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EVALUATION OF DENSITY SEPARATORS MODELS IN CANADIAN COAL PREPARATION PLANTS

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EVALUATION OF DENSITY SEPARATORS MODELS IN CANADIAN COAL PREPARATION PLANTS

by

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ABSTRACT

A program of sampling in Canadian washeries was carried out by the Coal Research Laboratory (CRL) over a period of two years to evaluate the performance of various density separators in these operations. Operating data on quality and recovery of products, separation efficiencies and cutpoints for each separator type are compared with data predicted using computer models developed from previously existing data.

The Heavy Medium Cyclone (HMC), Heavy Medium Bath (HMB) and jig models give predictions of recoveries and qualities of products for sizes coarser than 10 mm with absolute value of error mean less than 7 per cent, whereas predictions of the Water Only Cyclone (WOC) model for -10 mm fine coal are with absolute value of error mean less than 8 per cent. Although less accurate for -10 mm fine coal, the HMC and jig models produce predictions of recoveries with absolute value of error mean less than 2 per cent and quality of products predictions with absolute value of error mean less than 28 per cent which are generally acceptable because more liberation in the fine fractions appears to compensate for any difference in the accuracy of separation, resulting in good prediction of the clean product. However, ash content of the actual fine refuse was lower than predicted. The difference is attributed to the highly friable nature of Canadian coals which tends to cause an excessive loss of coal fines to the fine refuse.

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A comparison study based on quality and recovery of products is made between the Dutch State Mines (DSM) models and the modified models derived from recent results on the linearization of separation curve. The results obtained are statistically analyzed to test the influence of size range and operating conditions on the models predictions.

ÉVALUATION DE DIFFÉRENTS MODÈLES D'ÉPURATEURS OPÉRANT EN MILIEU LIQUIDE DENSE DANS LES USINES CANADIENNES DE PRÉPARATION DU CHARBON

par

A.I.A. Salama*, M.W. Mikhail** et L.C. Bird***

RÉSUMÉ

Le Laboratoire de recherche sur le charbon (LRC) a réalisé au cours d'une période de deux ans un programme d'échantillonnage dans les laveries canadiennes portant sur l'évaluation du rendement de divers épurateurs opérant en milieu liquide dense. Dans le présent résumé, on compare les données de fonctionnement ayant trait à la qualité et à la récupération des produits, à l'efficacité du procédé de séparation et du taillant de chaque épurateur aux données obtenues lors d'essais à partir de modèles informatiques élaborés d'après des données obtenues antérieurement.

Les épurateurs HMC (Heavy Medium Cyclone), HMB (Heavy Medium Bath) et les hydrotamis donnent des prévisions au sujet de la récupération et de la qualité des produits dont la granulométrie est supérieure à 10 mm avec une valeur absolue d'erreur moyenne de moins de 7% tandis que le modèle WOC (Water Only Cylone) donne des prévisions avec une valeur absolue d'erreur moyenne inférieure à huit p. cent pour les charbons fins de -10 mm. Bien que leurs prévisions soient moins précises dans le cas des charbons fins de -10 mm, le modèle HMC et les hydrotamis donnent des prévisions sur la récupération avec une valeur absolue d'erreur moyenne inférieure à deux p. cent et des prévisions quant à la qualité du produit avec une valeur absolue d'erreur moyenne inférieure à vingt-huit p. cent. En général, ces prévisions sont acceptables car la libération accrue des gaz que renferment les fractions fines semble compenser toute différence quant aux données relatives à l'épuration et se traduit par des prévisions justes sur la propreté du produit. Cependant, la teneur en cendres des déchets fins était inférieure aux prévisions.

Cette différence s'explique en raison de la nature extrêmement friable des charbons canadiens, caractéristique qui entraîne une perte excessive de fines de charbons.

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INTRODUCTION

Performance prediction models of density separators have been used for more than two decades in designing coal preparation plants (1,2,3). These models are based on data accumulated over several years from pilot plants and preparation plant operations in Europe, the United States and Canada. All the existing models are empirical, i.e., based on actual data and are generally considered characteristic of the density separator itself and independent of coal washability. It is possible that these models can be utilized in evaluating Canadian washeries or simulating new ones. This would have, however, to be verified by testing, as the construction of some of these models was based on limited data for specific coals. For example western Canadian coals are known to be more friable than European and American coals and degradation might change feed washability. This would in turn alter the quality of products from those predicted on the basis of actual raw feed (4).

The use of simulation models is essential at early stages of development to be able to estimate recoveries and quality of products during the exploration stage. Subsequent economic and feasibility studies depend to a great extent on model predictions for flowsheet design and optimization and for plant design along with material balance. Models are also useful for monitoring plant performance and then in correcting its operation and strategy and for optimizing its operation. Recent advances in minicomputers have speeded up the application of simulation models by eliminating tedious and time-consuming manual calculations.

Despite the advantages of simulation models, there are shortcomings in their application; a) simulation models are constructed by averaging sometimes scattered actual data accumulated over a long period of time, and b) operating conditions of a given plant may be different from those of plants on which the models were based.

This paper reviews and describes presently available models for the heavy medium cyclone (HMC), heavy medium bath (HMB), jig, and water only cyclone (WOC). The computer models of the HMC, HMB and jig employed in this paper are based on Dutch State Mines (DSM) equations (1). The WOC model was developed at the Coal Research Laboratory (CRL) from pilot plant data accumulated over the past fifteen years. Operating data from Canadian washeries on quality and recovery of products, separation efficiencies and cutpoints for each separator type are compared with data predicted using computer models. The influence of change in washability characteristics resulting from size degradation during washing is examined by comparing the quality and recovery of product predictions based on raw and reconstituted feeds. Recent work by Tromp on modified methods of assessing separating efficiency, are utilized to modify the DSM models of HMC, HMB, and jig (5). A comparison based on quality and recovery of products is made between the original and modified DSM models (incorporating Tromp's recent results).

DENSITY SEPARATORS

Coal beneficiation by density methods relies on the differences in relative density between the coal and the impurities mixed in the raw coals. In all cases the impurities are more dense than the clean coal. Density separators represent 88.3 per cent of the total washing capacity in Canadian washeries and include HMC, HMB, jig, WOC, and tables. The HMC is the most common separator followed in order by WOC, HMB and jig respectively. Tables are found in only one plant and are not included in this paper due to insufficient data. The commonly used density separators are briefly described as follows:

1. Heavy Medium Cyclone: the cyclone includes a cone with an angle of approximately 20°. The size range treated is usually between 50 and 0.5 mm. The HMC is the most efficient density separator and the one best suited for coals which are difficult to clean. It treats 42.2 per cent of the total raw coal washed in Canada.

2. Heavy Medium Bath: the HMB is a practical extension of the familiar float-sink test where the raw feed is introduced to the bath and high

density particles move down and low density particles float to the top of the bath. The size range treated is between 200 and 10 mm. The HMB treats 12 per cent of the total raw coal washed in Canada.

3. Water Only Cyclone: the WOC differs from the HMC in the cone angle. Instead of a 20° cone, a cone or tricon of angle(s) between 75-135°is used. In addition the WOC has a longer vortex finder. Two types of WOC are in use: the DSM cyclone with 75° cone and the Compound Water Cyclone (Tricon) (CWC). The size range treated (in Canada) is between 25 and 0.1 mm. The WOC treats 18.9 per cent of the total raw coal washed in Canada. Water Only Cyclone has proved to be the most practical process for the 0.6 - 0.15 mm size fraction.

4. Jig: jigging is one of the oldest separation methods in the field of coal preparation. With respect to methodology (simplicity) and economics (low cost) it is highly regarded in modern coal technology as is evident from the large percentage of coal in the world being presently cleaned by jigs. Presently, jigs application in Canada is limited and treat 12.1 per cent of total raw coal washed. The size range treated is between 127 and 10 mm.

PRINCIPLE OF DENSITY SEPARATOR MODELLING

In an ideal density separation, all particles of density less than the relative density of separation (d_p) report to the clean product while all particles of density higher than d_p report to the reject. If the percentage recovery of reject is plotted against the density, we obtain a step function with ordinate 0 per cent for d less than d_p and ordinate 100 per cent for d greater than d_p (Fig. 1). In practice, however, some of the clean coal reports (is misplaced) to the reject and some of the reject reports to the clean product. The plot of the percentage of feed reporting to the reject side against the mid-point of each density fraction, is called the separation curve (SC), distribution curve, error curve, or partition curve. This curve is considered to be a characteristic of the separator and is independent of the washability of the coal being processed. Two of the criteria derived from this curve which may be used to evaluate the efficiency of separation, are the probable error and the error area. The probable error (E_p) is defined as half the density interval between 25 and 75 percent ordinates of the curve (Fig. 1). Thus E_p is simply a measure of the average slope of the middle portion of the curve with no account being taken of the tails, those portions lying outside the 25 and 75 per cent ordinates. The error area is the area lying between the actual separation curve and the ideal separation step function (Fig. 1) and may be determined by employing curve fitting and numerical integration routines. The error area is considered a better measure of efficiency than the probable error because it takes into account the overall curve.

The separation curve may be influenced by the operating conditions of the separator and/or of the size of coal being washed. Consequently, the performance data for different separators of the same type under different operating conditions and for different cut points are recorded over a long period of time to accumulate sufficient data. Regression analysis of the accumulated data can be used to generate a representative separation curve (model) for a given density separator.

The density separator computer modelling method presented here simulates the representative separation curve (model) of the separator digitally on the computer (Fig. 2). To facilitate the digital simulation three approaches may be utilized: a) expressing the representative separation curve in a mathematical form, b) transforming the representative separation curve to straight lines, or c) using a curve-fitting routine. The Dutch State Mines employed the first approach to model the HMC, HMB, and jig separators (1). Their method started by specifying the relative density of separation and using empirical relationships to calculate the probable error as defined above. The calculated probable error and a set of partition factors based on accumulated data were used to determine the separation curve points (Fig. 3). The DSM approach was adopted in this paper by using their density separator models for the HMC, HMB and jig separations curves. The WOC data gathered from the pilot plant and operating plants by CRL over the past fifteen years were used to model the WOC curve. In all models a curve fitting routine (spline function) was employed to generate a continuous curve.

A computer program incorporating the DSM models for HMC, HMB, jig and the CRL model for WOC has been developed at CRL to simulate coal preparation operations. The computer program can simulate single or two-stage (with no circulation) operation or any combination of density separators. Program input data are the washability data of the feed and the relative density of separation (for each unit in case of two-stage operation). The program computes predictions of yield and ash contents of the clean product and refuse (Fig. 4). It is simple to operate and has been designed to allow application using a desk-top computer.

PREDICTED AND ACTUAL OPERATING DATA COMPARISON

Western Canadian coals are known to be more friable than European and American coals and it is possible that size degradation would change feed washability and, in turn, the quality predictions of products (4). To illustrate this problem, operating data from Canadian washeries were used to generate reconstituted feeds for different density separators, (Fig. 5). These feeds along with actual raw feeds were used for the predictions of product quality and recovery using the simulation models discussed above. The results calculated for the individual size fractions for the HMC, HMB, jig and WOC are compared with plant washing results in Tables 1 to 4. Table heading A denotes the actual result, P(AF) the predicted result based on actual raw feed and P(RF) predicted result based on reconstituted feed. It is worth noting that actual operating data were based on samples collected from operating plants with no attempt to tune up or optimize their operations. Examination of the results in Tables 1 to 4 and A-1 (mean and standard deviation columns) revelas that the absolute value of the mean of recoveries predictions based on reconsituted feed is less than 5 per cent while for predictions based on raw feed is less than 21 per cent. Also the standard deviation (a measure of spread) for recoveries predictions based on reconsistuted feed is less than 7 per cent while for predictions based on raw feed is less than 42 per cent. This indicates change in coal washability during washing as a result of size degradation.

For sizes coarser than 10mm the HMC, HMB, and jig simulation models give predictions (based on reconstituted feed) of recoveries and qualities of products with absolute value of error mean less than 7 per cent (Table A-3), whereas predictions of the the WOC model for -10 mm fine coal are with absolute value of error mean less than 8 per cent (Table A-4). Although less accurate for -10 mm fine coal, the HMC and jig models produce predictions of recoveries with absolute value of error mean less than 2 per cent and quality of products predictions with absolute value of error mean less than 28 per cent which are generally acceptable. The improvements in washabilities due to liberation in the fine fractions requires higher cutpoints where little near density materials is present appears to compensate for any difference in the accuracy of separation resulting in an acceptable prediction of the clean product. In some cases ash content of the actual fine refuse was lower than the predicted value, the difference being attributed to the highly friable nature of western Canadian coals which tend to cause excessive loss of fine coal to the fine refuse as finer particles are separated less efficiently than coarser ones. Also, the models of HMB, HMC and jig are based on relatively coarser average particle size than the actually washed Canadian coals and as a result the models indicated sharper separation.

MODIFIED MODELS

Recently, Tromp (5) has shown that the separation curve of a density separator can be represented by straight lines if the curve is redrawn using a new variable "Y" instead of the variable "partition number" (Fig. 6). The relationship between Y and the partition number is given as

$$Y = [130 \ln(50/\text{Probability factor})]^{1/1.2}$$

where

Probability factor = Partition no. if partition no. $\leq$ 50

Probability factor = 100 - Partition no. if partition no. $\geq$ 50

The coefficient 130 is a scaling factor. This method was employed to modify the partition factors developed by the Dutch State Mines for the HMC, HMB, and jig models. The modified models were incorporated into the CRL coal preparation simulation program. The reconstituted feed data were fed to the simulation program incorporating the modified models and the product quality and recovery data predicted are compared with the actual values and the Dutch State Mines models predictions. The results are summarized in Tables 5 to 8, where A denotes the actual result, P(RF) the predictions based on reconstituted feed using Dutch State Mines models and M(RF) the predictions based on reconstituted feed using modified Dutch State Mines models respectively. Examination of the results in Tables 5 to 7 reveals that the modified DSM and DSM models produce comparable predictions. Introduction of the modified models is made to verify Tromp's results (5) as well as to refine the DSM models.

Table 9 shows a comparison between actual and predicted performance for the 2 x 0.6 mm size fraction of fine coal treatment plants. The differences between predicted and actual results are larger than for coarser size fractions. Presence of excessive amounts of fines due to coal friability and attrition might be the cause of the greater differences. It is of interest to note that the differences in reject ash of highly friable coals (A and B) are larger than for non-friable coals (C, D and E), Table 9. It appears that prediction of separation for plus 2 mm coal is more reliable than for the finer fractions 2 x 0.6 mm depending on the cutpoint and near density material. Samples C and D are less friable but due to high near density material show relatively lower ash in reject than predicted.

A statistical study is carried out to examine the effect of particle size and operating conditions on the models predictions and is reported in Appendix A. The statistical methods utilized in this study are the F-test and t-test. The statistical study indicates that in general the prediction data do not yield evidence that the particle size nor the operating conditions have significant influence on the model predictions.

CONCLUSIONS

Computer models for HMC, HMB, jig and WOC density separators were evaluated by comparing predicted results with actual operating data from Canadian Coal preparation plants. The HMC, HMB, and jig models are based on DSM data and the WOC model is based on CRL data accumulated over the past fifteen years. These models can be used in single or two-stage operation with no circulation or in any combination. The HMC, HMB and jig models give accurate predictions for coal sizes coarser than 10 mm whereas the WOC model gives predictions which compare favorably for -10 mm fine coal. The modified and original Dutch State Mines models of the HMC, HMB, and jig separators produced comparable predictions.

The HMC, HMB and jig models predictions errors of recoveries and qualities of products are statistically analyzed and the predictions in general do not yield evidence that the particle size nor the operating conditions have significant influence on the models predictions.

The described computer models are simple to operate and have been designed to allow application using a desk top computer.

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Table 1 - Comparison between actual washing results (A) and predictions based on actual feed (P(AF)) and reconstituted feed (P(RF)) for heavy medium cyclone

Plant		+25.4 mm			25.4 x 12.7 mm			12.7 x 10 mm			10 x 2 mm			2 x 0.6 mm		
		A	P(AF)	P(RF)	A	P(AF)	P(RF)	A	P(AF)	P(RF)	A	P(AF)	P(RF)	A	P(AF)	P(RF)
A	Yield	76.1	71.1	76.9	76.4	69.4	72.7	79.4	67.5	73.4	76.6	68.7	76.3	65.6	64.7	67.4
	Clean Coal Ash	4.1	3.5	3.3	2.5	2.7	2.5	2.2	2.3	2.2	2.0	2.1	3.1	1.6	1.6	3.2
	Reject Ash	76.9	71.2	74.3	68.1	52.9	57.7	61.3	48.5	46.4	57.6	56.9	51.2	55.6	47.6	51.3
B	Yield	22.6	34.0	22.3	42.0	46.4	41.6	52.3	55.1	52.6	72.7	64.1	73.4	87.4	74.3	88.2
	Clean Coal Ash	9.4	9.6	10.4	10.4	9.2	9.8	7.5	8.0	7.0	5.4	6.8	6.0	5.2	5.8	6.1
	Reject Ash	75.6	70.9	77.0	70.9	70.6	76.3	72.2	67.4	74.4	66.1	66.9	73.7	64.7	61.6	75.5
C	Yield							78.5	80.0	79.1	77.1	81.6	77.3	82.4	84.1	83.6
	Clean Coal Ash							12.7	14.2	13.6	12.0	12.3	12.0	12.3	11.5	10.0
	Reject Ash							64.5	62.1	62.3	65.0	60.2	60.5	61.0	61.1	63.9
D	Yield	82.6	83.9	83.7	85.2	85.6	85.5	83.4	87.1	84.4						
	Clean Coal Ash	10.5	12.6	12.0	9.7	10.5	9.9	8.4	8.6	8.1						
	Reject Ash	70.1	67.7	72.3	64.4	59.8	64.1	61.1	62.8	61.7						

Table 2 - Comparison between actual washing results (A) and predictions based on actual feed (P(AF)) and reconstituted feed (P(RF)) for heavy medium bath

Plant		+50.8 mm			50.8 x 25.4 mm			25.4 x 10 mm		
		A	P(AF)	P(RF)	A	P(AF)	P(RF)	A	P(AF)	P(RF)
A	Yield	9.0	5.3	10.1	7.1	6.5	7.0	13.6	20.5	14.4
	Clean Coal Ash	7.5	8.1	8.1	7.9	6.5	6.5	8.3	7.5	8.0
	Reject Ash	81.7	83.1	81.3	80.1	79.4	76.9	71.0	71.8	71.1
B	Yield	48.8	54.0	54.2	63.3	68.5	61.8	76.2	71.3	77.0*
	Clean coal Ash	11.4	11.4	12.2	12.7	11.6	11.8	11.6	11.7	11.5
	Middling Ash	26.9	21.1	20.2	29.7	23.4	23.4	29.0	29.0	28.6
	Reject Ash	70.3	62.0	58.7	63.6	62.3	61.6	63.9	65.1	63.5

* size fraction is 25.4 x 12.7 mm

Table 3 - Comparison between actual washing results (A) and predictions based on actual feed (P(AF)) and reconstituted feed (P(RF)) performance data for jigs

Plant		+50.8 mm			50.8 x 25.4 mm			25.4 x 10 mm			10 x 2 mm			2 x 0.6 mm		
		A	P(AF)	P(RF)	A	P(AF)	P(RF)	A	P(AF)	P(RF)	A	P(AF)	P(RF)	A	P(AF)	P(RF)
A	Yield				76.6	76.7	77.4	87.2	87.6	87.8	87.8	90.0	88.8	88.3	91.9	89.9
	Clean Coal Ash				12.7	14.4	14.3	13.3	13.2	13.0	10.1	11.0	10.1	7.5	7.1	6.5
	Reject Ash				46.4	42.4	43.8	54.1	55.6	52.9	62.3	61.1	61.0	48.4	51.6	55.9
B	Yield	21.2	14.1	22.3	35.5	37.4	33.9	41.2	51.1	44.4						
	Clean Coal Ash	8.8	7.8	7.1	9.2	8.2	11.7	10.5	7.8	8.2						
	Reject Ash	68.1	77.5	75.5	70.8	75.1	74.2	58.6	72.2	73.1						

Table 4 - Comparison between actual washing results (A) and predictions based on actual feed (P(AF)) and reconstituted feed (P(RF)) for water only cyclone

Plant		+10 mm			10 x 3.2 mm			3.2 x 0.6 mm		
		A	P(AF)	P(RF)	A	P(AF)	P(RF)	A	P(AF)	P(RF)
A	Yield	98.4	99.0	99.1	91.7	88.8	88.5	83.1	82.6	81.3
	Clean Coal Ash	10.1	10.4	10.3	10.6	10.6	11.0	10.0	9.2	10.1
	Reject Ash	32.8	37.4	35.1	24.5	19.6	20.6	27.8	20.8	23.9
B	Yield	30.9	62.1	30.0	47.2	59.9	47.5	73.6	70.6	72.6
	Clean Coal Ash	11.2	7.6	7.9	8.0	7.4	8.9	7.9	8.4	7.1
	Reject Ash	75.1	63.1	74.3	51.3	35.9	52.5	23.7	27.3	23.2

Table 5 - Comparison between predicted results based on reconstituted feed using DSM models (P(RF)) and modified models (M(RF)) for heavy medium cyclone

Plant		+25.4 mm			25.4 x 12.7 mm			12.7 x 10 mm			10 x 2 mm			2 x 0.6 mm		
		A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)
A	Yield	76.1	76.9	76.8	76.4	72.7	72.1	79.4	73.4	72.6	76.6	76.3	75.6	65.6	67.4	66.8
	Clean Coal Ash	4.1	3.3	3.3	2.5	2.5	2.5	2.2	2.2	2.2	2.0	3.1	3.1	1.6	3.2	3.2
	Reject Ash	76.9	74.3	74.0	68.1	57.7	56.5	61.3	46.4	45.1	57.6	51.2	49.8	55.6	51.3	50.4
B	Yield	22.6	22.3	22.2	42.0	41.6	41.5	52.3	52.6	52.5	72.7	73.4	73.3	87.4	88.2	88.2
	Clean Coal Ash	9.4	10.4	10.3	10.4	9.8	9.8	7.5	7.0	7.0	5.4	6.0	5.9	5.2	6.1	6.1
	Reject Ash	75.6	77.0	77.0	70.9	76.3	76.2	72.2	74.4	74.3	66.1	73.7	73.5	64.7	75.5	75.7
C	Yield							78.5	79.1	78.9	77.1	77.3	77.2	82.4	83.6	83.5
	Clean Coal Ash							12.7	13.6	13.6	12.0	12.0	12.0	12.3	10.0	10.0
	Reject Ash							64.5	62.3	62.1	65.0	60.5	60.4	61.0	63.9	63.7
D	Yield	82.6	83.7	83.9*	85.2	85.5	85.5*	83.4	84.4	84.3*						
	Clean Coal Ash	10.5	12.0	12.0	9.7	9.9	9.9	8.4	8.1	8.1						
	Reject Ash	70.1	72.3	72.8	64.4	64.1	64.0	61.1	61.7	61.5						

*Size fractions are 50.8 x 25.4 mm, 25.4 x 10 mm and 10 x 0.6 mm respectively.

Table 6 - Comparison between predicted results based on reconstituted feed using DSM models (P(RF)) and modified models (M(RF)) for heavy medium bath

Plant		+50.8 mm			50.8 x 25.4 mm			25.4 x 10 mm		
		A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)
A	Yield	9.0	10.1	10.0	7.1	7.0	7.0	13.6	14.4	14.4
	Clean Coal Ash	7.5	8.1	8.1	7.9	6.5	6.5	8.3	8.0	8.0
	Reject Ash	81.7	81.3	81.2	80.1	76.9	76.8	71.0	71.1	71.1
B	Yield	48.8	54.2	54.0	63.3	61.8	61.8	76.2	77.0	76.9*
	Clean Coal Ash	11.4	12.2	12.2	12.7	11.8	11.8	11.6	11.5	11.5
	Middling As	26.9	20.2	20.3	29.7	23.4	23.4	29.0	28.6	29.0
	Reject Ash	70.3	58.7	58.7	63.6	61.6	61.6	63.9	63.5	63.6

*Size fraction 25.4 x 12.7 mm.

Table 7 - Comparison between predicted results based on reconstituted feed using DSM models (P(RF)) and modified models (M(RF)) for jigs

Plant		+50.8 mm			50.8 x 25.4 mm			25.4 x 10 mm			10 x 2 mm			2 x 0.6 mm		
		A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)
A	Yield				76.6	77.4	77.3	87.2	87.8	87.6	87.8	88.8	88.6	88.3	89.9	89.5
	Clean Coal Ash				12.7	14.3	14.3	13.3	13.0	13.0	10.1	10.1	10.1	7.5	6.5	6.4
	Reject Ash				46.4	43.8	43.7	54.1	52.9	52.2	62.3	61.0	59.9	48.4	55.9	54.6
B	Yield	21.2	22.3	22.7	35.5	33.9	34.0	41.2	44.4	44.3						
	Clean Coal Ash	8.8	7.1	7.1	9.2	11.7	11.5	10.5	8.2	8.2						
	Reject Ash	68.1	75.5	75.9	70.8	74.2	74.5	58.6	73.1	73.0						

Table 8 - Comparison between predicted results based on reconstituted feed using DSM models (P(RF)) and modified models (M(RF)) for the composite +0.6 mm size fraction

	Plant	A			B			C			D			E		
		A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)
HMC	Yield	76.2	73.9	73.2	59.5	60.1	60.0	79.0	77.4	77.3	79.7	79.5	79.4	83.9	84.0	84.0
	Clean Coal Ash	2.4	2.5	2.5	6.3	6.8	6.8	12.2	11.3	11.2	13.9	12.5	12.4	10.8	8.9	8.9
	Reject Ash	63.9	56.6	55.3	70.7	68.8	68.8	63.8	58.5	58.4	70.1	66.2	66.2	63.4	62.7	62.5
HMB	Yield	65.8	64.6	64.6*	10.6	12.0	12.0**	58.4	64.3	64.5**						
	Clean Coal Ash	12.0	11.3	11.3	8.0	7.8	7.8	12.4	12.5	12.5						
	Middling Ash	28.3	23.8	23.9												
	Reject Ash	65.4	61.4	61.4	76.5	74.6	74.6	56.3	55.7	55.9						
Jig	Yield	84.8	85.3	84.8	35.0	37.0	36.9									
	Clean Coal Ash	12.2	12.2	12.2	9.4	7.0	7.0									
	Reject Ash	52.5	50.7	50.3	68.1	74.6	74.5									

* Size fraction +12.7 mm
**Size fraction +10 mm

Table 9 - Comparison between HMC actual washing results (A) and predictions based on reconstituted feed using DSM models (P(RF)) and modified models (M(RF)) for size fraction 2 x 0.6 mm

Plant	A			B			C			D			E		
	A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)	A	P(RF)	M(RF)
Yield	88.4	92.1	92.0	87.4	88.2	88.2	90.5	92.8	92.8	82.4	83.6	83.5	65.6	67.4	66.8
Clean Coal Ash	9.7	10.4	10.4	5.2	6.1	6.1	8.2	6.9	6.9	12.3	10.0	10.0	1.6	3.2	3.2
Reject Ash	66.4	81.1	81.0	64.7	75.5	75.7	61.7	68.2	68.0	61.0	63.9	63.7	55.6	51.3	50.4

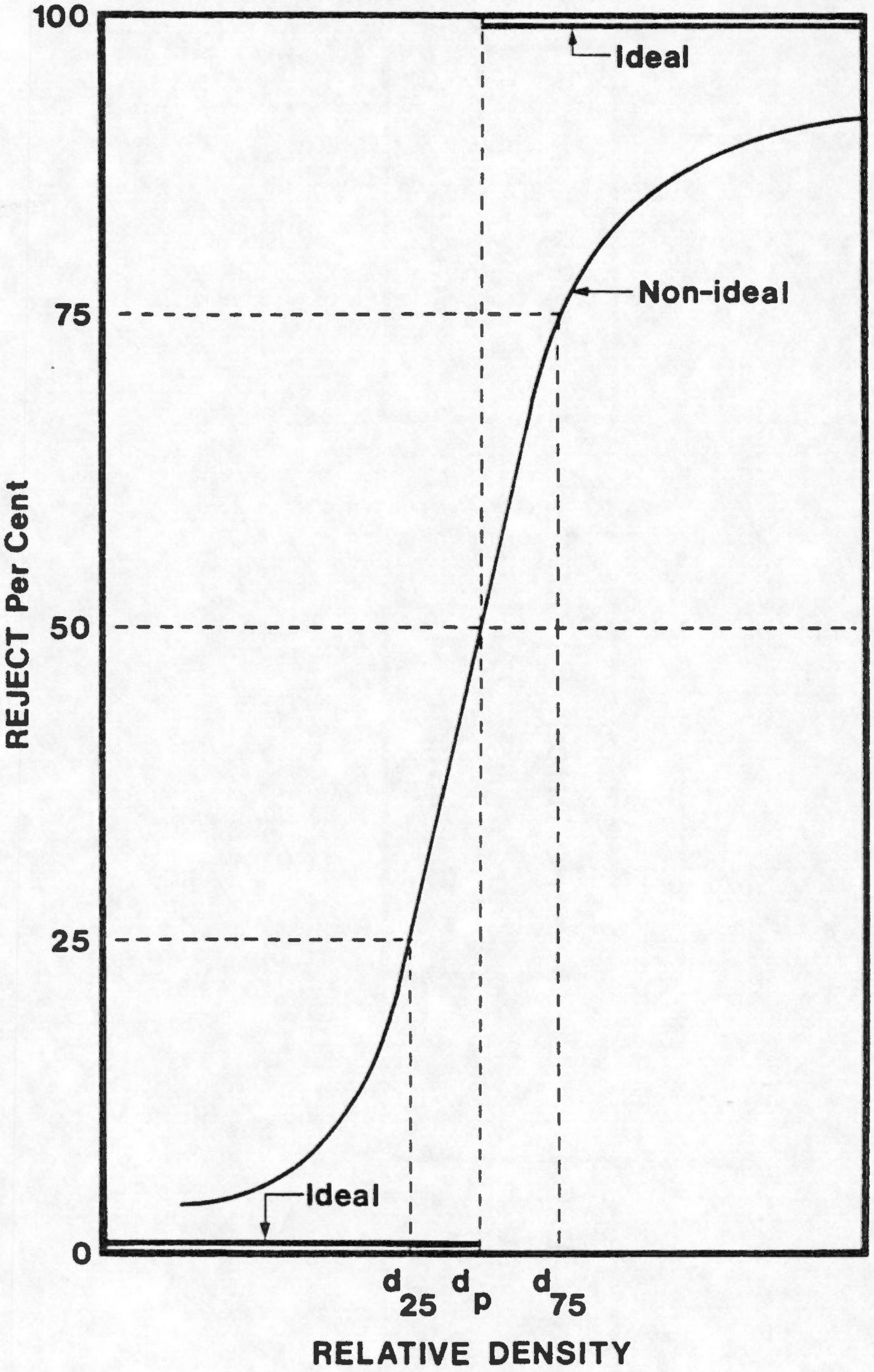

Fig. 1 - Ideal and nonideal separation curves

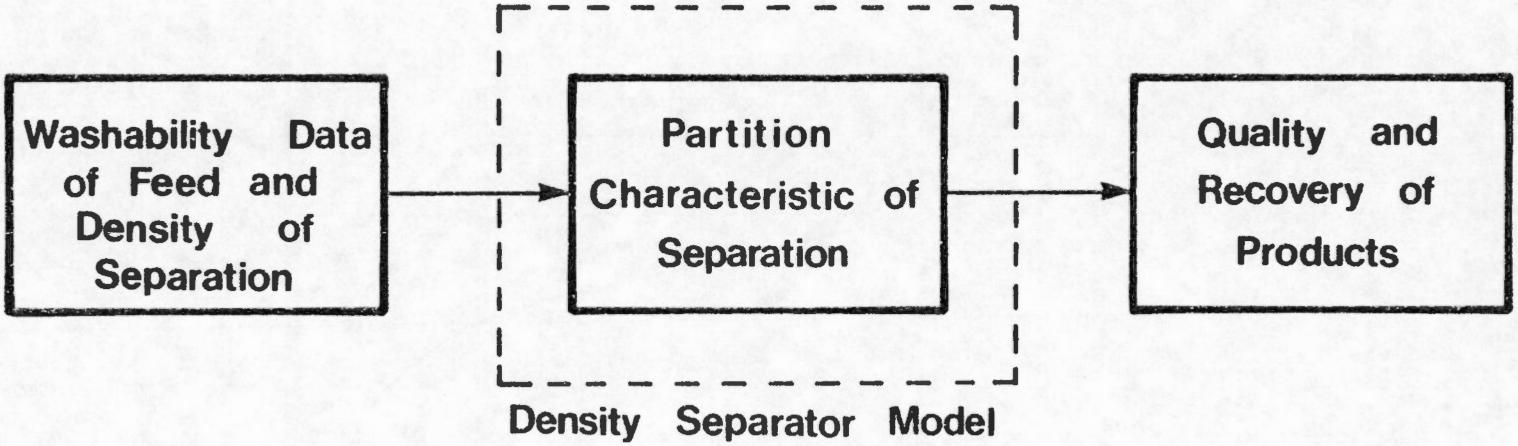

Fig. 2 - Density separator modelling principle

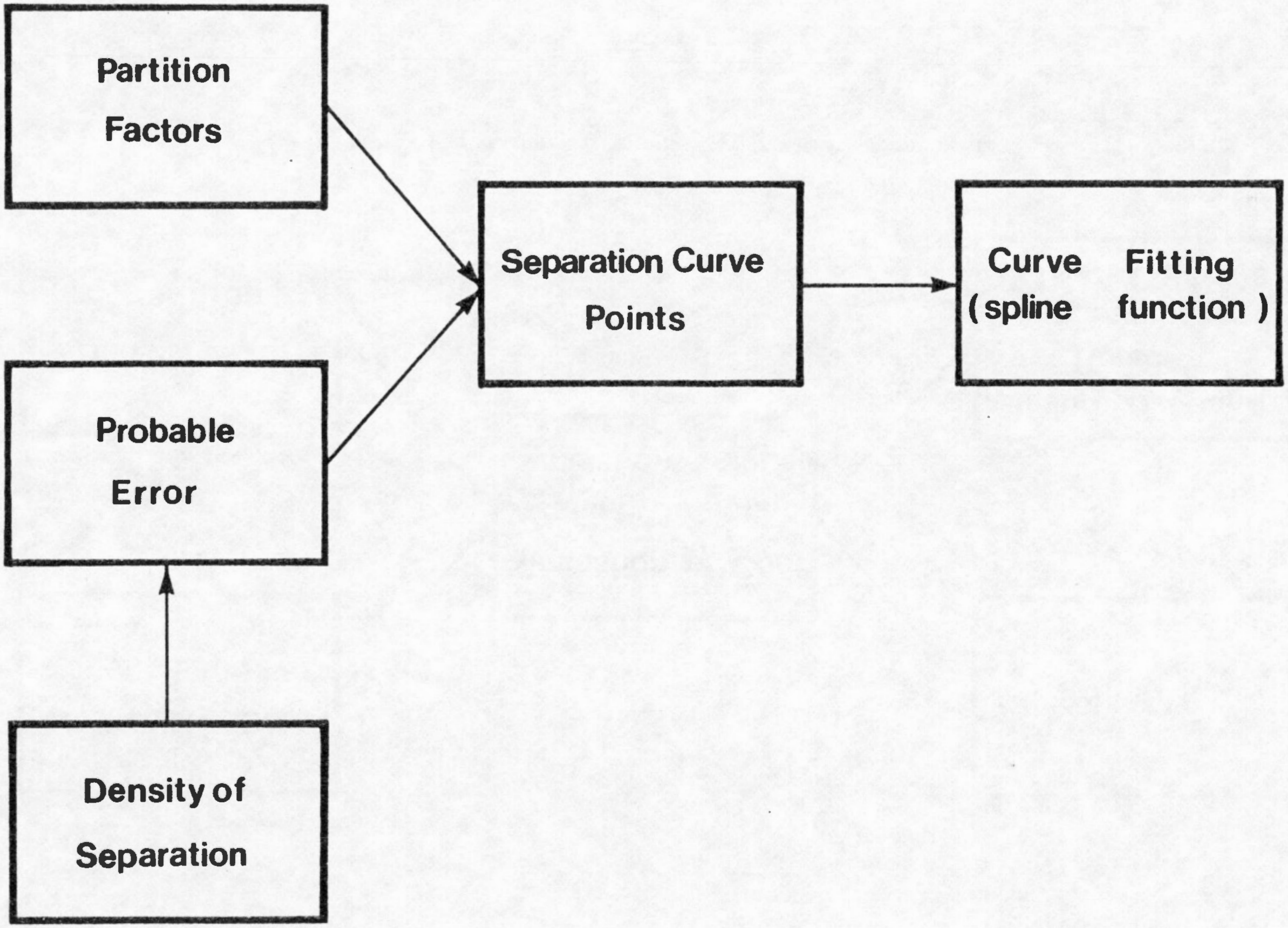

Fig. 3 - The Dutch State Mines approach for HMC, HMB and jig modelling

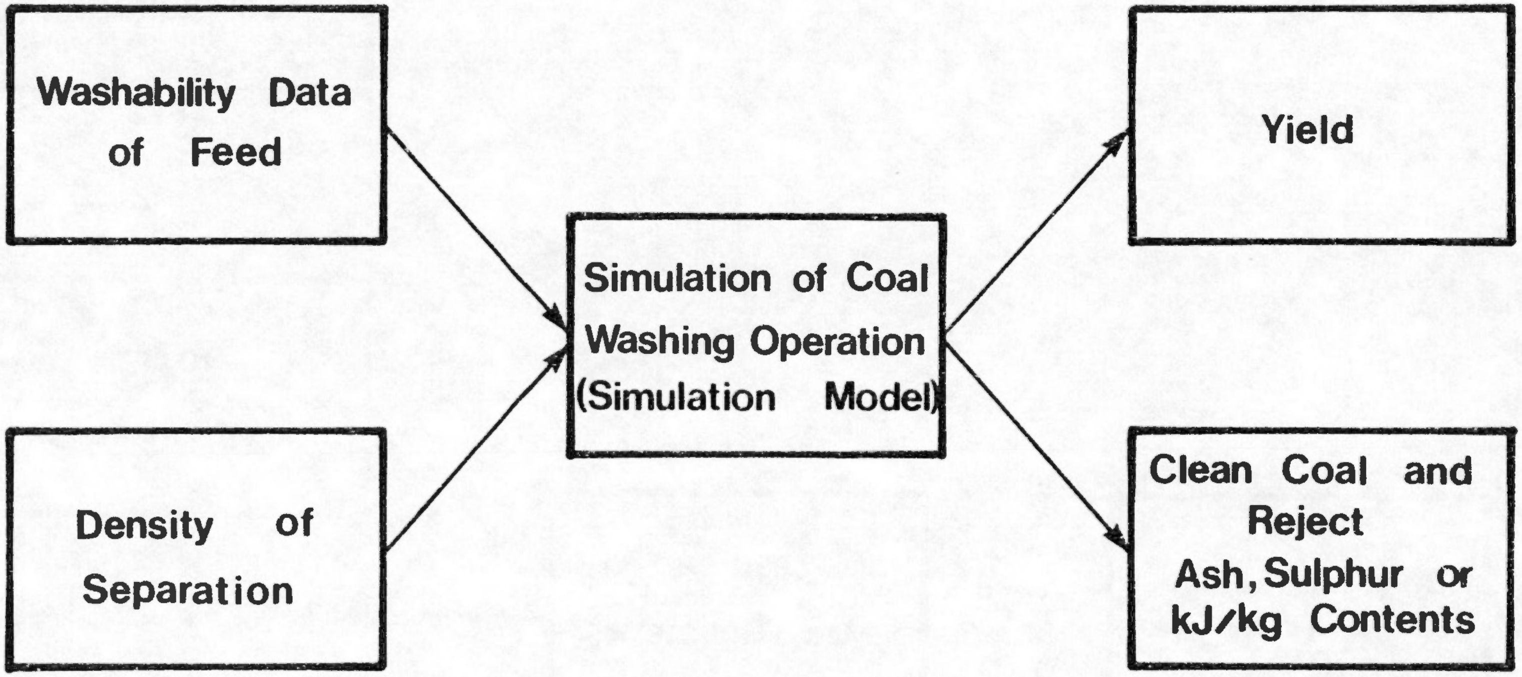

Fig. 4 - The CRL density separator simulation program

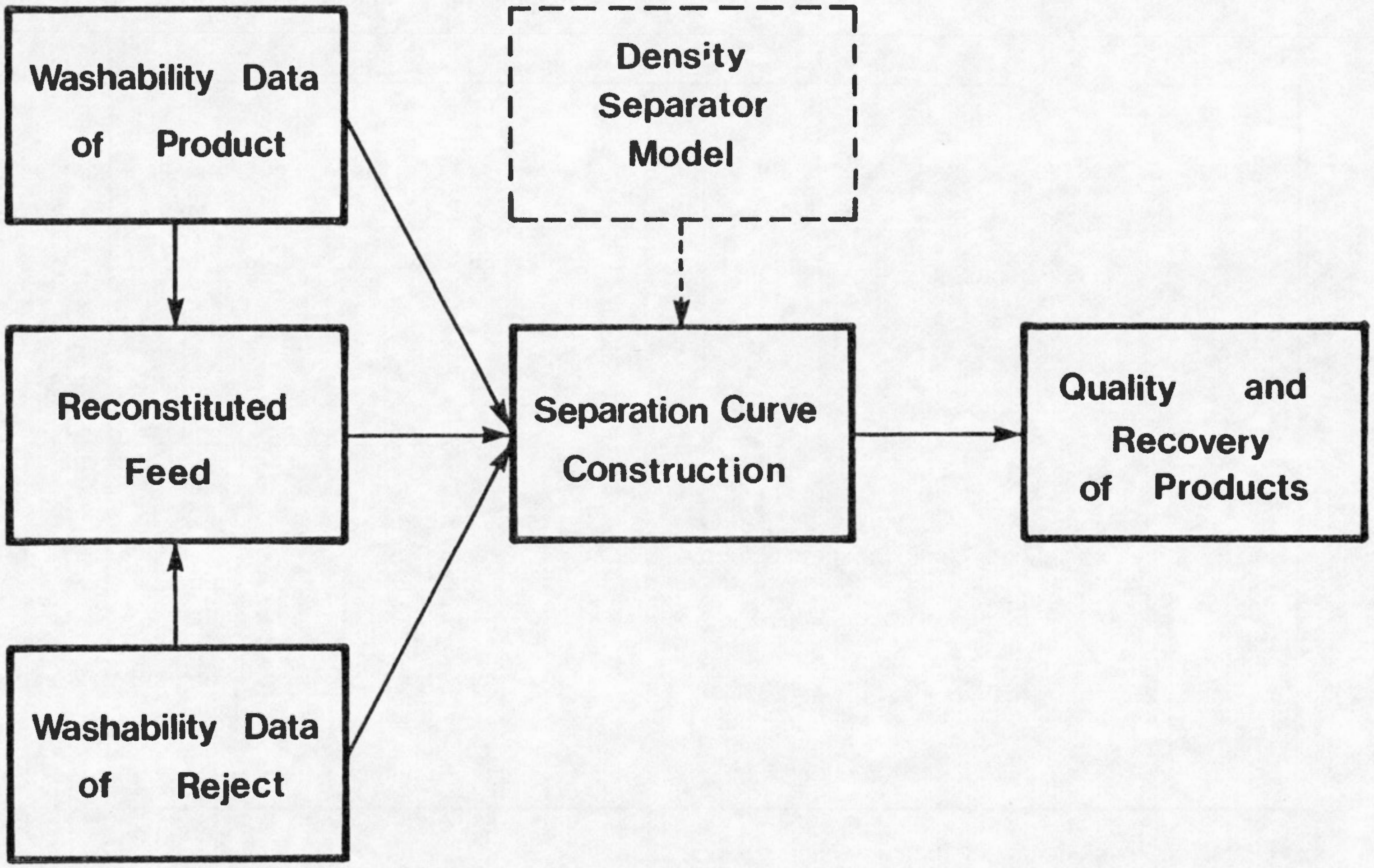

Fig. 5 - Density separator simulation program with reconstituted feed

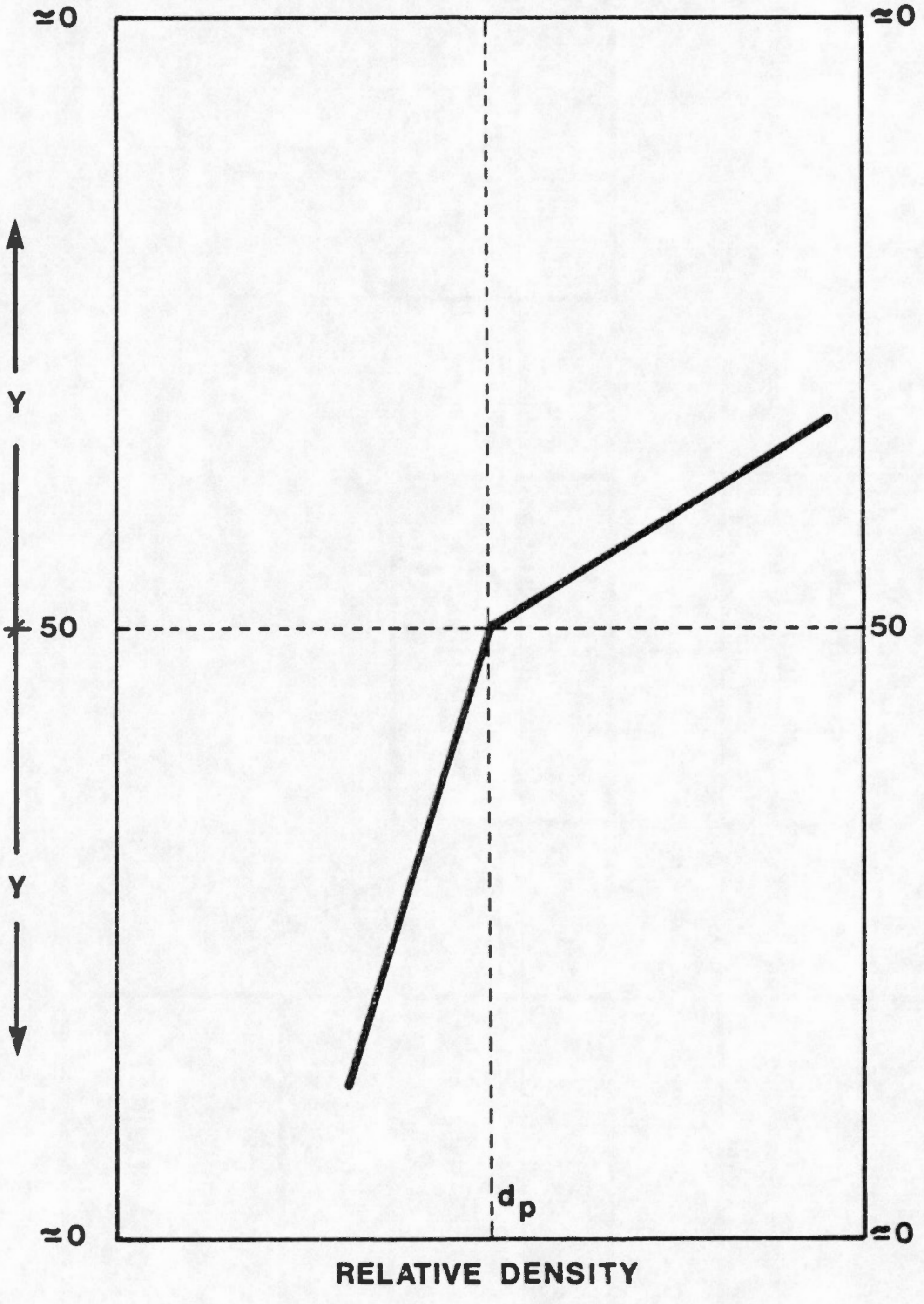

Fig. 6 - Transformed separation curve employing Tromp 's modified method of assesing separation efficiency (5)

APPENDIX A

APPENDIX A

STATISTICAL ANALYSIS OF THE MODELS PREDICTIONS

Two of the main variables which can affect the predictions of a model are the particle size variation and the operating conditions. Such influence can be tested using some of the statistical tools i.e. F-test and t-test. The details of these tests can be found in (6,7) and will not be presented here. To simplify the statistical analysis adopted in this paper two approaches are utilized: a) one way classification (size fraction), and b) two way classification (size fraction, operating conditions).

ONE WAY CLASSIFICATION (SIZE FRACTION)

In this case the size fraction variation is considered as the only variable and the product predictions errors of a unit as an observation. The models predictions of recoveries and qualities of products (based on actual and reconstituted feeds) and the actual operating data given in Tables 1 to 4 are used to calculate the predictions errors. These errors are then used to determine the F-ratios and are given in Table A-1, where AF and RF designate actual and reconstituted feeds respectively (6,7). For a given degrees of freedom, q_1 and q_2 and a level of significance α, the value $(F_{1-\alpha}(q_1, q_2))$ can be obtained using F-tables and are recorded in Table A-1 (6,7). Examination of the results reported in Table A-1 we find in all cases (based on reconsituted feed) except the HMB results that the F-ratio is less than $F_{0.9}(q_1, q_2)$. Consequently the null hypothesis of no size fraction effect can not be rejected i.e. sample does not yield evidence that there are significant differences between predictions means of the different size fractions. The scattered results of the HMB is attributed to the poor sampling and operating conditions (one plant was overloaded and the other handled coal with high clay content) as a result the actual operating data were not reliable.

Testing of the overall sample mean against the population mean (equal zero for a good model) is of some interest. This can be carried out using the t-test, first we determine the t-value and check whether it lies inside or outside the range $\pm t_{\alpha/2}$ (q),where q and α are the degree of freedom and level of significance respectively (6,7). The predictions errors based on the results in Tables 1 to 4 and used for the F-test, are used to determine the t-values and the results obtained are given in Table A-1. Again in all cases (based on reconstituted feed) we find that the t-value lies inside the range $\pm t_{\alpha/2}(q)$ which indicates the sample does not yield evidence that the population mean is different from zero.

The predictions of the DSM models and the modified DSM models (based on reconstituted feed) given in tables 5 to 7 are subjected to F-test and t-test and the results obtained are given in Table A-2. Examination of Tabe A-2 reveals that the predictions of the DSM and modified DSM models are comparable, which establishes some verification for the DSM models as well as Tromp's results (5).

The size fractions of +10 mm and -10 mm given in Tables 1 to 4 are used to generate predictions errors and the results are subjected to F-test and t-test and the results obtained are given in Tables A-3 and A-4 respectively. Examining Table A-3 we find that for the HMC and jig the sample of +10 mm does not yeild evidence that there is significant size fraction effect on the model predictions. The problem associated with the HMB results were discussed earlier. In Table A-4 we notice that for the WOC the sample of -10 mm does not yeild evidence that there is significant size fraction effect on the model predictions.

The results reported in Table 8 and 9 are used to generate predictions errors and the results are used in t-test and the results obtained are given in Table A-5. Examination of these results reveals that in general for the composite +0.6 mm size fraction the HMC, HMB and jig predictions errors sample does not yield evidence that the overall sample mean is different from 0. However, for the size fraction 2 x 0.6 mm, the HMC sample does yeild evidence that the overall sample mean is different from 0, i.e. significant effect of that size fraction on the HMC performance.

TWO WAY CLASSIFICATION (SIZE FRACTION, OPERATING CONDITIONS)

In this case the predictions results reported in Tables 1 to 4 can be viewed as the size range variation and plant variation (operating conditions) as the two variables affecting the model prediction and unit predictions as a single observation. The predictions errors generated from Tables 1 to 4 are used again to carry F-test (two way classification with single observation) and the results obtained are given in Tables A-6 and A-7, where SFE and OCE designate size fraction effect and operating conditions respectively. Examination of the results given in Tables A-6 and A-7 reveals that in general the sample does not yeild evidence that there are significant effects of the size range and operating conditions on the models predictions. The results of F-test on +10 mm size fractions and -10 mm size fractions are given in Tables A-8 and A-9 respectively, where again in general the sample does not yield evidence that there are significant effects of the size range and operating conditions on the model predictions.

Table A-1 - F-Test and t-test on results in Tables 1 to 4

Density Separator		F-ratio		$F_P(q_1,q_2)$	t-value		$\pm t_{\alpha/2}(q)$	Mean		St. Deviation	
		AF	RF		AF	RF		AF	RF	AF	RF
HMC	Yield	0.85	0.90	$F_{0\cdot5}(4,11)=0.89$	0.22	-0.32		0.83	-0.21	15.40	2.58
	Clean Coal Ash	0.41	0.84	$F_{0\cdot9}(4,11)=\underline{2.54}$	$\underline{1.84}$	1.39	$\pm t_{0\cdot05}(15)=\underline{\pm1.75}$	4.75	10.20	10.35	29.41
	Reject Ash	0.34	0.42	$F_{0\cdot95}(4,11)=3.36$	$\underline{-3.48}$	-0.56	$\pm t_{0\cdot025}(15)=\pm2.13$	-6.35	-1.44	7.29	10.20
HMB	Yield	0.68	$\underline{21.32}$	$F_{0\cdot5}(2,3)=0.88$	0.18	1.71		2.27	4.4	30.08	6.32
	Clean Coal Ash	3.48	$\underline{9.85}$	$F_{0\cdot9}(2,3)=\underline{5.46}$	-1.2	-0.61	$\pm t_{0\cdot05}(5)=\underline{\pm2.02}$	-4.52	-2.38	9.20	9.55
	Reject Ash	0.70	0.80	$F_{0\cdot95}(2,3)=9.55$	-0.79	-1.60	$\pm t_{0\cdot025}(5)=\pm2.57$	-1.67	-4.10	5.20	6.29
Jig	Yield	2.51	0.61	$F_{0\cdot5}(4,2)=1.21$	0.07	1.28		0.44	1.87	17.08	3.86
	Clean Coal Ash	0.37	2.67	$F_{0\cdot9}(4,2)=\underline{9.24}$	-0.91	-0.36	$\pm t_{0\cdot05}(6)=\pm1.94$	-4.54	-2.43	13.24	17.76
	Reject Ash	0.52	0.38	$F_{0\cdot95}(4,2)=19.25$	1.53	1.58	$\pm t_{0\cdot025}(6)=\pm2.45$	6.00	6.57	10.35	11.03
WOC	Yield										
	Clean Coal Ash	0.83	0.05	$F_{0\cdot5}(2,3)=0.88$	1.19	-2.00		20.10	-1.45	41.30	1.77
	Reject Ash	0.42	1.17	$F_{0\cdot9}(2,3)=\underline{5.46}$	-1.13	-0.61	$\pm t_{0\cdot05}(5)=\underline{\pm2.02}$	-6.38	-3.58	13.82	14.44
		0.75	0.81	$F_{0\cdot95}(2,3)=9.55$	-1.27	-1.07	$\pm t_{0\cdot025}(5)=\pm2.57$	-10.33	-3.97	19.89	9.10

Table A-2 - F-test and t-test on results in Tables 5 to 7

Density Separator		F-Ratio		$F_P(q_1,q_2)$	t-value		$\pm t_{\alpha/2}(q)$	Mean		St. Deviation	
		DSM	M.DSM		DSM	M.DSM		DSM	M.DSM	DSM	M.DSM
HMC	Yield	0.90	0.75	$F_{0.5}(4,11)=0.89$	-0.32	-0.77		-0.21	-0.54	2.58	2.81
	Clean Coal Ash	0.84	0.84	$F_{0.9}(4,11)=2.54$	1.39	1.36	$\pm t_{0.05}(15)=\pm 1.75$	10.20	10.03	29.41	29.41
	Reject Ash	0.42	0.39	$F_{0.95}(4,11)=3.36$	-0.56	-0.73	$\pm t_{0.025}(15)=\pm 2.13$	-1.44	-2.00	10.20	10.92
HMB	Yield	21.32	18.97	$F_{0.5}(2,3)=0.88$	1.71	1.69		4.4	4.13	6.32	5.98
	Clean Coal Ash	9.85	9.85	$F_{0.9}(2,3)=5.46$	-0.61	-0.61	$\pm t_{0.05}(5)=\pm 2.02$	-2.38	-2.38	9.55	9.55
	Reject Ash	0.80	0.83	$F_{0.95}(2,3)=9.55$	-1.61	-1.60	$\pm t_{0.025}(5)=\pm 2.57$	-4.10	-4.12	6.29	6.29
Jig	Yield	0.61	0.83	$F_{0.5}(4,2)=1.21$	1.28	1.31		1.87	2.01	3.86	4.08
	Clean Coal Ash	2.67	2.84	$F_{0.9}(4,2)=9.24$	-0.36	-0.45	$\pm t_{0.05}(6)=\pm 1.94$	-2.43	-2.94	17.76	17.30
	Reject Ash	0.38	0.32	$F_{0.95}(4,2)=19.25$	1.58	1.38	$\pm t_{0.025}(6)=\pm 2.45$	-6.57	5.84	11.03	11.18

Table A-3 - F-test and t-test on +10 mm size fraction in Tables 1 to 3

Density Separator		F-ratio		$F_P(q_1,q_2)$	t-value		$\pm t_{\alpha/2}(q)$	Mean		St. Deviation	
		AF	RF		AF	RF		AF	RF	AF	RF
HMC	Yield	0.74	0.37	$F_{0.5}(2,7)=0.77$	0.78	-0.98		4.39	-0.93	17.84	2.99
	Clean Coal Ash	0.18	0.07	$F_{0.9}(2,7)=\underline{3.26}$	1.16	-0.05	$\pm t_{0.05}(9)=\underline{\pm 1.83}$	3.76	-0.15	10.27	9.68
	Reject Ash	0.18	0.33	$F_{0.95}(2,7)=4.74$	$\underline{-2.94}$	-0.99	$\pm t_{0.025}(9)=\pm 2.26$	-7.52	-3.03	8.09	9.64
HMB	Yield	0.68	$\underline{21.32}$	$F_{0.5}(2,3)=0.88$	0.18	1.71		2.27	4.4	30.08	6.32
	Clean Coal Ash	3.48	$\underline{9.85}$	$F_{0.9}(2,3)=\underline{5.46}$	-1.2	-0.61	$\pm t_{0.05}(5)=\underline{\pm 2.02}$	-4.52	-2.38	9.20	9.55
	Reject Ash	0.70	0.80	$F_{0.95}(2,3)=9.55$	-0.79	-1.60	$\pm t_{0.025}(5)=\pm 2.57$	-1.67	-4.10	5.20	6.29
Jig	Yield	4.95	1.20	$F_{0.5}(2,2)=1.00$	-0.08	0.97		-0.70	2.04	20.77	4.71
	Clean Coal Ash	0.39	4.87	$F_{0.9}(2,2)=\underline{9.00}$	-1.09	-0.08	$\pm t_{0.05}(4)=\pm 2.13$	-7.08	-0.74	14.48	20.94
	Reject Ash	0.80	0.38	$F_{0.95}(2,2)=19.00$	1.40	1.22	$\pm t_{0.025}(4)=\pm 2.78$	7.46	6.52	11.93	12.00

Table A-4 - F-test and t-test on -10 mm size fractions in Tables 1 to 4

Density Separator		F-ratio		$F_p(q_1,q_2)$	t-value		$\pm t_{\alpha/2}(q)$	Mean		St. Deviation	
		AF	RF		AF	RF		AF	RF	AF	RF
HMC	Yield	0.01	4.42	$F_{0\cdot5}(1,4)=0.55$	-1.48	$\underline{2.31}$		-5.10	1.00	8.42	1.06
	Clean Coal Ash	1.08	0.08	$F_{0\cdot9}(1,4)=\underline{4.54}$	1.40	1.56	$\pm t_{0\cdot05}(5)=\underline{\pm2.02}$	6.40	27.45	11.23	43.07
	Reject Ash	0.60	0.47	$F_{0\cdot95}(1,4)=7.71$	-1.84	0.26	$\pm t_{0\cdot025}(5)=\pm2.57$	-4.40	1.22	5.86	11.45
WOC	Yield	0.88	0.03	$F_{0\cdot5}(1,2)=0.67$	0.64	-1.89		4.75	-1.63	14.84	1.72
	Clean Coal Ash	0.13	3.26	$F_{0\cdot9}(1,2)=\underline{8.53}$	-0.68	0.34	$\pm t_{0\cdot05}(3)=\underline{\pm2.35}$	-2.30	1.50	6.80	8.87
	Reject Ash	0.92	0.01	$F_{0\cdot95}(1,2)=18.51$	-1.46	-1.67	$\pm t_{0\cdot025}(3)=\pm3.18$	-15.00	-7.43	20.54	8.91

Table A-5 - t-test on results in Tables 8 and 9

Composite size fraction +0.6 mm

	HMC				HMB				Jig			
	DSM		M.DSM		DSM		M.DSM		DSM		M.DSM	
	t-value	$\pm t_{0.05}(4)$	t-value	$\pm t_{0.05}(4)$	t-value	$\pm t_{0.05}(2)$	t-value	$\pm t_{0.05}(2)$	t-value	$\pm t_{0.05}(1)$	t-value	$\pm t_{0.05}(1)$
Yield	-1.16		-1.31		1.57		1.58		1.24		1.00	
Clean Coal Ash	-0.98	2.13	-1.03	2.13	-1.31	2.92	-1.31	2.92	-1.00	6.31	-1.00	6.31
Reject Ash	-3.13		-2.92		-2.17		-1.95		0.47		0.38	

Size fraction 2 x 0.6 mm

	HMC			
	DSM		M.DSM	
	t-value	$\pm t_{0.05}(4)$	t-value	$\pm t_{0.05}(4)$
Yield	4.17		3.77	
Clean Coal Ash	0.83	2.13	0.83	2.13
Reject Ash	1.80		1.62	

Table A-6 - F-test on results in Table 1

Density Separator				F-ratio		$F_p(q_1,q_2)$
				A.F.	R.F.	
HMC	Size Fractions 1-3 Plants A,B,D	Yield	SFE	1.32	0.62	
			OCE	4.22	2.19	
		Clean Coal Ash	SFE	0.04	0.15	$F_{0.9}(2,4)=4.32$
			OCE	0.78	0.61	$F_{0.9}(2,4)=4.32$
		Reject Ash	SFE	0.30	0.98	
			OCE	4.00	7.21	
	Size Fractions 3-5 Plants A,B,C	Yield	SFE	0.08	1.25	
			OCE	1.56	0.78	
		Clean Coal Ash	SFE	1.13	0.83	$F_{0.9}(2,4)=4.32$
			OCE	2.29	2.58	$F_{0.9}(2,4)=4.32$
		Reject Ash	SFE	1.12	6.66	
			OCE	1.77	24.77	
	Size Fractions 1-5 Plants A, B	Yield	SFE	0.96	1.06	
			OCE	1.86	0.89	
		Clean Coal Ash	SFE	1.07	1.77	$F_{0.9}(4,4)=4.11$
			OCE	0.78	1.29	$F_{0.9}(1,4)=4.54$
		Reject Ash	SFE	1.51	1.68	
			OCE	6.61	27.77	

Table A-7 - F-test on results in Tables 2 to 4

Density Separator			F-ratio		$F_P(q_1,q_2)$
			A.F.	R.F.	
HMB	Yield	SFE	0.46	37.69	
		OCE	0.01	3.30	
	Clean Coal Ash	SFE	2.80	11.28	$F_{0.9}(2,2)=9.00$
		OCE	0.42	1.44	$F_{0.9}(1,2)=8.53$
	Reject Ash	SFE	0.71	0.79	
		OCE	1.05	0.96	
Jig	Yield	SFE	1.09	0.91	
		OCE	2.50	0.02	
	Clean Coal Ash	SFE	2236.11	3.50	$F_{0.9}(1,1)=39.86$
		OCE	6724.00	0.02	$F_{0.9}(1,1)=39.86$
	Reject Ash	SFE	25.00	1.99	
		OCE	37.92	5.11	
WOC	Yield	SFE	1.08	0.03	
		OCE	1.91	0.04	
	Clean Coal Ash	SFE	0.34	1.20	$F_{0.9}(2,2)=9.00$
		OCE	0.44	1.08	$F_{0.9}(1,2)=8.53$
	Reject Ash	SFE	0.50	0.77	
		OCE	0.00	0.86	

Table A-8 - F-test on +10 mm size fractions in Tables 1 to 3

Density Separator			F-ratio		$F_p(q_1,q_2)$
			A.F.	R.F.	
HMC	Yield	SFE	1.32	0.62	
		OCE	4.22	2.19	
	Clean Coal Ash	SFE	0.04	0.15	$F_{0.9}(2,4)=4.32$
		OCE	0.78	0.61	$F_{0.9}(2,4)=4.32$
	Reject Ash	SFE	0.30	0.98	
		OCE	4.00	7.21	
HMB	Yield	SFE	0.46	37.69	
		OCE	0.01	3.30	
	Clean Coal Ash	SFE	2.80	11.28	$F_{0.9}(2,2)=9.00$
		OCE	0.42	1.44	$F_{0.9}(1,2)=8.53$
	Reject Ash	SFE	0.71	0.79	
		OCE	1.05	0.96	
Jig	Yield	SFE	1.09	0.91	
		OCE	2.50	0.02	
	Clean Coal Ash	SFE	2236.11	3.50	$F_{0.9}(1,1)=39.86$
		OCE	6724.00	0.02	$F_{0.9}(1,1)=39.86$
	Reject Ash	SFE	25.00	1.99	
		OCE	37.92	5.11	

Table A-9 - F-test on -10 mm size fraction in Tables 1 and 4

Density Separator			F-ratio		$F_p(q_1,q_2)$
			A.F.	R.F.	
HMC	Yield	SFE	0.03	2.27	
		OCE	5.95	0.03	
	Clean Coal Ash	SFE	12.08	0.34	$F_{0.9}(1,2)=8.53$
		OCE	21.31	7.83	$F_{0.9}(1,2)=8.53$
	Reject Ash	SFE	0.40	7.21	
		OCE	0.34	29.78	
WOC	Yield	SFE	0.71	0.05	
		OCE	0.63	2.20	
	Clean Coal Ash	SFE	0.07	1.69	$F_{0.9}(1,1)=39.86$
		OCE	0.10	0.04	$F_{0.9}(1,2)=39.86$
	Reject Ash	SFE	0.63	0.16	
		OCE	0.36	22.83	