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### STUDIES OF PLUME RISE DURING NEUTRAL AND STABLE CONDITIONS IN CANADA

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STUDIES OF PLUME RISE DURING NEUTRAL AND  
STABLE CONDITIONS IN CANADA

By

H. Whaley\* and G. K. Lee\*\*

Abstract

Plume rise data obtained during six years of research by the Canadian Combustion Research Laboratory (CCRL) has been evaluated and compared to the two forms of the Briggs relationship for neutral and stable conditions. The data were obtained from ten sources in five geographic regions of Canada in spring, fall and winter.

It has been shown that the data can be represented by the Briggs form of equation, particularly for stable conditions. In neutral conditions the data suggest a proportionality constant of 0.87 and a levelling-off at 15 stack heights rather than a 1.6 constant and a levelling-off at 10 stack heights as suggested by Briggs. In stable conditions the Briggs levelling-off value of 2.9 for the dimensionless plume rise is in good agreement with the findings in this paper, but this occurs at a dimensionless downwind distance of 18.4 rather than 2.4. A slight variation in the maximum plume rise with the bulk Richardson number over the dispersion zone has been noted for stable conditions.

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Nomenclature

$c_p$	=	specific heat at constant pressure, kJ/kg °C
$F$	=	buoyancy flux = $gQ/\pi c_p T$ , $m^4/s$
$g$	=	gravitational constant, $m/s^2$
$h$	=	stack height, m
$L$	=	characteristic length for a buoyant plume = $F/\bar{U}^3$ , m
$Q$	=	heat emission from stack kJ/s
$R_b$	=	bulk Richardson number = $\frac{gZ_p^2}{4\bar{U}^2 T} \left( \frac{\partial \theta}{\partial Z} \right)$
$S$	=	stability parameter = $(g/T)(\partial \theta / \partial Z)$ , $s^{-2}$
$T$	=	absolute temperature of ambient air, K
$\bar{U}$	=	mean wind speed over the plume height $Z_p$ , m/s
$X$	=	downwind distance, km
$\bar{Y}, \bar{Z}$	=	horizontal and vertical co-ordinates of the centre of mass flow, at any given cross-wind plume traverse, m
$Z_p$	=	height of plume top above terrain, m
$\bar{\Delta Z}$	=	plume rise above stack top, m
$\frac{\partial \theta}{\partial Z}$	=	vertical potential temperature gradient, °C/m
$\rho$	=	density of ambient air, $kg/m^3$

## Introduction

Before the oil embargo of 1973, cheap, plentiful energy and progressively more stringent environmental controls led to regulations that compelled industry to utilize clean fuels such as natural gas and distillate oil in direct-fired combustion equipment. Consequently, the use of coal declined dramatically in Canada and many coal mines were closed. The advent of potential energy shortages, however, led to the realization that energy supplies were not unlimited and that increased coal use would be needed to stretch dwindling oil and gas reserves. It also focused attention on the often conflicting requirements of clean air quality criteria and efficient energy utilization.

It is against these developments that the Canadian Combustion Research Laboratory (CCRL) of the Canada Centre for Mineral and Energy Technology (CANMET), plume dispersion research program was developed. The main objective of this program was to provide atmospheric dispersion parameters that could be used with confidence by the energy processing industries and by environmental regulators.

## The CCRL Plume Dispersion Research Program

### Background

In 1969 it was clear that reliable information on dispersion parameters was not available in Canada (1). Therefore, a comprehensive research program was undertaken by CCRL to study the atmospheric dispersion of buoyant plumes emitted from tall stacks located in various geographic regions of Canada characterized by:

- (1) Flat terrain.
- (2) Land adjacent to large bodies of water.
- (3) Rolling terrain or mountain foothills.
- (4) Deep mountain valleys and river valleys.
- (5) Arctic and sub-arctic regions.

An immersion probing methodology was developed which utilizes helicopter- and automobile-carried instrumentation to obtain three-dimensional data on plume dispersion. In addition, atmospheric temperature and wind vector profiles in the surface boundary layer, within and above the dispersion zone, are determined at locations both near and remote from the source. This meteorological data together with the synoptic weather maps provide the background information necessary for parameter correlation. Full details of the methodology have been described elsewhere (2).

The program was jointly sponsored with industry and meteorological support was provided by the Atmospheric Environment Service (AES) of Environment Canada. To date over six years of research have been completed and studies have been conducted in all five geographic regions. Some comparative studies have been conducted on the same source during different seasons or during the same season when emissions have been reduced by pollutant control strategies. Table I lists the sources studied during the six-year period, together with their geographic location, relevant emission parameters and source configuration.

## The Derivation of Plume Rise Parameters

### The CCRL Program

In the CCRL program, it has been found that the voluminous data obtained by aerial probing techniques can be evaluated best numerically by a three-step procedure which employs the method of finite differences. This method, which is mathematically rigorous, eliminates any discrepancies introduced by a subjective approach and minimizes errors due to acquiring data by instruments that have short response times.

The first step in the method involves the reconstruction of three or more cross-wind sections of the plume to show spatial concentration isopleths that are plotted from continuous SO<sub>2</sub> measurements along accurately located cross-wind traverse lines in space. The second step consists of digitizing the spatial co-ordinates (y, z) of each SO<sub>2</sub> contour of the plume cross-section to establish the co-ordinates of the centre of pollutant mass flow,  $\bar{Y}$  and  $\bar{Z}$  and the standard deviations  $\sigma_y$  and  $\sigma_z$ . The data can then be used to reconstruct plan and side views of the plume on which the centre of pollutant mass  $\bar{Y}$  and  $\bar{Z}$  can be accurately located. Plume rise is determined from these plots which show the horizontal (X) and vertical (Z) variation of the plume axis with downwind distance. In addition, some measurements of plume rise were obtained from side views derived from variable altitude traverses along the axis of the plume.

In simple dispersion models the plume axis is usually a line in the mean wind direction at the effective height of emission. In a more complex model detailed measurements of the spatial variation of wind and temperature together with topographical information may be used to predict closely the complex variation in the plume axis that may occur in practice. The importance of an accurate prediction of the plume axis cannot be overestimated since it is a key parameter in the dispersion process which in turn influences ground-level impingement concentrations.

By definition the plume rise is that portion of the plume axis between the stack top and the levelling-off point. In general the term plume rise becomes inappropriate beyond about 2 km from the source because of differential dispersion rates and assimilation of plume buoyancy by the atmosphere. However, the plume axis may still be rising or falling in this region, and it is instructive to compare measured plume rise values at these intermediate downwind distances with the levelling-off values predicted in the literature (3). For convenience the term plume rise will be applied to all data discussed in this paper even though it may be beyond the levelling-off point.

### Neutral and Stable Plume Rise Parameters

A comprehensive analysis of plume rise under a variety of meteorological conditions has been published by Briggs (3). He concluded that the rise of buoyant plumes in stable and neutral conditions up to their levelling-off point can be represented by an equation of the form:

$$\overline{\Delta Z} = K \left( \frac{FX^2}{U^3} \right)^{1/3} \quad 1$$

where  $K = 1.6$

This equation reduces to the dimensionless relationship

$$\frac{\overline{\Delta Z}}{L} = K \left( \frac{X}{L} \right)^{2/3} \quad 2$$

where  $L = \frac{F}{U^3}$  is a characteristic length for a

a buoyant plume.

The levelling-off point in neutral conditions occurs at some downwind distance usually designated by the number of stack heights from the emission source. This results in the following modification to equation 2:

$$\frac{\overline{\Delta Z}}{L} = K \left( \frac{nh}{L} \right)^{2/3} \quad 3$$

where  $\overline{\Delta Z}$  is the final plume rise and  $h$  is the stack height. Briggs, in a simple analysis predicts a levelling-off at  $n = 10$ .

For stable conditions, equation 1 is valid up to  $X = 2.4 US^{-1/2}$ . Beyond this downwind distance, according to Briggs, the final plume rise is defined as:

$$\overline{\Delta Z} = 2.9 \left( \frac{F}{US} \right)^{1/3} \quad 4$$

$S$  is the stability parameter for normalizing stable plume-rise data.

In previous publications (4,-9) the authors have shown that the above relationships give useful and valid interpretations of measured plume-rise data. However, these data indicated that parameters other than the Briggs stability parameters (5) are required to interpret more fully the data for plume rise in stable conditions. The parameters used in this paper is a modified form of the bulk Richardson number suggested in the literature (10, 11, 12).

$$R_b = \frac{g Z_p^2}{4\bar{U}^2 T} \left( \frac{\partial \theta}{\partial Z} \right) \quad 5$$

From an examination of Equation 5 it is clear that the bulk Richardson number will increase as stability increases and that the dry adiabatic lapse rate corresponds to  $R_b = 0$ . Unstable conditions represent  $R_b < 0$  and the authors have categorized  $R_b > 0$  combined with potential temperature gradients higher than isothermal ( $> 5^\circ\text{C/km}$ ) as stable.

## Discussion of Measured Plume Rise Data

### Neutral Conditions

Previous papers by the authors (4-9) have utilized the Briggs relationship for neutral plume-rise evaluations. In these papers, which represent individual case studies mentioned in Table I, the 2/3 power law dependence of the plume rise data with downwind distance is endorsed; however, the value of the proportionality constant is usually lower than the 1.6 quoted by Briggs. Figure 1 is a plot of the CCRL data for the buoyant plume-rise region and the following equation is based on a regression analysis of the data

$$\left(\frac{\overline{\Delta Z}}{L}\right) = 0.87 \left(\frac{X}{L}\right)^{2/3} \quad 6$$

where  $X < 15$  h. An examination of these data show that levelling-off occurred at  $X = 15$  h rather than at  $X = 10$  h as suggested by Briggs; in addition the regression constant of 0.87 is much lower than the Briggs widely accepted value of 1.6. Figure 2 shows the plume-rise data at  $X > 15$  h correlated with the predicted levelling-off values from Equation 6, and the correspondance is seen to be excellent.

### Stable Conditions

It has already been noted that stable plumes have a tendency to level-off and then return back to the emission level at large downwind distances (4-9) as the plume buoyancy is assimilated by the atmosphere. Figure 3a shows the variation of the limiting plume rise with downwind distance under stable conditions. In this plot, excellent agreement with Briggs' limiting value of  $\overline{\Delta Z} (US/F)^{1/3} = 2.9$  can be observed, with only a slight decrease with increasing distance. However, the limiting downwind distance was at  $XS^{1/2}/U = 18.4$  rather than at 2.4 as predicted by Briggs.

The correlation of the limiting plume rise with bulk Richardson number was excellent and can be represented by:

$$\overline{\Delta Z} (US/F)^{1/3} = 2.46 R_b^{0.042} \quad 7$$

### Conclusions

1. Measured plume rise values obtained in six geographic regions of Canada have shown good agreement with the relationships suggested by Briggs for both neutral and stable conditions.
2. The Briggs equation for plume rise in neutral conditions in general overestimated the CCRL data by about 85%. The CCRL data, however, closely followed Briggs 2/3 power law dependence with downwind distance. It was also found that plumes tend to level-off at  $X = 15$  h rather than at  $X = 10$  h as suggested by Briggs.

3. The Briggs equation for stable conditions gave an excellent prediction of the final rise of the plumes, but in general this occurred at a dimensionless distance of 18.4 rather than at 2.4. It was found that the final plume rise could be correlated with the bulk Richardson number in the form of a 0.042 power law relationship. In stable conditions the bulk Richardson number, developed in the paper, appears to provide a more definitive classification than either S as defined by Briggs or classifications E and F as defined by Pasquill, for assessing the maximum buoyant plume rise.



List of Figure Captions

- Figure 1      Correlation of dimensionless plume rise with dimensionless downwind distance; neutral conditions, buoyant plume rise region.
- Figure 2      Plot of measured limiting dimensionless plume rise versus predicted values for neutral conditions.
- Figure 3      Variation of dimensionless plume rise with bulk Richardson number and dimensionless downwind, stable condition.

References

1. H. Whaley, "A Proposed Canadian Standard for Estimating Atmospheric Dispersion of Combustion Source Pollution From Chimneys," Atmospheric Environment, 3. 177 (1969).
2. H. Whaley, "The Derivation of Plume Dispersion Parameters From Measured Three-Dimensional Data," Atmospheric Environment, 8. 281 (1974).
3. G. A. Briggs, Plume Rise, U.S.A.E.C. TLD 25075, 38 (1968).
4. H. Whaley and G. K. Lee, "Plume Dispersion From a Thermal Power Station on the Shore of a Large Lake," J Inst Fuel:47:242 (1974).
5. H. Whaley and G. K. Lee "Plume Dispersion From a Lignite-Fired Power Station in Flat, Rural Terrain," ASME Winter Annual Meeting, New York, Paper No. 76-WA/APC-7 (1976).
6. H. Whaley and G. K. Lee, "Plume Dispersion in a Mountainous River Valley During Spring," J Air Pollut Control Assoc: 27, 10,10001 (1977).
7. H. Whaley and G. K. Lee, "An Assessment of Plume Dispersion Parameters Measured in Fall and Winter at a Tar-Sands Refinery Complex," J Air Pollut Control Assoc: 28, 6, 589 (1978).
8. H. Whaley and G. K. Lee, "The Behaviour of Buoyant Merging Plumes in the Rocky Mountain Foothills," Air Pollut Control Assoc 72nd Annual Technical Meeting: Paper 79-8.1 Cincinnati (1979) submitted for publication in J Air Pollut Control Assoc.
9. H. Whaley, G. K. Lee and J. G. Gainer, "Plume Dispersion From a Natural-Gas Sulphur Extraction Plant Under a Persistent Elevated Inversion," 13th World Gas Conference: Paper IGU-B10-76 London (1976).
10. S. R. Hanna, G. A. Briggs, J. Deardorff, B. A. Egan, F. A. Gifford and F. Pasquill, "AMS Workshop on Stability Classification and Sigma Curves - Summary of Recommendations," Bulletin American Meteorological Society 58, 12, 1305 (1977).
11. R. R. Braxler "Determination of Atmospheric Diffusion Parameters," Atmospheric Environment 10. 99 (1976).
12. R. E. Munn, "Biometeorological Methods" Academic Press, New York, 161 (1970).

TABLE I. EMISSION SOURCE DATA  
FROM THE CCRL PLUME RESEARCH PROGRAM

Source Type	Geographic Region*	Stack Heights, m	Total SO <sub>2</sub> Emission kg/s**	Total Heat Emission MJ/s
Coal-Fired Power Station	2,U F,S	1 152	5.0	220
		2 152		
		3 152		
		4 152		
Smelter	1,R F	1 152	50.0	936
		2 152		
		3 106		
Coal-Fired Power Station	1,R F,W	1 92	0.6	120
		2 92		
		3 56		
Refinery	4,A F,W	1 106	1.9	104 W 78 F
		2 106		
		3 106	6.5	
		4 76		
Sour Gas Plant	3,R S	1 152	2.7	125
			1.2**	118**
Sour Gas Plant	3,R	1 107	0.2 0.3**	21 5.2**
Smelter	4,R S	1 122	0.8	25
		2 122		
		3 87		
Sour Gas Plant	3,R S	1 135	1.8 23.0**	125 1250**
Sour Gas Plant	3,R S	1 98	0.5	12.6
Sour Gas Plant	3,R S	1 61	1.0	21.2

\* Legend

- |   |                |            |
|---|----------------|------------|
| 1. Flat terrain.                            | U = Urban      | F = Fall   |
| 2. Land adjacent to large bodies of water.  | R = Rural      | W = Winter |
| 3. Rolling terrain or foothills.            | A - Sub-arctic | S = Spring |
| 4. Shallow or deep river, mountain valleys. |                |            |

\*\*Repeat study, same season at same plant.

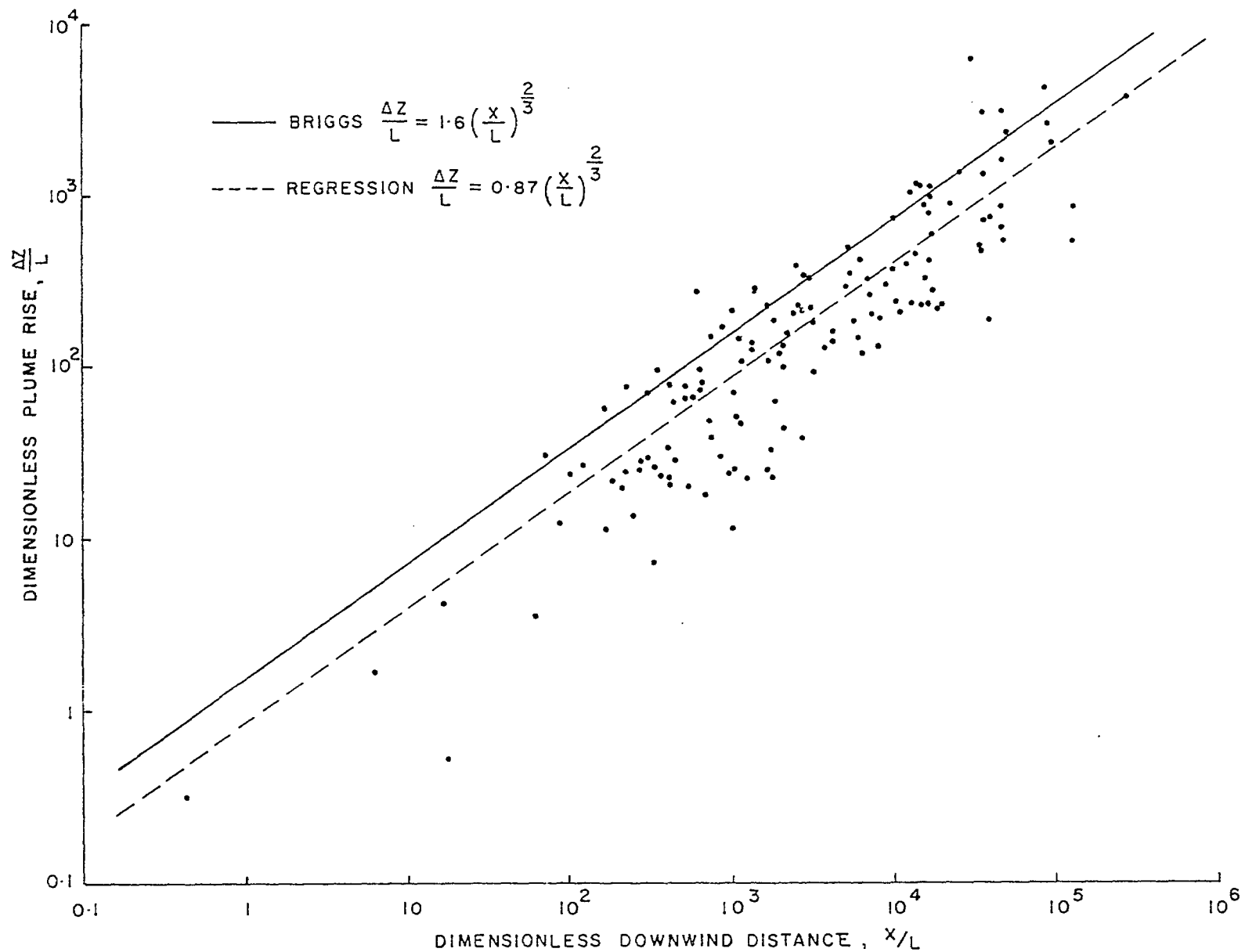


FIGURE 1. Correlation of dimensionless plume rise with dimensionless downwind distance; neutral conditions, buoyant plume rise region.

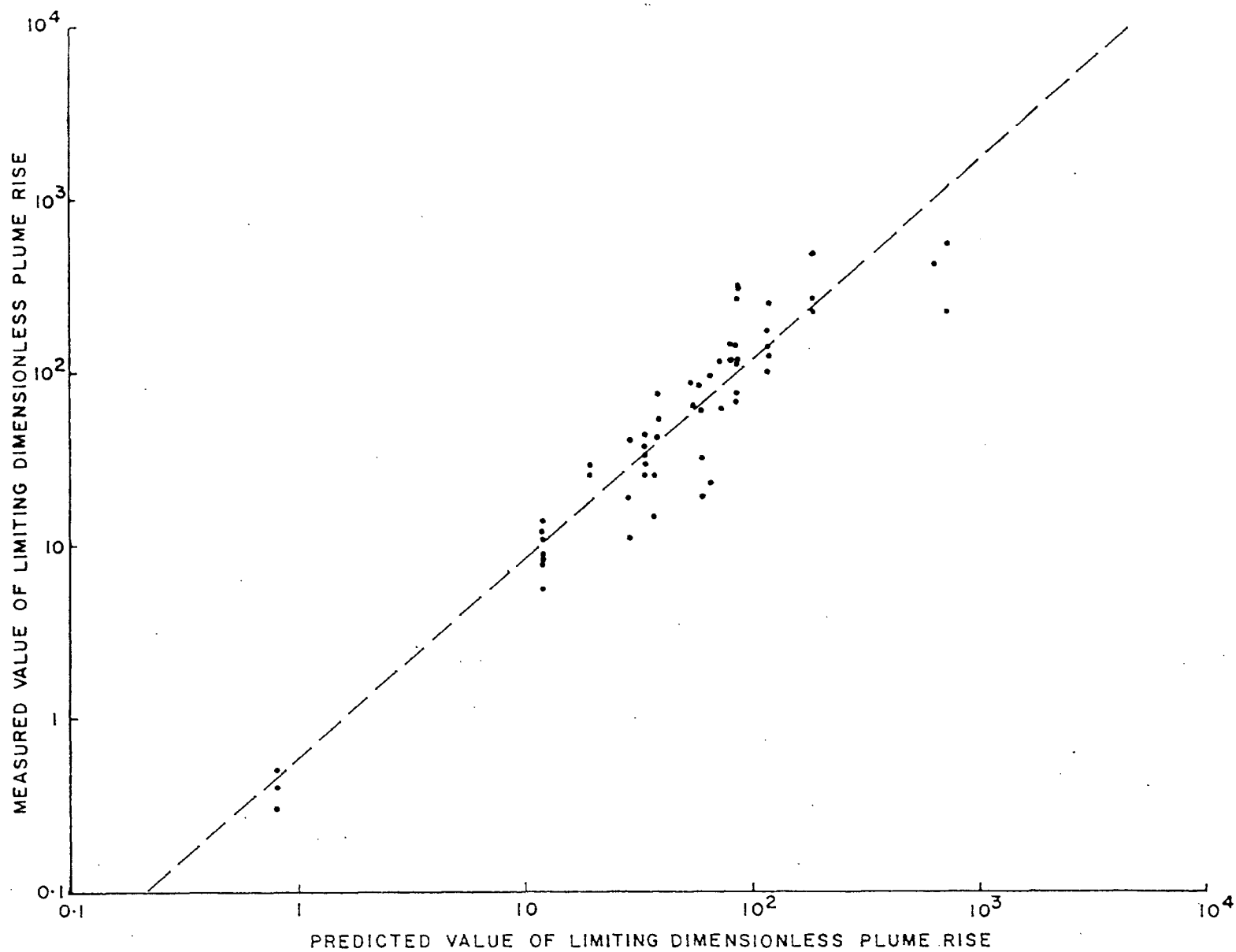
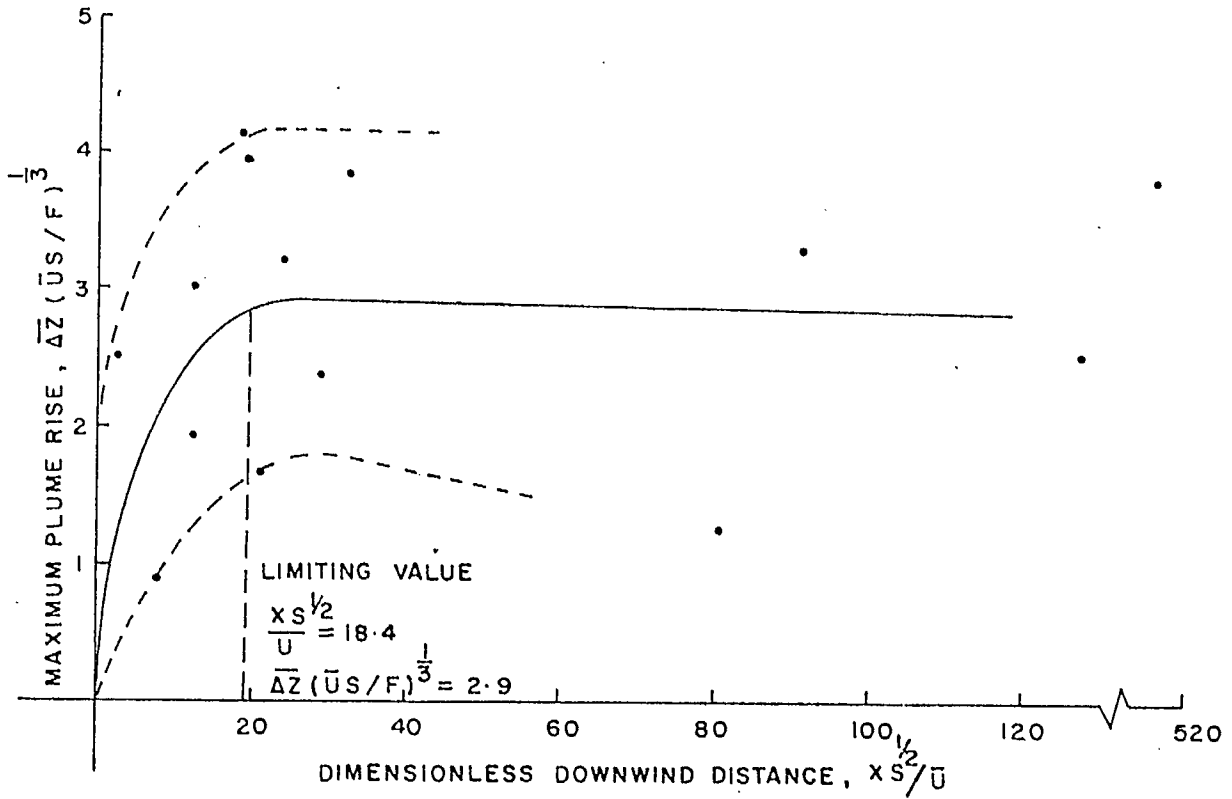
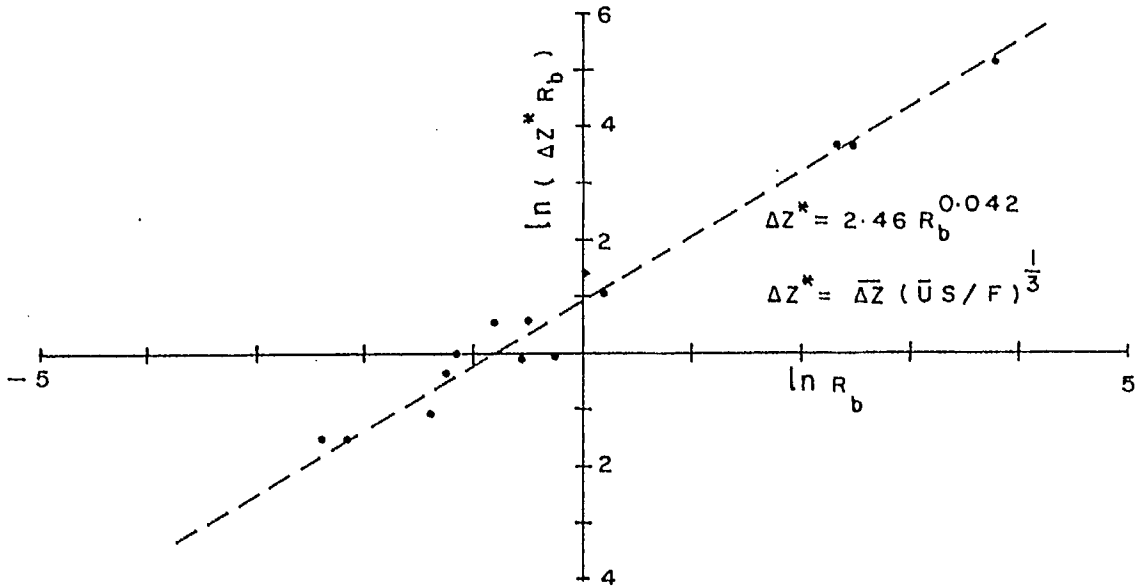


FIGURE 2. Plot of measured limiting dimensionless plume rise versus predicted values for neutral conditions.



a) Variation of limiting rise with downwind distance.



b) Variation of limiting plume rise with bulk Richardson number.

FIGURE 3. Variation of dimensionless plume rise with bulk Richardson number and dimensionless downwind, stable condition.