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SPOC

Simulated Processing of Ore and Coal

Chapter 2.2 Grinding Circuit Sampling

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The SPOC Manual

Chapter 2.2 Grinding Circuit Sampling

Grinding Circuit Sampling for Steady-State Modelling

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Editor: D. Laguitton

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THE SPOC MANUAL

The **SPOC*** manual consists of eighteen chapters, published separately. Their numbers and short titles are as follows:

1. Summary
2. Sampling Methodology
2.1 SAMBA Computer Program
2.2 Grinding Circuit Sampling
3. Material Balance
3.1 BILMAT Computer Program
3.2 MATBAL Computer Program
4. Modelling and Simulation
4.1 Industrial Ball Mill Modelling
5. Unit Models: Part A
5.1 Unit Models: Part B
5.2 Unit Models: Part C
6. Flowsheet Simulators
7. Model Calibration
7.1 STAMP Computer Program
7.2 FINDBS Computer Program
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*__S__imulated __P__rocessing of __O__re and __C__oal

FOREWORD

High energy costs and depleting ore reserves combine to make process evaluation and optimization a challenging goal in the 80's. The spectacular growth of computer technology in the same period has resulted in widely available computing power that can be distributed to the most remote mineral processing operations. The SPOC project, initiated at CANMET in 1980, has undertaken to provide Canadian industry with a coherent methodology for process evaluation and optimization assisted by computers. The SPOC Manual constitutes the written base of this methodology and covers most aspects of steady-state process evaluation and simulation. It is expected to facilitate industrial initiatives in data collection and model upgrading.

Creating a manual covering multidisciplinary topics and involving contributions from groups in universities, industry and government is a complex endeavour. The reader will undoubtedly notice some heterogeneities resulting from the necessary compromise between ideals and realistic objectives or, more simply, from oversight. Critiques to improve future editions are welcomed.

D. Laguitton
SPOC Project Leader
Canada Centre for Mineral and Energy Technology

AVANT-PROPOS

La croissance des coûts de l'énergie et l'appauvrissement des gisements ont fait de l'évaluation et de l'optimisation des procédés un défi des années 80 au moment même où s'effectuait la dissémination de l'informatique jusqu'aux concentrateurs les plus isolés. Le projet SPOC, a été lancé en 1980 au CANMET, en vue de développer pour l'industrie canadienne, une méthodologie d'application de l'informatique à l'évaluation et à l'optimisation des procédés minéralurgiques. Le Manuel SPOC constitue la documentation écrite de cette méthodologie et en couvre les différents éléments. Les retombées devraient en être une vague nouvelle d'échantillonnages et d'amélioration de modèles.

La rédaction d'un ouvrage couvrant différentes disciplines et rassemblant des contributions de groupes aussi divers que les universités, l'industrie et le gouvernement est une tâche complexe. Le lecteur notera sans aucun doute des ambiguïtés ou contradictions qui ont pu résulter de la diversité des sources, de la traduction ou tout simplement d'erreurs. La critique constructive est encouragée afin de parvenir au format et au contenu de la meilleure qualité possible.

D. Laguitton
Chef du projet SPOC,
Centre canadien de la technologie des minéraux et de l'énergie

ABSTRACT

This chapter of the SPOC manual presents a case study of sampling experiments conducted at the Brenda Mines grinding circuit for modelling purposes. The notions of flowsheet analysis, experimental design, sampling campaign planning and execution, and even computing of circuit steady-state material balance, are covered in a practical context that provides a most useful supplement to other chapters in which each topic is expanded.

RÉSUMÉ

Ce chapitre du manuel SPOC présente l'aspect pratique d'expériences d'échantillonnage du circuit de broyage de Brenda Mines. Les notions d'analyse de flowsheet, de plan d'expérience, de préparation et d'exécution de campagne d'échantillonnage ainsi que de calcul de bilan matière y sont passées en revue sous un angle appliqué. Ce texte constitue un complément précieux aux chapitres du manuel SPOC qui traitent de chacun de ces sujets avec plus de détails.

ACKNOWLEDGEMENTS

The SPOC project has benefited from such a wide range of contributions throughout the industry, the university, and the government sectors that a nominal acknowledgement would be bound to make unfair omissions. The main groups that contributed are: the various contractors who completed project elements; the Industrial Steering Committee members who met seven times to provide advice to the project leader; the various users of project documents and software who provided feedback on their experience; the CANMET Mineral Sciences Laboratories staff members who handled the considerable in-house task of software development, maintenance, and documentation; the EMR Computer Science Centre staff who were instrumental in some software development; and the CANMET Publications Section. Inasmuch as in a snow storm, every flake is responsible, their contributions are acknowledged.

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1. INTRODUCTION

For the past century, circuit sampling and data acquisition have been used by mineral process engineers for purposes such as performance evaluations, mass balancing for determining recoveries and efficiencies, and comparing circuit alternatives. These evaluations usually involved time-consuming calculations that were performed manually. Digital computers have almost eliminated computational problems associated with large data sets. Process engineers have therefore been encouraged to initiate ambitious data acquisition campaigns with multi-purpose objectives that include control strategy evaluation, model building, and circuit simulation.

Decisions by process engineers to construct mathematical models of their production equipment during normal operation have not been easy because of the employee-hours and computing time required. To minimize costs, data acquisition procedures must be carefully planned.

This chapter describes a reasonable approach for acquiring steady-state data around a production grinding circuit, where the data are to be used for model building. This approach involves:

- —an analysis of the existing circuit to define its current status;
- —a description of data acquisition plans suitable for raw data acquisition;
- —a methodology for planning circuit sampling and for obtaining standard laboratory measurements;
- —the analysis of raw data in preparation for statistical adjustment; and
- —the statistical adjustment of raw data.

SAMPLER RECORD SHEET

Sample location: ______________________________

Name of sampler: ______________________ Date: ______________

Run number: ______________ Shift: ______________ Start time: ______________

Sampling interval									
	1	2	3	4	5	6	7	8	9
Time:									
Approx. volume:									

Sample characteristics:

Fluid (yes/no):

Viscous (yes/no):

Settles out fast (yes/no):

Settles out slow (yes/no):

OBSERVATIONS:

2. ANALYSIS OF THE EXISTING CIRCUIT

Each grinding circuit must be analyzed individually:

- —to define and categorize variables;
- —to establish all mass balance relationships;
- —to assess equipment and circuit constraints; and
- —to evaluate the need for existing and projected instrumentation.

Preliminary analyses are important in general model building procedures (1,2,3,4,5) where data acquisition plans are to be followed.

2.1 DEFINING AND CATEGORIZING VARIABLES

Figure 1 shows a typical rod mill, ball mill, and cyclone circuit, indicating all input-output streams around major process units. Solid lines represent dry solids flows; dotted lines represent water and/or aqueous reagent flows. Dry reagents added at any point are assumed to dissolve rapidly with negligible influence on the mass balance. Likewise, the contribution of liquid reagent flows to the water balance is taken to be negligible. However, these assumptions should be checked for each circuit.

All water and solids flows must be identified. These include gland water from pump seals and feed spout seals, water sprayed onto mill discharges, floor clean-up water, and dust collector water, solids added to pump boxes, and any other extraneous sources of flows. The contribution of these flows to the water-solids balance must be evaluated and included as necessary. Remember that fresh feed solids to the circuit have a moisture content!

A first step in the analysis is to assign symbols and units to variables associated with all flows as shown in Figure 1. Variables normally include mass, volume, density, per cent solids, and size fractions at various key points. Tables 1 and 2 summarize all symbols, stream definitions, and units, and indicate if measurement of the flow involves sampling and if a specific instrument has to be

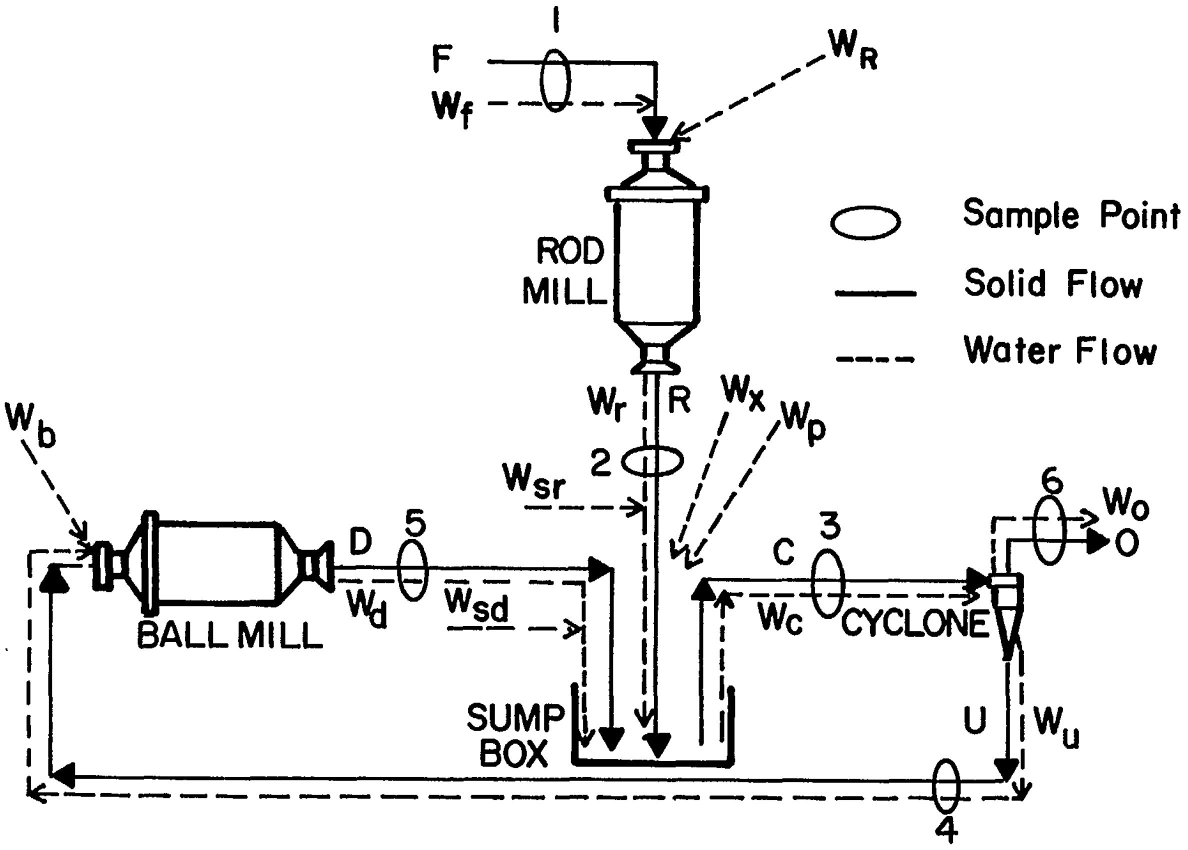

Fig. 1 – Grinding circuit schematic showing all flows and sample points (see Table 1 and Table 2 for definitions)

mounted in the circuit. Subsequent steps involve identification of input, output, and disturbance variables.

2.1.1 Input (Manipulable) Variables

Input variables are independent variables that are considered to be important in models of process units and circuit stability. They must be manipulable with minimum down time; they must be measurable and should be controllable. They can be associated with the circuit as a whole or with individual process units, or both. The two general categories of input variables are operating and design variables.

Circuit operating variables are those that are altered to maintain circuit stability in accordance with a control objective. From Table 1 and Figure 1, circuit operating variables are the fresh feed rate of wet solids to the rod mill, $F + W_f$, and the water addition rates, W_R, W_p, W_b, W_{sr}, W_{sd}, and W_x. Sometimes rod and ball loads, which are replenished at regular intervals to maintain constant loading, are classed as circuit operating variables together with replenishment sizes of rods and balls. In the example, rod mill feed, $F + W_f$, is a controlled variable in a feedback loop where the mass flow rate is detected by a weightometer and manipulated by altering the belt speed. Mass flow is recorded continuously. W_f must be calculated from the measured moisture content and the recorded mass flow. However, W_R, W_p, and W_b are controlled variables in corresponding feedback loops and are also recorded continuously. Flow rates are detected by orifice plates/dp cells or flowmeters and manipulated with automatic valves. W_{sr} and W_{sd} are manipulated by manual adjustment of valve openings and are measured manually. They are usually held constant and may be combined with W_x which incorporates all extra water to the pump box. W_x is measured and held constant during the data acquisition phase.

Rod mill operating variables include F, W_f, W_R, and f_i. Of these, the feed size distribution, denoted as f_i for wt % retained on the ith screen, and W_f are not easily manipulated. However, they can be estimated from samples. For convenience the variable W_f is sometimes added to W_R. Note that changes in F, W_f, and W_R will alter Q_f, P_f, and ρ_f. The latter two may be determined from samples.

Ball mill operating variables include U, W_u, W_b, and u_i. Of these, U, W_u, and u_i are manipulated by altering the circuit operating variables. The u_i are determined by sampling and screening and U and W_u are determined from a circuit balance and sampling. Changes in U and W_u cause changes in Q_u, P_u, and ρ_u. The latter two are found from sampling.

Pump box operating variables include R, W_r, W_{sr}, W_p, W_x, D, W_d, and W_{sd}. These are manipulated by altering the circuit variables. R, W_f, D, and W_d are estimated from a circuit balance, samples, or both. The variables P_d, P_r, ρ_d, and ρ_r are measured from samples.

Cyclone operating variables include C, W_c, and c_i, where each is manipulated by altering circuit variables. These variables can be estimated, together with P_c and ρ_c, from a circuit balance based on samples or from instruments that detect Q_c and ρ_c.

Figure 1 shows that solids specific gravities ρ_{sf}, ρ_{sr}, ρ_{sc}, ρ_{so}, ρ_{su}, and ρ_{sd}, are measured. In many cases these are constant and are essentially the same as ρ_{sf}. Changes in these values may be classed as disturbances associated with variations in mineralogical composition and circuit operation.

Design variables are associated with process equipment and include factors such as mill speed, mill diameter, and pump box pump speed. Usually these variables cannot be manipulated readily. Referring to Table 1, circuit design variables that can be manipulated with minimum down time are the vortex and spigot diameters of cyclone classifiers and the number of cyclones employed. Others, such as liner-lifter type and degree of liner wear may be altered but at greater cost. Most equipment design variables are held constant.

2.1.2 Output (Performance) Variables

Output variables are dependent variables important in models of process units and in circuit performance. Their values depend on input variables, on residence times, and on general requirements to maintain circuit stability.

Circuit output variables relate to circuit performance. Referring to Table 2, which summarizes output variables, circuit outputs are W_o, O, and o_i. W_o and O together with P_o, o_i, and ρ_o are estimated from circuit balances, circuit inputs, or sample measurements.

Rod mill output variables include W_r, R, and r_i and are estimated from a steady-state balance or from sampling mill discharge. P_r and ρ_r are determined in the same way.

Ball mill output variables include W_d, D, and d_i and are estimated from a mass balance or from sample measurements that involve determining P_d and ρ_d.

Cyclone classifier output variables include W_o, O, o_i, U, W_u, and u_i. These are estimated from a mass balance or from sample measurements that involve determining P_o, P_u, ρ_o, and ρ_u.

Pump box output variables are cyclone inputs just as grinding mill outputs are pump box inputs. In circuits, interrelationships where outputs of one process unit are inputs to another are not uncommon. Generally, pump boxes are ignored at steady state. However, when the pump box level is fluctuating uncontrollably, caution should be exercised. Causes must be identified and the disturbance eliminated.

Table 1 – Summary of input variables for typical grinding circuit

Symbol	Definition	Units	How determined
	Rod Mill:		
F	solids feed rate	t/h	REC
W_f	solids moisture feed rate	t/h	REC/SAM/MEA/CAL
W_R	water feed rate	t/h	REC
f_i	feed wt % of size i	%	SAM/MEA
Q_f	volume feed rate, wet solids	m³/h	REC/CAL
P_f	feed % moisture	%	SAM/MEA
ρ_f	feed slurry specific gravity	g/cm³	SAM/MEA
	Ball Mill:		
U	solids feed rate	t/h	SAM/CAL
W_u	cyclone underflow water rate	t/h	SAM/MEA/CAL
W_b	fresh water feed rate	t/h	REC
u_i	feed wt % of size i	%	SAM/MEA
Q_u	volume feed rate, wet solids	m³/h	SAM/CAL
P_u	cyclone underflow % solids	%	SAM/MEA
ρ_u	slurry specific gravity	g/cm³	SAM/MEA
	Pump Box:		
R	rod mill solids discharge rate	t/h	REC/CAL
W_r	rod mill water discharge rate	t/h	REC/CAL
W_{sr}	rod mill discharge spray water	t/h	SAM/MEA
W_p	pump box water addition rate	t/h	REC
W_x	extra circuit water rate	t/h	SAM/MEA
Q_r	rod mill discharge volume rate	m³/h	REC/CAL
P_r	rod mill discharge % solids	%	SAM/MEA
Q_d	ball mill discharge volume rate	m³/h	SAM/MEA
P_d	ball discharge % solids	%	SAM/MEA
ρ_r	slurry specific gravity, RMD	g/cm³	SAM/MEA
ρ_d	slurry specific gravity, BMD	g/cm³	SAM/MEA
	Cyclopac:		
C	solids feed rate	t/h	SAM/CAL/REC
W_c	water feed rate	t/h	SAM/CAL/REC
c_i	feed wt % of size i	%	SAM/MEA or CAL
Q_c	feed volume flow rate	m³/h	REC
P_c	feed wt % solids	%	REC/CAL or SAM/MEA
ρ_c	feed slurry specific gravity	g/cm³	REC or SAM/MEA
V	diameter of vortex	cm	MEA
S	diameter of spigot	cm	MEA
N	number of cyclones	—	MEA

REC = recorder tracing
SAM = stream sample
MEA = measured in some way from samples
CAL = calculated from mass balance

NOTE: Specific gravities of solids at key sample points are either constant and equal to that of fresh feed or are classed as disturbances that must be measured at sample points. These are ρ_{sf}, ρ_{sr}, ρ_{sc}, ρ_{sd}, ρ_{su}, and ρ_{so} (see Figure 1).

Table 2 – Summary of output variables for typical grinding circuit

Symbol	Definition	Units	How determined*
	Rod Mill:		
W_r	discharge water rate	t/h	REC/SAM/CAL
R	discharge solids rate	t/h	REC/CAL
r_i	discharge wt % of size i	%	SAM/MEA
Q_r	volume flow of discharge slurry	m^3/h	REC/CAL/SAM/MEA
P_r	discharge % solids by weight	%	SAM/MEA
ρ_r	discharge slurry density	g/cm^3	SAM/MEA
	Ball Mill:		
W_d	discharge water rate	t/h	CAL/SAM/MEA
D	discharge solids rate	t/h	CAL/SAM/MEA
d_i	discharge wt % of size i	%	SAM/MEA
Q_d	volume flow of discharge slurry	m^3/h	CAL/SAM/MEA
P_d	discharge % solids by weight	%	SAM/MEA
ρ_d	discharge slurry density	g/cm^3	SAM/MEA
	Pump Box:		
W_c	discharge water rate	t/h	CAL/SAM/MEA
C	discharge solids rate	t/h	CAL/SAM/MEA/REC
c_i	discharge wt % of size i	%	SAM/CAL
Q_c	volume flow of discharge slurry	m^3/h	REC
P_c	discharge % solids by weight	%	REC/CAL
ρ_c	discharge slurry density	g/cm^3	REC
	Cyclones:		
W_o	overflow water rate	t/h	CAL/SAM/MEA
O	overflow solids rate	t/h	CAL
o_i	overflow wt % of size i	%	SAM/MEA
W_u	underflow water rate	t/h	SAM/MEA/CAL
U	underflow solids rate	t/h	SAM/MEA/CAL
u_i	underflow wt % of size i	%	SAM/MEA
P_o	overflow % solids by weight	%	SAM/MEA
P_u	underflow % solids by weight	%	SAM/MEA
ρ_o	overflow slurry density	g/cm^3	SAM/MEA
ρ_u	underflow slurry density	g/cm^3	SAM/MEA

*Abbreviations as in Table 1.

2.1.3 Disturbance Variables

Disturbance variables are inputs that occur in either a random or cyclic manner and that disturb outputs. For example, sudden changes in ore hardness or mineralogical composition are disturbances that influence all outputs. Cyclic disturbances are associated with factors such as equipment wear and upstream operation (e.g., periodic crushing plant shutdowns — see Reference 6). Disturbances should be measured if possible and should at least be identified.

Disturbances identified for the circuit shown in Figure 1 include hardness of fresh feed and clay content. Fresh feed specific gravity remains constant at 2.65. Rod mill feed size distribution varies periodically during the day, and is dependent on operating procedures used in the secondary crushing plant, on the amount of primary fines added to fine ore bins, and on a slow, longer-term influence associated with the mining plan. Rod and ball charges are maintained regularly. Quantity and quality shows some cyclic variation. Power draws are recorded to permit calculation of operating work indices. Bond work indexes are also measured.

Slurry viscosity, related to slurry density and particle size/shape composition, may be classed as a disturbance. When viscosity becomes high the performance of process units may be adversely affected. Measurements of viscosity are difficult.

Liner-lifter, cyclone, and pump wear conditions also influence performance variables in a cyclic manner. Finally, the process may be disturbed by instrument malfunction and operator interference (i.e., washing the basement floor).

2.2 MASS BALANCE RELATIONSHIPS

Because data are to be at steady state, essential mass balance relationships must be established for clarity. With reference to Figure 1, these relationships are given in Table 3.

Obviously, volume balances that incorporate other variables may be written as shown. For these, it is essential that units be consistent throughout. Suitable conversion factors may be incorporated as necessary. An essential characteristic of all balances is that "what comes in must go out at steady state".

2.3 EQUIPMENT AND CIRCUIT CONSTRAINTS

A constraint is a restriction associated with input or output variables. For example, a rod mill may not be able to handle a type of fresh feed in excess of some value (which is dependent on several factors). This value will cause the onset of coarse particle rejection from the discharge trunnion, and a rod tangle would be imminent. Constraints associated with equipment limitations and ore properties must therefore be evaluated to avoid problems during the data acquisition phase. In addition, constraints should be quantified to serve as warnings when models are extrapolated beyond their range of applicability. Important equipment constraints are identified in Table 4. Obviously, when constraints are exceeded during data acquisition phases, the circuit should immediately be brought to a stable region.

Circuit constraints exist because of downstream treatment requirements. Thus, the amount of material coarser than a specific size and the per cent solids in the cyclone overflow may be constrained between limits. For example, when cyclone overflow contains more than a critical amount (say 33% +65 mesh), material handling problems may occur downstream and recovery will fall drastically. When cyclone overflow % solids is less than a critical value (say 35%), a sanding problem may be initiated in rougher flotation cells. Constraints of this nature that involve both upper and lower limits on circuit outputs will serve as indirect constraints on circuit inputs. This restricts the range over which input variables may be studied.

2.4 NEED FOR INSTRUMENTATION

As grinding circuits have not been instrumented to the same degree, the extent of automation must be evaluated. If a circuit exhibits frequent instability, additional instrumentation can usually be justified on economic grounds. Instrumented circuits speed process investigations and, if maintained and calibrated properly, increase the reliability of the data.

Because output variables are influenced by inputs and disturbances, feedback control loops are often used for stabilization, where the output variable is controlled. A controlled variable is an output variable associated with either process units, the circuit, or both. It is measured with on-line instrumentation or sensors and is controlled to a setpoint in a feedback control loop. A controller in the loop compares the measurement with the setpoint. The deviation from setpoint manipulates an input variable in a direction that reduces the deviation to zero.

Where the measurements are useful to operators, on-line instruments are often used to measure variables continuously. The recorder tracing is usually of significant value to model building. Variables measured and recorded for the circuit shown in Figure 1 were: cyclone overflow % +65 mesh (with PSM 100), cyclone overflow per cent solids (with PSM 100), cyclone feed slurry volume flowrate (with magnetic flowmeter), and cyclone feed density (with gamma gauge).

Table 5 lists controlled variables and relevant sensors for the circuit shown in Figure 1.

Table 3 – Mass balance relationships

Location	Balance
	Total Solids:
Around rod mill	$F = R$
Around pump box	$C = R + D$
Around cyclones	$C = O + U$
Around ball mill	$U = D$
Circuit	$F = O$
	Total Water:
Around rod mill	$W_r = W_R + W_f$
Around pump box	$W_c = W_r + W_{sr} + W_x + W_{sd} + W_p + W_d$
Around cyclones	$W_c = W_o + W_u$
Around ball mill	$W_d = W_u + W_b$
Circuit	$W_o = W_f + W_R + W_x + W_{sr} + W_{sd} + W_p + W_b$
	Size Fractions, $i = 1,2,...,n$:
Around pump box	$Rr_i + Dd_i = Cc_i$
Around cyclones	$Cc_i = Oo_i + Uu_i$
	Mass Conservation:
	$\Sigma f_i = \Sigma r_i = \Sigma d_i = \Sigma c_i = \Sigma o_i = \Sigma u_i = 1$

Table 4 – Equipment constraints

Constrained Variable	Reasons for Constraint
Rod Mill	
Power draw	Loss in power draw implies mill overload or underload.
Ore feed rate	When too low, mill efficiency decreases.
Pulp viscosity	Particle rejection, rod tangle, feed spout blockage may occur when these attain high critical values.
Pulp density	
Ball Mill	
Power draw	Loss in power draw implies mill overload or underload.
Slurry feed rate	If too high, the mill overloads or steel balls will be carried out of the discharge trunnion.
Pulp viscosity	
Pulp density	
Pump Box	
Level	Overflows or empties resulting from disturbances and incorrect combination of circuit variables; measure of flow rate.
Cyclones	
Roping	Result from disturbances or incorrect combination of circuit variables or wrong size of spigot or obstructions.
Viscosity	If too high, detrimentally influences efficiency because of changes in properties of fresh ore feed and wrong combination of circuit variables.

Table 5 – Controlled variables and sensors

Controlled variable	Manipulated by:	Detected by:
Ore feed rate	Belt speed	Weightometer
Rod mill water	Automatic valve assembly	Orifice plate/dp cell
Pump box water	Automatic valve assembly	Orifice plate/dp cell
Ball mill water	Automatic valve assembly	Orifice plate/dp cell

3. EXPERIMENTAL DESIGNS FOR DATA ACQUISITION

Experimental designs are data acquisition plans. These plans are tables of pre-determined levels (target values) of input variables (factors) that are constructed to minimize the number of runs* in an experiment. The equipment must generate the information desired and yet avoid unnecessary computational and statistical effort. Desirable features of experimental designs have been documented (7,8). A good design provides the information necessary for assessing individual and joint influences of factors on responses in the presence of random error. One-factor-at-a-time experiments are not involved; the levels of the factors are manipulated simultaneously and jointly.

Experimental designs are recommended for all types of data gathering campaigns in concentrators, even if the plan cannot be followed directly during a campaign. Because the manipulation of all factors, both simultaneously and jointly, is involved, important features of stable circuit operation are discerned. Factorial designs are recommended for mineral processing circuits (9). However, before a suitable design is proposed for the grinding circuit shown in Figure 1, it is useful to review the general methodology involved in constructing two-level factorial designs. The reader is also referred to Chapter 4 of the SPOC Manual for further discussion of experimental design (10).

3.1 CONSTRUCTION OF TWO-LEVEL FACTORIAL DESIGNS

General factorial designs are those that would permit the evaluation of the effects of all possible combinations of factor levels on responses. For example, suppose that m_1 levels of a factor, X_1, m_2 levels of a factor, X_2, and m_3 levels of a factor, X_3, are to be studied in an experiment. All possible combinations of levels would result in a plan that contains $m_1 \times m_2 \times m_3$ runs.

When $m_1 = m_2 = m_3 = 2$, the design is referred to as a two-level factorial with $2 \times 2 \times 2 = 8$ runs. In general, designs contain 2^n runs, where n is the number of factors and the 2 refers to the number of levels studied for each factor.

Designs of the 2^n type have advantages in that they involve a minimum number of runs for up to four factors. Design construction is simplified by using the 2^{j-1} rule and coded levels (1). For example, suppose the effect of changes in spigot diameter, V_1, pump box water, V_2, and rod mill feed rate, V_3 are to be evaluated in a grinding circuit. All other inputs are to be held constant. Factor changes chosen for study are $7.5 \text{ cm} \leq V_1 \leq 10 \text{ cm}$; $1000 \text{ l/s} \leq V_2 \leq 1400 \text{ l/s}$; $330 \text{ t/h} \leq V_3 \leq 430 \text{ t/h}$. Code these upper and lower factor levels using the relationship $X_j = (V_j - C_j)/B_j$, where $C_j = 1/2$(low level of jth factor + high level of jth factor) = centre point of jth factor, $B_j = 1/2$(high level of jth factor − low level of jth factor) = range above or below the centre point of jth factor, and X_j is the coded level of the jth factor V_j. Thus:

$$X_1 = (V_1 - 8.75)/1.25$$

$$X_2 = (V_2 - 1200)/200$$

$$X_3 = (V_3 - 380)/50$$

Note that if $V_1 = 7.5$ cm then $X_1 = -1$ and if $V_1 = 10$ cm then $X_1 = +1$. In general, each coded low level of a factor is −1; each coded high level of a factor is +1. It is now convenient to construct the design using coded levels.

Construct a table with the first column heading as the 'run' column. This column will have $2^n = 2^3 = 8$ rows or elements, namely, 1, 2,..., 8. Next, $n = 3$ additional columns are necessary, where j in 2^{j-1} takes values of $j = 1, j = 2,..., j = n$ and this is used to write the coded levels for the n columns as shown in Table 6.

Table 6 – Factorial design for hydrocyclone experiment

Run	X_1 = Spigot diameter	X_2 = Pump Box water	X_3 = Rod Mill feed
1	−1	−1	−1
2	+1	−1	−1
3	−1	+1	−1
4	+1	+1	−1
5	−1	−1	+1
6	+1	−1	+1
7	−1	+1	+1
8	+1	+1	+1
9	0	0	0
10	0	0	0
11	0	0	0
12	0	0	0

Let j = 1 so that $2^{1-1}=1$. Under column $X_j = X_1$, put down one (i.e., $2^{1-1}=1$) minus 1, followed by 1 plus 1, and repeat this pattern of alternating −1 and +1 until run number $2^n = 8$ is reached.

---

* *A run is the procedure of adjusting factors to levels called for by the plan, waiting for steady state, and then measuring output variables (responses).*

Then let j = 2 so that $2^{2-1} = 2$. Under column $X_j = X_2$, put down two (i.e., $2^{2-1} = 2$) minus ones followed by two plus ones and repeat the pattern of alternating groups of (-1's) followed by ($+1$'s) until run number 8 is reached.

Then let j = 3 so that $2^{3-1} = 4$. Under column $X_j = X_3$, put down four (i.e., $2^{3-1} = 4$) minus ones followed by four plus ones and repeat the pattern of alternating groups of (-1's) followed by ($+1$'s) until run number $2^n = 8$ is reached. The procedure is repeated for j = 1, 2,..., n.

The resulting table represents the data acquisition plan with input variables (factors) in coded levels. It is now an easy task to rewrite the table in terms of real levels of the factors.

Note that four 'centre point' runs have been added to Table 6. At this point, each $X_j = 0$; in real units each $V_j = C_j$. One of the main purposes of centre point runs is that they permit the estimation of error variances associated with repetitive determinations of responses. Error variances are necessary for statistical purposes, such as data adjustment, and are best determined by replicating the complete design. Each run in the table of runs is repeated at least once. However, more work is obviously involved in replicating the design.

After the design has been constructed, the order in which runs are performed should be randomized to ensure that errors involving measurements and level adjustments are randomly distributed over the experiment. In some situations, especially when design variables are factors, it may be necessary to bias the run order to avoid costly downtime.

The calculations and the statistical analyses of results on completion of an experiment have been discussed in detail (1,2,7,8,10) and do not need to be repeated. At this stage, it is useful to remember that data acquired in accordance with a design must be 'adjusted' for reasons described in later sections. Consequently, orthogonality may be sacrificed.

3.2 CONSTRUCTION OF FRACTIONAL FACTORIAL DESIGNS

Fractional factorials of the two-level class are $(1/2)^p$ fractions or subsets of full factorials. The number of runs involved is 2^{n-p}, where n is the number of factors and p is the number of principal generators. 2^{n-p} designs are useful for several reasons (1,11). In particular, they reduce the number of runs to be performed, although something will be lost as a consequence. For example, with seven factors a full 2^7 design will contain 128 runs. However, a 2^{7-3} subset will involve only 16 runs and has a resolution of IV (a resolution of IV means that main effects are clear of two-factor effects; although two-factor effects may be confounded with each other).

Designs of resolution V are probably best for mineral processes, which means that main effects and two-factor effects are not confounded with each other. Table 7 shows the number of runs required for resolution V designs involving more than four variables. Clearly, 64 runs seems excessive from a practical viewpoint. Hence, resolution IV designs may be an alternative when six or more input variables are to be studied.

Table 7 – Number of runs for resolution V designs

Number of runs	Number of factors	p	2^{-p}
16	5	1	1/2
32	6	1	1/2
64	7	1	1/2
64	8	2	1/4
128	9	2	1/4
128	10	3	1/8
128	11	4	1/16

Methods for constructing 2^{n-p} designs have been described (1,7,11). Briefly, the resolution is selected, p of the factors are confounded with interactions (related to principal generators) that involve the other n − p factors, signs of coded levels for the p factors are deter-

Table 8 – Factor ranges*

Factor	Symbol	Units	Low level	Centre point	High level
RMF	V_1	STPH	260	285	310
PBW	V_2	USGPM	950	1025	1100
V	V_3	INCH	9	10	11
S	V_4	INCH	5.5	5.75	6
BMW	V_5	USGPM	0	50	100
RMW	V_6	USGPM	310	350	390

* The original (non-SI) units are given.

mined from coded levels of n − p factors in standard order (shown in 3.1), the defining relation is obtained from p principal generators, and the alias structure for the design is found from the defining relation. The latter is useful for interpreting results. Methodology is given in the next section.

3.3 CONSTRUCTION OF EXPERIMENTAL DESIGN FOR GRINDING CIRCUIT

Referring to Figure 1 and Table 1, it is clear that there are six controlled circuit inputs (factors), namely, rod mill feed rate of dry solids (RMF) = V_1, pump box water flow (PBW) = V_2, cyclone vortex diameter (V) = V_3, cyclone spigot diameter (S) = V_4, ball mill water flow (BMW) = V_5, and rod mill water flow (RMW) = V_6. A careful analysis of past operating data and process knowledge indicated that the factor ranges shown in Table 8 should permit stable circuit operation in various combinations.

A 2^6 design will involve 64 runs, so that a fractional design seems desirable. To obtain resolution V, 32 runs are necessary. However, some of the two-factor effects, such as ℓ_{45}, may not be significant in the presence of normal process fluctuations. Hence, a reasonable approach would be to construct a 2^{6-2} fractional design with 16 runs and perform the runs. The results can then be analyzed to find significant aliases. A second subset of 16 runs can then be selected to 'break' the alias structure and separate corresponding two-factor effects confounded with each other. In this case, 32 runs would be performed if necessary. Clearly, this idea of sequential experimentation (in this example, four subsets of 16 runs each constitute the full factorial design) is useful to keep the number of runs to a minimum.

Using coded levels, a full factorial design in terms of the first 6 − 2 = 4 factors is constructed in standard order. Factor X_5 is confounded with $X_1X_2X_3$ (a 3-factor term); factor X_6 is confounded with $X_2X_3X_4$. This implies that ℓ_{123} and ℓ_{234} are unimportant, an assumption that could be incorrect.

$X_1X_2X_3$ and $X_2X_3X_4$ are associated with principal generators. Column elements under X_5 are set equal to the row products of elements under the X_1, X_2 and X_3 columns. X_6 elements are found in an analogous manner. The resulting design is shown in Table 9, where coded levels of factors are used.

To determine the alias structure for the design, the defining relation must be determined first. Multiply both sides of $X_5 = X_1X_2X_3$ by X_5 and both sides of $X_6 = X_2X_3X_4$ by X_6 to obtain the principal generators (p). This gives

$$(X_5)^2 = (X_6)^2 = X_1X_2X_3X_5 = X_2X_3X_4X_6 = I$$

an incomplete defining relation.

Elements under a column I are always +1; elements under $(X_5)^2$ and $(X_6)^2$ (this means each element is squared) are also +1. The defining relation which is

Table 9 – Factorial design

Run	X_1	X_2	X_3	X_4	$X_5 = X_1X_2X_3$	$X = X_2X_3X_4$
1	−1	−1	−1	−1	−1	−1
2	1	−1	−1	−1	1	−1
3	−1	1	−1	−1	1	1
4	1	1	−1	−1	−1	1
5	−1	−1	1	−1	1	1
6	1	−1	1	−1	−1	1
7	−1	1	1	−1	−1	−1
8	1	1	1	−1	1	−1
9	−1	−1	−1	1	−1	1
10	1	−1	−1	1	1	1
11	−1	1	−1	1	1	−1
12	1	1	−1	1	−1	−1
13	−1	−1	1	1	1	−1
14	1	−1	1	1	−1	−1
15	−1	1	1	1	−1	1
16	1	1	1	1	1	1
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0

Note: Four centre point runs have been added to estimate error variances.

used to find the alias structure, must now be determined. To be complete it must contain $2^p = 2^2 = 4$ terms, including I. This is accomplished by taking products of the principal generators, first one at a time, then two at a time, and so on to p at a time.

One at a time gives $X_1X_2X_3X_5$ and $X_2X_3X_4X_6$; and two at a time gives $(X_1X_2X_3X_5)\ (X_2X_3X_4X_6) = X_1X_4X_5X_6$. Hence the complete defining relation is:

$$I = X_1X_2X_3X_5 = X_2X_3X_4X_6 = X_1X_4X_5X_6$$

and from this the alias structure may be determined.

Of immediate interest are all main effects and two-factor effects. A total of 15 effects may be estimated (i.e., $2^{n-p} - 1$ in total) excluding the mean. Of interest are six main effects and as many two-factor effects as can be estimated.

To find aliases for the main effect of factor X_1, multiply the defining relation by X_1. The result is $X_1 = X_2X_3X_5 = X_1X_2X_3X_4X_6 = X_4X_5X_6$.

This means that the effect, $\hat{L}_1$, estimated from the fractional design is really estimating the algebraic sum of effects associated with the full design, i.e., $\hat{L}_1 = \hat{\ell}_1 + \hat{\ell}_{235} + \hat{\ell}_{12346} + \hat{\ell}_{456}$.

Similar operations using X_2, X_3, X_4, X_5 and X_6 would lead to corresponding aliases. For two factor aliases, analogous operations are performed using two factor terms. For example, if the defining relation is multiplied on both sides by X_1X_2, the result will be $X_1X_2 = X_3X_5 = X_1X_3X_4X_6 = X_2X_4X_5X_6$.

This means that the effect, $\hat{L}_{12}$, associated with the fractional design is really the algebraic sum of the following effects associated with the full factorial design: $\hat{L}_{12} = \hat{\ell}_{12} + \hat{\ell}_{35} + \hat{\ell}_{1346} + \hat{\ell}_{2456}$. In this way, an alias structure of interest can be formulated. Further details can be reviewed in the literature (1,7,11).

After a suitable data acquisition plan has been devised, preliminary circuit runs should be performed to gain familiarity with circuit operation. Combinations of factor levels dictated by the design, but suspected to cause circuit instability, should be tested and, if necessary, factor ranges should be reformulated. At this stage, procedures for sampling the circuit in accordance with the design are developed.

4. SAMPLING THE CIRCUIT IN ACCORDANCE WITH THE DESIGN

Before a sampling campaign is initiated in a production environment, it is important that on-site technical personnel, such as the chief metallurgist, maintain effective control of the campaign. Management must be convinced that temporary production losses may be required to achieve a better understanding of the process. A planning team, which includes the sampling team, should be formed and advice or support from operators should be solicited and encouraged. A senior operator and the instrumentation supervisor should be enlisted as members of the team. If a data acquisition campaign is to be successful, operators must feel that they understand why their environment is to be systematically (but responsibly) disturbed for a period. Execution of the sampling campaign will probably involve manipulation of the process in a direction completely contrary to operating policy.

Scheduling the campaign must be coordinated with the maintenance department to ensure that mill liners-lifters are in good condition, classification and pumping equipment have been checked and overhauled, instrumentation has been checked and calibrated, and the upstream and/or downstream projects do not interfere with the campaign. In addition, the team should inquire about the current or projected mining sequence to assess the influence of potential ore changes during the sampling period.

A sampling team should be prepared to work long, hard, night-shift hours to complete the data acquisition plan in the shortest possible time. The detrimental effects of unforeseen disturbances are thus minimized.

Planners of a good campaign will ensure that: redundant measurements will be obtained; sample locations, sample sizes and sampling techniques have been carefully reviewed; an adequate sampling team has been formed and individual responsibilities have been defined; record sheets have been devised; and laboratory measurements are standardized.

4.1 IMPORTANCE OF REDUNDANT MEASUREMENTS

Referring to Figure 1, it is apparent that a minimum amount of information is required to balance the circuit. Because raw data are subject to error, an adjustment procedure will be necessary. The procedures recommended here will work best when superfluous data points (i.e., data not necessary for balancing) are acquired. For example, consider a balance on mass around cyclone classifiers. At steady state,

$$C = O + U$$

Suppose that $\tilde{C}$, $\tilde{O}$, and $\tilde{U}$ are measured, where the symbol ~ indicates a measured value. In general,

$$\tilde{C} \neq \tilde{O} + \tilde{U} \qquad \text{Eq 1}$$

The left side of the expression does not equal the right side, because each measured data point is subject to error. It is tempting to drop one measured quantity in Equation 1, say $\tilde{C}$, and calculate it from the other two, e.g., $\hat{C} = \tilde{O} + \tilde{U}$ where ^ means calculated. However, this is arbitrary.

Clearly, one of the three measured quantities in Equation 1 is 'redundant' for balancing purposes; yet, all three may be entitled to at least some weight. Balancing methods that can permit all measured data points to contribute are described in later sections.

For the circuit shown in Figure 1, planners selected the following set of data points for measurement purposes.

$\tilde{F}$, $(\tilde{W}_R + \tilde{W}_f)$, $\tilde{Q}_c$, $\tilde{W}_b$, $(\tilde{W}_{sr} + \tilde{W}_{sb} + \tilde{W}_x) = \tilde{W}_X$, $\tilde{W}_p$, $\tilde{P}_c$, $\tilde{P}_o$, $\tilde{P}_u$, $\tilde{P}_d$, $\tilde{r}_i$, $(i = 1,n)$, $\tilde{d}_i$ $(i = 1,n)$ $\tilde{u}_i$ $(1 = 1,n)$ and $\tilde{o}_i$ $(1 = 1,n)$.

Thus, information necessary for a mass balance is not only sufficient — there is redundancy.

4.2 SAMPLE LOCATIONS

The number of sample locations depends on the ease of access to them and on the set of data points to be measured. If the sample is necessary, access must be ensured. Referring to Figure 1, six locations are shown. However, to sample the cyclone feed line a special sample pipe and valve assembly must be devised and installed. The potential down time for installation and the potential unreliability of the sample (taken from a 75 cm line with flows of the order of 400 l/s) led to rejection of this location. In addition, a pump box sample to represent cyclone feed is notoriously unreliable because there is poor mixing and particle segregation. Consequently, the planners selected five sample locations after first verifying that the cyclone feed sample was unnecessary for a steady state balance. Exact locations and reasons they were selected are given in Table 10.

Planners checked sensor maintenance and calibration schedules to ensure that recorded signals were reliable. All sensors were recalibrated, chart pens were checked and data logging equipment was inspected.

4.3 SAMPLE SIZES

Methods for estimating sample size (i.e., mass or volume) around a given circuit have been reviewed (12), where a simplification of the Gy formula (13) was proposed for estimating sample size. For practical purposes, the amount of sample in a cut is directly proportional to the flow rate of the stream and cutter width, but inversely proportional to speed. The total number of cuts judged to be representative depends on several factors, including the size distribution of the particles in the stream, the stream density and viscosity, and the characteristics of particle segregation in the stream. Circuit fluctuations become extremely important. Because of this complexity, a reasonable approach for estimating sample size is provided below.

Over the years most concentrator personnel have established sampling procedures that have been accepted internally for various purposes. Assume that within a sampling period (performing a run) the composite sample sizes listed in Table 11 are consistent with past practice.

Fix circuit inputs at center point levels and perform a run according to procedures outlined in later sections, where sample volumes are consistent with Table 11. Measure all data points. A day later, repeat the run, again measuring all data points. Calculate error variances associated with measured data points, each with one degree of freedom. Adjust the two data sets (see section on Data Adjustment) and determine the differences $\tilde{M}_{ij} - \tilde{M}_{ij}$ for the ith point and the $j = 1,2$ sets. Here $\tilde{M}_{ij}$ is a measured value and $\hat{M}_{ij}$ is the corresponding adjusted value. Suppose results are as follows:

Repeat run j	Measured, $\tilde{M}_{ij}$	Adjusted, $\hat{M}_{ij}$	Difference, E_{ij}
1	36.9	36.4	0.5
2	36.5	36.2	0.3

Calculate the "variance" of differences as ${}_1V_i = 1/2(E_{i1} - E_{i2})^2$ with one degree of freedom to give 0.020.

Table 10 – Sampling locations

Sample	Location	Selected because:
RM feed	Feed belt	Easy to stop; readily shovelled into sample drums; adequate space for work area.
RM discharge	Discharge trunnion	Easy to access and cut; high % solids ensures stream is well mixed; cutter design reliable and in use for years.
BM discharge	Discharge trunnion	Easy to access and cut; high % solids ensures stream is well mixed; cutter design reliable and in use for years.
Cyclone underflow	Directly below spigot	Easy to access and cut; stream not as well mixed but space adequate to cut whole stream.
Cyclone overflow	Head assay sample stream at bulk conditioner	Reliability of this sample stream had been verified by previous investigations.

Table 11 – Composite sample sizes

Sample	Approx size (kg dry solids)	Approx slurry vol (L)
RM feed	35	19
RM discharge	8	6
BM discharge	8	6
Cyclone underflow	8	6
Cyclone overflow	3	6

Next, increase all sample sizes by some factor (say 1.3) and repeat the complete procedure as outlined. Choosing an extreme case, assume that the following result is obtained:

Repeat run j	Measured, $\tilde{M}_{ij}$	Adjusted, $\hat{M}_{ij}$	Difference, E_{ij}
1	36.7	36.8	−0.1
2	36.6	36.8	−0.2

From this, the variance of differences is ${}_2V_i = 0.005$. Now, by means of the F-Statistic, (11) determine whether ${}_1V_i$ is significantly larger than ${}_2V_i$. If so, the small sample size is judged inadequate and larger sizes should be obtained.

For the circuit herein described, planners chose sample sizes listed in Table 11. The modified Gy formula (12) can also be used to determine sample size.

Remember that the loss of sample volume in small tonnage circuits may create noticeable circuit imbalance downstream of a sample point. In other words, do not take more than is judged necessary.

4.4 SAMPLING TECHNIQUES

Techniques for obtaining circuit samples have been reasonably well discussed (12-14). The planners should review the literature to ensure that cutters are adequately designed and that correct techniques will be employed by samplers. Install automatic cutters if possible, or employ automatic injector samplers. Of course, special process investigations may involve several flow streams that are not normally monitored. Consequently, the design and installation of automatic cutters may not be justified. Considerable practice may be required to cut representative samples from high tonnage process streams. It is imperative that samplers be assigned to specific sampling points and instructed not to modify their techniques thereafter. In other words, establish a procedure and stick to it!

4.5 SAMPLING TEAM AND RESPONSIBILITIES

The number of people involved in sampling a grinding circuit will depend on the number of sample points and their locations around the circuit. Elevation and ease of access to the location must be considered. The minimum sampling team might consist of a team leader, an assistant, and a lab technician.

The team leader assumes responsibility for manipulating circuit inputs, monitors the circuit closely, and makes decisions that ensure circuit stability before actual sampling. The team leader is responsible for all record sheets employed and must instruct team members about their duties before a run. Contact must be made with operations personnel to ensure that upstream problems will not arise and that floor operators are cooperating. Adverse downstream disturbances created by the run can be minimized by cooperative operations personnel who have been advised of grinding process changes.

Samplers are responsible for locating their sample buckets, wash water hoses, cutters, etc., and actual sample procurement. They should be instructed on matters such as cut frequency, sample size, and time of sampling. Good sampling techniques must be employed and records maintained.

The lab technician assumes responsibility for identifying sample buckets and assigning them to samplers before a run. At the end of a run, samplers must transport samples to the laboratory, where the technician assumes responsibility for them. It is possible that a portion of the grinding floor will have to be allocated for storing and treating samples to avoid interference with routine laboratory functions. If samples are stored for more than an hour, they should be covered to avoid loss of moisture.

4.6 RECORD SHEETS

Careful records must be maintained for each run. Appendix A contains examples of record sheets maintained by the team leader, the instrument records person and the samplers. The lab technician will normally have record sheets for documenting density, size, and specific gravity measurements.

Normally, record sheets are designed for each situation. The design is dependent on the type of circuit, location of sensors, and whether the measurement is recorded manually or automatically.

The team leader must assume responsibility for record sheets at the end of each run. Before a run the leader coordinates special measurements such as mill water flows and extra pump box water flows.

4.7 PERFORMING RUNS

The experimental design to be followed is shown in Section 3.3 in standard order. To minimize downtime (and hence loss in production) runs nine to 16 (higher level of vortex finder) can be placed in one group with two centre points; the other group will comprise runs one to eight with two centre points. Each group can be broken into two subgroups. One subgroup will comprise runs at the lower level of spigot diameter (four runs with one centre point); the other subgroup will comprise runs at the higher level of spigot diameter (four runs with one centre point). Within a subgroup, the run order should be randomized.

Standard routines for rod and ball charging are followed. Thus, the ball mill is charged every eight hours; the rod mill is rodded daily at 7 p.m. Ideally, runs should be performed on the midnight shift to avoid problems associated with scheduled maintenance. It is possible to complete as many as two runs per shift.

At least an hour should be allowed for the circuit to stabilize after the variables are brought to target levels. Before this, all manual flow measurements should be complete and samplers and sample buckets in position. During stabilization of the circuit, the team leader maintains communications to ensure that pump box level is stable and that cyclones are not roping. Visual observation by the team will be important at this stage.

Composite samples should be taken after steady state is reached for approximately 60 min (one sample every 10 min to total seven samples as a composite). A rod mill feed sample may be taken at the end of the sample period and at the beginning (20 kg lots to total about 40 kg)*. This is possible because feed characteristics appear to be extremely stable. In other situations, special gate samplers may have to be installed to obtain samples at more frequent intervals.

Remember that a sample is a stream loss. If the stream is an input to a process unit, then a disturbance has been introduced. If the sample size is large in relation to the flow and/or equipment capacity, the disturbance will be felt and time must be allotted to smooth the disturbance. Samples should be taken in the order, downstream-to-upstream, with sampling time intervals chosen carefully. For the circuit being considered, a sample order was not critical. Do not accept questionable circuit stability, sample cuts, or instrument readings. At the slightest suggestion of questionable data, a run should be dropped immediately and repeated. Do not begrudge some wasted time. Extra work to obtain high quality data is well worth the effort.

4.8 LABORATORY MEASUREMENTS

For each sample, standard procedures should be devised to measure variables such as per cent solids, pulp density, solids specific gravity, and size distributions.

Per Cent Solids/Pulp Densities: Normally, sample buckets have been tared so that the weight of slurry can be measured readily. The sample is then filtered and dried to find the dry solids weight. Per cent solids can then be calculated. If the sample volume has been measured (i.e., use sample buckets with calibrated volume marks), pulp density can likewise be found.

Solids Specific Gravity: A Beckman air pycnometer or its equivalent is most convenient for measuring dry solids specific gravity. Ensure that standard sample splitting techniques worked out by the metallurgical lab are used to obtain a small lot for measurements.

Size Distributions: A sieve series should be selected for screening so that a definite size ratio is followed. The Canadian sieve series is recommended. For the circuit of interest, the square root of two ratio was used except for the coarse sizes. Some points were therefore determined by an interpolation method described later (15).

If samples are coarser than about 6 mm, a Gilson screen or its equivalent may be used for screening to this size (rod mill feed for example). Minus 6 mm material should be riffled to approximately 300 g and wet screened on a 4.5 mm sieve. The oversize should be dried and dry screened in Ro-Taps or their equivalent. Generally, a good metallurgical lab will have developed a screening procedure unique to the ore types it encounters daily. Standard sample splitting techniques taught to all mineral processors should be reviewed. Keep duplicates for checks!

**If sampled at the beginning, delay circuit sampling until the circuit reaches a new steady state.*

5. PREPARATION OF RAW DATA FOR STATISTICAL ADJUSTMENT

Measured data points should be neatly tabulated and scanned for glaring errors. When plotted on a cumulative per cent finer than size basis, size distributions should exhibit smooth curvature. Linearity may be observed on log-log plots. Duplicate samples may sometimes have to be screened to eliminate obvious discrepancies.

For ease of model building, it is useful to convert all size information to a common sieve ratio such as two or its square root. For example, if 53 micrometres (270 mesh) is the smallest size of interest, then all other sieves larger than 53 micrometres would increase in size by $(2)^{i-1}$, $i = 2,3,\ldots, n$. Here, i is an integer that represents the ith sieve from the 53 micrometre sieve. The largest size through which all particles will pass is the nth. For the circuit of interest, sieve sizes are given in Table 12.

Thus all material passes the 54 400 micrometre sieve, but at least some may be retained by the 27 200 micrometre sieve. If any of the screens are not used for screening samples, the weight per cent retained on the missing screens may be estimated from plots of raw data or by a computer interpolation method (15).

Final raw data must be arranged in a format acceptable for computer programs written for data adjustment.

Table 12 – Sieve sizes

i	Micrometre
pan	−53
1	53
2	106
3	212
4	425
5	850
6	1 700
7	3 400
8	6 800
9	13 600
10	27 200
11	54 400

6. STATISTICAL ADJUSTMENT OF RAW DATA

Steady-state material balances calculated from data measured at various locations around a grinding circuit are necessary for model building. It is often possible to calculate material balances by several independent procedures when excess measurement information, i.e., redundant data, is available. Clearly, if the data were collected without error, a theoretical condition, all material balances calculated from redundant data would be in agreement. Realistically, various sources of error are present, so that the results of material balances that are determined from available optional procedures differ. Consequently, statistical procedures that take measurement errors into account have been formulated. These have been summarized (16,17).

Two procedures are discussed here. One uses a direct search routine to obtain least squares estimates of all measured data points. The other takes advantage of the fact that estimates of almost all data points can be found analytically by means of linear regression. The rest are determined by search routines. The resultant saving in computer time is significant, although the mathematics is considerably more complicated.

6.1 DIRECT SEARCH METHODS ADJUSTMENT

The adjustment of raw data by means of direct search methods (16,18) involves selection of symbols for all flows and measurements throughout the circuit; tabulation of measured data points; writing all mass balance relationships; selection of search variables; showing how to calculate measured data points from search variables; construction of the objective function; and choosing a suitable search method and preparing the computer program.

6.1.1 Symbol Selection

Symbols selected for all variables are listed in Tables 1 and 2. Figure 1 associates symbols with circuit locations. Note that symbols used for cyclone underflow are identical to those for ball mill feed. Symbols were deemed unnecessary for the latter because a sample of ball mill feed was not taken near the feed trunnion.

6.1.2 Tabulation of Measured Data Points

Table 13 lists measured data* for a typical run. Rod mill water, $\tilde{W}_r$, is really the sum of $\tilde{W}_f + \tilde{W}_R$ which were measured. Note that water addition rates are expressed in units of STPH, because these readings are control computer output.

Adjustment of data by direct searches places no restrictions on the measurement. Thus variables can be expressed as volume flow rates, slurry densities, per cent solids, mass flows, and weight per cent retained on size (even cumulative weight per cent passing). The actual measured variable will appear in the objective function for adjustment.

6.1.3 Mass Balance Relationships

Mass balance relationships, sometimes referred to as restrictions or constraints that must be satisfied at steady state, are given in Section 2.2. These are modi-

Table 13 – Measured data for a typical run

$\tilde{F}$ = 258.88 STPH
$\tilde{W}_b$ = 3.23 STPH
$\tilde{P}_o$ = 41.59%
$\tilde{P}_r$ = 76.54%

$\tilde{W}_r$ = 78.7 STPH
$\tilde{W}_p$ = 237.74 STPH
$\tilde{P}_u$ = 75.88%
$\tilde{W}_X$ = 34.39 STPH

$\tilde{Q}_c$ = 3970 USGPM
$\tilde{P}_c$ = 62.48%
$\tilde{P}_d$ = 75.6%

Size, i	$\tilde{f}_i$	$\tilde{d}_i$	$\tilde{o}_i$	$\tilde{u}_i$
1	.04	0	0	.01
2	.71	.08	0	.40
3	7.56	1.22	.16	4.04
4	19.09	6.14	.36	13.18
5	20.21	21.62	1.83	28.30
6	15.55	30.04	18.35	29.33
7	10.69	16.22	22.46	11.81
8	6.59	7.47	14.36	4.49
9	19.55	17.20	42.48	8.42

SG of solids = 2.65

**Instrument calibration errors and other constant errors were detected and used to correct original readings (see 6.1.6).*

fied because W_f and W_R have been combined and W_{sr}, W_x, and W_{sd} have been combined. The modified balance is given in Table 14.

6.1.4 Selection of Search Variables

Search variables are the minimum number of variables listed in Table 14 that are necessary for determining balances as shown. Selection of search variables is arbitrary. However, the speed of convergence is influenced by the variables.

There are six total solids variables and four equations. Thus $6 - 4 = 2$ of the variables are artibrary. Select $\underline{F}$ and $\underline{U}$, where $\underline{\ \ }$ indicates a search variable. For total water, $8 - 3 = 5$ variables are arbitrary. Select $\underline{W}_u$, $\underline{W}_b$, $\underline{W}_o$, $\underline{W}_r$, and $\underline{W}_X$ as search variables. For size fractions conservation, an additional 45 variables are introduced with 21 equations involved. Hence $45 - 21 = 24$ of the variables are arbitrary. Select $\underline{o}_i$, $\underline{u}_i$, and $\underline{d}_i$ ($i = 1,2,\ldots,8$ for each) as the search variables.

From the above, it is apparent that a minimum of 31 search variables must be known to determine a balance for the circuit. Each data point given in Table 13 can therefore be calculated from some suitable combination of the search variables, although not all data points will be required in a specific combination.

6.1.5 Calculation of Data Points from Search Variables

Data points are calculated from some combination of search variables. These expressions must be written in preparation for the objective function. For the data set of interest, computations are as follows, where ˆ indicates a data point calculated from search variables:

$\hat{F} = \underline{F}\ \ \hat{W}_r = \underline{W}_r \quad \hat{W}_u = \underline{W}_u \quad \hat{W}_b = \underline{W}_b \quad \hat{W}_X = \underline{W}_X$

$\hat{W}_p = \underline{W}_o - \underline{W}_X - \underline{W}_r - \underline{W}_b$

$\hat{Q}_c = 3.99(((\underline{F} + \underline{U})/\rho_{sc}) + \underline{W}_o + \underline{W}_u)$ where

$\rho_{sc} = 2.65 = \rho_{sf}$

$\hat{P}_c = 100(\underline{F} + \underline{U})/(\underline{F} + \underline{U} + \underline{W}_o + \underline{W}_u)$

$\hat{P}_o = 100(\underline{F})/(\underline{F} + \underline{W}_o)$

$\hat{P}_r = 100(\underline{F})/(\underline{F} + \underline{W}_r) \quad \hat{P}_u = 100(\underline{U})/(\underline{U} + \underline{W}_u)$

$\hat{P}_d = 100(\underline{U})/(\underline{U} + \underline{W}_u + \underline{W}_b) \quad \hat{o}_i = \underline{o}_i\ i = 1,8$

$\hat{u}_i = \underline{u}_i\ i = 1,8 \quad \hat{d}_i = \underline{d}_i\ i = 1,8$

$\hat{r}_i = \underline{o}_i + (\underline{U}/\underline{F})(\underline{u}_i - \underline{d}_i)\ i = 1,8$

$\hat{r}_9 = 100 - \Sigma\hat{r}_i\ i = 1,8 \quad \hat{o}_9 = 100 - \Sigma\hat{o}_i\ i = 1,8$

$\hat{u}_9 = 100 - \Sigma\hat{u}_i\ i = 1,8 \quad \hat{d}_9 = 100 - \Sigma\hat{d}_i\ i = 1,8$

The calculated data points will appear in the objective function with their measured counterparts.

Table 14 – Modified balance relationships

	Number of new variables	Number of independent equations
Total solids		
$F = R$	2	1
$C = R + D$	2	1
$C = O + U$	2	1
$U = D$	0	1
Total:	6	4
Total water		
$W_d = W_u + W_b$	3	1
$W_c = W_u + W_o$	2	1
$W_c = W_r + W_X + W_p + W_d$	3	1
Total:	8	3
Size fractions conservation		
$Rr_i + Dd_i = Cc_i\ i = 1,8$	24	8
$Cc_i = Oo_i + Uu_i\ i = 1,8$	16	8
$c_9 = 100 - \Sigma c_i\ i \neq 9$	1	1
$d_9 = 100 - \Sigma d_i\ i \neq 9$	1	1
$r_9 = 100 - \Sigma r_i\ i \neq 9$	1	1
$o_9 = 100 - \Sigma o_i\ i \neq 9$	1	1
$u_9 = 100 - \Sigma u_i\ i \neq 9$	1	1
Total:	45	21

Note: W_X is all extra circuit water including W_{sr} and W_{sd}.

6.1.6 Construction of Objective Function

To construct the objective function, it is convenient to transfer to alternate symbols. Let m_i refer to variables such as total solids, water, and per cent solids; let M_{ij} refer to the ith weight per cent retained on size in the jth stream. Then

$$m_1 = F \quad m_2 = W_r \quad m_3 = W_u \quad m_4 = W_b \quad m_5 = W_X$$

$$m_6 = W_p \quad m_7 = Q_c \quad m_8 = P_c \quad m_9 = P_u \quad m_{10} = P_d$$

$$m_{11} = P_r \quad m_{12} = P_o$$

$$\begin{array}{llll}
M_{11} = o_1 & M_{12} = u_1 & M_{13} = d_1 & M_{14} = r_1 \\
M_{12} = o_2 & M_{22} = u_2 & M_{23} = d_2 & M_{24} = r_2 \\
\cdot \quad \cdot & \cdot \quad \cdot & \cdot \quad \cdot & \cdot \quad \cdot \\
\cdot \quad \cdot & \cdot \quad \cdot & \cdot \quad \cdot & \cdot \quad \cdot \\
M_{91} = o_9 & M_{92} = u_9 & M_{93} = d_9 & M_{94} = r_9
\end{array}$$

Using this, an objective function can be written to satisfy the least squares criterion as*:

$$J = \sum_{i=1}^{12} \frac{(\tilde{m}_i - \hat{m})^2}{S_i^2} + (1/9) \sum_{j=1}^{4} \left[\sum_{i=1}^{9} \frac{(\tilde{M}_{ij} - \hat{M}_{ij})^2}{S_{ij}^2} \right]$$

where ~ indicates measured data point and ˆ indicates data point calculated from search variables, S_i^2 is the error variance associated with $\tilde{m}_i$ and calculated from repeat runs, and S_{ij}^2 is the error variance associated with $\tilde{M}_{ij}$ and calculated from repeat runs.

There are four repeat runs (centre points) as part of the data acquisition plan, so that each variance has three degrees of freedom.

For example, suppose that for the four centre point runs, $\tilde{P}_o$ was measured as $\tilde{P}_{o1} = 34.2$, $\tilde{P}_{o2} = 35.7$, $\tilde{P}_{o3} = 35.4$, $\tilde{P}_{o4} = 35$. If $\bar{P}_o = (34.2 + 35.7 + 35.4 + 35)/4 = 35.00$ = the mean, the variance S_{12}^2 is calculated as:

$$\begin{aligned} S_{12}^2 &= [(34.2 - 35)^2 + (35.7 - 35)^2 \\ &\quad + (35.4 - 35)^2 + (35 - 35)^2]/3 = .4225 \end{aligned}$$

which is the variance that is associated with $\tilde{m}_{12}$ in the objective function.

As there are 48 data points, 48 variances must be calculated from the repeat runs. The higher the degree of freedom, the greater the accuracy of adjusted data. Hence perform as many repeats as possible. Try to replicate the design as discussed in Section 3.1. Remember, a repeat run involves a repeat of the whole procedure in the performance of a run.

Error variance associated with a data point arises from all possible sources of error associated with a run. These include bias error, error associated with normal process fluctuations, and sampling or measurement error (12,18). When bias error exists, a method has been proposed (15,18) to estimate its value from adjusted data. Raw data are then corrected for the bias, and the corrected raw data become the raw data set to be adjusted.

6.1.7 Choosing a Search Method

A search method is defined here as a computer procedure that finds that unique set of search variables that will minimize the objective function of Section 6.1.6. If this unique set was known for a given run, then all data points calculated from this set would constitute the adjusted data for the run. The problem is to find this unique set of search variable values.

One way would be to guess their values, calculate the objection function, guess again, recalculate the objective function, and compare it with the previous one. The smallest objective function is then the best of the two. However, with 31 search variables, finding a minimum objective function might take hundreds of years! Direct search methods are an obvious replacement for guessing.

Direct search methods have been reviewed (1,2). At least two have been used to adjust grinding circuit data, namely, the Simplex search routine (18) and a version of the White search routine (19).

The Simplex method will invariably converge, but the routine becomes extremely slow near the minimum. Kelly (20) suggested a combination of the Simplex search with Powells search without derivatives (21). Thus the Simplex approaches the minimum rapidly. Near the minimum, the search is transferred to Powell routine which finds the minimum rapidly.

The White method combines a step search with a quadratic search with a unique search along a vector to speed convergence by a factor of ten. It has been employed to search successfully for 101 search variables around a grinding circuit (22) at costs from $4 to $8 per run.

The SPOC Project (23) now makes search routines available to users. If the search method is adequately documented, programming the adjustment problem as formulated in preceding sections is not tedious. A generalized program that uses the above method for data adjustment is not available. Programs are written for each specific circuit.

**The factor 1/9 is the weight assigned to each of the nine size fractions that resulted from a single sample. The sample receives a weight of one, but the nine portions are weighted 1/9 each.*

Using the Simplex search routine, a program (18) was constructed to adjust 47 sets of data for the circuit described here (see Fig. 1). For the data set shown in Section 6.1.2 results are shown in Table 15. Note that adjustments are, in general, minimal. This was invariably found to be true.

6.2 BILMAT OR MATBAL ADJUSTMENT

BILMAT (17) and MATBAL (24) are user-friendly material balance programs developed as part of the SPOC project. Similar programs are available from various sources [i.e., Wiegel's MATBAL (25)].

The SPOC adjustment programs have two major advantages. First, a knowledge of computer programming, matrix algebra, and mass balance procedures is unnecessary. The user merely enters circuit and raw data that has been written in a standard format. Second, most of the variables used in the objective function are calculated analytically rather than searched. For the data given in Table 13, least squares estimates of all but one search variable can be found by regression. Computing time is therefore significantly reduced.

Minor disadvantages of BILMAT and MATBAL are that linear regression involves matrix inversion which is numerically difficult in some cases (ill-conditioned matrices) and some generality is sacrificed for simplicity, e.g., the flexibility of the user-programmed method (6.2) in terms of physical units may be lost. However, this is a practical and not a theoretical limitation.

Complete user-oriented documentation and source programs specific to BILMAT and MATBAL are available (17,24) at nominal cost.

6.3 COMPARISON OF BILMAT AND DIRECT SEARCHES

When BILMAT is used to adjust circuit data, results are virtually identical with those obtained by means of the Simplex direct search method. For example, the data set shown in Table 13 was adjusted by means of BILMAT. Results are shown in Table 16.

Note that BILMAT adjusted data points are almost identical with those obtained by the search method (Table 15). Minor differences are because BILMAT used raw data rounded to two figures after the decimal. The cost of a BILMAT adjustment was about 1/20 of the cost of a Simplex adjustment. Clearly, except for round off errors, the two methods must give identical answers. Cost therefore becomes a most important selection criterion.

Another useful feature of BILMAT type programs is shown in Table 16. Note that relative standard deviations are listed. BILMAT contains routines that calculate some statistics that are useful for assessing raw data reliability. The reader is referred to the SPOC Manual (17) for further information.

Table 15 – Sample material balance results by the SEARCH method

	Macroscopic variables		
Variable	Measured value	Pred value	Difference
RMFTPH	258.88	258.84	0.04
RMWTPH	78.7	78.7	0
CFGPM	3970	3875.91	94.09
CFPS	62.48	62.47	0.01
BMWTPH	3.23	3.23	0
PBWTPH	237.74	237.74	0
COPS	41.59	41.64	–0.05
CUPS	75.88	75.87	0.01
BMDPS	75.6	75.62	–0.02
RMDPS	76.54	76.68	–0.14
EXTRA PB H20	34.69	43.12	–8.73

Size analysis			
Pred RMD	Measured RMD	Pred BMD	Measured BMD
0.04	0.04	0	0.00
0.71	0.71	0.15	0.08
7.56	7.56	1.41	1.22
19.11	19.09	6.48	6.14
20.21	20.21	21.68	21.62
15.55	15.55	30.18	30.04
10.68	10.69	16.02	16.22
6.58	6.59	7.27	7.47
19.55	19.55	16.83	17.2

RSS,RMD = 3.333094E-4 RSS,BMD = 0.3929342

Pred CO/F	Measured CO/F	Pred CU/F	Measured CU/F
0	0.00	0.01	0.01
0	0.00	0.4	0.4
0.16	0.16	4.02	4.04
0.36	0.36	13.1	13.18
1.82	1.83	28.17	28.3
18.34	18.35	29.19	29.33
22.46	22.46	11.86	11.81
14.38	14.36	4.51	4.49
42.48	42.48	8.74	8.42

RSS,CO/F = 8.06413E-4 RSS,CU/F = 0.1456997

	Adjusted circuit balance			
	RMD	BMD	COF	CUF
Water:	78.7	236.22	362.8	232.99
Solids:	258.84	732.73	258.84	732.71
% Solids:	76.88	75.62	41.64	75.87

Table 16 – BILMAT results

Macroscopic variables

		Pulp flow rates				Water flow rates		
Streams	Measured values	Estimated values	Residence values	Relative St. dev.	Measured values	Estimated values	Residence values	Relative St. dev.
RM.FEED		258.83				0.00		
RM.DIS		337.53				78.70		
CYC.FD.	1626.00	1589.57	2.24	7.00		596.45		
CYC.OF.		621.63				362.81		
CYC.UF.		967.94				233.65		
BM.DIS.		971.17				236.88		
RM.WTR.		78.70			78.70	78.70	.00	.07
SUMPWTR		237.74			237.74	237.74	.00	.05
BM.WTR.		3.23			3.23	3.23	.08	5.00
EXTRA W		43.13			35.22	43.13	22.46	26.00

		Solid flow rates				Pulp per cent solids		
Streams	Measured values	Estimated values	Residence values	Relative St. dev.	Measured values	Estimated values	Residence values	Relative St. dev.
RM.FEED	258.88	258.83	.02	.20		0.00		
RM.DIS.		258.83			76.54	76.68	.19	.45
CYC.FD.		993.12			62.48	62.48	.00	.47
CYC.OF.		258.83			41.59	41.64	.11	.44
CYC.UF		734.29			75.88	75.86	.02	1.04
BM.DIS.		734.29			75.60	75.61	.01	1.22

Size analysis

		RM.DIS.				CYC.OF.		
Streams	Measured values	Estimated values	Residence values	Relative St. dev.	Measured values	Estimated values	Residence values	Relative St. dev.
+6730	.04	.04	2.90	75.00	0.00	0.00	100.00	.01
3360	.71	.74	4.90	70.00	0.00	0.00	100.00	.01
1680	7.56	7.56	.02	1.98	.16	.16	.06	18.75
850	19.09	19.10	.03	1.86	.36	.36	.20	26.35
425	29.21	20.21	.01	1.76	1.83	1.82	.68	27.92
212	15.55	15.55	.01	1.72	18.35	18.33	.11	3.23
106	10.69	10.68	.14	7.79	22.46	22.46	.01	1.46
53	6.59	6.58	.14	4.99	14.36	14.40	.28	6.51
−53	19.55	19.53	.09	4.12	42.48	42.47	.01	1.64

		CYC.UF.				BM.DIS.		
Streams	Measured values	Estimated values	Residence values	Relative St. dev.	Measured values	Estimated values	Residence values	Relative St. dev.
+6730	.01	.01	32.80	300.00	0.00	0.00	100.00	.01
3360	.40	.40	.09	7.50	.08	.14	71.37	483.00
1680	4.04	4.02	.42	7.43	1.22	1.41	15.91	88.66
850	13.18	13.10	.61	6.19	6.14	6.50	5.78	30.90
425	28.30	28.16	.50	7.72	21.62	21.68	.26	13.45
212	29.33	29.20	.45	5.60	30.04	30.18	.45	7.54
106	11.81	11.86	.39	7.75	16.22	16.01	1.30	10.13
53	4.49	4.51	.38	6.01	7.47	7.26	2.77	11.98
−53	8.42	8.73	3.63	23.90	17.20	16.82	2.21	11.44

7. SUMMARY AND CONCLUSIONS

The acquisition of steady state data around a grinding circuit involves an analysis of the existing circuit, the construction of experimental designs for data acquisition, sampling of the circuit in accordance with the design, preparing raw data for statistical adjustment, and adjustment of the raw data by direct search methods or generalized material balance programs such as BILMAT.

A careful circuit analysis includes defining and categorizing variables, particularly input, output, and disturbance variables, establishing mass balance relationships for the circuit, defining equipment and circuit constraints, and establishing the need for further instrumentation.

Experimental designs recommended for data acquisition are the factorials such as the two-level class. Fractional factorials are useful when more than four factors are involved in the design.

When sampling the circuit in accordance with a design, redundant measurements are necessary. Sample locations, samples sizes, and sampling techniques must be reviewed. Special samplers may be necessary. A planning and sampling team, with responsibilities carefully defined, develops record sheets for each run involved. After performing a run, it is important to employ standard laboratory measurements.

Raw data are plotted and reviewed for faults. Duplicate samples may be necessary to resolve discrepancies. Interpolation methods may sometimes be necessary for obtaining the desired data.

To adjust raw data, symbols are selected, raw data points are tabulated, mass balance relations are reestablished, search variables are selected, data points are calculated from search variables, an objective function that involves measured and calculated data points is constructed, and a search method is selected to find the unique set of search variables that will make the objective function a minimum.

General adjustment programs such as BILMAT or MATBAL are extremely useful and efficient because the user does not need to spend a great deal of time on circuit analysis. Adjusted data points determined by BILMAT and MATBAL are virtually identical with those determined by the method described in Section 6.1.

The practical approach described here for acquiring steady state data around a production grinding circuit has been tested and judged to be satisfactory.

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APPENDIX A

SAMPLE RECORD SHEETS

APPENDIX A

SAMPLE RECORD SHEETS

TEAM LEADER RECORD SHEET

Run Number: __________ Date: ____________________________ Shift: ______________________________

Time rod mill was rodded: ____

No. of rods: ____

Time ball mill was charged: ____

Wt. charge: ____

Ball mill discharge spray flow rate: ____

Rod mill discharge spray flow rate: ____

Extra circuit water flow rate: ____

Reagent flow rate: ____

Gland water flow rates: RM: ____

BM: ____

Pump box level at start: ____

Time that variables were manipulated: ____

____

Variable	Desired value	Actual value	Variable that caused constraint
Rod mill feed rate			
Pump box water rate			
Ball mill water rate			
Rod mill water rate			
Spigot diameter			
Vortex diameter			

Rod mill stable? ______

Ball mill stable? ______

Cyclones roping? ______

Pump box level at end: ______

Time to stabilize circuit: ______

Samplers instructed? ______

Sample buckets marked? ______

Time to start sampling: ______

Time to stop sampling: ______

Record sheets picked up? ______

COMMENTS:

INSTRUMENT RECORD SHEET

Run Number: Date: Shift:

Start time: End time:

	Value of variable		
	Start	Middle	End
Rod mill feed rate, T/h			
Rod mill water, L/s			
Ball mill water, L/s			
Pump box water, L/s			
Pump box level, m			
% +65 mesh, PSM			
% solids, PSM			
Cyclone feed flow, L/s			
Cyclone feed density, g/cm^3			
Rod mill power, kW			
Ball mill power, kW			
Pump power draw, kW			
Pump speed, rpm			
	Hi	Med	Low
Cyclone inlet pressure, kPa			

OBSERVATIONS:

SAMPLER RECORD SHEET

Sample location: ______________________________

Name of sampler: ______________________ Date: ______________

Run number: ______________ Shift: ______________ Start time: ______________

Sampling interval									
	1	2	3	4	5	6	7	8	9
Time:									
Approx. volume:									

Sample characteristics:

Fluid (yes/no):

Viscous (yes/no):

Settles out fast (yes/no):

Settles out slow (yes/no):

OBSERVATIONS:

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