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THE DETERMINATION OF MEASURED DISPERSION PARAMETERS  
FOR PLUMES FROM TALL STACKS IN CANADA

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## THE DETERMINATION OF MEASURED DISPERSION PARAMETERS FOR PLUMES FROM TALL STACKS IN CANADA

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### SUMMARY

An immersion probing methodology which utilizes synchronized helicopter- and automobile-carried instrumentation has been developed which provides three-dimensional data on plume dispersion together with atmospheric temperature and wind vector profiles within the dispersion zone.

A rigorous numerical method utilizing finite differences to determine plume axis elevations and downwind distances together with horizontal and vertical standard deviations of plume spread from the measured plume concentration profiles is described. Some comparisons of measured values are made with accepted methods of prediction which illustrate the kind of discrepancies that can arise in the absence of reliable measured data.

The derivation of plume dispersion parameters from regionally measured data significantly improves the precision of dispersion modelling for specifying stack heights, for selecting new plant locations and for predicting ground-level impingement profiles. The often conflicting requirements of energy conservation and environmental impact are then more readily resolved.

### INTRODUCTION

In the 1960's when energy was cheap and supposedly plentiful, environmental concerns resulted in legislation that compelled fuel users to burn clean fuels and the use of coal in direct-fired equipment declined in Canada. Then in the early seventies the energy crisis led to the realization that energy supplies were not unlimited and that conservation measures were needed to stretch out dwindling reserves. This led to a re-evaluation of the use of coal and recognition of the fact that some resolution of the often conflicting requirements of clean environment and energy conservation was needed. It was against these developments that the plume dispersion research program of the Canadian Combustion Research Laboratory (CCRL) of the Department of Energy, Mines and Resources was developed. The objective of this program was to provide atmospheric dispersion parameters that could be used with confidence by both energy processing industries

and by environmental control authorities for optimizing plant site and chimney height selection with emission and ambient criteria.

In 1969 it was clear that reliable information on dispersion parameters was not available in Canada (Whaley 1969). Therefore, a comprehensive research program was undertaken by CCRL to study the atmospheric dispersion of buoyant plumes emitted from tall stacks located in various geographic regions of Canada characterized by:

- (1) flat terrain;
- (2) land adjacent to large bodies of water;
- (3) rolling terrain or mountain foothills;
- (4) deep mountain valleys and river valleys;
- (5) Arctic and Sub-arctic regions.

The program was sponsored jointly with industry and meteorological support was provided by the Atmospheric Environment Service of Environment Canada. To date over 6 years of research have been completed and studies have been conducted in all 5 geographic regions. Some comparative studies have been conducted on the same source during different seasons or during the same season when emissions have been reduced by pollutant control strategies.

#### THE CCRL PLUME DISPERSION RESEARCH PROGRAM

##### Dispersion Modelling Background

Most dispersion models, being derived from statistical or physical principles, are gaussian in nature. In such a model the gases emitted from a stack become distributed across the plume in a gaussian or normal distribution. Thus, in three-dimensions, if axial diffusion is neglected, the model represents a bivariate normal distribution in the plane normal to the plume axis. A gaussian distribution may be completely defined by its standard deviation if represented in dimensionless form. Therefore, in the bivariate three-dimensional model, the horizontal and vertical standard deviations  $\sigma_y$  and  $\sigma_z$ , respectively have been used to report plume dispersion spread parameters in the literature. The most commonly used simple dispersion model is represented by the equation:

$$C_{xyz} = \frac{E \times 10^6}{2\pi\sigma_y\sigma_zU} \exp \left[ -1/2 \left( \frac{y}{\sigma_y} \right)^2 \right] \left\{ \exp \left[ -1/2 \left( \frac{z-h}{\sigma_z} \right)^2 \right] + \exp \left[ -1/2 \left( \frac{z+H}{\sigma_z} \right)^2 \right] \right\} \quad (1)$$

in which total reflection from the ground plane is assumed and  $\sigma_y$  and  $\sigma_z$  are horizontal and vertical standard deviations of plume spread respectively at that particular downwind distance.

Pioneers of the concept of using standard deviations of plume spread were Pasquill (1961, 1962) who used angular values and Gifford (1961) who converted these to linear dimensions and developed the well known graphical representation.

#### The CCRL Plume Dispersion Methodology

It is clear from the above examination of simple plume dispersion models that, in order to be able to determine  $\sigma_y$  and  $\sigma_z$ , complete concentration profiles of some gaseous or particulate constituent within the plume are required in the plane perpendicular to the plume axis. In order to define a plume, 3 or 4 downwind cross-sections should be so determined at distances dependent on the strength of emission of the particular gaseous or particulate constituent being measured. In addition to this it must be recognized that the atmosphere can change rapidly and the studies must be completed during relatively steady meteorological conditions to be meaningful. Using an immersion probing methodology a number of field research studies were conducted to obtain factual information on the dispersion of plumes from both single and multiple stacks under a variety of meteorological, topographical and seasonal conditions. In each study the plume was monitored in three-dimensions for relevant pollution parameters using co-ordinated helicopter- and vehicle-mounted instruments. The instrument package used comprised a continuous  $SO_2$  analyser with 2-1/2-second response together with continuous, instantaneous measurements of plume temperature, ambient temperature, dew-point depression and pressure-height. The sample probe was mounted externally on the helicopter or automobile well forward of any vehicle interference such as rotor downwash. Continuous tracking of the aerial probe to within 3 metres was accomplished by using a radar navigational system. Each flight was planned to last 1-1/2 to 2 hours in order to minimize the risk of meteorological changes. In general the following procedure was followed to obtain quantitative data on plume characteristics:

- (1) Temperature soundings were taken about 1 kilometre upwind and about 2 kilometres downwind of the source prior to the commencement of the plume measurements. In selected cases temperature soundings were made at the farthest downwind traverse to measure any axial variation in vertical temperature gradient.

- (2) At least 3 cross-wind traverses were flown through the plume at distances determined largely by the strength of the  $\text{SO}_2$  emission and the atmospheric stability. Each traverse consisted of a series of level passes through the plume at height intervals of between 25 and 250 metres depending on the thickness of the plume. Concurrently with the helicopter flights, automobiles equipped with similar instrumentation were used to traverse areas of ground-level plume impingement wherever access permitted. In some cases during very stable conditions when the plume was very wide horizontally and thin vertically a series of vertical sawtooth passes was used to define the plume cross-section. When there was no automobile access beneath the plume, low-level passes were made by helicopter.

Prior to and during each study, vertical profiles of temperature, wind speed and direction were measured near the source using radiosonde and pilot balloons. In some instances, additional information was obtained from tower-mounted meteorological instruments and stationary ground-level  $\text{SO}_2$  monitors.

The plume dispersion data, after reduction, were plotted as two-dimensional  $\text{SO}_2$  contour maps in the cross-wind (y, z) direction (Figure 1). These maps which incorporate some degree of data interpretation were used to construct plan and side views of the plume showing the maximum projected horizontal and vertical spread of selected isopleths (Figures 2 and 3). The meteorological data obtained, together with synoptic weather maps, provide the background information necessary for further interpretation.

Although data have been accumulated in all of the five aforementioned geographic regions, this paper can only give examples from some of the studies and refer the reader to the references for a more detailed analysis of each study (Whaley and Lee, 1971, 1972 etc).

## THE DERIVATION OF DISPERSION PARAMETERS

### Estimation Methods

When using ground-based scanning techniques such as photography, lidar or cospec to study plume behaviour it is very difficult to make precise measurements of the standard deviation of plume spread, particularly at large downwind distances. In particular, a photograph is very dependent on plume visibility which in turn is dependent on many other factors such as particulate or aerosol content, nature and

size distribution of particulates or aerosols, amount of sunlight and other factors. In such a case, lacking precise concentration profile data, a normal distribution is assumed and spread parameters derived by simple calculation. However, this procedure is limited by the fact that many concentration profiles are non-gaussian particularly on a short term basis or close to the point of emission. Because of the imprecise nature of the available means of calculation a more rigorous numerical method was developed to estimate dispersion parameters from the CCRL plume dispersion research program (Whaley 1974).

#### The CCRL Finite Difference Method

In the CCRL program, it has been found that the voluminous data obtained by aerial probing techniques can be evaluated best numerically by a three-step procedure which employs the method of finite differences. This method, which is mathematically rigorous, eliminates any discrepancies introduced by a subjective approach and minimizes errors due to acquiring data by instruments that have short response times.

The first step in the method involves the reconstruction of three or more cross-wind sections of the plume to show spatial concentration isopleths that are plotted from continuous  $\text{SO}_2$  measurements along accurately located cross-wind traverse lines in space. The second step consists of digitizing the spatial co-ordinates  $(y, z)$  of each  $\text{SO}_2$  contour of the plume cross-section to establish the co-ordinates of the centre of pollutant mass flow,  $\bar{y}$  and  $\bar{z}$ , and the standard deviations  $\sigma_y$  and  $\sigma_z$  according to the equations given below.

The mass of pollutant per unit length of plume is:

$$q = \int \int_A C \, dy \, dz. \quad (2)$$

the centre of pollutant mass flow is the first moment:

$$\bar{y} = \frac{1}{q} \int \int_A Cy \, dy \, dz. \quad (3)$$

and the variance is the second moment about the axis

$$\sigma_y^2 = \frac{1}{q} \int \int_A C(y - \bar{y})^2 \, dy \, dz \quad (4)$$

Similarly for the vertical dimensions,

$$\bar{z} = \frac{1}{q} \int \int_A Cz \, dz \, dy. \quad (5)$$

and

$$\sigma_z^2 = \frac{1}{q} \int \int_A C(z - \bar{z})^2 dz. dy. \quad (6)$$

If these integral equations are translated into finite difference form then (2) becomes

$$Q = \Sigma \Delta C \Sigma Y \Delta Z \quad (7)$$

and the co-ordinates of the centre of pollutant mass (3) and (5) respectively become

$$\bar{Y} = \frac{1}{2Q} \Sigma \Delta C \Sigma Y^2 \Delta Z \quad (8)$$

$$\bar{Z} = \frac{1}{2Q} \Sigma \Delta C \Sigma Z^2 \Delta Y \quad (9)$$

Likewise the variances (4) and (6) respectively become

$$\sigma_y^2 = \frac{1}{3Q} \Sigma \Delta C \Sigma (Y - \bar{Y})^2 \Delta Z \quad (10)$$

and

$$\sigma_z^2 = \frac{1}{3Q} \Sigma \Delta C \Sigma (Z - \bar{Z})^2 \Delta Y \quad (11)$$

In the third step, these equations are codified for the computer and used in conjunction with digital input from the cross-wind SO<sub>2</sub> contour maps to derive values for critical parameters used in plume rise and dispersion computations.

#### Corrections for Traversing and Source Effects

For comparison with published data on single stack emissions some corrections must be applied to the derived values of the dispersion parameters  $\sigma_y$  and  $\sigma_z$  from line or small area sources. Basically, the approach used by CCRL which is given in detail in Whaley (1974) is to estimate an equivalent standard deviation of the source. Corrections can then be made easily using angular similarity. However, it should be noted that, other than the traverse angle cosine correction to the horizontal spread,  $\sigma_y$ , it is the uncorrected measured values which must be used in computations for the particular source being studied. A suggestion for correction of measured values of  $\bar{Z}$  and  $\sigma_z$  for ground impingement effects has been made by Rowe (1974).

## COMPARISON OF MEASURED PARAMETERS WITH ACCEPTED METHODS OF PREDICTION

### Plume Axis

The plume axis has many interpretations. In a simple model, the plume axis in the horizontal plane is a line in the mean wind direction. In the vertical plane it is a line at the effective height of emission. In a more complex model, detailed wind vector and temperature data may generate complex variations in the spatial location of the plume axis (Lanz et al. 1972). However, for three-dimensional plume dispersion data such as that generated by the CCRL program, the plume axis is represented by the centre of pollutant mass  $\bar{Y}$  and  $\bar{Z}$ . In conjunction with a plan view such as that shown in Figure 3, the plume axis also defines the downwind distance against which all measured dispersion parameters are plotted. An overall representation of the plume is shown in Figure 4.

It can be seen that the derived value of  $\bar{Z}$ , the vertical co-ordinate of the centre of pollutant mass, represents the height of the plume axis above the reference plane,  $z = 0$ . Therefore,  $\Delta\bar{Z}$ , the plume axis elevation above the stack, can be defined by:

$$\Delta\bar{Z} = \bar{Z} - (h_s + Z_g) \quad (12)$$

Some examples of measured values of  $\Delta\bar{Z}$  compared with values calculated from the accepted formulae of Briggs (1969) are given in Table I. It should be noted that none of the values given in Table I for neutral conditions exceed the levelling-off point suggested by Briggs in his analysis.

### Plume Standard Deviations

Table II shows comparisons of some measured  $\sigma_y$  and  $\sigma_z$  values with those obtained after Pasquill (1961, 1962) and Gifford (1961) and represented conveniently by Bowne (1974) for rural terrain. These results are only presented as examples and no conclusions are drawn about the deviations from Pasquill. Such deviations are the subject of continuing study by CCRL of the plume dispersion data and the references listed contain the evaluations that have been made to date.

### Applications of Derived Parameters

The concentration of industrial processes in certain geographic regions has created a growing need to ensure that stack-emitted pollutants become sufficiently dispersed in the atmosphere to meet legislated or other ambient air quality



requirements. Thus, the acquisition of replicate factual data on plume dispersion under a variety of conditions will permit the derivation of parameters for the statistical validation of plume dispersion models. These derived parameters, particularly  $Z$  and  $\sigma_z$  which significantly influence the precision of ground-level concentration estimates, will enable more meaningful assessments to be made in the following important areas:

- (1) assessment of heat processes to minimize heat losses in stack gases and to reduce energy penalties due to pollution control equipment;
- (2) selection of adequate stack heights to avoid excessive pollutant levels at receptor locations;
- (3) optimization of the locations of new plant sites in relation to existing industrial plants to prevent environmental degradation.

#### CONCLUSIONS

It is possible to determine accurate values of plume dispersion parameters from detailed cross-wind concentration profiles. Such numerically derived values effectively eliminate errors that can be caused by subjective or arbitrary interpretation of measured data.

The derivation of reliable plume dispersion parameters that are specific to a particular geographic region will increase the confidence in the application of such parameters in similar geographic regions. Factual input data, in addition to providing the needed validation of atmosphere dispersion models, will enable better resolution of the economic costs and energy penalties associated with pollution control measures such as emission levels, tall stacks, exit gas temperature and pollutant removal.

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#### NOTATION

A	cross-sectional area of plume ( $m^2$ )
C	concentration of tracer (ppm $SO_2$ )
E	emission rate of tracer ( $Sm^3/s$ $SO_2$ )
h	effective height of emission, $\bar{Z} - Z_g$ (m)
$h_s$	height of stack above ground at $x = 0$ (m)
q, Q	pollutant mass flows defined in (2) and (7) (ppm- $m^2$ )
U	mean wind speed over dispersion zone (m/s)
x, y, z	three-dimensional cartesian co-ordinates in downwind, cross-wind and vertical dimensions respectively (km, m)
$\bar{y}$ , $\bar{Y}$ and $\bar{z}$ , $\bar{Z}$	co-ordinates of centre of pollutant mass flow in cross-wind and vertical dimensions respectively (km, m)
$Z_g$	height of ground elevation above $z = 0$ at measuring location (m)
$\Delta C$ , $\Delta Y$ and $\Delta Z$	finite difference forms of above defined variables
$\overline{\Delta Z}$	elevation of plume axis above stack top, $\bar{Z} - h_s - Z_g$ (m)
$\sigma_y$ , $\sigma_z$	standard deviations of plume spread in the cross-wind and vertical dimensions respectively (m)

TABLE I

Comparison of Measured Plume Axis Elevations  
With Calculated Values (after Briggs, 1969)

Source Type	1	1	2	2	3	4	5
Stack Height (m)	152	152	91	91	66	106	122
Heat Flux (mcal/s)	50.8	29.4	24.0	34.0	11.9	16.6	4.6
Ambient Temp. (°C)	29.0	9.8	7.5	-18.2	8.0	6.6	19.6
Potential Temp. Grad. (°C/100 m)	0.12	1.65	-0.09	3.30	0.10	3.81	0.07
Wind Speed (m/s)	11.9	11.8	9.7	7.8	6.0	2.5	3.7
Atmospheric Stability	Neutral	Stable	Neutral	Stable	Neutral	Stable	Neutral
Axial Distance (km)	0.66	1.30	0.55	1.10	0.33	2.05	1.07
Measured Plume Axis Elev. (m)	66	87	90	166	154	223	25
Briggs Value (m)	113	157	109	150	47	164	74
Source Type	<ol style="list-style-type: none"> <li>1. Land adjacent to large body of water, urban</li> <li>2. Flat terrain, rural</li> <li>3. Foothills, rural</li> <li>4. Shallow river valley, rural, sub-arctic</li> <li>5. Deep mountain valley, rural</li> </ol>						

TABLE II

Comparison of Derived Plume Standard Deviations With Predicted Values (after Bowne 1976 and Pasquill 1961, 1962)

Source Type	Atmospheric Stability	Pasquill Stability	Downwind Distance (km)	Measured $\sigma_y$ $\sigma_z$ (m)	Predicted $\sigma_y$ $\sigma_z$ (m)
1	Neutral	D	0.66	37.4	47.3
				18.0	22.6
			3.50	189	215
				55.0	71.0
1	Stable	E	6.10	517	357
				184	104
			1.30	91.7	64.9
				45.4	25.0
1	Stable	E	4.10	314	182
				38.4	50.3
			12.50	1147	494
				65.2	88.9
2	Neutral	D	0.60	91.0	43.3
				85.2	21.1
			1.20	203	81.4
				106	34.8
2	Stable	F	3.00	407	187
				148	64.2
			1.10	193	34.6
				68.8	14.8
2	Stable	F	2.70	274	78.1
				77.6	25.1
			6.00	606	161
				59.0	37.2
3	Neutral	D	0.33	300	25.2
				91.2	13.2
			3.22	499	200
				89.3	67.2
3	Neutral	D	5.11	510	304
				303	90.3
			2.05	549	60.8
				25.9	21.6
4	Stable	F	13.24	2778	330
				18.9	51.6
			28.17	3330	653
				22.6	67.5
5	Neutral	D	1.07	255	73.3
				94.0	32.2
			3.75	638	229
				178	74.3
5	Neutral	D	5.50	913	325
				281	95.6

DATE : 24-4-72  
TIME : 1516 - 1636 MST  
ALL ISOPLETHS IN PPM SO<sub>2</sub>

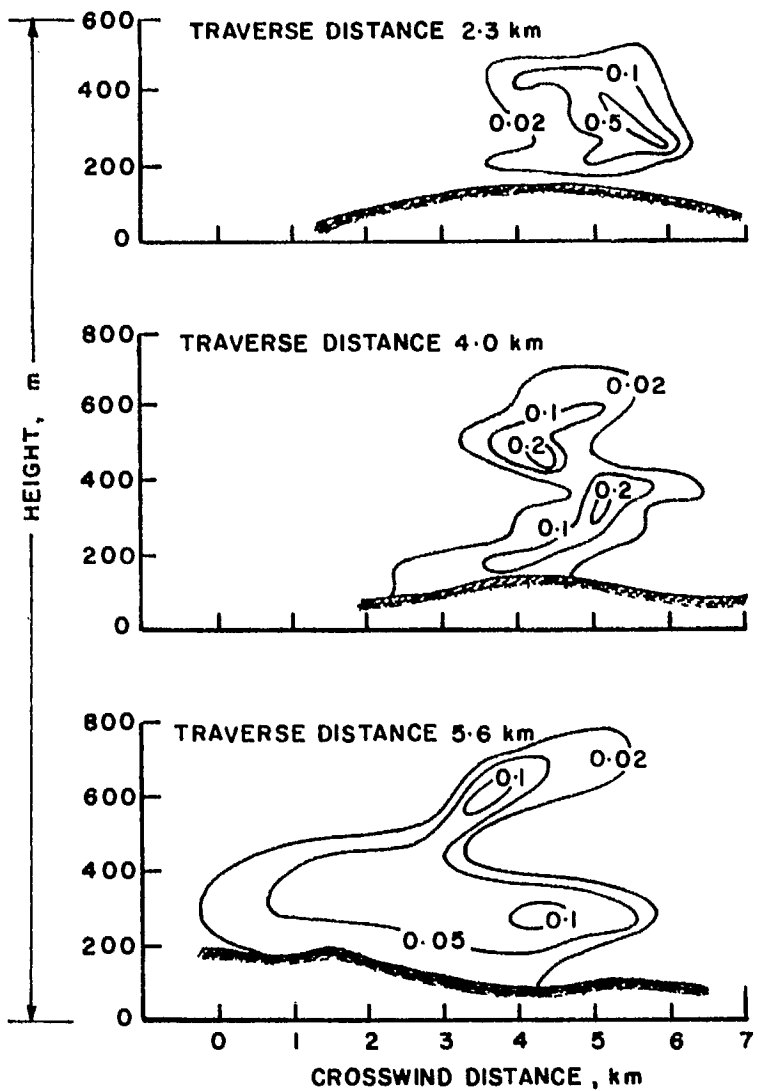


FIGURE 1. Measured Vertical Cross-wind Sections of a Typical Plume

DATE : 24-4-72

TIME : 1516-1636 MST

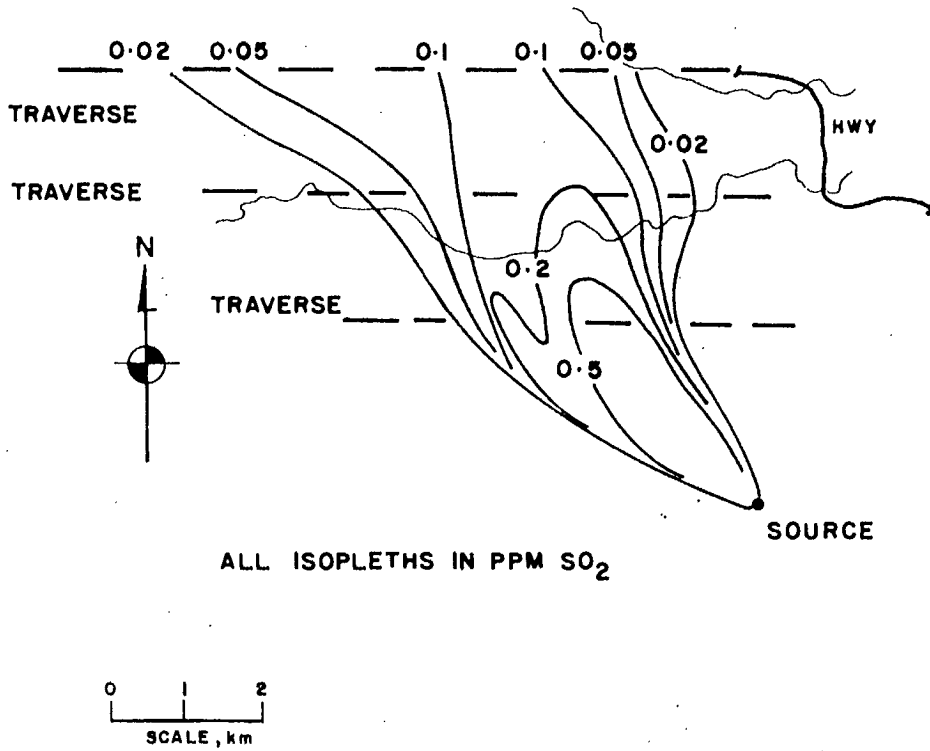


FIGURE 2. Derived Plan View of a Typical Plume

DATE : 24 - 4 - 72  
TIME : 1516 - 1636 MST  
BEARING : 350°

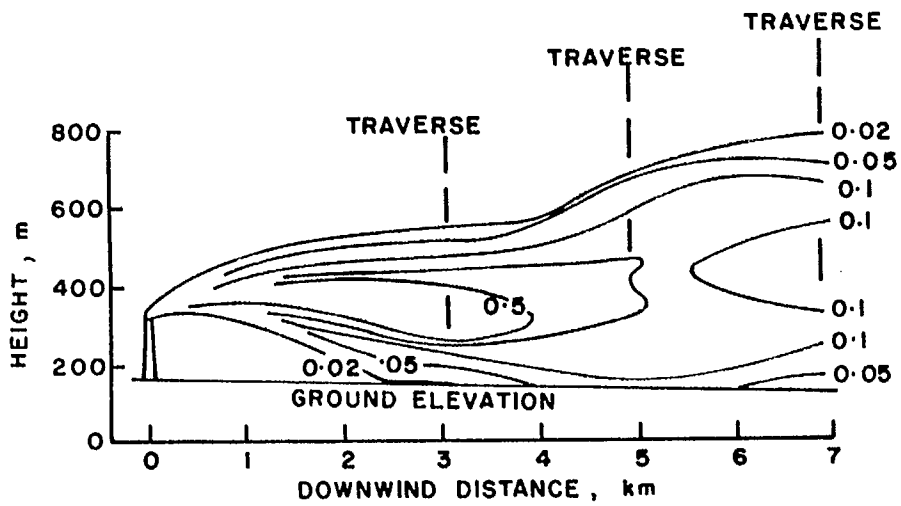


FIGURE 3. Derived Side View of a Typical Plume



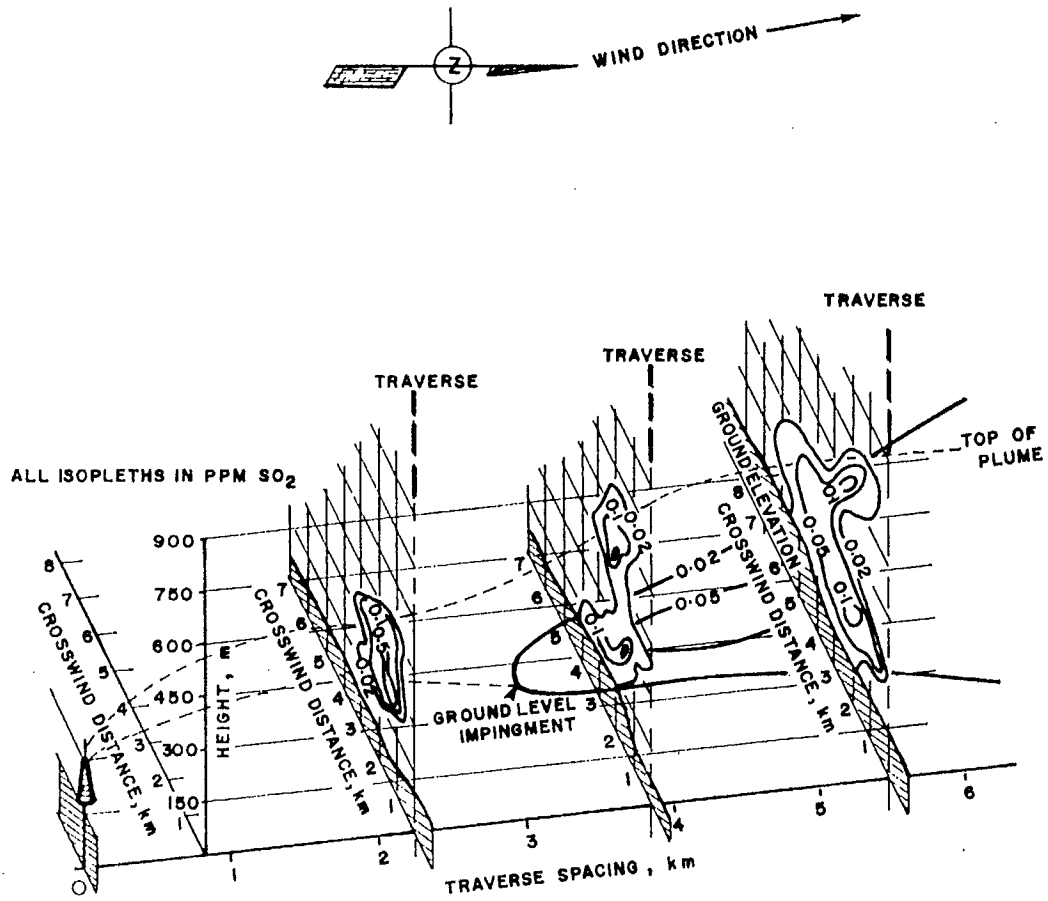


FIGURE 4. Pictorial View of a Typical Plume



**NDB**  
**ARCTIC BAY**  
Subject to unscheduled  
outages without NOTAM  
1750

**NDB**  
**POND INLET**  
Subject to unscheduled  
outages without  
NOTAM  
6440

**NDB**  
**IGLOOLIK**  
Subject to unscheduled  
outages without NOTAM  
830

**NDB**  
**YOR NDB**  
**ALL BEACH**

**NDB**  
**LONGSTAFF BLUFF**

**NDB**  
**CLYDE RIVER**

**NDB**  
**DEWAR LAKES**

**DEWIZ**

GV 40°E

GV 20°E

88°W

84°W

80°W

76°W

72°W

68°W

4100

3900

6250

6440

5900

5900

73°

71°

69°