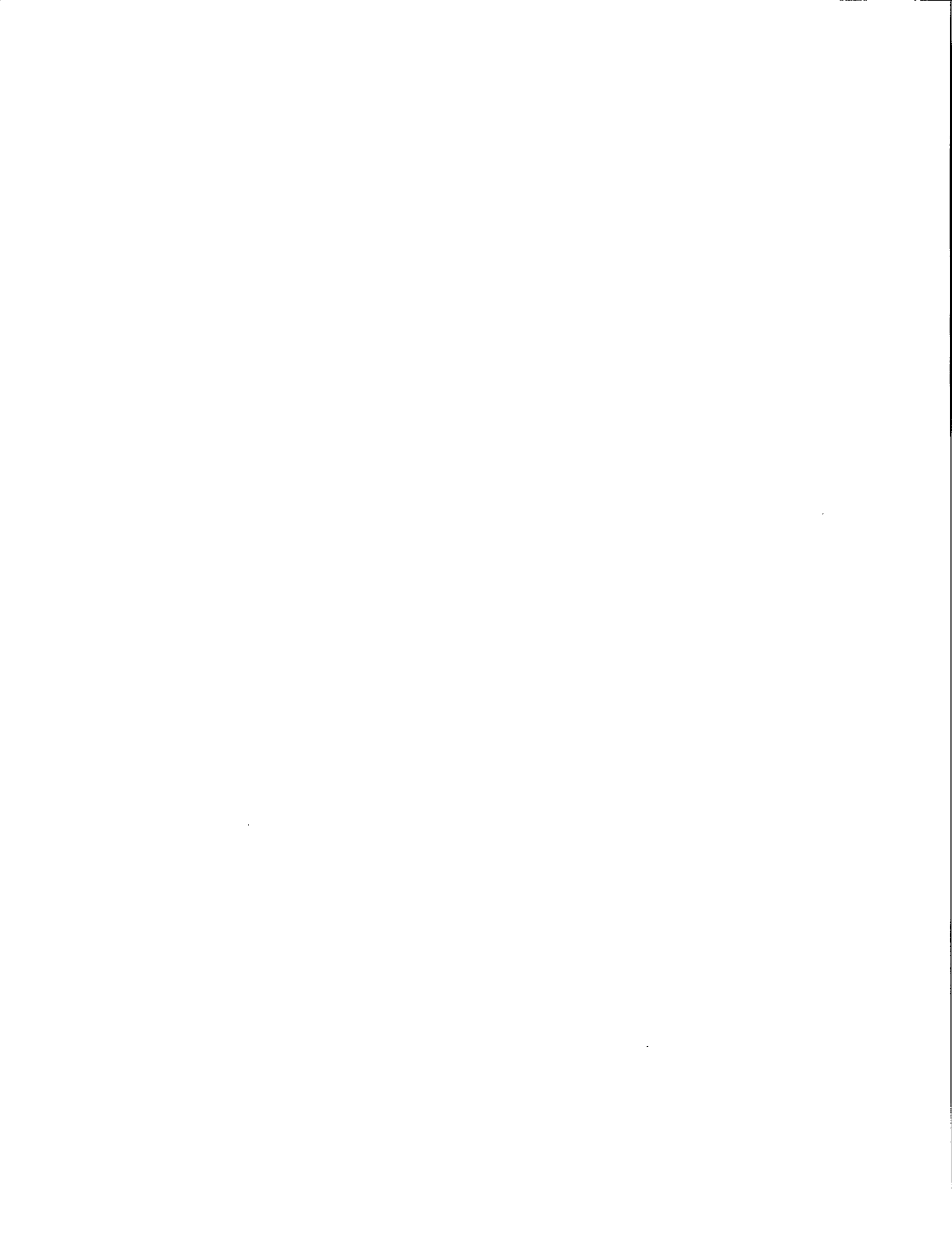
$$\alpha = M_i \underline{D} \underline{V} \underline{D} M_i^T$$

$$\underline{x} = \text{diag } x_{ik}$$

$$\underline{D} = \text{diag } d_i$$

$$\underline{V} = \text{diag } V_{ik}^{-1}$$

APPENDIX G
EDIMAT AND EDISIV SAMPLE RUNS



1. EDIMAT SAMPLE RUN

INTERACTIVE PROGRAM EDIMAT
TO EDIT MAIN DATA FILE OF BILMAT

CHOOSE OPTION :

C = CREATION OF NEW DATA FILE
M = MODIFICATION OF EXISTING DATA FILE
L = DESCRIPTIVE LIST OF EXISTING DATA FILE

OPTION (C/M/L) =C

CREATION OF DATA FILE

ENTER A TITLE (MAXIMUM 72 CHARACTERS) :
MASS BALANCE ON A CYCLONE CLASSIFIER

SELECTION OF OPTIONS

HAVE YOU MEASURED SOME FLOW RATES (Y/N) ?Y
HAVE YOU MEASURED SOME PULP PERCENT SOLIDS (Y/N) ?Y
IS STREAM #1 A DRY STREAM (Y/N) ?N
DO YOU NEED NETWORK 3 (Y/N) ?Y
DO YOU NEED NETWORK 4 (Y/N) ?Y
DO YOU NEED NETWORK 5 (Y/N) ?N
DO YOU NEED NETWORK 8 (Y/N) ?Y
FOR LEVEL 4 DATA SELECT SUBOPTIMAL SOLUTION
NUMBER (1 OR 2) AS EXPLAINED IN SECTION 2.4.1.2
OF CHAPTER 3.1 : 2
DO YOU WISH TO CONSTRAIN FLOW RATES TO POSITIVE VALUES (Y/N) ?N
TO SET FIRST ESTIMATE VALUES, CHOOSE OPTION :
0 = USER WILL ENTER HIS OWN ESTIMATES
1 = PROGRAM WILL USE NETWORK 3 DATA TO COMPUTE ESTIMATES
2 = PROGRAM WILL USE NETWORK 5 DATA TO COMPUTE ESTIMATES

1

DATA INPUT AND CALCULATION PATH OPTIONS:

INTEGER VECTOR OF OPTIONS: 1 1 1 1 0 0 0 0 1 1 0 1

OPTION MEANING:

- 1 SOME FLOWRATES HAVE BEEN MEASURED.
- 2 SOME PULP PERCENT SOLIDS HAVE BEEN MEASURED.
- 3 DATA ON NETWORK 3 ARE AVAILABLE.
- 4 DATA ON NETWORK 4 ARE AVAILABLE.
- 9 DATA ON NETWORK 8 ARE AVAILABLE.
- 10 SUB-OPTIMAL SOLUTION NO. 2 IS SELECTED.
- 12 ACTIVATE BILMAT3 CALCULATION OF THESE FIRST ESTIMATE USING NETWORK 3 DATA.

IS THIS OK (Y / N) ?Y

NETWORKS 1 AND 2 CHARACTERISTICS

ENTER THE FOLLOWING VALUES :

- NUMBER OF NODES OF NETWORK 1
- NUMBER OF STREAMS OF NETWORK 1
- NUMBER OF NODES OF NETWORK 2
- NUMBER OF STREAMS OF NETWORK 2

1 3 1 3

ENTER THE LABELS OF THE 3 STREAMS OF NETWORK 1

(ONE PER LINE), MAX.12 CHAR. :

CYCLONE FEED
OVERFLOW
UNDERFLOW

ENTER FOR EACH STREAM, SOURCE AND DESTINATION NODE NUMBERS

EXAMPLE: STREAM 3: 2 0

MEANS STREAM 3 COMES FROM NODE 2
AND GOES TO NO NODE

STREAM 1: 0 1

STREAM 2: 1 0

STREAM 3: 1 0

INCIDENCE MATRIX OF NETWORK #2

NETWORK MATRIX IS BUILT

NETWORK 3 CHARACTERISTICS AND DATA

ENTER THE NUMBER OF NODES,
OF STREAMS AND OF COMPONENTS:
1 3 4

THIS NETWORK

- 1- USES ONLY NODES ALREADY
DESCRIBED FOR NETWORK #1
- 2- REQUIRES TO DEFINE NEW
NODES

CHOOSE CASE (1/2) ?1

NETWORK MATRIX IS BUILT

ENTER THE MATRIX OF MEASURED VALUES (4 ROWS BY 3 COLUMNS (I.E. 1 ROW
PER COMPONENT))

24 0 3.75
26.5 9.4 33.4
24.5 34.6 18.7
25 56 10.4

ARE ALL RELATIVE STANDARD DEVIATIONS EQUAL (Y/N) ?N

ENTER THE MATRIX OF RELATIVE STANDART DEVIATIONS
(4 ROWS BY 3 COLUMNS (I.E. 1 ROW PER COMPONENT))

999.999 0 5.33
999.999 6.54 5.37
1.1 5.34 5.66
1.1 5.23 6.39

ENTER THE LABELS OF THE 4 COMPONENTS
(ONE PER LINE, MAX. 8 CHAR.) :

150/ 75
75/ 37
37/ 26
26/0

NETWORK 4 CHARACTERISTICS AND DATA

ENTER THE NUMBER OF NODES,
OF STREAMS AND OF COMPONENTS:
1 3 3

THIS NETWORK

- 1- USES ONLY NODES ALREADY
DESCRIBED FOR NETWORK #1
- 2- REQUIRES TO DEFINE NEW
NODES

CHOOSE CASE (1/2) ?1

NETWORK MATRIX IS BUILT

ENTER THE MATRIX OF MEASURED VALUES (3 ROWS BY 3 COLUMNS (I.E. 1 ROW PER COMPONENT))

50.8 38.5 89.7
82.1 27.7 .7
27.1 33.8 9.6

ARE ALL RELATIVE STANDARD DEVIATIONS EQUAL (Y/N) ?N

ENTER THE MATRIX OF RELATIVE STANDART DEVIATIONS
(3 ROWS BY 3 COLUMNS (I.E. 1 ROW PER COMPONENT))

10 5 5
10 5 5
10 5 5

ENTER THE LABELS OF THE 3 COMPONENTS
(ONE PER LINE, MAX. 8 CHAR.) :

26/ 18
18/ 9
- 9

NETWORK 8 CHARACTERISTICS AND DATA

ENTER THE NUMBER OF NODES,
OF STREAMS AND OF COMPONENTS:
1 3 3

THIS NETWORK

- 1- USES ONLY NODES ALREADY DESCRIBED FOR NETWORK #1
- 2- REQUIRES TO DEFINE NEW NODES

CHOOSE CASE (1/2) ?1

NETWORK MATRIX IS BUILT

...FOR COMPONENT NO. 1 OF NETWORK 3 :

ENTER THE MATRIX OF MEASURED VALUES (3 ROWS BY 3 COLUMNS (I.E. 1 ROW PER COMPONENT))

6.3 43.1 5.4
16.2 23.8 16.8
43 0 49

ARE ALL RELATIVE STANDARD DEVIATIONS EQUAL (Y/N) ?N

ENTER THE MATRIX OF RELATIVE STANDART DEVIATIONS
(3 ROWS BY 3 COLUMNS (I.E. 1 ROW PER COMPONENT))
5 5 5
2 2 2
8 8 8

...FOR COMPONENT NO. 2 OF NETWORK 3 :

ENTER THE MATRIX OF MEASURED VALUES (3 ROWS BY 3 COLUMNS (I.E. 1 ROW
PER COMPONENT))
6.1 26.6 3.4
16.2 28.4 14.4
47 13.4 50.4

ARE ALL RELATIVE STANDARD DEVIATIONS EQUAL (Y/N) ?N

ENTER THE MATRIX OF RELATIVE STANDART DEVIATIONS
(3 ROWS BY 3 COLUMNS (I.E. 1 ROW PER COMPONENT))
5 5 5
2 2 2
8 8 8

...FOR COMPONENT NO. 3 OF NETWORK 3 :

ENTER THE MATRIX OF MEASURED VALUES (3 ROWS BY 3 COLUMNS (I.E. 1 ROW
PER COMPONENT))
6.4 10.9 1
16.3 21.2 10.8
44 9.7 53.8

ARE ALL RELATIVE STANDARD DEVIATIONS EQUAL (Y/N) ?N

ENTER THE MATRIX OF RELATIVE STANDART DEVIATIONS
(3 ROWS BY 3 COLUMNS (I.E. 1 ROW PER COMPONENT))
5 5 5
2 2 2
8 8 8

...FOR COMPONENT NO. 4 OF NETWORK 3 :

ENTER THE MATRIX OF MEASURED VALUES (3 ROWS BY 3 COLUMNS (I.E. 1 ROW PER COMPONENT))

6.2 7.4 1.1
16.1 17.5 10.1
44.4 42.9 56.7

ARE ALL RELATIVE STANDARD DEVIATIONS EQUAL (Y/N) ?N

ENTER THE MATRIX OF RELATIVE STANDART DEVIATIONS
(3 ROWS BY 3 COLUMNS (I.E. 1 ROW PER COMPONENT))

5 5 5
2 2 2
8 8 8

ENTER THE LABELS OF THE 3 COMPONENTS
(ONE PER LINE, MAX. 8 CHAR.) :

SIO2
FES2
PBS

FLOW RATE MEASUREMENTS

ENTER THE NUMBER OF PULP,
OF SOLID AND OF WATER FLOW RATE
MEASUREMENTS :

3 3 3

ENTER FOR THE 3 MEASUREMENTS OF PULP FLOWRATES,
STREAM NUMBER, MEASURED VALUES
AND STD. DEVIATION.
(ONE ROW PER MEASUREMENT)

1 201 10
2 120 15
3 73 10

ENTER FOR THE 3 MEASUREMENTS OF SOLID FLOWRATES,
STREAM NUMBER, MEASURED VALUES
AND STD. DEVIATION.
(ONE ROW PER MEASUREMENT)

1 98 10
2 35 10
3 56 10

ENTER FOR THE 3 MEASUREMENTS OF WATER FLOWRATES,
STREAM NUMBER, MEASURED VALUES
AND STD. DEVIATION.
(ONE ROW PER MEASUREMENT)
1 110 10
2 82 10
3 19 10

ENTER THE NUMBER OF PULP PERCENT SOLID MEASUREMENTS : 3

ENTER FOR THE 3 MEASUREMENTS OF PULP PERCENT SOLID
STREAM NUMBER, MEASURED VALUES
AND STD. DEVIATION.
(ONE ROW PER MEASUREMENT)
1 48.7 6
2 29 6
3 76.6 6

SET UP OF ITERATIVE ALGORITHM(S)

FOR POWELL ALGORITHM ENTER ESCALE VALUE,
MAXIMUM NUMBER OF ITERATIONS AND PRINTING
PARAMETER : 1 5 3

FOR THE COGGINS ALGORITHM, ENTER :
AN ESTIMATE FOR THE STREAM #1 ORE FLOW RATE
A TOLERANCE (IN %)
AND A PRINTING OPTION (4 IS SUGGESTED) :
98 1 4

DO YOU WANT A DATA FILE LISTING (Y/N) ?N

DO YOU WANT TO MODIFY YOUR NEW FILE (Y/N) ?Y

CHANGE OPTIONS:

- 1. - TITLE AND PROGRAM OPTIONS
 - 1.1 - TITLE
 - 1.2 - PROGRAM OPTIONS
- 2. - NETWORK 1 & 2:
 - 2.2 - STREAM LABELS
 - 2.3 - NETWORK #1 MATRIX
 - 2.4 - COMMON STREAMS
 - 2.5 - NETWORK #2 MATRIX
- 3./8. - CORRESPONDING TO NETWORK 3 TO 8:
 - .1 - DIMENSION
 - .2 - STREAMS COMMON WITH NETWORK 1
 - .3 - NETWORK INCIDENCE MATRIX
 - .4 - MEASURED VALUES
 - .5 - RELATIVE STANDARD DEVIATIONS
 - .6 - COMPONENT LABELS

N.B. GIVEN THAT NETWORK 8 HAS SOME SUB-SECTIONS (COMPONENTS OF NET. #3), IT'S POSSIBLE TO REPETE THE '8.4' AND '8.5' OPTIONS, IF YOU FOLLOW THE COMPONENTS ORDER.
- 9. - FLOW RATES:
 - .1 - NUMBER OF EACH TYPE OF FLOW RATE
 - .2 - DATA FOR PULP FLOW RATES
 - .3 - DATA FOR SOLID FLOW RATES
 - .4 - DATA FOR WATER FLOW RATES
- 10. - PULP PERCENT SOLID
 - .1 - NUMBER OF MEASURED VALUES
 - .2 - MEASURED VALUES
- 11. - CONTROLLING VARIABLES:
 - .1 - FIRST ESTIMATES FOR POWELL ALGORITHM
 - .2 - CONTROLLING PARAMETERS
 - .3 - CONTROLLING PARAMETERS FOR COGGINS ALGORITHM
- 12. - END OF EDITION.

LAST SELECTION, : 0.0

WHICH OPTION DO YOU WANT ?
(0 = MENU, 12 = END) : 1.1

OLD TITLE :
MASS BALANCE ON A CYCLONE CLASIFIER

ENTER NEW TITLE (MAXIMUM 72 CHARACTERS) :
SAMPLE RUN2 : MASS BALANCE ON A CYCLONE CLASSIFIER

LAST SELECTION : 1.1

WHICH OPTION DO YOU WANT ?
(0 = MENU, 12 = END) : 3.4

CHANGE THE DATA FOR NETWORK # 3

NUMBER OF COMPONENTS : 4
NUMBER OF STREAMS : 3
DATA:
24.000 0.000 3.750
26.500 9.400 33.400
24.500 34.600 18.700
25.000 56.000 10.400

WHICH MODIFICATION MODE DO YOU WANT ?

1 - VALUE BY VALUE
2 - ROW BY ROW
3 - COLUMN BY COLUMN
4 - ENTIRE MATRIX
5 - STOP

ENTER THE NUMBER : 1

ENTER THE COMPONENT NO. AND THE
STREAM NO. YOU WANT TO CHANGE
AND THEN THE NEW VALUE : 1 3 37.5

ANOTHER CHANGE (Y/N) ?N

WHICH MODIFICATION MODE DO YOU WANT ?

1 - VALUE BY VALUE
2 - ROW BY ROW
3 - COLUMN BY COLUMN
4 - ENTIRE MATRIX
5 - STOP

ENTER THE NUMBER : 5
LAST SELECTION : 3.4

WHICH OPTION DO YOU WANT ?
(0 = MENU, 12 = END) : 3.6

CHANGE IN THE LABELS OF THE NETWORK # 3 COMPONENTS

NUMBER OF COMPONENTS : 4
VECTOR OF LABELS (8 CHAR. MAX.) :
1 : 150/ 75 2 : 75/ 37 3 : 37/ 26 4 : 26/0

ENTER THE LABEL NO. TO BE CHANGE, AND
(ON THE NEXT LINE) THE NEW LABEL:

4

- 26

ANOTHER CHANGE (Y/N) ?N
LAST SELECTION : 3.6

WHICH OPTION DO YOU WANT ?
(0 = MENU, 12 = END) : 9.1

NUMBER OF PULP, SOLID AND WATER FLOW RATE MEASUREMENTS : 3 3 3

ENTER 3 NEW VALUES :
3 0 0

LAST SELECTION : 9.1

WHICH OPTION DO YOU WANT ?
(0 = MENU, 12 = END) : 9.3

THE SOLID FLOWRATE DATA IS :

| STREAM NO., | FLOWRATE, | STD. DEV. |
|-------------|-----------|-----------|
| 1 | 98.000 | 10.000 |
| 2 | 35.000 | 10.000 |
| 3 | 56.000 | 10.000 |

*** SOLID FLOWRATES DELETED
LAST SELECTION : 9.3

WHICH OPTION DO YOU WANT ?
(0 = MENU, 12 = END) : 12

YOUR NEW IZ VECTOR OR DIMENSIONS REQUIRES
THAT YOU CHOOSE OPTION 9.4

THE WATER FLOWRATE DATA IS :

| STREAM NO., | FLOWRATE, | STD. DEV. |
|-------------|-----------|-----------|
| 1 | 110.000 | 10.000 |
| 2 | 82.000 | 10.000 |
| 3 | 19.000 | 10.000 |

*** WATER FLOWRATES DELETED
LAST SELECTION : 9.4

WHICH OPTION DO YOU WANT ?
(0 = MENU, 12 = END) : 12

DO YOU WANT A DATA FILE LISTING (Y/N) ?N
DO YOU WANT TO MODIFY YOUR NEW FILE (Y/N) ?N
*** END OF EDITING ***

YOUR NEW DATA FILE IS : TAPE5

THE BILMAT3 PROGRAM IS NOW READY TO RUN.

2. EDISIV SAMPLE RUN

SENSITIVITY ANALYSIS DATA SELECTION

THE SENSITIVITY ANALYSIS PROGRAM UTILIZES THE MASS BALANCE RESULTS AND THE ERROR MODEL OF THE MEASURED VALUES TO COMPUTE THE VARIANCES OF THE MASS BALANCE RESULTS, IN OTHER WORDS TO COMPUTE THEIR RELIABILITY. NOT ALL THE DATA CAN BE USED. HERE IS THE LIST OF DATA THAT ARE PROCESSABLE BY THE SENSITIVITY ANALYSIS PROGRAM:

*** ON NETWORK 5, THE 7 COMPONENTS
*** PLUS THE MEASURED SOLID FLOW RATES
*** YOU MIGHT ALSO USE SOME OF THE
MEASURED PULP FLOWRATES

OTHER SOURCES OF DATA ARE NOT PROCESSABLE BY THE PRESENT VERSION OF THIS PROGRAM.

THIS EDITOR, NAMED EDISIV, GIVES YOU THE OPPORTUNITY TO MAKE A SELECTION AMONG THE PROCESSABLE DATA. IF YOU SELECT ALL THE DATA, THE PROGRAM WILL EVALUATE THE RELIABILITY OF THE MASS BALANCE RESULTS AS THEY ARE. IF YOU DISCARD SOME DATA, THE PROGRAM WILL EVALUATE WHAT WOULD BE THE RELIABILITY OF THE RESULTS AS IF THE ONLY SELECTED DATA WERE USED IN THE MASS BALANCE COMPUTATION. YOU ALSO HAVE THE OPPORTUNITY TO ALTER THE ERROR MODEL ON THE MEASUREMENTS, TO SEE BEFOREHAND WHETHER AN IMPROVEMENT IN THE SAMPLE ANALYSIS WOULD RESULT IN ANY PROFITABLE IMPROVEMENT OF THE RELIABILITY THE MASS BALANCE RESULTS.

NETWORK 5 DATA

LIST OF AVAILABLE COMPONENTS :

1 : NIOBIUM
2 : MAGNESM
3 : SILICON
4 : PHOSPHOR
5 : CALCIUM
6 : IRON
7 : SULPHUR

HOW MANY COMPONENTS DO YOU WISH TO USE ? 7

DO YOU WISH TO MODIFY ANY STANDART DEVIATION ? (Y/N) N

SOLID FLOWRATES DATA :

LIST OF AVAILABLE VALUES :

| | | | | | |
|--------|----|---|--------------|---|--------|
| STREAM | 1 | : | FEED | = | 14.200 |
| STREAM | 8 | : | CLEANER2CONC | = | 1.497 |
| STREAM | 11 | : | CLEANER4CONC | = | .352 |
| STREAM | 13 | : | CLEANER5CONC | = | .295 |
| STREAM | 14 | : | SULPHUR CONC | = | .121 |
| STREAM | 15 | : | MAGNTIC CONC | = | .353 |

HOW MANY VALUES DO YOU WISH TO USE ? 6

*** END OF EDISIV ***

SENSITIVITY ANALYSIS
DATA SELECTION

THE SENSITIVITY ANALYSIS PROGRAM UTILIZES THE MASS BALANCE RESULTS AND THE ERROR MODEL OF THE MEASURED VALUES TO COMPUTE THE VARIANCES OF THE MASS BALANCE RESULTS, IN OTHER WORDS TO COMPUTE THEIR RELIABILITY. NOT ALL THE DATA CAN BE USED. HERE IS THE LIST OF DATA THAT ARE PROCESSABLE BY THE SENSITIVITY ANALYSIS PROGRAM:

*** ON NETWORK 3, THE 4 COMPONENTS

*** ON NETWORK 5, THE 3 COMPONENTS

*** YOU MIGHT ALSO USE SOME OF THE MEASURED PULP FLOWRATES

OTHER SOURCES OF DATA ARE NOT PROCESSABLE BY THE PRESENT VERSION OF THIS PROGRAM.

THIS EDITOR, NAMED EDISIV, GIVES YOU THE OPPORTUNITY TO MAKE A SELECTION AMONG THE PROCESSABLE DATA. IF YOU SELECT ALL THE DATA, THE PROGRAM WILL EVALUATE THE RELIABILITY OF THE MASS BALANCE RESULTS AS THEY ARE. IF YOU DISCARD SOME DATA, THE PROGRAM WILL EVALUATE WHAT WOULD BE THE RELIABILITY OF THE RESULTS AS IF THE ONLY SELECTED DATA WERE USED IN THE MASS BALANCE COMPUTATION. YOU ALSO HAVE THE OPPORTUNITY TO ALTER THE ERROR MODEL ON THE MEASUREMENTS, TO SEE BEFOREHAND WHETHER AN IMPROVEMENT IN THE SAMPLE ANALYSIS WOULD RESULT IN ANY PROFITABLE IMPROVEMENT OF THE RELIABILITY OF THE MASS BALANCE RESULTS.

NETWORK 3 DATA

LIST OF AVAILABLE COMPONENTS :

1 : 150/ 75
2 : 75/ 37
3 : 37/ 26
4 : - 26

HOW MANY COMPONENTS DO YOU WISH TO USE ? 4

DO YOU WISH TO MODIFY ANY STANDART DEVIATION ? (Y/N) N

NETWORK 5 DATA

LIST OF AVAILABLE COMPONENTS :

1 : COPPER
2 : CALCIUM
3 : COBALT

HOW MANY COMPONENTS DO YOU WISH TO USE ? 3

DO YOU WISH TO MODIFY ANY STANDART DEVIATION ? (Y/N) Y

CHOOSE OPTION :

1 : LIST THE TABLE OF VALUES
2 : ROW BY ROW MODIFICATION
3 : COLUMN BY COLUMN MODIFICATION
4 : VALUE BY VALUE MODIFICATION
5 : STOP.

1

ROW 1 : COPPER
ROW 2 : CALCIUM
ROW 3 : COBALT

3.000 3.000 3.000
12.000 12.000 12.000
8.000 8.000 8.000

CHOOSE OPTION :

1 : LIST THE TABLE OF VALUES
2 : ROW BY ROW MODIFICATION
3 : COLUMN BY COLUMN MODIFICATION
4 : VALUE BY VALUE MODIFICATION
5 : STOP.

2

ENTER ROW NUMBER AND 3 NEW VALUES : 2 15 15 15

ANOTHER ONE (Y/N)? N

CHOOSE OPTION :

- 1 : LIST THE TABLE OF VALUES
- 2 : ROW BY ROW MODIFICATION
- 3 : COLUMN BY COLUMN MODIFICATION
- 4 : VALUE BY VALUE MODIFICATION
- 5 : STOP.

1

ROW 1 : COPPER
ROW 2 : CALCIUM
ROW 3 : COBALT

3.000 3.000 3.000
15.000 15.000 15.000
8.000 8.000 8.000

CHOOSE OPTION :

- 1 : LIST THE TABLE OF VALUES
- 2 : ROW BY ROW MODIFICATION
- 3 : COLUMN BY COLUMN MODIFICATION
- 4 : VALUE BY VALUE MODIFICATION
- 5 : STOP.

5

SOLID FLOWRATES DATA :

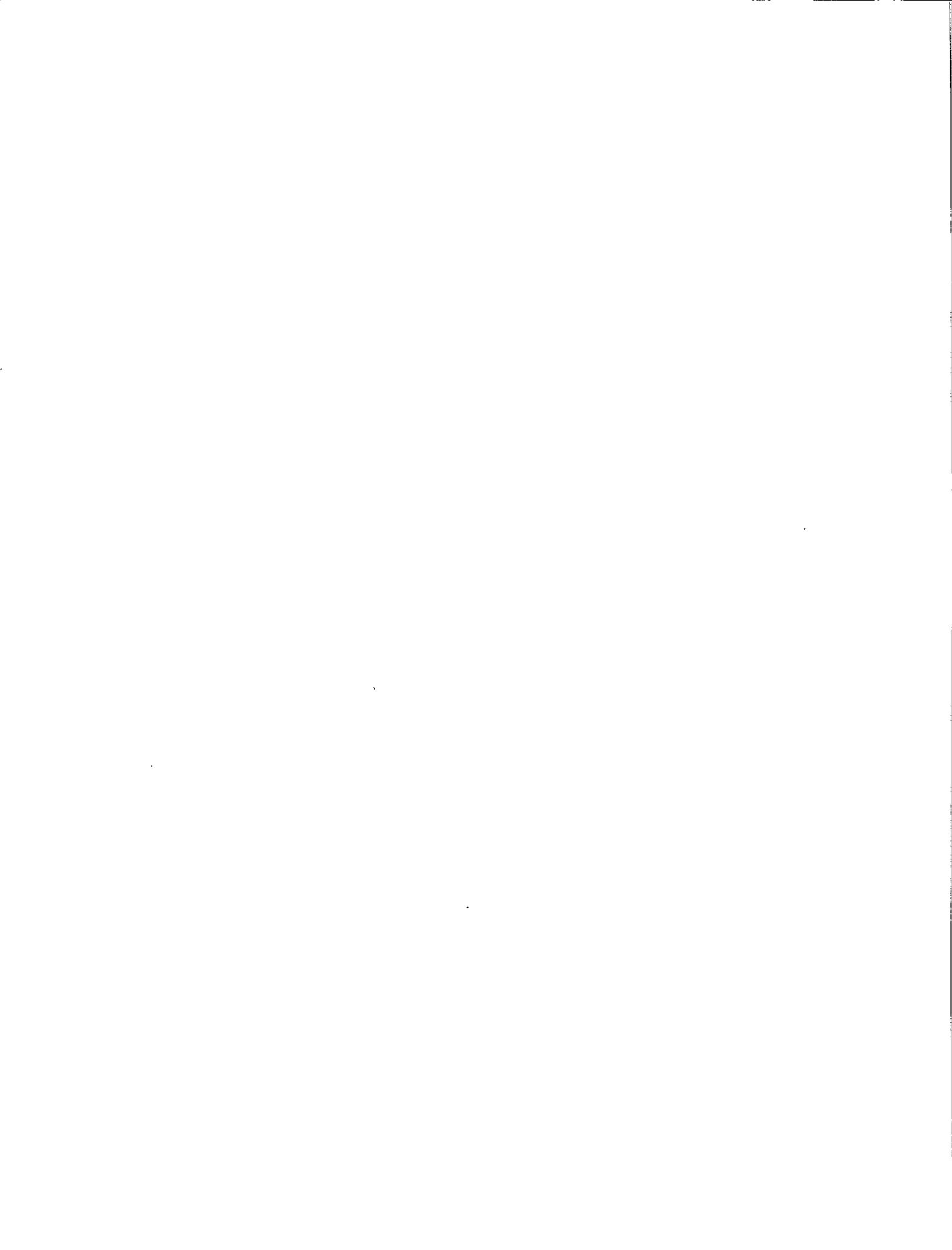
LIST OF AVAILABLE VALUES :

STREAM 1 : CYCLONE FEED = 97.887
STREAM 2 : OVERFLOW = 34.800
STREAM 3 : UNDERFLOW = 55.918

HOW MANY VALUES DO YOU WISH TO USE ? 3

*** END OF EDISIV ***

APPENDIX H
PYROCHLORE FLOTATION CIRCUIT/SAMPLE RUN 1



| | | | |
|--------|--------|--------|---|
| 14 | .590 | 10.000 | |
| 15 | 3.170 | 20.000 | |
| 1 | 14.200 | 5.000 | |
| 17 | 1.330 | 10.000 | |
| 18 | .540 | 10.000 | |
| 19 | .980 | 10.000 | |
| 20 | .250 | 10.000 | |
| 21 | 3.500 | 50.000 | |
| 1 | 42.890 | 10.000 | |
| 2 | 36.220 | 10.000 | |
| 3 | 39.470 | 10.000 | |
| 4 | 23.080 | 10.000 | |
| 5 | 33.060 | 10.000 | |
| 6 | 22.210 | 10.000 | |
| 7 | 20.510 | 10.000 | |
| 8 | 34.900 | 10.000 | |
| 9 | 32.170 | 10.000 | |
| 10 | 16.500 | 20.000 | |
| 11 | 22.540 | 10.000 | |
| 12 | 4.440 | 50.000 | |
| 13 | 31.350 | 10.000 | |
| 14 | 20.530 | 10.000 | |
| 15 | 11.130 | 10.000 | |
| 16 | 29.790 | 10.000 | |
| | .813 | 2.000 | |
| | .187 | 2.000 | |
| | .059 | 2.000 | |
| | .128 | 2.000 | |
| | .100 | 2.000 | |
| | .077 | 2.000 | |
| | .105 | 2.000 | |
| | .065 | 2.000 | |
| | .037 | 2.000 | |
| | .034 | 2.000 | |
| | .006 | 2.000 | |
| | .028 | 2.000 | |
| | .009 | 2.000 | |
| | .030 | 2.000 | |
| | 1.011 | 2.000 | |
| | 1.015 | 3 | |
| 14.200 | 3.000 | 1.060 | 4 |

TITLE : MASS BALANCE ON A FLOTATION CIRCUIT

DATA INPUT AND CALCULATION PATH OPTIONS:

INTEGER VECTOR OF OPTIONS: 1 1 0 0 0 1 0 0 0 0 0 0

OPTION MEANING:

- 1 SOME FLOWRATES HAVE BEEN MEASURED.
- 2 SOME PULP PERCENT SOLIDS HAVE BEEN MEASURED.
- 6 DATA ON NETWORK 5 ARE AVAILABLE.
- 12 READ USER'S FIRST ESTIMATES FOR THE POWELL ALGORITHM.

NUMBER OF NODES IN NETWORK #1 : 7
NUMBER OF STREAMS IN NETWORK #1 : 21
NUMBER OF NODES IN NETWORK #2 : 7
NUMBER OF STREAMS IN NETWORK #2 : 16

LABELS TO IDENTIFY THE STREAMS OF NETWORK #1 :

| | | | |
|-------------------|-------------------|-------------------|-------------------|
| 1 : FEED | 2 : ROUGHER TAIL | 3 : ROUGHER CONC | 4 : CLEANER1TAIL |
| 5 : CLEANER1CONC | 6 : CLEANER2TAIL | 7 : CLEANER3TAIL | 8 : CLEANER2CONC |
| 9 : CLEANER3CONC | 10 : CLEANER4TAIL | 11 : CLEANER4CONC | 12 : CLEANER5TAIL |
| 13 : CLEANER5CONC | 14 : SULPHUR CONC | 15 : MAGNTIC CONC | 16 : TAILINGS |
| 17 : CLEANER1 H2O | 18 : CLEANER3 H2O | 19 : CLEANER4 H2O | 20 : CLEANER5 H2O |
| 21 : ROUGHER H2O | | | |

MATRIX OF NETWORK 1 :

| | | |
|----------------|-------------|-----------|
| STREAM NB. 1: | FROM NODE 0 | TO NODE 1 |
| STREAM NB. 2: | FROM NODE 1 | TO NODE 7 |
| STREAM NB. 3: | FROM NODE 1 | TO NODE 2 |
| STREAM NB. 4: | FROM NODE 2 | TO NODE 7 |
| STREAM NB. 5: | FROM NODE 2 | TO NODE 3 |
| STREAM NB. 6: | FROM NODE 3 | TO NODE 7 |
| STREAM NB. 7: | FROM NODE 4 | TO NODE 3 |
| STREAM NB. 8: | FROM NODE 3 | TO NODE 4 |
| STREAM NB. 9: | FROM NODE 4 | TO NODE 5 |
| STREAM NB. 10: | FROM NODE 5 | TO NODE 4 |

STREAM NB. 11: FROM NODE 5 TO NODE 6
 STREAM NB. 12: FROM NODE 6 TO NODE 5
 STREAM NB. 13: FROM NODE 6 TO NODE 0
 STREAM NB. 14: FROM NODE 0 TO NODE 7
 STREAM NB. 15: FROM NODE 0 TO NODE 7
 STREAM NB. 16: FROM NODE 7 TO NODE 0
 STREAM NB. 17: FROM NODE 0 TO NODE 2
 STREAM NB. 18: FROM NODE 0 TO NODE 4
 STREAM NB. 19: FROM NODE 0 TO NODE 5
 STREAM NB. 20: FROM NODE 0 TO NODE 6
 STREAM NB. 21: FROM NODE 0 TO NODE 1

THE NUMBERS OF THE STREAMS OF NETWORK #1
 COMMON WITH NETWORK #2 :

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

MATRIX OF NETWORK 2 :

STREAM NB. 1: FROM NODE 0 TO NODE 1
 STREAM NB. 2: FROM NODE 1 TO NODE 7
 STREAM NB. 3: FROM NODE 1 TO NODE 2
 STREAM NB. 4: FROM NODE 2 TO NODE 7
 STREAM NB. 5: FROM NODE 2 TO NODE 3
 STREAM NB. 6: FROM NODE 3 TO NODE 7
 STREAM NB. 7: FROM NODE 4 TO NODE 3
 STREAM NB. 8: FROM NODE 3 TO NODE 4
 STREAM NB. 9: FROM NODE 4 TO NODE 5
 STREAM NB. 10: FROM NODE 5 TO NODE 4
 STREAM NB. 11: FROM NODE 5 TO NODE 6
 STREAM NB. 12: FROM NODE 6 TO NODE 5
 STREAM NB. 13: FROM NODE 6 TO NODE 0
 STREAM NB. 14: FROM NODE 0 TO NODE 7
 STREAM NB. 15: FROM NODE 0 TO NODE 7

STREAM NB. 16: FROM NODE 7 TO NODE 0

NETWORK # 5 :

NUMBER OF NODES : 7
NUMBER OF STREAMS : 16
NUMBER OF COMPONENTS: 7

NUMBERS OF THE STREAMS COMMON WITH NETWORK #1 :

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

MATRIX OF NETWORK 5 :

STREAM NB. 1: FROM NODE 0 TO NODE 1
STREAM NB. 2: FROM NODE 1 TO NODE 7
STREAM NB. 3: FROM NODE 1 TO NODE 2
STREAM NB. 4: FROM NODE 2 TO NODE 7
STREAM NB. 5: FROM NODE 2 TO NODE 3
STREAM NB. 6: FROM NODE 3 TO NODE 7
STREAM NB. 7: FROM NODE 4 TO NODE 3
STREAM NB. 8: FROM NODE 3 TO NODE 4
STREAM NB. 9: FROM NODE 4 TO NODE 5
STREAM NB. 10: FROM NODE 5 TO NODE 4
STREAM NB. 11: FROM NODE 5 TO NODE 6
STREAM NB. 12: FROM NODE 6 TO NODE 5
STREAM NB. 13: FROM NODE 6 TO NODE 0
STREAM NB. 14: FROM NODE 0 TO NODE 7
STREAM NB. 15: FROM NODE 0 TO NODE 7
STREAM NB. 16: FROM NODE 7 TO NODE 0

MEASURED VALUES OF THE COMPONENTS MASS FRACTIONS
(1 ROW PER COMPONENT)

| | | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1.320 | .090 | 4.910 | .240 | 6.800 | .670 | 1.370 | 8.840 | 15.300 | 3.320 |
| 25.720 | 3.700 | 36.270 | .520 | .360 | .180 | | | | |
| 14.120 | 14.410 | 9.870 | 10.280 | 9.070 | 10.950 | 8.110 | 6.040 | 3.330 | 5.450 |
| 1.620 | 4.560 | .880 | .130 | 3.110 | 13.280 | | | | |
| 3.580 | 1.390 | 14.850 | 9.240 | 19.400 | 24.590 | 26.030 | 20.030 | 9.730 | 15.360 |
| 4.040 | 12.640 | 3.900 | .720 | 1.470 | 3.400 | | | | |
| 3.680 | 2.390 | 5.890 | 8.580 | 4.220 | 4.610 | 2.570 | 1.710 | 1.000 | 1.220 |
| .300 | .490 | .290 | .020 | .550 | 3.250 | | | | |
| 29.900 | 33.200 | 16.450 | 25.960 | 11.470 | 11.530 | 7.510 | 7.270 | 6.850 | 5.990 |

| | | | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--|
| 7.150 | 5.030 | 8.630 | .790 | 7.520 | 29.550 | | | | | |
| 5.380 | 3.690 | 11.820 | 7.330 | 13.690 | 12.800 | 15.610 | 17.030 | 23.110 | 24.690 | |
| 22.930 | 27.890 | 18.060 | 44.970 | 58.560 | 7.020 | | | | | |
| .820 | .100 | 3.420 | .300 | 4.200 | 2.000 | 4.400 | 6.000 | 15.800 | 14.800 | |
| 10.200 | 17.100 | 14.000 | 54.800 | .510 | .760 | | | | | |

RELATIVE STANDARD DEVIATIONS (%) ASSOCIATED WITH THE MEASURED VALUES

| | | | | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 20.000 | 20.000 | 20.000 | 20.000 | 20.000 | 20.000 | 20.000 | 20.000 | 20.000 | 20.000 | 20.000 |
| 20.000 | 20.000 | 20.000 | 20.000 | 20.000 | 20.000 | | | | | |
| 6.000 | 6.000 | 6.000 | 6.000 | 6.000 | 6.000 | 6.000 | 6.000 | 6.000 | 6.000 | 6.000 |
| 6.000 | 6.000 | 6.000 | 6.000 | 6.000 | 6.000 | | | | | |
| 30.000 | 30.000 | 6.000 | 6.000 | 6.000 | 6.000 | 30.000 | 30.000 | 30.000 | 30.000 | 30.000 |
| 30.000 | 30.000 | 6.000 | 30.000 | 30.000 | 30.000 | | | | | |
| 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 |
| 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | | | | | |
| 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 |
| 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | 10.000 | | | | | |
| 5.000 | 5.000 | 5.000 | 5.000 | 5.000 | 5.000 | 5.000 | 5.000 | 5.000 | 5.000 | 5.000 |
| 5.000 | 5.000 | 5.000 | 5.000 | 5.000 | 5.000 | | | | | |
| 8.000 | 8.000 | 8.000 | 8.000 | 8.000 | 8.000 | 8.000 | 8.000 | 8.000 | 8.000 | 8.000 |
| 8.000 | 8.000 | 8.000 | 8.000 | 8.000 | 8.000 | | | | | |

LABELS TO IDENTIFY COMPONENTS:

| | | | | |
|-------------|-------------|-------------|--------------|-----------|
| 1 : NIOBIUM | 2 : MAGNESM | 3 : SILICON | 4 : PHOSPHOR | 5 : CALCI |
| 6 : IRON | 7 : SULPHUR | | | |

NUMBER OF PULP FLOW RATE MEASUREMENTS : 5
 STREAM NB., MEASURED VALUE, STD. DEV.:

| | | |
|----|-------|--------|
| 8 | 4.290 | 20.000 |
| 11 | 1.560 | 20.000 |
| 13 | .940 | 20.000 |
| 14 | .590 | 10.000 |
| 15 | 3.170 | 20.000 |

NUMBER OF SOLID FLOW RATE MEASUREMENTS : 1
 STREAM NB., MEASURED VALUE, STD. DEV.:

| | | |
|---|--------|-------|
| 1 | 14.200 | 5.000 |
|---|--------|-------|

NUMBER OF WATER FLOW RATE MEASUREMENTS : 5
 STREAM NB., MEASURED VALUE, STD. DEV.:

| | | |
|----|-------|--------|
| 17 | 1.330 | 10.000 |
| 18 | .540 | 10.000 |
| 19 | .980 | 10.000 |
| 20 | .250 | 10.000 |
| 21 | 3.500 | 50.000 |

NUMBER OF PULP PERCENT SOLID MEASUREMENTS : 16
 STREAM NB., MEASURED VALUE, STD. DEV.:

| | | |
|---|--------|--------|
| 1 | 42.890 | 10.000 |
| 2 | 36.220 | 10.000 |
| 3 | 39.470 | 10.000 |
| 4 | 23.080 | 10.000 |
| 5 | 33.060 | 10.000 |
| 6 | 18.620 | 10.000 |

| | | |
|----|--------|--------|
| 7 | 17.480 | 20.000 |
| 8 | 34.900 | 10.000 |
| 9 | 32.170 | 10.000 |
| 10 | 12.380 | 20.000 |
| 11 | 22.540 | 10.000 |
| 12 | 4.440 | 50.000 |
| 13 | 31.350 | 10.000 |
| 14 | 20.530 | 10.000 |
| 15 | 11.130 | 10.000 |
| 16 | 29.790 | 10.000 |

ESTIMATE AND PRECISION OF RELATIVE ORE FLOWRATES (FOR POWELL ALGO.)
 (REFERENCE STREAM IS #1)

| | |
|-------|-------|
| .813 | 2.000 |
| .187 | 2.000 |
| .059 | 2.000 |
| .128 | 2.000 |
| .100 | 2.000 |
| .077 | 2.000 |
| .105 | 2.000 |
| .065 | 2.000 |
| .037 | 2.000 |
| .034 | 2.000 |
| .006 | 2.000 |
| .028 | 2.000 |
| .009 | 2.000 |
| .030 | 2.000 |
| 1.011 | 2.000 |

CONTROLLING PARAMETERS : 1.0 15 3

ESTIMATE AND PRECISION OF REFERENCE STREAM FLOWRATE, AND
 CONTROL PARAMETERS OF COGGINS ALGO : 14.200 3.000 4

*** END OF DATA FILE LISTING ***

2. RESULTS PRINTOUT (OUTPUT FILE)

MASS BALANCE ON A FLOTATION CIRCUIT

VECTOR OF INDEPENDENT STREAMS :

1 2 15 4 11 6 7 14 9

VECTOR OF DEPENDENT STREAMS :

3 5 8 10 12 13 16

POWELL ALGORITHM

N = 8 MAXIT = 15 ESCALE = 1.00

STARTING VALUES (X) REQUIRED ACCURACIES (E)

| | |
|---------------|---------------|
| .81300000E+00 | .16260000E-01 |
| .30000000E-01 | .60000000E-03 |
| .59000000E-01 | .11800000E-02 |
| .34000000E-01 | .68000000E-03 |
| .10000000E+00 | .20000000E-02 |
| .77000000E-01 | .15400000E-02 |
| .90000000E-02 | .18000000E-03 |
| .65000000E-01 | .13000000E-02 |

POWELL ALGORITHM

ITERATION 7 146 FUNCTION EVALUATIONS F = .42726266E+02
 ESTIMATES : .81406 .02936 .06255 .03099 .09875 .07534 .00863 .05727

END OF FUNCTION MINIMIZATION

FUNCTION VALUE FOR NETWORK 5 DATA : 36.7839

 36.7839

FUNCTION VALUE FOR NETWORKS 1 & 2 DATA : 5.9423

 42.7263

RELATIVE ORE FLOW RATES :

STREAMS RELATIVE FLOW RATES

| | |
|--------------|--------|
| FEED | 1.0000 |
| ROUGHER TAIL | .8141 |
| ROUGHER CONC | .1859 |
| CLEANER1TAIL | .0625 |
| CLEANER1CONC | .1234 |
| CLEANER2TAIL | .0988 |
| CLEANER3TAIL | .0753 |
| CLEANER2CONC | .1000 |
| CLEANER3CONC | .0573 |
| CLEANER4TAIL | .0326 |
| CLEANER4CONC | .0310 |
| CLEANER5TAIL | .0064 |
| CLEANER5CONC | .0246 |
| SULPHUR CONC | .0086 |
| MAGNTIC CONC | .0294 |
| TAILINGS | 1.0133 |

MASS BALANCE RESULTS

ESTIMATES OF NETWORKS 1 & 2 VARIABLES

| STREAMS | PULP FLOW RATES | | | | WATER FLOW RATES | | | |
|--------------|-----------------|---------------|---------------|---------------|------------------|---------------|---------------|---------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV |
| FEED | 0.00 | 33.53 | 100.00 | 0.00 | 0.00 | 19.85 | 100.00 | 0.00 |
| ROUGHER TAIL | 0.00 | 30.44 | 100.00 | 0.00 | 0.00 | 19.30 | 100.00 | 0.00 |
| ROUGHER CONC | 0.00 | 7.08 | 100.00 | 0.00 | 0.00 | 4.54 | 100.00 | 0.00 |
| CLEANER1TAIL | 0.00 | 3.49 | 100.00 | 0.00 | 0.00 | 2.64 | 100.00 | 0.00 |
| CLEANER1CONC | 0.00 | 4.95 | 100.00 | 0.00 | 0.00 | 3.26 | 100.00 | 0.00 |
| CLEANER2TAIL | 0.00 | 5.73 | 100.00 | 0.00 | 0.00 | 4.38 | 100.00 | 0.00 |
| CLEANER3TAIL | 0.00 | 4.85 | 100.00 | 0.00 | 0.00 | 3.82 | 100.00 | 0.00 |
| CLEANER2CONC | 4.29 | 4.07 | 5.09 | 20.00 | 0.00 | 2.70 | 100.00 | 0.00 |
| CLEANER3CONC | 0.00 | 2.44 | 100.00 | 0.00 | 0.00 | 1.66 | 100.00 | 0.00 |
| CLEANER4TAIL | 0.00 | 2.67 | 100.00 | 0.00 | 0.00 | 2.23 | 100.00 | 0.00 |
| CLEANER4CONC | 1.56 | 1.82 | 16.65 | 20.00 | 0.00 | 1.40 | 100.00 | 0.00 |
| CLEANER5TAIL | 0.00 | 1.05 | 100.00 | 0.00 | 0.00 | .97 | 100.00 | 0.00 |
| CLEANER5CONC | .94 | 1.02 | 8.21 | 20.00 | 0.00 | .68 | 100.00 | 0.00 |
| SULPHUR CONC | .59 | .58 | 1.29 | 10.00 | 0.00 | .46 | 100.00 | 0.00 |
| MAGNTIC CONC | 3.17 | 3.51 | 10.86 | 20.00 | 0.00 | 3.11 | 100.00 | 0.00 |
| TAILINGS | 0.00 | 43.75 | 100.00 | 0.00 | 0.00 | 29.89 | 100.00 | 0.00 |
| CLEANER1 H2O | 0.00 | 1.36 | 100.00 | 0.00 | 1.33 | 1.36 | 2.25 | 10.00 |
| CLEANER3 H2O | 0.00 | .55 | 100.00 | 0.00 | .54 | .55 | .96 | 10.00 |
| CLEANER4 H2O | 0.00 | 1.00 | 100.00 | 0.00 | .98 | 1.00 | 1.85 | 10.00 |
| CLEANER5 H2O | 0.00 | .25 | 100.00 | 0.00 | .25 | .25 | .71 | 10.00 |
| ROUGHER H2O | 0.00 | 3.99 | 100.00 | 0.00 | 3.50 | 3.99 | 14.06 | 50.00 |

| STREAMS | SOLID FLOW RATES | | | | PULP PERCENT SOLIDS | | | |
|--------------|------------------|---------------|---------------|---------------|---------------------|---------------|---------------|---------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV |
| FEED | 14.20 | 13.68 | 3.67 | 5.00 | 42.89 | 40.80 | 4.87 | 10.00 |
| ROUGHER TAIL | 0.00 | 11.14 | 100.00 | 0.00 | 36.22 | 36.58 | 1.00 | 10.00 |
| ROUGHER CONC | 0.00 | 2.54 | 100.00 | 0.00 | 39.47 | 35.92 | 8.99 | 10.00 |
| CLEANER1TAIL | 0.00 | .86 | 100.00 | 0.00 | 23.08 | 24.50 | 6.16 | 10.00 |
| CLEANER1CONC | 0.00 | 1.69 | 100.00 | 0.00 | 33.06 | 34.11 | 3.18 | 10.00 |
| CLEANER2TAIL | 0.00 | 1.35 | 100.00 | 0.00 | 22.21 | 23.59 | 6.21 | 10.00 |
| CLEANER3TAIL | 0.00 | 1.03 | 100.00 | 0.00 | 20.51 | 21.25 | 3.62 | 10.00 |
| CLEANER2CONC | 0.00 | 1.37 | 100.00 | 0.00 | 34.90 | 33.59 | 3.74 | 10.00 |
| CLEANER3CONC | 0.00 | .78 | 100.00 | 0.00 | 32.17 | 32.09 | .26 | 10.00 |
| CLEANER4TAIL | 0.00 | .45 | 100.00 | 0.00 | 16.50 | 16.69 | 1.16 | 20.00 |
| CLEANER4CONC | 0.00 | .42 | 100.00 | 0.00 | 22.54 | 23.30 | 3.36 | 10.00 |
| CLEANER5TAIL | 0.00 | .09 | 100.00 | 0.00 | 4.44 | 8.24 | 85.57 | 50.00 |
| CLEANER5CONC | 0.00 | .34 | 100.00 | 0.00 | 31.35 | 33.14 | 5.70 | 10.00 |
| SULPHUR CONC | 0.00 | .12 | 100.00 | 0.00 | 20.53 | 20.26 | 1.31 | 10.00 |
| MAGNTIC CONC | 0.00 | .40 | 100.00 | 0.00 | 11.13 | 11.43 | 2.70 | 10.00 |
| TAILINGS | 0.00 | 13.86 | 100.00 | 0.00 | 29.79 | 31.68 | 6.35 | 10.00 |

ESTIMATES OF NETWORK 5 MASS FRACTIONS

| STREAMS | FEED | | | | ROUGHER TAIL | | | |
|----------|--------------|---------------|---------------|----------------|--------------|---------------|---------------|----------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| NIOBIUM | 1.32 | .98 | 25.49 | 20.00 | .09 | .09 | 3.32 | 20.00 |
| MAGNESM | 14.12 | 13.64 | 3.39 | 4.00 | 14.41 | 14.58 | 1.17 | 4.00 |
| SILICON | 3.58 | 3.98 | 11.31 | 30.00 | 1.39 | 1.27 | 8.28 | 30.00 |
| PHOSPHOR | 3.68 | 3.18 | 13.62 | 10.00 | 2.39 | 2.61 | 9.09 | 10.00 |
| CALCIUM | 29.90 | 29.96 | .19 | 4.00 | 33.20 | 33.10 | .29 | 4.00 |
| IRON | 5.38 | 5.27 | 2.13 | 5.00 | 3.69 | 3.78 | 2.38 | 5.00 |
| SULPHUR | .69 | .64 | 6.67 | 8.00 | .10 | .10 | .57 | 8.00 |

| STREAMS | ROUGHER CONC | | | | CLEANER1TAIL | | | |
|----------|--------------|---------------|---------------|----------------|--------------|---------------|---------------|----------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| NIOBIUM | 4.91 | 4.88 | .57 | 20.00 | .24 | .24 | .69 | 20.00 |
| MAGNESM | 9.87 | 9.54 | 3.37 | 4.00 | 10.28 | 10.41 | 1.24 | 4.00 |
| SILICON | 14.85 | 15.85 | 6.73 | 6.00 | 9.24 | 9.09 | 1.58 | 6.00 |
| PHOSPHOR | 5.89 | 5.68 | 3.55 | 10.00 | 8.58 | 8.94 | 4.25 | 10.00 |
| CALCIUM | 16.45 | 16.19 | 1.58 | 4.00 | 25.96 | 26.17 | .82 | 4.00 |
| IRON | 11.82 | 11.78 | .37 | 5.00 | 7.33 | 7.36 | .44 | 5.00 |
| SULPHUR | 3.42 | 3.02 | 11.61 | 8.00 | .30 | .30 | .47 | 8.00 |

| STREAMS | CLEANER1CONC | | | | CLEANER2TAIL | | | |
|----------|--------------|---------------|---------------|----------------|--------------|---------------|---------------|----------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| NIOBIUM | 6.80 | 7.23 | 6.39 | 20.00 | .67 | .69 | 2.54 | 20.00 |
| MAGNESM | 9.07 | 9.10 | .29 | 4.00 | 10.95 | 11.15 | 1.81 | 4.00 |
| SILICON | 19.40 | 19.27 | .65 | 6.00 | 24.59 | 23.12 | 5.97 | 6.00 |
| PHOSPHOR | 4.22 | 4.03 | 4.59 | 10.00 | 4.61 | 4.96 | 7.62 | 10.00 |
| CALCIUM | 11.47 | 11.13 | 2.98 | 4.00 | 11.53 | 11.87 | 2.97 | 4.00 |
| IRON | 13.69 | 14.01 | 2.37 | 5.00 | 12.80 | 12.73 | .56 | 5.00 |
| SULPHUR | 4.20 | 4.40 | 4.82 | 8.00 | 2.00 | 2.06 | 3.15 | 8.00 |

| STREAMS | CLEANER3TAIL | | | | CLEANER2CONC | | | |
|----------|--------------|---------------|---------------|----------------|--------------|---------------|---------------|----------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| NIOBIUM | 1.37 | 1.36 | .58 | 20.00 | 8.84 | 9.28 | 4.93 | 20.00 |
| MAGNESM | 8.11 | 7.92 | 2.38 | 4.00 | 6.04 | 6.18 | 2.35 | 4.00 |
| SILICON | 26.03 | 25.68 | 1.34 | 30.00 | 20.03 | 20.30 | 1.36 | 30.00 |
| PHOSPHOR | 2.57 | 2.35 | 8.57 | 10.00 | 1.71 | 1.84 | 7.57 | 10.00 |
| CALCIUM | 7.51 | 7.31 | 2.65 | 4.00 | 7.27 | 7.52 | 3.40 | 4.00 |
| IRON | 15.61 | 15.84 | 1.49 | 5.00 | 17.03 | 16.66 | 2.16 | 5.00 |
| SULPHUR | 4.40 | 4.18 | 5.02 | 8.00 | 6.00 | 6.54 | 9.08 | 8.00 |

| STREAMS | CLEANER3CONC | | | | CLEANER4TAIL | | | |
|----------|-----------------|------------------|------------------|-------------------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| NIOBIUM | 15.30 | 16.28 | 6.40 | 20.00 | 3.32 | 3.29 | .79 | 20.00 |
| MAGNESM | 3.33 | 3.41 | 2.44 | 4.00 | 5.45 | 5.33 | 2.27 | 4.00 |
| SILICON | 9.73 | 10.11 | 3.87 | 30.00 | 15.36 | 14.83 | 3.48 | 30.00 |
| PHOSPHOR | 1.00 | .88 | 12.46 | 10.00 | 1.22 | 1.33 | 8.66 | 10.00 |
| CALCIUM | 6.85 | 6.90 | .80 | 4.00 | 5.99 | 5.97 | .40 | 4.00 |
| IRON | 23.11 | 22.53 | 2.52 | 5.00 | 24.69 | 25.07 | 1.53 | 5.00 |
| SULPHUR | 15.80 | 14.68 | 7.09 | 8.00 | 14.80 | 15.36 | 3.78 | 8.00 |

| STREAMS | CLEANER4CONC | | | | CLEANER5TAIL | | | |
|----------|-----------------|------------------|------------------|-------------------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| NIOBIUM | 25.72 | 27.37 | 6.42 | 20.00 | 3.70 | 3.69 | .19 | 20.00 |
| MAGNESM | 1.62 | 1.63 | .48 | 4.00 | 4.56 | 4.55 | .28 | 4.00 |
| SILICON | 4.04 | 5.19 | 28.35 | 30.00 | 12.64 | 10.34 | 18.18 | 30.00 |
| PHOSPHOR | .30 | .32 | 6.68 | 10.00 | .49 | .48 | 2.23 | 10.00 |
| CALCIUM | 7.15 | 7.50 | 4.92 | 4.00 | 5.03 | 4.99 | .71 | 4.00 |
| IRON | 22.93 | 21.07 | 8.12 | 5.00 | 27.89 | 28.45 | 2.02 | 5.00 |
| SULPHUR | 13.50 | 14.40 | 6.66 | 8.00 | 17.10 | 16.80 | 1.73 | 8.00 |

| STREAMS | CLEANER5CONC | | | | SULPHUR CONC | | | |
|----------|-----------------|------------------|------------------|-------------------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| NIOBIUM | 36.27 | 33.47 | 7.71 | 20.00 | .52 | .52 | .12 | 20.00 |
| MAGNESM | .88 | .88 | .53 | 4.00 | .13 | .13 | .00 | 4.00 |
| SILICON | 3.90 | 3.86 | 1.12 | 6.00 | .72 | .72 | .03 | 30.00 |
| PHOSPHOR | .29 | .28 | 3.78 | 10.00 | .02 | .02 | .00 | 10.00 |
| CALCIUM | 8.63 | 8.15 | 5.59 | 4.00 | .79 | .79 | .00 | 4.00 |
| IRON | 18.06 | 19.17 | 6.12 | 5.00 | 44.97 | 45.04 | .15 | 5.00 |
| SULPHUR | 14.00 | 13.78 | 1.58 | 8.00 | 54.80 | 54.10 | 1.27 | 8.00 |

| STREAMS | MAGNTIC CONC | | | | TAILINGS | | | |
|----------|-----------------|------------------|------------------|-------------------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| NIOBIUM | .36 | .36 | .28 | 20.00 | .18 | .17 | 4.75 | 20.00 |
| MAGNESM | 3.11 | 3.11 | .01 | 4.00 | 13.28 | 13.53 | 1.89 | 4.00 |
| SILICON | 1.47 | 1.47 | .18 | 30.00 | 3.40 | 3.89 | 14.33 | 30.00 |
| PHOSPHOR | .55 | .55 | .02 | 10.00 | 3.25 | 3.15 | 3.20 | 10.00 |
| CALCIUM | 7.52 | 7.52 | .00 | 4.00 | 29.55 | 29.59 | .13 | 4.00 |
| IRON | 58.56 | 58.96 | .68 | 5.00 | 7.02 | 6.82 | 2.83 | 5.00 |
| SULPHUR | .51 | .51 | .04 | 8.00 | .76 | .78 | 2.07 | 8.00 |

END OF BILMAT3 PROGRAM

MASS BALANCE ON A FLOTATION CIRCUIT

SENSITIVITY ANALYSIS

STANDARD DEVIATION OF RELATIVE FLOW RATES
 =====

| STREAM | RELATIVE FLOW RATE | STANDARD DEVIATION | RELATIVE ST. DEV. |
|--------------|-----------------------|-----------------------|----------------------|
| FEED | 1.000 | 0.000 | 0.000 |
| ROUGHER TAIL | .814 | .016 | 1.942 |
| ROUGHER CONC | .186 | .016 | 8.503 |
| CLEANER1TAIL | .063 | .010 | 15.349 |
| CLEANER1CONC | .123 | .010 | 8.482 |
| CLEANER2TAIL | .099 | .010 | 9.720 |
| CLEANER3TAIL | .075 | .013 | 16.741 |
| CLEANER2CONC | .100 | .014 | 13.638 |
| CLEANER3CONC | .057 | .007 | 12.120 |
| CLEANER4TAIL | .033 | .005 | 16.740 |
| CLEANER4CONC | .031 | .003 | 9.551 |
| CLEANER5TAIL | .006 | .001 | 15.434 |
| CLEANER6CONC | .025 | .002 | 9.302 |
| SULPHUR CONC | .009 | .001 | 13.049 |
| MAGNTIC CONC | .029 | .005 | 18.502 |
| TAILINGS | 1.013 | .006 | .594 |

STANDARD DEVIATION OF SOLID FLOW RATES

=====

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | |
|--------------|-----------|--------------------|-----------|--------------------|-----------|--------------------|
| | FLOW RATE | STANDARD DEVIATION | FLOW RATE | STANDARD DEVIATION | RESID. | STANDARD DEVIATION |
| FEED | 14.200 | .710 | 13.680 | .666 | .520 | .246 |
| ROUGHER TAIL | 0.000 | 0.000 | 11.136 | .592 | 0.000 | 0.000 |
| ROUGHER CONC | 0.000 | 0.000 | 2.544 | .245 | 0.000 | 0.000 |
| CLEANER1TAIL | 0.000 | 0.000 | .856 | .137 | 0.000 | 0.000 |
| CLEANER1CONC | 0.000 | 0.000 | 1.688 | .162 | 0.000 | 0.000 |
| CLEANER2TAIL | 0.000 | 0.000 | 1.351 | .147 | 0.000 | 0.000 |
| CLEANER3TAIL | 0.000 | 0.000 | 1.031 | .175 | 0.000 | 0.000 |
| CLEANER2CONC | 1.497 | .449 | 1.368 | .190 | .129 | .407 |
| CLEANER3CONC | 0.000 | 0.000 | .783 | .098 | 0.000 | 0.000 |
| CLEANER4TAIL | 0.000 | 0.000 | .446 | .076 | 0.000 | 0.000 |
| CLEANER4CONC | .352 | .105 | .424 | .043 | -.072 | .097 |
| CLEANER5TAIL | 0.000 | 0.000 | .087 | .014 | 0.000 | 0.000 |
| CLEANER5CONC | .295 | .088 | .337 | .033 | -.042 | .082 |
| SULPHUR CONC | .121 | .024 | .118 | .016 | .003 | .018 |
| MAGNTIC CONC | .353 | .106 | .402 | .075 | -.049 | .075 |
| TAILINGS | 0.000 | 0.000 | 13.862 | .674 | 0.000 | 0.000 |

STANDARD DEVIATION OF NETWORK 5 DATA

=====

COMPONENT : NIOBIUM

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | RES. | STANDARD DEVIATION | |
| FEED | 1.320 | .264 | .984 | .094 | .337 | .247 | 1 |
| ROUGHER TAIL | .090 | .018 | .093 | .017 | -.003 | .006 | 0 |
| ROUGHER CONC | 4.910 | .982 | 4.882 | .465 | .028 | .865 | 0 |
| CLEANER1TAIL | .240 | .048 | .242 | .048 | -.002 | .003 | 0 |
| CLEANER1CONC | 6.800 | 1.360 | 7.234 | .697 | -.434 | 1.168 | 0 |
| CLEANER2TAIL | .670 | .134 | .687 | .127 | -.017 | .043 | 0 |
| CLEANER3TAIL | 1.370 | .274 | 1.362 | .273 | .008 | .026 | 0 |
| CLEANER2CONC | 8.840 | 1.768 | 9.276 | 1.058 | -.436 | 1.416 | 0 |
| CLEANER3CONC | 15.300 | 3.060 | 16.279 | 1.592 | -.979 | 2.613 | 0 |
| CLEANER4TAIL | 3.320 | .664 | 3.294 | .660 | .026 | .070 | 0 |
| CLEANER4CONC | 25.720 | 5.144 | 27.372 | 2.489 | -1.652 | 4.502 | 0 |
| CLEANER5TAIL | 3.700 | .740 | 3.693 | .740 | .007 | .019 | 0 |
| CLEANER5CONC | 36.270 | 7.254 | 33.475 | 3.083 | 2.795 | 6.566 | 0 |
| SULPHUR CONC | .520 | .104 | .521 | .104 | -.001 | .002 | 0 |
| MAGNTIC CONC | .360 | .072 | .361 | .072 | -.001 | .004 | 0 |
| TAILINGS | .180 | .036 | .172 | .018 | .008 | .031 | 0 |

COMPONENT : MAGNESM

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | RES. | STANDARD DEVIATION | |
| FEED | 14.120 | .565 | 13.641 | .303 | .479 | .477 | 1 |
| ROUGHER TAIL | 14.410 | .576 | 14.578 | .373 | -.168 | .440 | 0 |
| ROUGHER CONC | 9.870 | .395 | 9.538 | .197 | .332 | .342 | 0 |
| CLEANER1TAIL | 10.280 | .411 | 10.408 | .392 | -.128 | .126 | 1 |
| CLEANER1CONC | 9.070 | .363 | 9.097 | .250 | -.027 | .263 | 0 |
| CLEANER2TAIL | 10.950 | .438 | 11.148 | .318 | -.198 | .301 | 0 |
| CLEANER3TAIL | 8.110 | .324 | 7.917 | .267 | .193 | .184 | 1 |
| CLEANER2CONC | 6.040 | .242 | 6.182 | .200 | -.142 | .136 | 1 |
| CLEANER3CONC | 3.330 | .133 | 3.411 | .123 | -.081 | .051 | 1 |
| CLEANER4TAIL | 5.450 | .218 | 5.326 | .204 | .124 | .078 | 1 |
| CLEANER4CONC | 1.620 | .065 | 1.628 | .063 | -.008 | .016 | 0 |
| CLEANER5TAIL | 4.560 | .182 | 4.547 | .181 | .013 | .026 | 0 |
| CLEANER5CONC | .880 | .035 | .875 | .035 | .005 | .004 | 1 |
| SULPHUR CONC | .130 | .005 | .130 | .005 | -.000 | .000 | 0 |
| MAGNTIC CONC | 3.110 | .124 | 3.110 | .124 | .000 | .001 | 0 |
| TAILINGS | 13.280 | .531 | 13.531 | .299 | -.251 | .439 | 0 |

COMPONENT : SILICON

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | RES. | STANDARD DEVIATION | |
| FEED | 3.580 | 1.074 | 3.985 | .367 | -.405 | 1.009 | 0 |
| ROUGHER TAIL | 1.390 | .417 | 1.275 | .383 | .115 | .165 | 0 |
| ROUGHER CONC | 14.850 | .891 | 15.850 | .561 | -1.000 | .692 | 1 |
| CLEANER1TAIL | 9.240 | .554 | 9.094 | .547 | .146 | .091 | 1 |
| CLEANER1CONC | 19.400 | 1.164 | 19.274 | .722 | .126 | .913 | 0 |
| CLEANER2TAIL | 24.590 | 1.475 | 23.121 | .962 | 1.469 | 1.118 | 1 |
| CLEANER3TAIL | 26.030 | 7.809 | 25.682 | 5.595 | .348 | 5.448 | 0 |
| CLEANER2CONC | 20.030 | 6.009 | 20.303 | 4.217 | -.273 | 4.281 | 0 |
| CLEANER3CONC | 9.730 | 2.919 | 10.106 | 1.964 | -.376 | 2.159 | 0 |
| CLEANER4TAIL | 15.360 | 4.608 | 14.826 | 3.440 | .534 | 3.066 | 0 |
| CLEANER4CONC | 4.040 | 1.212 | 5.185 | .674 | -1.145 | 1.008 | 1 |
| CLEANER5TAIL | 12.640 | 3.792 | 10.342 | 3.208 | 2.298 | 2.021 | 1 |
| CLEANER5CONC | 3.900 | .234 | 3.856 | .232 | .044 | .031 | 1 |
| SULPHUR CONC | .720 | .216 | .720 | .216 | .000 | .000 | 0 |
| MAGNTIC CONC | 1.470 | .441 | 1.467 | .441 | .003 | .005 | 0 |
| TAILINGS | 3.400 | 1.020 | 3.887 | .364 | -.487 | .953 | 0 |

COMPONENT : PHOSPHOR

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | RES. | STANDARD DEVIATION | |
| FEED | 3.680 | .368 | 3.179 | .160 | .501 | .331 | 1 |
| ROUGHER TAIL | 2.390 | .239 | 2.607 | .191 | -.217 | .143 | 1 |
| ROUGHER CONC | 5.890 | .589 | 5.681 | .314 | .209 | .498 | 0 |
| CLEANER1TAIL | 8.580 | .858 | 8.945 | .777 | -.364 | .365 | 0 |
| CLEANER1CONC | 4.220 | .422 | 4.026 | .272 | .194 | .323 | 0 |
| CLEANER2TAIL | 4.610 | .461 | 4.961 | .337 | -.351 | .315 | 1 |
| CLEANER3TAIL | 2.570 | .257 | 2.350 | .176 | .220 | .187 | 1 |
| CLEANER2CONC | 1.710 | .171 | 1.839 | .131 | -.129 | .110 | 1 |
| CLEANER3CONC | 1.000 | .100 | .875 | .062 | .125 | .078 | 1 |
| CLEANER4TAIL | 1.220 | .122 | 1.326 | .102 | -.106 | .066 | 1 |
| CLEANER4CONC | .300 | .030 | .320 | .019 | -.020 | .023 | 0 |
| CLEANER5TAIL | .490 | .049 | .479 | .047 | .011 | .013 | 0 |
| CLEANER5CONC | .290 | .029 | .279 | .023 | .011 | .017 | 0 |
| SULPHUR CONC | .020 | .002 | .020 | .002 | 0.000 | .000 | 0 |
| MAGNTIC CONC | .550 | .055 | .550 | .055 | -.000 | .000 | 0 |
| TAILINGS | 3.250 | .325 | 3.146 | .159 | .104 | .283 | 0 |

COMPONENT : CALCIUM

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | RES. | STANDARD DEVIATION | |
| FEED | 29.900 | 1.196 | 29.958 | .677 | -.058 | .986 | 0 |
| ROUGHER TAIL | 33.200 | 1.328 | 33.102 | .840 | .098 | 1.029 | 0 |
| ROUGHER CONC | 16.450 | .658 | 16.190 | .510 | .260 | .416 | 0 |
| CLEANER1TAIL | 25.960 | 1.038 | 26.174 | .977 | -.214 | .351 | 0 |
| CLEANER1CONC | 11.470 | .459 | 11.129 | .289 | .341 | .356 | 0 |
| CLEANER2TAIL | 11.530 | .461 | 11.873 | .354 | -.343 | .295 | 1 |
| CLEANER3TAIL | 7.510 | .300 | 7.311 | .239 | .199 | .182 | 1 |
| CLEANER2CONC | 7.270 | .291 | 7.517 | .183 | -.247 | .226 | 1 |
| CLEANER3CONC | 6.850 | .274 | 6.905 | .156 | -.055 | .225 | 0 |
| CLEANER4TAIL | 5.990 | .240 | 5.966 | .219 | .024 | .098 | 0 |
| CLEANER4CONC | 7.150 | .286 | 7.502 | .190 | -.352 | .214 | 1 |
| CLEANER5TAIL | 5.030 | .201 | 4.994 | .200 | .036 | .022 | 1 |
| CLEANER5CONC | 8.630 | .345 | 8.148 | .236 | .482 | .252 | 1 |
| SULPHUR CONC | .790 | .032 | .790 | .032 | -.000 | .000 | 0 |
| MAGNTIC CONC | 7.520 | .301 | 7.520 | .301 | .000 | .002 | 0 |
| TAILINGS | 29.550 | 1.182 | 29.589 | .667 | -.039 | .976 | 0 |

COMPONENT : IRON

| STREAM | MEASURED | | ADJUSTED | | RES. | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|--------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | | RES. | STANDARD DEVIATION | |
| FEED | 5.380 | .269 | 5.265 | .157 | .115 | .219 | 0 | |
| ROUGHER TAIL | 3.690 | .185 | 3.778 | .162 | -.088 | .088 | 0 | |
| ROUGHER CONC | 11.820 | .591 | 11.777 | .322 | .043 | .496 | 0 | |
| CLEANER1TAIL | 7.330 | .367 | 7.362 | .360 | -.032 | .068 | 0 | |
| CLEANER1CONC | 13.690 | .685 | 14.014 | .391 | -.324 | .562 | 0 | |
| CLEANER2TAIL | 12.800 | .640 | 12.729 | .476 | .071 | .428 | 0 | |
| CLEANER3TAIL | 15.610 | .780 | 15.843 | .648 | -.233 | .435 | 0 | |
| CLEANER2CONC | 17.030 | .851 | 16.662 | .503 | .368 | .687 | 0 | |
| CLEANER3CONC | 23.110 | 1.156 | 22.529 | .658 | .581 | .950 | 0 | |
| CLEANER4TAIL | 24.690 | 1.235 | 25.068 | 1.069 | -.378 | .618 | 0 | |
| CLEANER4CONC | 22.930 | 1.147 | 21.069 | .625 | 1.861 | .961 | 1 | |
| CLEANER5TAIL | 27.890 | 1.395 | 28.454 | 1.364 | -.564 | .291 | 1 | |
| CLEANER5CONC | 18.060 | .903 | 19.165 | .727 | -1.105 | .536 | 2 | |
| SULPHUR CONC | 44.970 | 2.249 | 45.039 | 2.247 | -.069 | .077 | 0 | |
| MAGNTIC CONC | 58.560 | 2.928 | 58.960 | 2.894 | -.400 | .443 | 0 | |
| TAILINGS | 7.020 | .351 | 6.822 | .274 | .198 | .220 | 0 | |

COMPONENT : SULPHUR

| STREAM | MEASURED | | ADJUSTED | | RES. | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | | RES. | STANDARD DEVIATION | |
| FEED | .690 | .055 | .644 | .040 | .046 | .038 | 1 | |
| ROUGHER TAIL | .100 | .008 | .101 | .008 | -.001 | .001 | 0 | |
| ROUGHER CONC | 3.420 | .274 | 3.023 | .163 | .397 | .220 | 1 | |
| CLEANER1TAIL | .300 | .024 | .301 | .024 | -.001 | .001 | 0 | |
| CLEANER1CONC | 4.200 | .336 | 4.403 | .215 | -.203 | .258 | 0 | |
| CLEANER2TAIL | 2.000 | .160 | 2.063 | .150 | -.063 | .056 | 1 | |
| CLEANER3TAIL | 4.400 | .352 | 4.179 | .322 | .221 | .142 | 1 | |
| CLEANER2CONC | 6.000 | .480 | 6.545 | .327 | -.545 | .351 | 1 | |
| CLEANER3CONC | 15.800 | 1.264 | 14.680 | .646 | 1.120 | 1.086 | 1 | |
| CLEANER4TAIL | 14.800 | 1.184 | 15.360 | 1.052 | -.560 | .543 | 1 | |
| CLEANER4CONC | 13.500 | 1.080 | 14.399 | .615 | -.899 | .887 | 1 | |
| CLEANER5TAIL | 17.100 | 1.368 | 16.805 | 1.337 | .295 | .292 | 1 | |
| CLEANER5CONC | 14.000 | 1.120 | 13.779 | .738 | .221 | .843 | 0 | |
| SULPHUR CONC | 54.800 | 4.384 | 54.104 | 4.171 | .696 | 1.350 | 0 | |
| MAGNTIC CONC | .510 | .041 | .510 | .041 | .000 | .000 | 0 | |
| TAILINGS | .760 | .061 | .776 | .053 | -.016 | .031 | 0 | |

STATISTICAL ANALYSIS ON THE RESIDUALS
 =====
 (RESIDUALS GREATER THAN 3 SIGMA EXCEPTED)

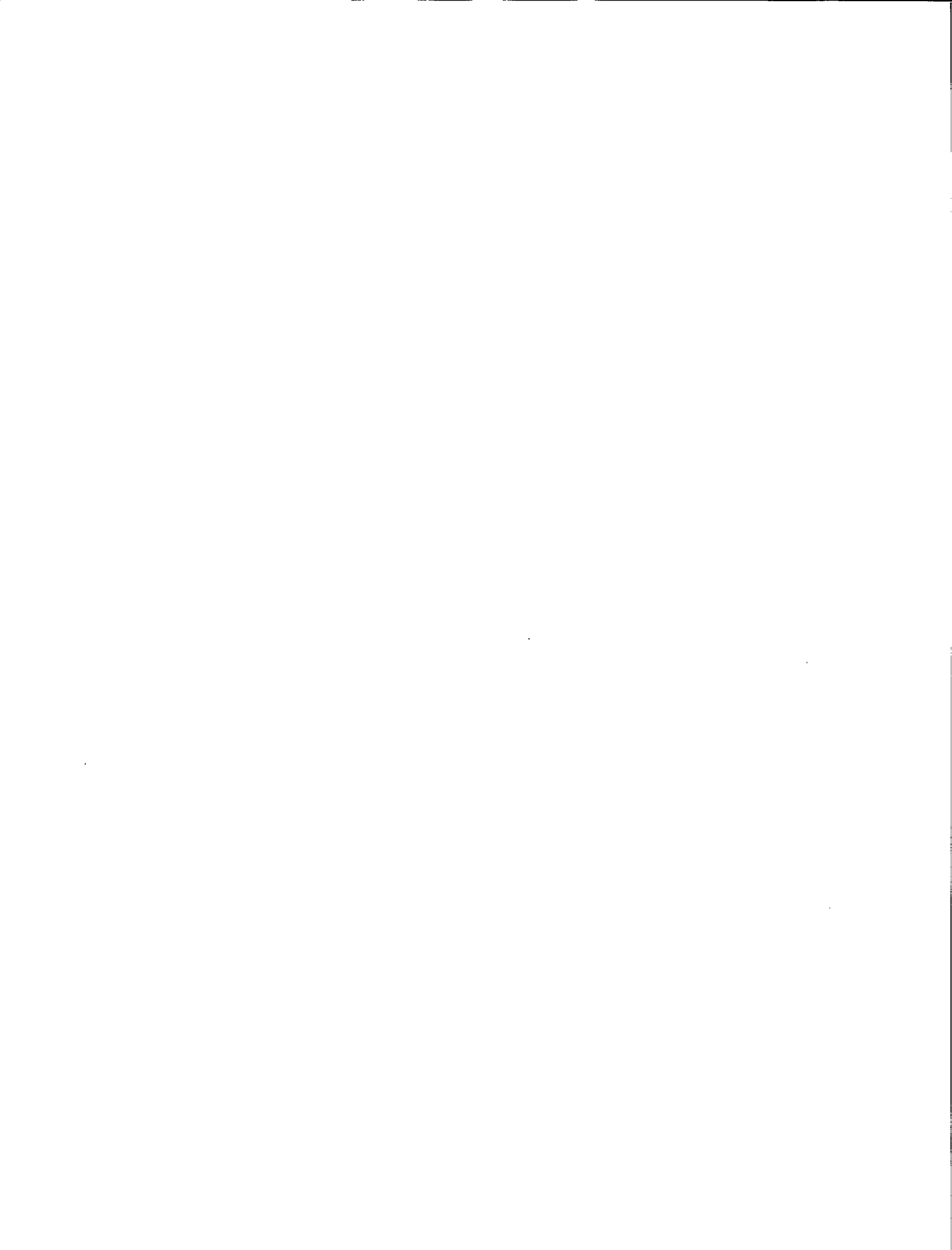
AVERAGE OF REDUCED RESIDUALS ON EACH STREAM

| STREAM | REAL AVERAGE | ABSOLUTE AVERAGE |
|--------------|--------------|------------------|
| FEED | 73.76 | 86.89 |
| ROUGHER TAIL | -36.89 | 59.50 |
| ROUGHER CONC | 35.67 | 76.95 |
| CLEANER1TAIL | -28.39 | 74.09 |
| CLEANER1CONC | -1.98 | 50.44 |
| CLEANER2TAIL | -42.43 | 84.72 |
| CLEANER3TAIL | 67.22 | 82.53 |
| CLEANER2CONC | -67.20 | 82.51 |
| CLEANER3CONC | 12.16 | 80.39 |
| CLEANER4TAIL | -12.17 | 80.40 |
| CLEANER4CONC | -51.20 | 106.51 |
| CLEANER5TAIL | 51.14 | 106.44 |
| CLEANER5CONC | 54.14 | 113.08 |
| SULPHUR CONC | -9.41 | 24.14 |
| MAGNTIC CONC | -8.88 | 24.83 |
| TAILINGS | -1.38 | 45.44 |

AVERAGE OF REDUCED RESIDUALS FOR EACH COMPONENT

| STREAM | REAL AVERAGE | ABSOLUTE AVERAGE |
|----------|--------------|------------------|
| NIOBIUM | -1.11 | 40.42 |
| MAGNESM | 3.24 | 86.97 |
| SILICON | 21.95 | 71.71 |
| PHOSPHOR | -.65 | 103.32 |
| CALCIUM | 11.85 | 76.47 |
| IRON | -26.49 | 86.07 |
| SULPHUR | 10.57 | 103.19 |

END OF SENSITIVITY ANALYSIS



APPENDIX I

HYDROCYCLONE CLASSIFICATION OF MINERALS WITH VARIOUS SPECIFIC GRAVITIES/SAMPLE RUN 2



1. DATA (INPUT FILE)

```

SAMPLE RUN 2 : MASS BALANCE ON A CYCLONE CLASSIFIER
CYCLONE FEEDOVERFLOW UNDERFLOW
1-1-1
1 2 3
1-1-1
1 2 3
1-1-1
24.000 0.000 37.500 26.500 9.400 33.400 24.500 34.600 18.700 25.000
56.000 10.400
999.999 0.000 5.330999.999 6.5400 5.370 11.000 5.340 5.660 11.000
5.230 6.390
150/ 75 75/ 37 37/ 26 - 26
1 2 3
1-1-1
50.800 38.500 89.700 22.100 27.700 0.700 27.100 33.800 9.600
10.000 5.000 5.000 10.000 5.000 5.000 10.000 5.000 5.000
26/ 18 18/ 9 - 9
1 2 3
1-1-1
6.100 10.100 3.700 16.200 19.200 18.700 46.600 36.200 47.000
3.000 3.000 3.000 12.000 12.000 12.000 8.000 8.000 8.000
COPPER CALCIUM COBALT
1 2 3
1-1-1
6.300 43.100 5.400 16.200 23.800 16.800 43.000 0.000 49.000
5.000 5.000 5.000 2.000 2.000 2.000 8.000 8.000 8.000

6.100 26.600 3.400 16.200 28.400 14.400 47.000 13.400 50.400
5.000 5.000 5.000 2.000 2.000 2.000 8.000 8.000 8.000

6.400 10.900 1.000 16.300 21.200 10.800 44.000 9.700 53.800
5.000 5.000 5.000 2.000 2.000 2.000 8.000 8.000 8.000

6.200 7.400 1.100 16.100 17.500 10.100 44.400 42.900 56.700
5.000 5.000 5.000 2.000 2.000 2.000 8.000 8.000 8.000
SI02 FES2 PBS
1201.000 10.000
2120.000 10.000
3 73.000 10.000
1 48.700 6.000
2 29.000 6.000
3 76.600 6.000
.355 2.000
.645 2.000
1.010 3
98.000 1.000 4
1 1 1 1 0 1 0 0 1 1 1 0
1 3 1 3
1 3 4
1 3 3
1 3 3
1 3 3
3 0 0
3

```

TITLE : SAMPLE RUN 2 : MASS BALANCE ON A CYCLONE CLASSIFIER

DATA INPUT AND CALCULATION PATH OPTIONS:

INTEGER VECTOR OF OPTIONS: 1 1 1 1 0 1 0 0 1 1 1 0

OPTION MEANING:

1 SOME FLOWRATES HAVE BEEN MEASURED.
2 SOME PULP PERCENT SOLIDS HAVE BEEN MEASURED.
3 DATA ON NETWORK 3 ARE AVAILABLE.
4 DATA ON NETWORK 4 ARE AVAILABLE.
6 DATA ON NETWORK 5 ARE AVAILABLE.
9 DATA ON NETWORK 8 ARE AVAILABLE.
10 SUB-OPTIMAL SOLUTION NB. 2 IS SELECTED.
11 POSITIVE VALUES ASSIGNED TO RELATIVE ORE FLOW RATES
DURING MASS BALANCE CALCULATIONS.
12 READ USER'S FIRST ESTIMATES FOR THE POWELL ALGORITHM.

NUMBER OF NODES IN NETWORK #1 : 1
NUMBER OF STREAMS IN NETWORK #1 : 3
NUMBER OF NODES IN NETWORK #2 : 1
NUMBER OF STREAMS IN NETWORK #2 : 3

LABELS TO IDENTIFY THE STREAMS OF NETWORK #1 : ,
1 : CYCLONE FEED 2 : OVERFLOW 3 : UNDERFLOW

MATRIX OF NETWORK 1 :

STREAM NB. 1: FROM NODE 0 TO NODE 1
STREAM NB. 2: FROM NODE 1 TO NODE 0
STREAM NB. 3: FROM NODE 1 TO NODE 0

THE NUMBERS OF THE STREAMS OF NETWORK #1
COMMON WITH NETWORK #2 :
1 2 3

MATRIX OF NETWORK 2 :

STREAM NB. 1: FROM NODE 0 TO NODE 1
STREAM NB. 2: FROM NODE 1 TO NODE 0
STREAM NB. 3: FROM NODE 1 TO NODE 0

NETWORK # 3 :

NUMBER OF NODES : 1
NUMBER OF STREAMS : 3
NUMBER OF COMPONENTS: 4

NUMBERS OF THE STREAMS COMMON WITH NETWORK #1 :

1 2 3

MATRIX OF NETWORK 3 :

STREAM NB. 1: FROM NODE 0 TO NODE 1

STREAM NB. 2: FROM NODE 1 TO NODE 0

STREAM NB. 3: FROM NODE 1 TO NODE 0

MEASURED VALUES OF THE COMPONENTS MASS FRACTIONS
(1 ROW PER COMPONENT)

24.000 0.000 37.500
26.500 9.400 33.400
24.500 34.600 18.700
25.000 56.000 10.400

RELATIVE STANDARD DEVIATIONS (%) ASSOCIATED WITH THE MEASURED VALUES

999.999 0.000 5.330
999.999 6.540 5.370
11.000 5.340 5.660
11.000 5.230 6.390

LABELS TO IDENTIFY COMPONENTS:

1 : 150/ 75 2 : 75/ 37 3 : 37/ 26 4 : - 26

NETWORK # 4 :

NUMBER OF NODES : 1
NUMBER OF STREAMS : 3
NUMBER OF COMPONENTS: 3

NUMBERS OF THE STREAMS COMMON WITH NETWORK #1 :

1 2 3

MATRIX OF NETWORK 4 :

STREAM NB. 1: FROM NODE 0 TO NODE 1

STREAM NB. 2: FROM NODE 1 TO NODE 0

STREAM NB. 3: FROM NODE 1 TO NODE 0

MEASURED VALUES OF THE COMPONENTS MASS FRACTIONS
(1 ROW PER COMPONENT)

50.800 38.500 89.700
22.100 27.700 .700
27.100 33.800 9.600

RELATIVE STANDARD DEVIATIONS (%) ASSOCIATED WITH THE MEASURED VALUES

10.000 5.000 5.000
10.000 5.000 5.000
10.000 5.000 5.000

LABELS TO IDENTIFY COMPONENTS:

1 : 26/ 18 2 : 18/ 9 3 : - 9

NETWORK # 5 :

NUMBER OF NODES : 1
NUMBER OF STREAMS : 3
NUMBER OF COMPONENTS: 3

NUMBERS OF THE STREAMS COMMON WITH NETWORK #1 :

1 2 3

MATRIX OF NETWORK 5 :

STREAM NB. 1: FROM NODE 0 TO NODE 1

STREAM NB. 2: FROM NODE 1 TO NODE 0

STREAM NB. 3: FROM NODE 1 TO NODE 0

MEASURED VALUES OF THE COMPONENTS MASS FRACTIONS
(1 ROW PER COMPONENT)

6.100 10.100 3.700
16.200 19.200 18.700
46.600 36.200 47.000

RELATIVE STANDARD DEVIATIONS (%) ASSOCIATED WITH THE MEASURED VALUES

3.000 3.000 3.000
12.000 12.000 12.000
8.000 8.000 8.000

LABELS TO IDENTIFY COMPONENTS:

1 : COPPER 2 : CALCIUM 3 : COBALT

NETWORK # 8 :

NUMBER OF NODES : 1
NUMBER OF STREAMS : 3
NUMBER OF COMPONENTS: 3

NUMBERS OF THE STREAMS COMMON WITH NETWORK #1 :

1 2 3

MATRIX OF NETWORK 8 :

STREAM NB. 1: FROM NODE 0 TO NODE 1

STREAM NB. 2: FROM NODE 1 TO NODE 0

STREAM NB. 3: FROM NODE 1 TO NODE 0

FOR COMPONENT NB. 1 ... 150/ 75 :

MEASURED VALUES OF THE COMPONENTS MASS FRACTIONS
(1 ROW PER COMPONENT)

6.300 43.100 5.400
16.200 23.800 16.800
43.000 0.000 49.000

RELATIVE STANDARD DEVIATIONS (%) ASSOCIATED WITH THE MEASURED VALUES

5.000 5.000 5.000
2.000 2.000 2.000
8.000 8.000 8.000

FOR COMPONENT NB. 2 ... 75/ 37 :

MEASURED VALUES OF THE COMPONENTS MASS FRACTIONS
(1 ROW PER COMPONENT)

6.100 26.600 3.400
16.200 28.400 14.400
47.000 13.400 50.400

RELATIVE STANDARD DEVIATIONS (%) ASSOCIATED WITH THE MEASURED VALUES

5.000 5.000 5.000
2.000 2.000 2.000
8.000 8.000 8.000

FOR COMPONENT NB. 3 ... 37/ 26 :

MEASURED VALUES OF THE COMPONENTS MASS FRACTIONS
(1 ROW PER COMPONENT)

6.400 10.900 1.000
16.300 21.200 10.800
44.000 9.700 53.800

RELATIVE STANDARD DEVIATIONS (%) ASSOCIATED WITH THE MEASURED VALUES

5.000 5.000 5.000
2.000 2.000 2.000
8.000 8.000 8.000

FOR COMPONENT NB. 4 ... - 26 :

MEASURED VALUES OF THE COMPONENTS MASS FRACTIONS
(1 ROW PER COMPONENT)

| | | |
|--------|--------|--------|
| 6.200 | 7.400 | 1.100 |
| 16.100 | 17.500 | 10.100 |
| 44.400 | 42.900 | 56.700 |

RELATIVE STANDARD DEVIATIONS (%) ASSOCIATED WITH THE MEASURED VALUES

| | | |
|-------|-------|-------|
| 5.000 | 5.000 | 5.000 |
| 2.000 | 2.000 | 2.000 |
| 8.000 | 8.000 | 8.000 |

LABELS TO IDENTIFY COMPONENTS:

1 : SI02 2 : FES2 3 : PBS

NUMBER OF PULP FLOW RATE MEASUREMENTS : 3
STREAM NB., MEASURED VALUE, STD. DEV.:

| | | |
|---|---------|--------|
| 1 | 201.000 | 10.000 |
| 2 | 120.000 | 10.000 |
| 3 | 73.000 | 10.000 |

NUMBER OF SOLID FLOW RATE MEASUREMENTS : 0

NUMBER OF WATER FLOW RATE MEASUREMENTS : 0

NUMBER OF PULP PERCENT SOLID MEASUREMENTS : 3
STREAM NB., MEASURED VALUE, STD. DEV.:

| | | |
|---|--------|-------|
| 1 | 48.700 | 6.000 |
| 2 | 29.000 | 6.000 |
| 3 | 76.600 | 6.000 |

ESTIMATE AND PRECISION OF RELATIVE ORE FLOWRATES (FOR POWELL ALGO.)
(REFERENCE STREAM IS #1)

| | |
|------|-------|
| .400 | 2.000 |
| .600 | 2.000 |

CONTROLLING PARAMETERS : 1.0 10 3

ESTIMATE AND PRECISION OF REFERENCE STREAM FLOWRATE, AND
CONTROL PARAMETERS OF COGGINS ALGO : 98.000 1.000 4

*** END OF DATA FILE LISTING ***

2. RESULTS PRINTOUT (OUTPUT FILE)

SAMPLE RUN 2 : MASS BALANCE ON A CYCLONE CLASSIFIER

VECTOR OF INDEPENDENT STREAMS :
1 2

VECTOR OF DEPENDENT STREAMS :
3

POWELL ALGORITHM

N = 1 MAXIT = 10 ESCALE = 1.00

STARTING VALUES (X) REQUIRED ACCURACIES (E)
.35500000E+00 .71000000E-02

POWELL ALGORITHM

ITERATION 2 8 FUNCTION EVALUATIONS F = .24715776E+02
ESTIMATES : .35852

END OF FUNCTION MINIMIZATION

| | |
|--|---------|
| FUNCTION VALUE FOR NETWORK 3 DATA : | .3788 |
| FUNCTION VALUE FOR NETWORK 4 DATA : | .0652 |
| FUNCTION VALUE FOR NETWORK 5 DATA : | 1.8994 |
| FUNCTION VALUE FOR NETWORK 8 DATA : | 21.7442 |
| | ----- |
| | 24.0876 |
| FUNCTION VALUE FOR NETWORKS 1 & 2 DATA : | .6281 |
| | ----- |
| | 24.7158 |

RELATIVE ORE FLOW RATES :

| STREAMS | RELATIVE FLOW RATES |
|--------------|---------------------|
| CYCLONE FEED | 1.0000 |
| OVERFLOW | .3585 |
| UNDERFLOW | .6415 |

MASS BALANCE RESULTS

ESTIMATES OF NETWORKS 1 & 2 VARIABLES

| STREAMS | PULP FLOW RATES | | | | WATER FLOW RATES | | | |
|--------------|-----------------|---------------|---------------|---------------|------------------|---------------|---------------|---------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV |
| CYCLONE FEED | 201.00 | 194.04 | 3.46 | 10.00 | 0.00 | 100.32 | 100.00 | 0.00 |
| OVERFLOW | 120.00 | 117.09 | 2.43 | 10.00 | 0.00 | 83.49 | 100.00 | 0.00 |
| UNDERFLOW | 73.00 | 76.95 | 5.42 | 10.00 | 0.00 | 16.84 | 100.00 | 0.00 |

| STREAMS | SOLID FLOW RATES | | | | PULP PERCENT SOLIDS | | | |
|--------------|------------------|---------------|---------------|---------------|---------------------|---------------|---------------|---------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV |
| CYCLONE FEED | 0.00 | 93.72 | 100.00 | 0.00 | 48.70 | 48.30 | .83 | 6.00 |
| OVERFLOW | 0.00 | 33.60 | 100.00 | 0.00 | 29.00 | 28.70 | 1.05 | 6.00 |
| UNDERFLOW | 0.00 | 60.12 | 100.00 | 0.00 | 76.60 | 78.12 | 1.99 | 6.00 |

ESTIMATES OF NETWORK 3 MASS FRACTIONS

| STREAMS | CYCLONE FEED | | | | OVERFLOW | | | |
|---------|--------------|---------------|---------------|----------------|--------------|---------------|---------------|----------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| 150/ 75 | 24.00 | 24.07 | .30 | 1000.00 | 0.00 | .00 | 100.00 | .00 |
| 75/ 37 | 26.50 | 24.82 | 6.35 | 1000.00 | 9.40 | 9.42 | .21 | 6.54 |
| 37/ 26 | 24.50 | 24.47 | .11 | 11.00 | 34.60 | 34.79 | .54 | 5.34 |
| - 26 | 25.00 | 26.64 | 6.55 | 11.00 | 56.00 | 55.79 | .37 | 5.23 |

| STREAM | UNDERFLOW | | | |
|---------|--------------|---------------|---------------|----------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| 150/ 75 | 37.50 | 37.53 | .07 | 5.33 |
| 75/ 37 | 33.40 | 33.42 | .06 | 5.37 |
| 37/ 26 | 18.70 | 18.71 | .05 | 5.66 |
| - 26 | 10.40 | 10.34 | .56 | 6.39 |

ESTIMATES OF NETWORK 4 LINKED MASS FRACTIONS

| STREAMS | CYCLONE FEED | | | | OVERFLOW | | | |
|---------|--------------|---------------|---------------|----------------|--------------|---------------|---------------|----------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| 26/ 18 | 13.53 | 13.64 | .78 | 20.32 | 21.48 | 21.44 | .20 | 10.25 |
| 18/ 9 | 5.89 | 5.64 | 4.20 | 20.32 | 15.45 | 15.60 | .94 | 10.25 |
| - 9 | 7.22 | 7.36 | 1.97 | 20.32 | 18.86 | 18.76 | .55 | 10.25 |

| STREAM | UNDERFLOW | | | |
|--------|--------------|---------------|---------------|----------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| 26/ 18 | 9.28 | 9.28 | .00 | 11.43 |
| 18/ 9 | .07 | .07 | .01 | 11.43 |
| - 9 | .99 | .99 | .04 | 11.43 |

ESTIMATES OF NETWORK 4 MASS FRACTIONS

| STREAMS | CYCLONE FEED | | | | OVERFLOW | | | |
|---------|-----------------|------------------|------------------|-------------------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| 26/ 18 | 50.80 | 51.19 | .78 | 10.00 | 38.50 | 38.42 | .20 | 5.00 |
| 18/ 9 | 22.10 | 21.17 | 4.20 | 10.00 | 27.70 | 27.96 | .94 | 5.00 |
| - 9 | 27.10 | 27.63 | 1.97 | 10.00 | 33.80 | 33.62 | .55 | 5.00 |

| STREAM | UNDERFLOW | | | |
|--------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| 26/ 18 | 89.70 | 89.70 | .00 | 5.00 |
| 18/ 9 | .70 | .70 | .01 | 5.00 |
| - 9 | 9.60 | 9.60 | .04 | 5.00 |

ESTIMATES OF NETWORK 5 MASS FRACTIONS

| STREAMS | CYCLONE FEED | | | | OVERFLOW | | | |
|---------|-----------------|------------------|------------------|-------------------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| COPPER | 6.10 | 6.03 | 1.15 | 3.00 | 10.10 | 10.17 | .68 | 3.00 |
| CALCIUM | 16.20 | 17.75 | 9.57 | 12.00 | 19.20 | 18.42 | 4.06 | 12.00 |
| COBALT | 46.60 | 44.28 | 4.98 | 8.00 | 36.20 | 36.70 | 1.39 | 8.00 |

| STREAM | UNDERFLOW | | | |
|---------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| COPPER | 3.70 | 3.72 | .45 | 3.00 |
| CALCIUM | 18.70 | 17.38 | 7.08 | 12.00 |
| COBALT | 47.00 | 48.51 | 3.22 | 8.00 |

ESTIMATES OF NETWORK 8 MASS FRACTIONS

FOR COMPONENT 150/ 75

| STREAMS | CYCLONE FEED | | | | OVERFLOW | | | |
|---------|-----------------|------------------|------------------|-------------------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| SiO2 | 6.30 | 5.78 | 8.24 | 5.00 | 43.10 | 43.10 | .00 | 5.00 |
| FeS2 | 16.20 | 16.49 | 1.78 | 2.00 | 23.80 | 23.80 | .00 | 2.00 |
| PBS | 43.00 | 45.61 | 6.07 | 8.00 | 0.00 | - .00 | 100.00 | 8.00 |

| STREAM | UNDERFLOW | | | |
|--------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| SIO2 | 5.40 | 5.78 | 7.06 | 5.00 |
| FES2 | 16.80 | 16.49 | 1.85 | 2.00 |
| PBS | 49.00 | 45.61 | 6.92 | 8.00 |

FOR COMPONENT 75/ 37

| STREAMS | CYCLONE FEED | | | | OVERFLOW | | | |
|---------|-----------------|------------------|------------------|-------------------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| SIO2 | 6.10 | 6.39 | 4.73 | 5.00 | 26.60 | 25.85 | 2.81 | 5.00 |
| FES2 | 16.20 | 16.26 | .39 | 2.00 | 28.40 | 28.37 | .09 | 2.00 |
| PBS | 47.00 | 46.12 | 1.87 | 8.00 | 13.40 | 13.41 | .07 | 8.00 |

| STREAM | UNDERFLOW | | | |
|--------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| SIO2 | 3.40 | 3.32 | 2.28 | 5.00 |
| FES2 | 14.40 | 14.36 | .30 | 2.00 |
| PBS | 50.40 | 51.27 | 1.73 | 8.00 |

FOR COMPONENT 37/ 26

| STREAMS | CYCLONE FEED | | | | OVERFLOW | | | |
|---------|-----------------|------------------|------------------|-------------------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| SIO2 | 6.40 | 6.20 | 3.15 | 5.00 | 10.90 | 11.20 | 2.74 | 5.00 |
| FES2 | 16.30 | 16.17 | .80 | 2.00 | 21.20 | 21.31 | .53 | 2.00 |
| PBS | 44.00 | 34.76 | 20.99 | 8.00 | 9.70 | 9.93 | 2.36 | 8.00 |

| STREAM | UNDERFLOW | | | |
|--------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| SIO2 | 1.00 | 1.00 | .24 | 5.00 |
| FES2 | 10.80 | 10.83 | .26 | 2.00 |
| PBS | 53.80 | 60.57 | 12.59 | 8.00 |

| STREAMS | CYCLONE FEED | | | | OVERFLOW | | | |
|---------|-----------------|------------------|------------------|-------------------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| SIO2 | 6.20 | 6.00 | 3.30 | 5.00 | 7.40 | 7.62 | 2.96 | 5.00 |
| FES2 | 16.10 | 15.84 | 1.63 | 2.00 | 17.50 | 17.73 | 1.33 | 2.00 |
| PBS | 44.40 | 45.59 | 2.68 | 8.00 | 42.90 | 42.07 | 1.94 | 8.00 |

| STREAM | UNDERFLOW | | | |
|--------|-----------------|------------------|------------------|-------------------|
| | MEAS. VALUES | ESTIM. VALUES | RESID. VALUES | RELAT. ST.DEV. |
| SIO2 | 1.10 | 1.10 | .15 | 5.00 |
| FES2 | 10.10 | 10.13 | .25 | 2.00 |
| PBS | 56.70 | 56.22 | .85 | 8.00 |

END OF BILMAT3 PROGRAM

SAMPLE RUN 2 : MASS BALANCE ON A CYCLONE CLASSIFIER

SENSITIVITY ANALYSIS

STANDARD DEVIATION OF RELATIVE FLOW RATES
 =====

| STREAM | RELATIVE FLOW RATE | STANDARD DEVIATION | RELATIVE ST. DEV. |
|--------------|-----------------------|-----------------------|----------------------|
| CYCLONE FEED | 1.000 | 0.000 | 0.000 |
| OVERFLOW | .355 | .026 | 7.295 |
| UNDERFLOW | .645 | .026 | 4.015 |

STANDARD DEVIATION OF SOLID FLOW RATES
 =====

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | |
|--------------|-----------|-----------------------|-----------|-----------------------|-----------|-----------------------|
| | FLOW RATE | STANDARD DEVIATION | FLOW RATE | STANDARD DEVIATION | RESID. | STANDARD DEVIATION |
| CYCLONE FEED | 97.887 | 15.662 | 93.813 | 8.680 | 4.074 | 13.036 |
| OVERFLOW | 34.800 | 5.568 | 33.304 | 3.786 | 1.496 | 4.083 |
| UNDERFLOW | 55.918 | 8.947 | 60.510 | 6.260 | -4.592 | 6.392 |

STANDARD DEVIATION OF NETWORK 3 DATA
 =====

COMPONENT : 150 / 75

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | RES. | STANDARD DEVIATION | |
| CYCLONE FEED | 24.000 | 240.000 | 24.210 | 1.614 | -.209 | 239.995 | 0 |
| OVERFLOW | 0.000 | 0.000 | 0.000 | .000 | 0.000 | .000 | 0 |
| UNDERFLOW | 37.500 | 1.999 | 37.522 | 1.999 | -.022 | .011 | 2 |

COMPONENT : 75 / 37

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | RES. | STANDARD DEVIATION | |
| CYCLONE FEED | 26.500 | 265.000 | 24.919 | 1.331 | 1.581 | 264.997 | 0 |
| OVERFLOW | 9.400 | .615 | 9.418 | .615 | -.018 | .001 | 0 |
| UNDERFLOW | 33.400 | 1.794 | 33.417 | 1.794 | -.017 | .008 | 2 |

COMPONENT : 37 / 26

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | RES. | STANDARD DEVIATION | |
| CYCLONE FEED | 24.500 | 2.695 | 24.416 | .967 | .084 | 2.516 | 0 |
| OVERFLOW | 34.600 | 1.848 | 34.774 | 1.799 | -.174 | .420 | 0 |
| UNDERFLOW | 18.700 | 1.058 | 18.714 | 1.028 | -.014 | .250 | 0 |

COMPONENT : - 26

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | RES. | STANDARD DEVIATION | |
| CYCLONE FEED | 25.000 | 2.750 | 26.477 | 1.449 | -1.477 | 2.337 | 0 |
| OVERFLOW | 56.000 | 2.929 | 55.808 | 2.774 | .192 | .941 | 0 |
| UNDERFLOW | 10.400 | .665 | 10.347 | .659 | .053 | .088 | 0 |

STANDARD DEVIATION OF NETWORK 5 DATA
 =====

COMPONENT : COPPER

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | RES. | STANDARD DEVIATION | |
| CYCLONE FEED | 6.100 | .183 | 6.015 | .154 | .085 | .099 | 0 |
| OVERFLOW | 10.100 | .303 | 10.183 | .287 | -.083 | .097 | 0 |
| UNDERFLOW | 3.700 | .111 | 3.720 | .108 | -.020 | .024 | 0 |

COMPONENT : CALCIUM

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | RES. | STANDARD DEVIATION | |
| CYCLONE FEED | 16.200 | 1.944 | 17.747 | 1.264 | -1.547 | 1.477 | 1 |
| OVERFLOW | 19.200 | 2.304 | 18.429 | 2.183 | .771 | .737 | 1 |
| UNDERFLOW | 18.700 | 2.244 | 17.371 | 1.850 | 1.329 | 1.270 | 1 |

COMPONENT : COBALT

| STREAM | MEASURED | | ADJUSTED | | RESIDUALS | | OUTLIER |
|--------------|----------|--------------------|----------|--------------------|-----------|--------------------|---------|
| | WEIGHT | STANDARD DEVIATION | WEIGHT | STANDARD DEVIATION | RES. | STANDARD DEVIATION | |
| CYCLONE FEED | 46.600 | 3.728 | 44.310 | 2.161 | 2.291 | 3.038 | 0 |
| OVERFLOW | 36.200 | 2.896 | 36.691 | 2.822 | -.491 | .651 | 0 |
| UNDERFLOW | 47.000 | 3.760 | 48.503 | 3.188 | -1.503 | 1.993 | 0 |

STATISTICAL ANALYSIS ON THE RESIDUALS
 =====
 (RESIDUALS GREATER THAN 3 SIGMA EXCEPTED)

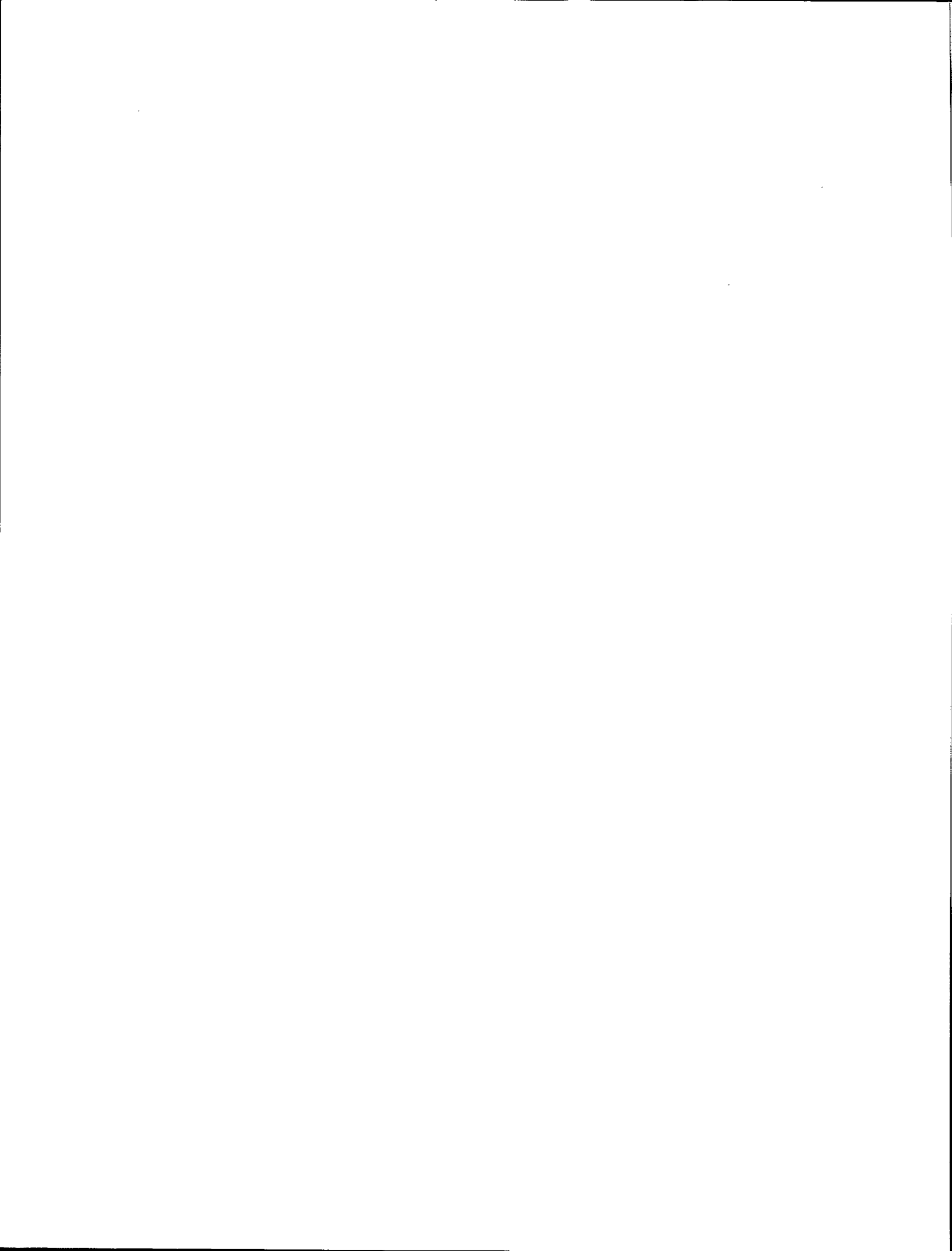
AVERAGE OF REDUCED RESIDUALS ON EACH STREAM

| STREAM | REAL AVERAGE | ABSOLUTE AVERAGE |
|--------------|--------------|------------------|
| CYCLONE FEED | -.38 | 47.61 |
| OVERFLOW | -11.11 | 46.85 |
| UNDERFLOW | -60.46 | 107.67 |

AVERAGE OF REDUCED RESIDUALS FOR EACH COMPONENT

| STREAM | REAL AVERAGE | ABSOLUTE AVERAGE |
|---------|--------------|------------------|
| 150/ 75 | -100.17 | 100.17 |
| 75/ 37 | -110.18 | 110.77 |
| 37/ 26 | -14.64 | 16.88 |
| - 26 | 5.92 | 48.06 |
| COPPER | -28.69 | 85.99 |
| CALCIUM | 34.89 | 104.68 |
| COBALT | -25.13 | 75.40 |

END OF SENSITIVITY ANALYSIS



APPENDIX J
AN EXAMPLE OF REDUNDANCY ANALYSIS

AN EXAMPLE OF REDUNDANCY ANALYSIS

Let us consider the flowsheet of Figure J.1, which could be a simplified flotation circuit. The measurements performed are indicated on the figure. Since the ore feed rate is assumed to be the concentrator feed, a supplementary node is added before flotation feed. The water feed represents all the additions of water performed before flotation.

To analyze the redundancy of a mass balance problem it is useful to evaluate the following:

- the number of variables defined in the studied circuit;
- the number of measured variables;

- the number of material conservation equations between the defined variables;
- the number of definition constraints of the supplementary variables.

A problem is globally redundant if the number of unknown variables (number of defined variables — number of measured variables) is lower than the sum of material conservation equations and definition constraints.

For the flowsheet of Figure J.1, Table J.1 gives the variables defined for the circuit.

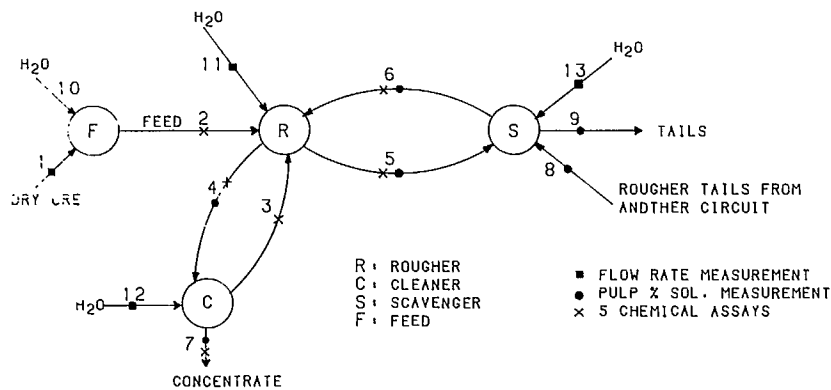


Fig. J.1 – A simplified hypothetical flotation network

Table J.1 – Number and type of measured values for each stream of the network described in Figure J.1

| Stream | Flowrates | | | | Chemical assays | Size distribution |
|--------|-----------|-------|------|----------|-----------------|-------------------|
| | Ore | Water | Pulp | % Solids | | |
| 1 | X | | | | X | |
| 2 | X | X | | X | X | |
| 3 | X | X | | X | X | |
| 4 | X | X | | X | X | |
| 5 | X | X | | X | X | |
| 6 | X | X | | X | X | |
| 7 | X | X | | X | X | |
| 8 | X | X | | X | X | |
| 9 | X | X | | X | X | |
| 10 | | X | | | | |
| 11 | | X | | | | |
| 12 | | X | | | | |
| 13 | | X | | | | |
| Total | 9 | 12 | 0 | 8 | 45 | 0 |

Seventy-four variables are considered in this circuit. Among these, 40 have been measured (the underlined ones in the table), which leads to 34 unknown variables.

Now let us determine the number of equations generated by the mass conservation laws for each node. Table J.2 indicates that 28 such equations must be written:

Table J.2 – Number and type of mass conservation equations for each node of the network described in Figure J.1

| Node | Chemical components | | Water conservation |
|------|---------------------|------------------|--------------------|
| | Ore conservation | Ore conservation | |
| R | 5 | 1 | 1 |
| S | 5 | 1 | 1 |
| C | 5 | 1 | 1 |
| F | 5 | 1 | 1 |

The eight per cent solids variables are related to water and ore flow rates, therefore eight definition equations must be added to the system as constraints. Finally, there are 36 (28 + 8) equations for the 34 unknown variables. Considered as a whole, the system is redundant. However, it is hybrid — being overdefined for some variables, exactly defined or underdefined for others.

When considering the solids and chemical components around nodes R and C, there are twelve conservation equations and five relative ore flow rates. Consequently, the system is redundant with respect to the five flow rates, d_3 to d_7 . Now there are six conservation equations for solids and chemical components around node S and

ten chemical assays and two flow rates (d_8 and d_9) unknown. This part of the system is underdefined.

The three water conservation equations around node R, C, and F contain three unknown variables: the pulp per cent solids on branches 2 and 3 and the water flow rate on branch 10. As a consequence, this part of the system is exactly defined.

Around the node S, grouping the two conservation equations of solids and water, there are two unknowns d_8 and d_9 , which can be exactly calculated. However, this does not remove the underdefinition of the system with respect to chemical assays on branches 8 and 9.

To summarize, the system is:

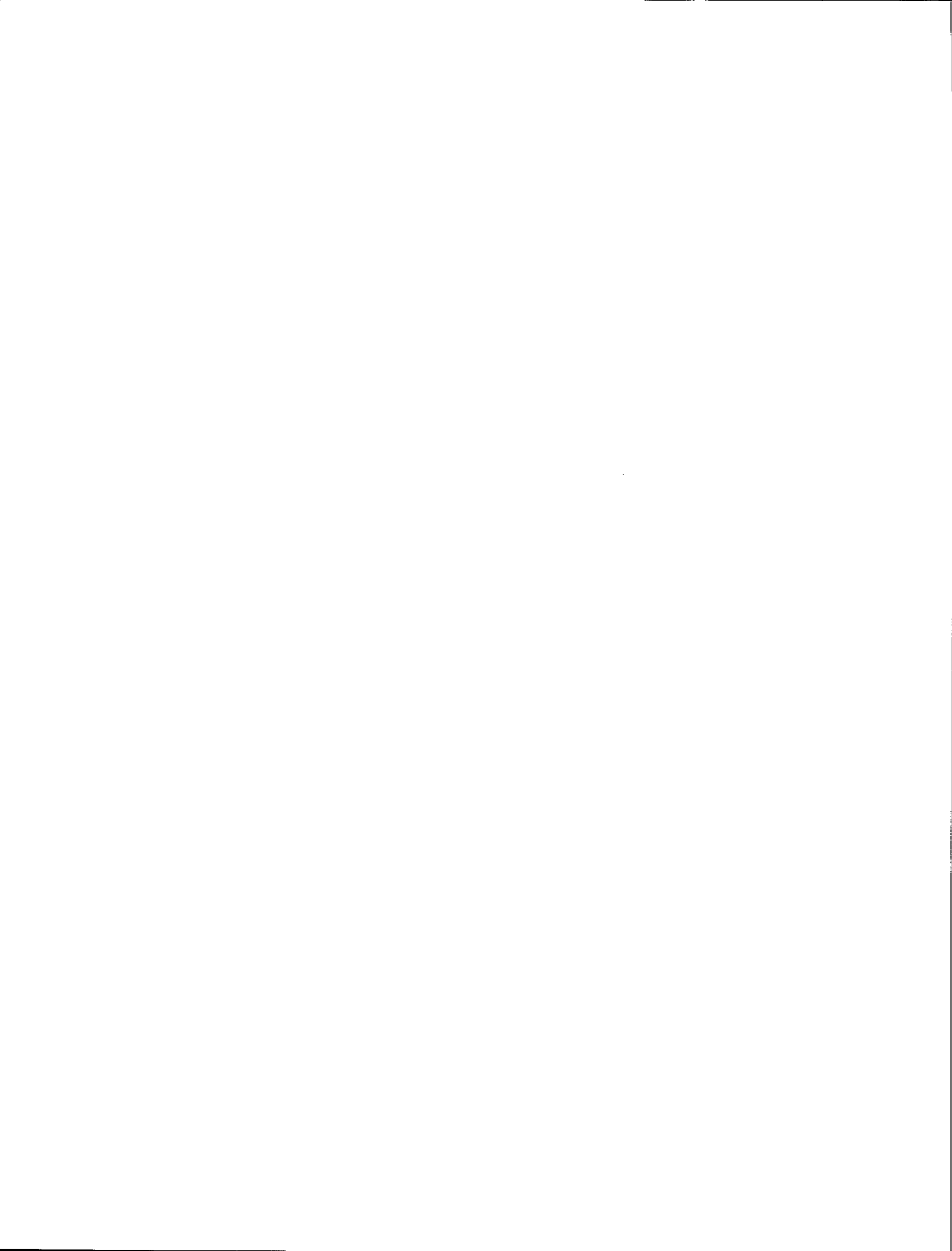
- overdefined for ore flow rates on streams 3, 4, 5, 6 and 7;
- exactly defined for ore flow rates on streams 1, 2, 8 and 9, for per cent solids on streams 2 and 3, and for water flow rate on stream 10;
- underdefined for chemical assays on streams 8 and 9.

It follows from the above that the only redundant data are the chemical assays on streams 2 to 7. All other data will not be corrected by BILMAT, their residuals having null values.

If stream 8 or 9 had been analyzed instead of 3 or 4 or 5 or 6, the underdefinition for chemical assays on streams 8 and 9 would have disappeared. If one pulp flow rate had been measured, the problem would have been redundant with respect to water flow rates.

APPENDIX K

A PROCEDURE TO RUN BILMAT UNDER NOS/BE OPERATING SYSTEM FROM A CDC CYBER



A PROCEDURE TO RUN BILMAT UNDER NOS/BE OPERATING SYSTEM FROM A CDC CYBER

```

.PROC,BILMAT3,DATFIL= ,STATION=DC,IDY=#ID.
RETURN,EDIMAT,EDISIV,PREMAT,PRESIV,BIL3,LGO,SENSIV.
RETURN,TAPE2,TAPE3,TAPE5,TAPE7,TAPE8,ZDGRTA,ZDGRTB.
CONNECT,INPUT,OUTPUT.
REMARK.%LP 80 24
DAYFILE,OFF.
IUSE,BILMAT3,FFLIPF,SPOC.
SET,R1=0.
SET,R2=0.
.*
.* START EXTERNAL LOOP
.*
WHILE,R1.LE.1,OUTLOOP.
IFE,R1.EQ.0,TILINT1.
SET,R1=1.
.*
.* IF A DATA FILE ALREADY EXISTS, COPY IT ON TAPE5.
.* OR EXECUTE EDIMAT
.*
IFE,$DATFIL$.NE.0,TILINT2.
REWIND,DATFIL.
COPY,DATFIL,TAPE5.
REWIND,TAPE5,DATFIL.
COPY,DATFIL,ZDGRTA.
REWIND,DATFIL,ZDGRTA.
REMARK. ....
REMARK.
REMARK. DO YOU WISH TO USE THE EDIMAT PROGRAM ?
IFNO,TILINT3.
SKIP,TOPREMAT.
.*
ENDIF,TILINT3.
ENDIF,TILINT2.
ENDIF,TILINT1.
.*
.* EXECUTE EDIMAT
.*
IGET,EDIMAT.BIN.
EDIMAT.
RETURN,TAPE7,ZDGRTA,EDIMAT.
REWIND,TAPE5.
REQUEST,ZDGRTA,*PF.
COPY,TAPE5,ZDGRTA.
REWIND,TAPE5,ZDGRTA.
.*
.* SAVE TAPES OR EXECUTE PREMAT
.*
REMARK. ....
REMARK.
REMARK. DO YOU WISH TO CATALOG YOUR DATA FILE ?
IFNO,TILSAVE1.
SKIP,TOPREMAT.
.*
ENDIF,TILSAVE1.
CATALOG,ZDGRTA,BIL3DAT,ID=IDY,RP=30.
REMARK.
REMARK. THE FILE NAME IS ;BIL3DAT , ID=IDY
REMARK. N.B. IT'S CATALOG FOR 30 DAYS.
REWIND,ZDGRTA.
ENDIF,TOPREMAT.
REMARK. ....
REMARK.
REMARK. DO YOU WISH TO RUN THE BILMAT3 PROGRAM ?

```

```

IFNO,TILBIL1.
SKIP,TOEDISIV.
.*
ENDIF,TILBIL1.
REWIND,TAPE5.
.*
.* EXECUTE PREMAT
.*
IGET,PREMAT.BIN,BIL3.BIN.
PREMAT.
REWIND,TAPE5,TAPE3.
.*
.* COMPILE BILMAT MAIN PROGRAM
.*
FTN,I=TAPE3,L=0.
RETURN,PREMAT,TAPE3,NEW.
.*
.* COMBINE MAIN PROGRAM AND
.* SUBROUTINES IN ONE FILE
.*
REWIND,LGO,BIL3.
COPYBF,LGO,NEW.
SKIPB,NEW,1.
COPYBF,BIL3,NEW.
RETURN,LGO,BIL3,TAPE6,TAPE7.
.*
.* EXECUTE BILMAT
.*
NEW.
REWIND,TAPE6,TAPE5.
COPY,TAPE6.
.*
.* SEND OUTPUT FILE TO REMOTE PRINTER ?
.*
REMARK. ....
REMARK.
REMARK. DO YOU WISH A HARD COPY OF THIS OUTPUT ?
IFNO,TILBATCH1.
SKIP,TOBIL2.
.*
ENDIF,TILBATCH1.
REWIND,TAPE6.
BATCH,TAPE6,PRINT,STATION.
ENDIF,TOBIL2.
.*
.* SAVE TAPE5 OR EXECUTE EDISIV
.*
RETURN,TAPE6,NEW,ZDGRTB,TAPE7.
SET,R2=0.
REMARK. ....
REMARK.
REMARK. DO YOU WISH TO CATALOG THE MASS BALANCE RESULT
REMARK. FOR A FUTURE SENSITIVITY ANALYSIS ?
IFNO,TILSAVE2.
SKIP,TOSENS2.
.*
ENDIF,TILSAVE2.
REQUEST,ZDGRTB,*PF.
COPY,TAPE5,ZDGRTB.
REWIND,TAPE5,ZDGRTB.
CATALOG,ZDGRTB,ANALDAT,ID=IDY,RP=30.
REMARK.
REMARK. THE FILE NAME IS : ANALDAT , ID=IDY
REMARK. N.B. IT'S CATALOG FOR 30 DAYS.

```



```

SKIP,TOSENS2.
ENDIF,TOEDISIV.
.*
.* SENSITIVITY ANALYSIS ?
.*
REMARK. ....
REMARK.
REMARK. DO YOU WISH TO RUN THE SENSIV PROGRAM ?
REMARK. (N.B. YOU MUST HAVE A MATERIAL BALANCE RESULT DATA FILE )
IFNO,TOSENS2.
SKIP,TOENDSIV.
.*
ENDIF,TOSENS2.
WHILE,R2.EQ.O,INLOOP.
IGET,EDISIV.BIN,PRESIV.BIN,SENSIV.BIN.
.*
.* EXECUTE EDISIV
.*
EDISIV.
REWIND,TAPE5,TAPE7,TAPE15.
.*
.* EXECUTE PRESIV
.*
PRESIV.
REWIND,TAPE5,TAPE7,TAPE3.
.*
.* COMPILE SENSIV MAIN PROGRAM
.*
FTN,I=TAPE3,L=O.
RETURN,EDISIV,PRESIV,TAPE3,NEW,TAPE5.
.*
.* COMBINE MAIN PROGRAM AND
.* SUBROUTINES IN ONE FILE
.*
REWIND,LGO,SENSIV.
COPYBF,LGO,NEW.
SKIPB,NEW,1.
COPYBF,SENSIV,NEW.
RETURN,SENSIV,LGO,TAPE6.
REWIND,NEW.
.*
.* EXECUTE SENSIV
.*
NEW.
REWIND,TAPE6,TAPE15.
COPY,TAPE6.
.*
.* SEND OUTPUT FILE TO REMOTE PRINTER ?
.*
REMARK. ....
REMARK.
REMARK. DO YOU WISH A HARD COPY OF THIS OUTPUT ?
IFNO,TILBATCH2.
SKIP,TOSENS3.
.*
ENDIF,TILBATCH2.
REWIND,TAPE6.
BATCH,TAPE6,PRINT,STATION.
ENDIF,TOSENS3.
RETURN,TAPE6,NEW.
.*
.* ANOTHER SENSITIVITY ANALYSIS ?
.*

```

```

REMARK. ....
REMARK.
REMARK. DO YOU WISH TO PERFORM ANOTHER SENSITIVITY ANALYSIS
REMARK. ON THOSE DATA ?
IFNO,TILANOTHR1.
SKIP,TOENDSIV.
.*
ENDIF,TILANOTHR1.
COPY,TAPE15,TAPE5.
REWIND,TAPE5.
RETURN,TAPE15,TAPE7.
ENDW,INLOOP.
.*
.* THE END ... UNLESS YOU WANT ANOTHER MASS BALANCE ?
.*
ENDIF,TOENDSIV.
RETURN,TAPE5,TAPE15.
REMARK. ....
REMARK.
REMARK. DO YOU WISH TO PERFORM ANOTHER MATERIAL BALANCE
REMARK. ON THOSE DATA ?
IFNO,TILANOTHR2.
COPDF,XX.
IUSE.
DAYFILE,ON.
RETURN,TAPE2,TAPE3,TAPE5,TAPE7,TAPES8,TAPE15,EDIMAT,EDISIV.
RETURN,PREMAT,BIL3,LGO,PRESIV,SENSIV,XX,ZDGRTA,ZDGRTB.
.*
REVERT.
.*
.*
ENDIF,TILANOTHR2.
REWIND,ZDGRTA.
COPY,ZDGRTA,TAPE5.
REWIND,TAPE5,ZDGRTA.
.*
ENDW,OUTLOOP.
.*
REMARK.
REMARK.
REMARK.
REMARK.
REMARK.
REVERT.
.*
.* ERROR RECOVERY SECTION: IN CASE OF FATAL ERROR
.* PRINT JOB DAYFILE AND CLEAR FILE STACK OF ALL
.* FILES EXCEPT TAPE6 WHICH CONTAINS THE RESULTS
.* UNTIL THE ERROR OCCURS
.*
EXIT(U).
COPDF,XX.
REWIND,XX.
COPY,XX.
IUSE.
DAYFILE,ON.
RETURN,TAPE2,TAPE3,TAPE5,TAPE7,TAPES8,TAPE15,XX.
RETURN,EDIMAT,PREMAT,BIL3,LGO,NEW.
RETURN,EDISIV,PRESIV,SENSIV.
REVERT(ABORT)

```


SER 85-1/3.1E C212sp
622(21) (c.1)
BILMAT, computer program for
material balance data adjustment /
D. Hodouin. -- (The SPOC manual.
Chapter 3.1, BILMAT computer pro-
gram). --

c1985. x, 140 p. (c.1)

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