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FOREWORD

This Information Circular was first presented at the region II Technical Conference of the Engineering Institute of Canada held at Saskatoon on October 31 and November 1, 1966. An abbreviated form of this Circular will be published in the Transactions of the Institute in July 1967.

This Information Circular is the unabridged paper as first submitted to the Engineering Institute of Canada for presentation at the Saskatoon meeting and subsequent publication, and is, therefore, not in the usual format of the Mines Branch publications.

H.M. Woodrooffe,
Chief,
Mineral Processing Division.

AVANT-PROPOS

Cette circulaire d'information a d'abord été présentée à la Conférence technique de la Région II de l'institut canadien des ingénieurs qui a eu lieu à Saskatoon les 31 octobre et 1er novembre 1966. Un résumé de cette circulaire sera publié dans les Transactions de l'Institut en juillet 1967.

Cette circulaire d'information est le document intégral tel qu'il a d'abord été soumis à l'Institut canadien des ingénieurs pour être présenté à la réunion de Saskatoon et publié par la suite; et n'a donc pas le format habituel des publications de la Direction des mines.

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Mines Branch Information Circular IC 191

PROBLEMS ASSOCIATED WITH DETERMINING THE TENSILE STRENGTH OF CONCRETE

by

V.M. Malhotra*

ABSTRACT

This paper critically examines the various direct and indirect methods for determining the tensile strength of concrete. The direct methods considered are the classical uniaxial tension tests, the modified direct tension test due to Todd, and the very recent methods in which thick steel plates are glued by means of epoxies to the ends of concrete specimens, which are then broken in tension. The inherent problems of parasitic stresses due to clamping and misalignment in these tests are outlined and discussed.

The indirect methods examined vary from bending tests, first proposed around 1904, to the cylinder and cube splitting tension tests, advanced in 1940 and 1960 respectively, and the ring tensile test proposed in 1965. The errors introduced in these tests due to the assumptions based upon the Hooke's Law of Linear Stress-strain Proportionality are outlined, and an attempt has been made to correct the strength values obtained in these tests to derive the "true" tensile strength of the concrete.

The reproducibility of the strength-test results for the various methods is given, and relationships have been attempted between the different types of strength.

The advantages and disadvantages of both direct and indirect tension test methods are given. The most common methods are illustrated by photographs or line drawings and over 100 pertinent references are listed.

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Circulaire d'information IC 191

de la Direction des Mines

PROBLÈMES CONNEXES À LA DÉTERMINATION DE LA RÉSISTANCE DU BÉTON À LA TRACTION

par

V.M. Malhotra*

RÉSUMÉ

L'auteur fait un examen critique des diverses méthodes directes et indirectes de détermination de la résistance du béton à la traction. Les méthodes directes étudiées sont les essais traditionnels de traction uniaxiale, l'essai modifié de traction directe de Todd, et les méthodes très récentes qui consistent à coller d'épaisses plaques d'acier à l'aide de résines époxydes aux extrémités des éprouvettes de béton qui sont alors brisés sous la traction. Les problèmes inhérents des contraintes parasitaires causées par le serrage et le mauvais alignement dans ces essais sont exposés et examinés avec soin.

L'auteur fait aussi l'examen de diverses méthodes indirectes: essais de flexion, d'abord proposés en 1904; essais de traction par rupture de cylindres et de cubes, proposés en 1940 et 1960 respectivement, et l'essai de traction sur anneau proposé en 1965. Les erreurs introduites dans ces essais à cause des hypothèses fondées sur la loi de Hooke sur la proportionnalité linéaire entre l'effort et la déformation sont exposées et l'auteur tente de corriger les valeurs de résistance obtenues dans ces essais afin d'en dériver la résistance "réelle" du béton à la traction.

L'auteur indique les possibilités de reproduire les résultats des essais de résistance obtenus par les diverses méthodes et tente d'établir des relations entre les différents types de résistance.

Il expose les avantages et les désavantages des méthodes d'essais de traction directes et indirectes, et illustre à l'aide de photos ou de graphiques les méthodes les plus courantes, en énumérant plus de 100 références pertinentes.

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INTRODUCTION

Tensile strength is one of the fundamental properties of concrete and a knowledge of it is essential for an understanding of the behaviour of concrete structures. The designers use the tensile strength of concrete to resist shear, shrinkage and temperature stresses. Unlike the cylinder/cube compression test, which has been standardized in almost every country, the tensile strength tests of concrete can be made by a number of different methods; these methods can be broadly classified into three types: direct, flexure, and indirect.

In direct methods, specimens in the form of long cylinders, prisms or briquettes are pulled in a testing machine until failure occurs. In North America such tests were first reported around 1904. In flexure tests, the tensile strength of concrete is estimated by determining the modulus of rupture of test beams. There is a considerable literature, dating back to 1906, covering this method. In the indirect methods, test cylinders, cubes and rings have been tested to estimate the tensile strength of concrete; these methods are relatively new and have only been used since around 1940.

This paper describes in detail the various methods used for the determination of tensile strength and discusses the testing problems associated with each of them.

DIRECT TENSION TEST METHODS

Earlier Test Methods

In direct tension test methods, concrete test specimens in the form of long cylinders, prisms, or briquettes are pulled in a testing machine until failure occurs. The gripping of the specimens is generally achieved either by truncated cones or by steel reinforcement embedded into concrete specimens. In North America such tests were first reported in detail by Talbot (1)* in 1904 and, since then, various investigators have used this form of the test with and without modification (2-22).

Recent Test Methods

1. Todd's Procedure

The most promising type of direct tension test since 1904 is the one proposed by Todd (23, 24, 25, 26) in 1955. In this method the test specimen consists of a 4-in.-diameter concrete cylinder, 12 in. long. A reinforcing bar, 1 in. in diameter, is embedded in the concrete and has an electrical resistance strain-gauge attached at its centre; the gauge is protected by a hollow tube 3 3/4 in. long, 1 1/2 in. in diameter and 1/10 in. thick. The hollow tube in the central region of the cylinder ensures failure at this reduced section. Eccentricity is compensated by applying an external moment over the middle third of the specimen, tested vertically; dead weights, carried over pulleys, are used for this purpose. The roller-mirror extensometers are used to measure strains over the central

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* The numbers in parentheses refer to the list of references appended to this paper.

portion of the cylinders, and when all extensometers show the same strain the eccentricity is eliminated.

2. Gluing Steel Plates to the Ends of Specimens with Epoxies

In the most recent form of the direct test, thick steel plates are glued with epoxies to the ends of concrete test-specimens, which are then broken in tension (27, 28, 29). The test procedure followed by the École Polytechnique, Montreal (29), which is typical of these forms of tests, is given below:

> After 24 days, 6 x 12-in. test-cylinders are removed from the moist-curing room. The ends of the cylinders are then sawed so that the sawn pieces do not exceed 1/4 in. in thickness. This is done to remove excessive mortar from the ends of the cylinder specimens. The cylinders are then dried in the laboratory air (temperature 72°F, relative humidity 40 to 50%) for one day. A machined plate 2 in. thick is glued with an epoxy (Chrysler's Cycleweld) to each sawn end of the 6 x 12-in. concrete cylinders, which are then cured in the laboratory air for one more day. Following this, the cylinders are returned to the moist-curing room for two more days to attain saturation of the concrete. Just before testing, special jigs with spherical seats connected to rods 1 1/2 in. in diameter are bolted to the end plates. The spherical seats help to achieve a nearly "true" axial stress in the specimen when the rods are held in the jaws of a universal testing machine.

Types of Direct Tension Methods Being Used by Various Laboratories

The International Union of Testing and Research Laboratories for Materials and Construction (RILEM) recently conducted an international survey of the types of direct-tension methods being used by various laboratories (21). The methods reported in the survey were as follows:

- (i) Gripping by steel reinforcement embedded into concrete specimens. (Five laboratories)
- (ii) Clamping concrete specimens by wings or truncated cones. (Four laboratories)
- (iii) Clamping concrete specimens by lateral grips. (One laboratory)
- (iv) Gluing thick steel plates to the ends of concrete specimens with epoxies. (Six laboratories)

Calculation of Direct Tensile Stress

The ultimate direct tensile strength is independent of the stress-strain relationship in the concrete. The direct tensile stress is simply the load divided by the cross-sectional area. As long as uniaxial loading is achieved (paradoxically, a condition almost impossible to satisfy), the actual tensile stress is determined.

Reproducibility of Direct Tensile Strength Test Results

The reproducibility of the direct tension test results depends to a large extent upon the type of method used and upon to what degree the extraneous stresses are eliminated. Table 1 compares the reproducibility of the direct tension test results as reported by several investigators.

TABLE 1

Reproducibility of Direct Tension Test Results

Test Specimen, Type and Size	Average Within-Batch Coefficient of Variation, per cent			
	Wright 1 (England)	Humphreys 2 (England)	Mines Branch 3 (Canada)	Komloš 4 (Czechoslovakia)
	Ref. 66	Ref. 16	Unpublished data	Personal Communication
4 x 18 in. cyl.	7.0	-	-	-
5 x 33 in. cyl.	-	8.4	-	-
6 x 12 in. cyl.	-	-		-
12 in. Long Briquette with 3 x 2 in. cross section at neck (approximate size only)	-	-	-	5.2

1. One water/cement ratio of 0.50 (by weight) used. Results based on 32 specimens tested at 28 days. Maximum size aggregate used was 3/4 in. river gravel.
2. One water/cement ratio of 0.44 (by weight) was used. Results based on 36 specimens tested at 28 days. Maximum aggregate was 3/8 in. river gravel.
3. Water/cement ratio varied from 1.03 to 0.31 (by weight). Results based on the average of 10 mixes with three specimens per mix. Tests carried out at Ecole Polytechnique, Montreal, employing the method in which thick steel plates are glued to the ends of concrete specimens. Maximum aggregate size was 3/8 in. crushed gravel.
4. Three water/cement ratios in the range 0.62 to 0.34 (by weight) were used. Results based on the average of the results of 26 mixes with three specimens per mix. Maximum aggregate was 1/2 in. river gravel. Test method used was one in which self centering clamps are employed.

Relationship Between Direct Tensile Strength and Compressive Strength

The general relationship between tensile and compressive strength of concrete is shown in Table 2. It is seen that the direct tensile strength ranges from 7 to 11, and averages about 10 per cent of the compressive strength; the higher the compressive strength, the lower the relative tensile strength.

Limitations of Direct Tension Tests

The problems associated with the direct tension test methods are well known. The classic direct tension tests used by Talbot and others are burdened with misalignment and clamping stresses. Eccentricities are known to produce major effects on the stresses, regardless of the specimen size and shape. Because of the stresses introduced due to gripping, there is a tendency for the specimens to break near the ends. This problem is often overcome by reducing the section of the central portion of the test specimen. The methods in which steel plates are glued with epoxies to the ends of test specimens eliminate stresses due to gripping, but offer no solution for the eccentricity problem. The test proposed by Todd is the only one which claims to eliminate parasitic problems of both clamping and misalignment; however, this test is slow and requires skilled

TABLE 2

Relationship Between Direct Tensile Strength and Compressive Strength of Concrete*

Compressive Strength, psi	Direct Tensile Strength, psi	Ratio of Direct Tensile Strength to Compressive Strengths, per cent
6 x 12 in. cylinders	6 x 18 in. cylinders	
1000	110	11.0
2000	200	10.0
3000	275	9.2
4000	340	8.5
5000	400	8.0
6000	460	7.7
7000	520	7.4
8000	580	7.2
9000	630	7.0

* From Reference 5.

Coarse aggregate: Elgin gravel with 1½ in. max. size.
Fine aggregate: Elgin sand.

operators and the use of relatively sophisticated techniques, Furthermore, all direct tension test methods require expensive universal testing machines and are too time-consuming. This explains why these tests are not used on a routine basis and are not yet standardized.

Because extraneous stresses are introduced in the specimens during testing, the tensile strengths obtained by the direct tension test methods are usually 10 to 30 per cent lower than the "true" tensile strength of concrete (25). This however, depends on the type of the test and the strength level of concrete.

Figures 1 to 6 show the direct tension tests being carried out by different methods.

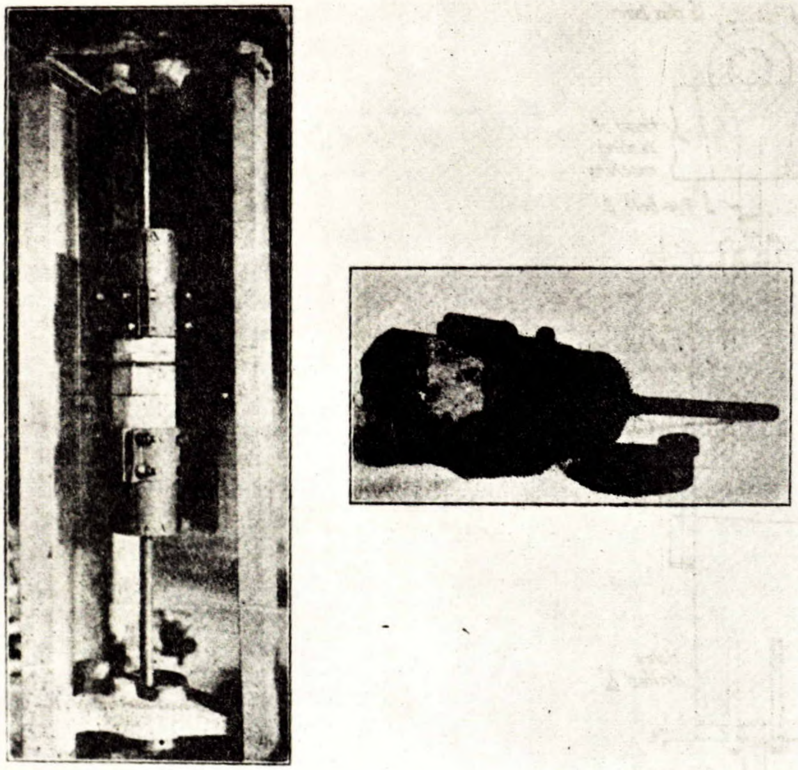


Fig. 1.

Direct tension test method using cylindrical specimen and circular type grips. (After Gonnerman and Shuman, 1928, Ref. 5)

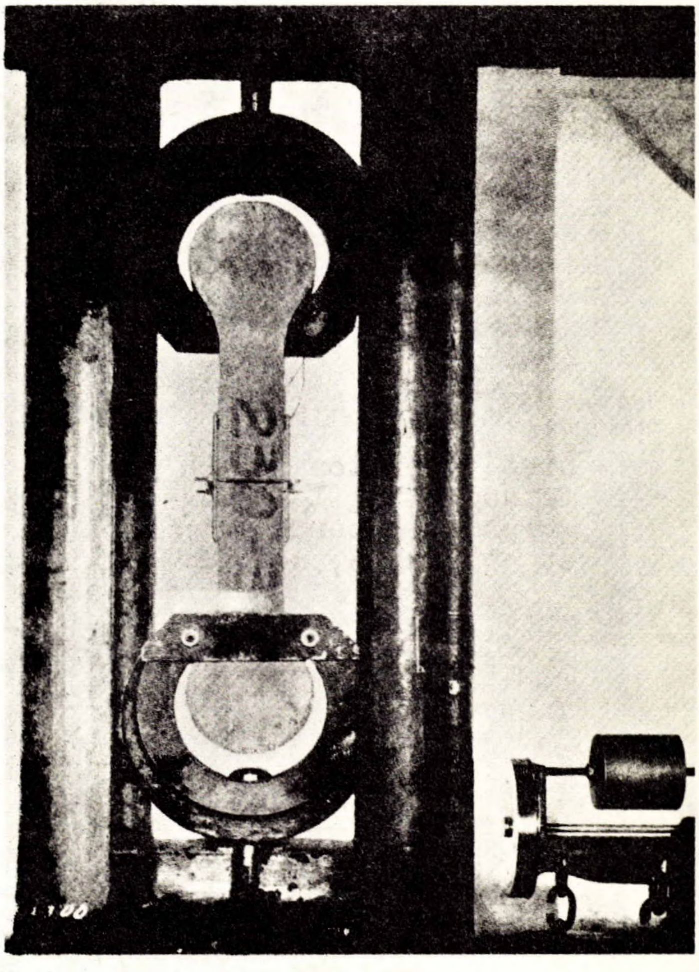


Fig. 2.

Direct tension test method using a briquette-type specimen. (After Johnson, 1928, Ref. 7)

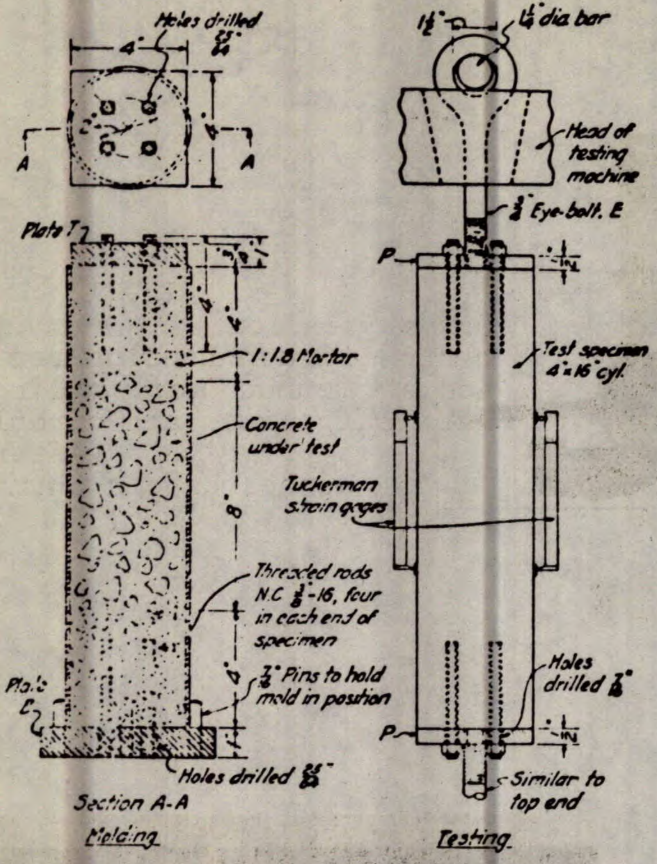
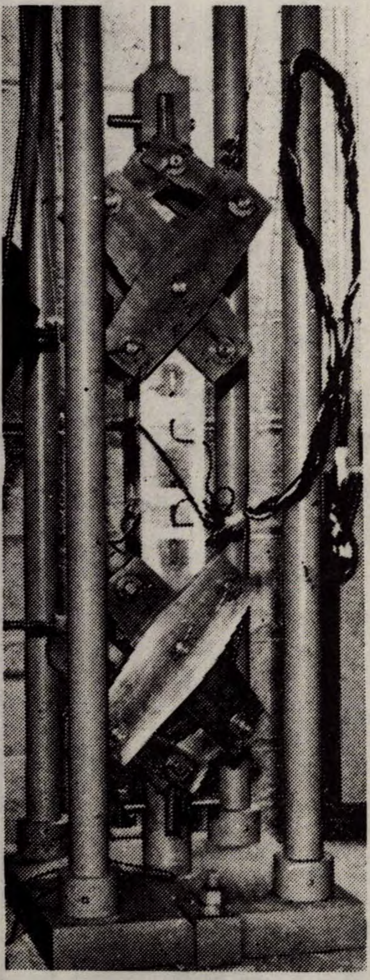


Fig. 3.

Direct tension test method using a cylindrical specimen with embedded reinforcement for gripping purposes. (After Schuman and Tucker, 1943, Ref. 12)

Fig. 4.

Direct tension test method using lateral grips. (After Newman et al., 1963, Ref. 22)



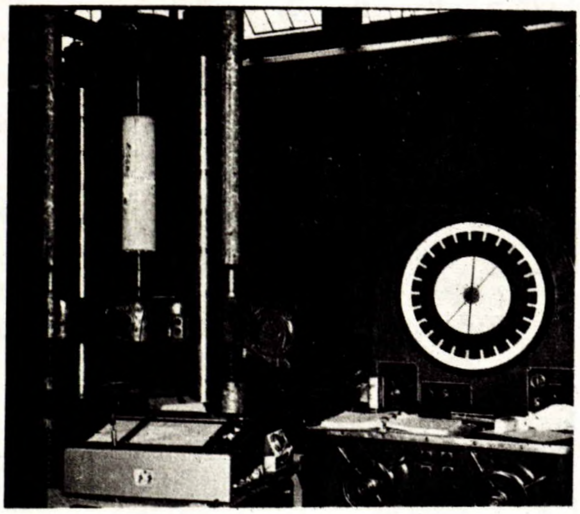
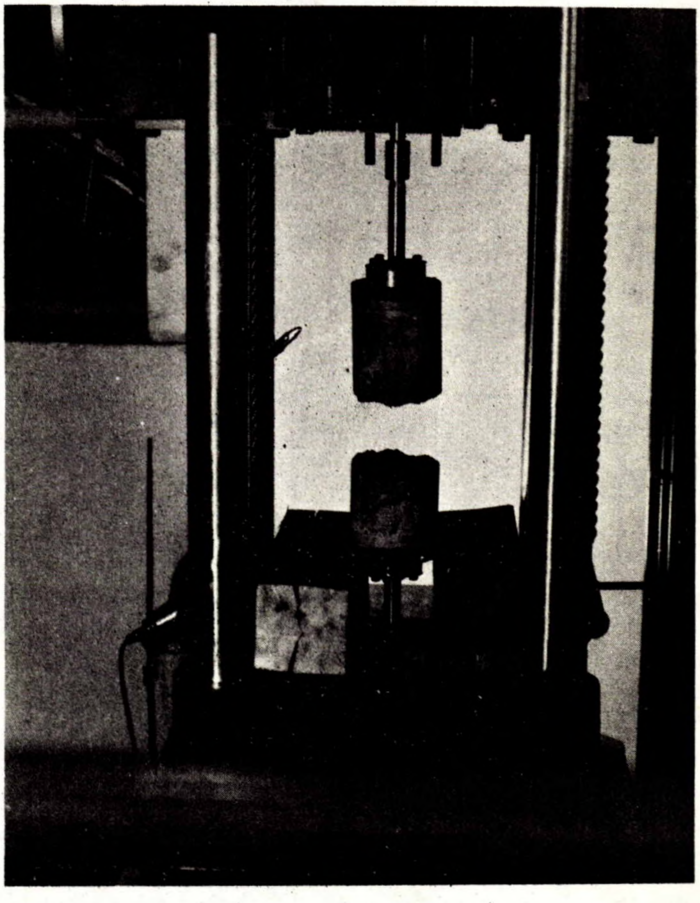


Fig. 5.

Direct tension test method as proposed by Todd. (After Ledbetter and Thompson, 1965, Ref. 26)

Fig. 6.

Direct tension test method using thick steel plates glued to the ends of a concrete cylinder with epoxies. (After Ecole Polytechnique, Canada, 1965, Ref. 29)



INDIRECT TENSION TEST METHODS

Because of the problems associated with determining the direct tensile strength of concrete, several types of indirect tension tests are being used for evaluating this property. These are:

1. Flexure Test
2. Cylinder-Splitting Tension Test
3. Cube- and Prism-Splitting Tension Tests
4. Torsion Tension Test
5. Ring Tension Test

1. Flexure Test

The most common plain concrete structure subjected to flexure is the highway pavement. The flexure strength of concrete for pavements is commonly evaluated by means of bending tests on beams, i.e., by the modulus of rupture from which an estimate of the "true" tensile strength of concrete is obtained.

There is considerable literature, dating back to 1904, covering flexure test methods. Early investigators included Talbot (1), Sabin (2), Feret (30), and Fuller and Thompson (31). These were followed by the classic studies of Abrams in 1922 (32), Gonnerman and Shuman in 1928 (5), Schuman and Tucker in 1943 (12), and others (33-58). The studies of Wright (39, 47, 49, 55), Reagel and Willis (35), Blakey and Beresford (49, 46, 48), and Walker and Bloem (50, 51) deserve special mention and have contributed greatly to the understanding of the flexural strength of concrete.

The American Society for Testing and Materials (ASTM) adopted the flexural strength (simple beam with third-point loading) as one of its Standards in 1938 (59) and since then the Standard has been revised several times. In 1959 the ASTM issued another Standard for flexural strength of simple beams with centre-point loading (60).

Calculation of Modulus of Rupture

The modulus of rupture may be defined as the tensile stress developed by beam action, assuming that the stress and deformation are directly proportional to the distance from the neutral axis of the beam. The modulus of rupture of a rectangular beam freely supported at the end is "determined by the following relations:

For Third-Point Loading

$$R = \frac{WL}{bd^2}, \qquad \text{(Eq. 1)}$$

For Centre-Point Loading

$$R = \frac{3\,WL}{2\,bd^2}, \qquad \text{(Eq. 2)}$$

where R = modulus of rupture of concrete, psi,
W = total load applied, lb,
L = span length, in.,
b = average width of beam, in.,
d = average depth of beam, in.

Reproducibility of Flexural Strength Test Results

The reproducibility of the flexural strength test results is given in Table 3. In laboratory test results the coefficient of variation ranges from about 5 to 8 per cent; however, the size of the beam, the maximum size of the aggregate used, and the strength level of concrete could greatly affect it.

Relationship Between Direct Tensile Strengths and Flexural Strengths

The flexure test does not measure the "true" tensile strength of concrete but determines what is known as the modulus of rupture. The results obtained are considerably higher than the "true" tensile strength, because the formulas for modulus of rupture assume a straight-line stress-strain distribution which is known to be incorrect. Gonnerman and Shuman's data (5) show that the direct tensile strength is about 50 to 60 per cent of the modulus of rupture values (Table 4). Blakey and Beresford (40) carried out extensive strain gauge instrumentation of the beam specimens and concluded that tensile stress-strain distribution in a beam is of a parabolic form instead of being linear; the correct modulus of rupture is 0.735 of the value calculated by normal elastic theory. Pincus and Gesund (25) suggest a correction factor of 0.70. It should be pointed out that these correction factors would vary with

TABLE 3

Reproducibility of Flexural Strength Test Results

Test Specimen, Type and size	Average Within-Batch Coefficient of Variation, Per Cent					
	Wright, England[1]	Efsen and Glarbo, Denmark [2]	Rüsch and Vigerust, Germany [3]	Ramesh and Chopra, India [4]	Kenis, U.S.A.[5]	Malhotra and Zoldners Canada [6]
	Ref. 66	Ref. 68	Ref. 70	Ref. 76	Ref. 81	Ref. 109
3½ x 4 x 16 in. beams	-	-	-	-	-	5.1
4 x 4 x 16 in. beams	6.0	-	-	-	-	-
4 x 4 x 20 in. beams	-	-	4.5	3.7	-	-
21 in. span (size of beams not given)	-	6.8	-	-	-	-
6 x 6 x 21 in. beams	-	-	-	-	7.4	-

1. One water/cement ratio of 0.50 by weight used. Results are based on 32 specimens tested at 28 days.
2. Water/cement ratios varied from 0.33 to 1.19. Results based on seven test specimens tested at 14 days (one day moist, three days in water, and ten days laboratory curing). A rapid-hardening portland cement was used.
3. No water/cement ratios given, but design strengths were 4300, 6400, and 8500 psi in compression at 28 days. Results are based on mean of three parallel tests.
4. Water/cement ratios varied from 0.40 to 0.60. Results are based on 15 specimens tested at 28 days.
5. Water/cement ratios and number of specimens per test not given. Results represent average values including 7, 14 and 28 strengths.
6. Water/cement ratios varied from 0.37 to 1.03. Results based on 12 specimens per test, tested at 28 days.

TABLE 4

Relationship Between Direct Tensile Strength and Flexural Strength *

Compressive Strength, psi 6 x 12 in. cylinders	Direct Tensile Strength, psi 6 x 18 in. cylinders	Flexural Strength, psi 7 x 10 x 38 in. beams	Ratio of Direct Tensile to Flexural Strength, per cent
1000	110	230	48
2000	200	375	53
3000	275	485	57
4000	340	580	59
5000	400	675	59
6000	460	765	60
7000	520	855	61
8000	580	930	62
9000	630	1010	63

* After reference 5. All strengths are for 28 days. Maximum aggregate size: 1½ in.

the type and maximum size of aggregate used and with the strength level of concrete.

Limitations of Flexure Test

In addition to the fact that the modulus of rupture overestimates considerably the "true" ultimate tensile stress, this test has several other limitations. The flexure strength varies in accordance with whether the beam is loaded centrally or at the third-points of the span, and with the depth-span ratio, and, furthermore, it is greatly affected by the moisture condition of the specimen. Also, there is wide disagreement among research workers as to the degree of reproducibility of the flexural strength test results in the field. Table 5 shows the effect of depth of specimen on the measured flexural strength of concrete.

Figures 7 to 10 show concrete beams under flexure test using cantilever transverse testing apparatus and under centre-point and third-point loading methods.

TABLE 5

Effect of Depth of Specimen on Measured Flexural Strength *

Depth of Beam, in.	Average Modulus of Rupture **, psi			
	Laboratory 1	Laboratory 2	Laboratory 3	Laboratory 4
4	841	912	820	872
6	802	976	815	835
8	765	849	786	764
10	741	789	779	745

* After Reagen and Willis (Ref. 35)

** Data are based upon a round robin programme carried out by four laboratories in the U.S.A. The results are average values for all widths of beam and all lengths of span. The widths of the beams were 4, 6, 8 and 10 in. and the lengths of the beams were 20, 26, 32 and 38 in. Coarse aggregate used was 1 in. max. size Burlington limestone; fine aggregate was river sand.

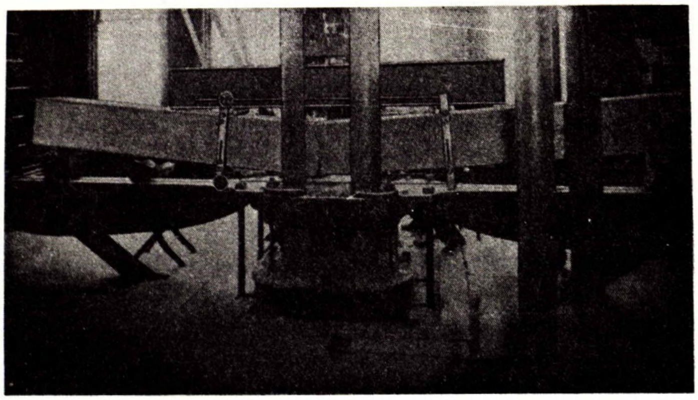
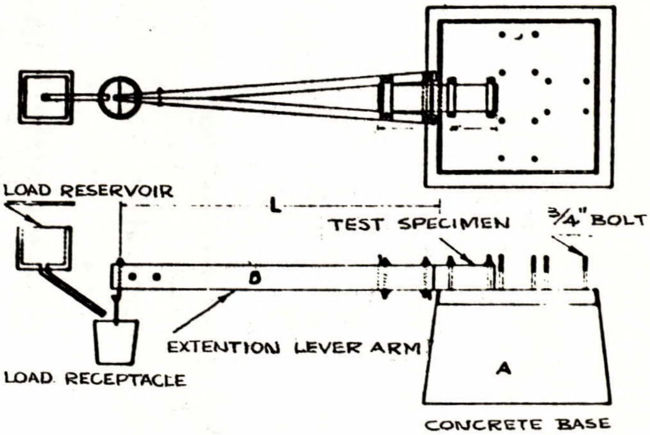


Fig. 7.

Concrete beam under flexure test by third-point loading method. (After Talbot, 1904; Ref. 1)

Fig. 8.

Cantilever transverse testing apparatus for flexure test. (After Clemmer and Burggraf, 1924, Ref. 33)



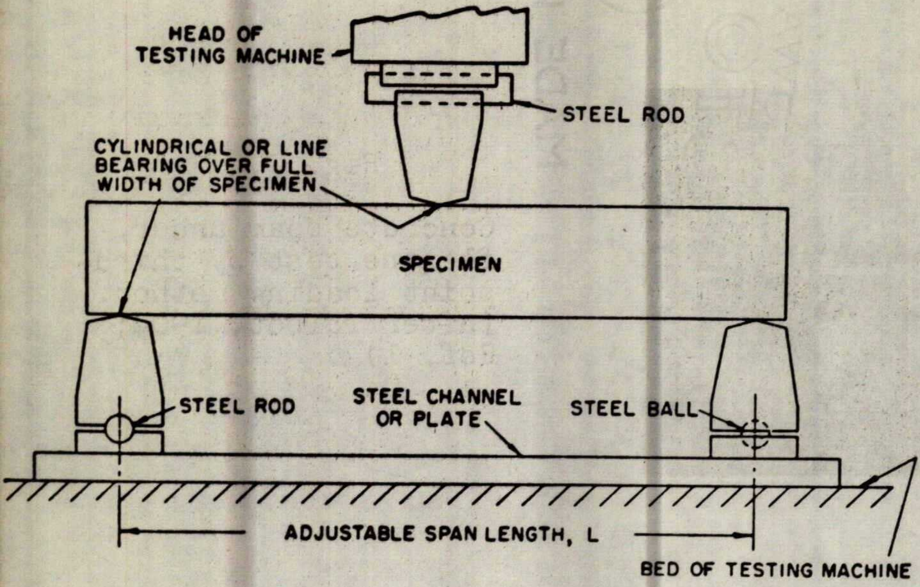


Fig. 9.

A diagrammatic view of apparatus for flexure test of concrete by ASTM centre-point loading method C 293-64.

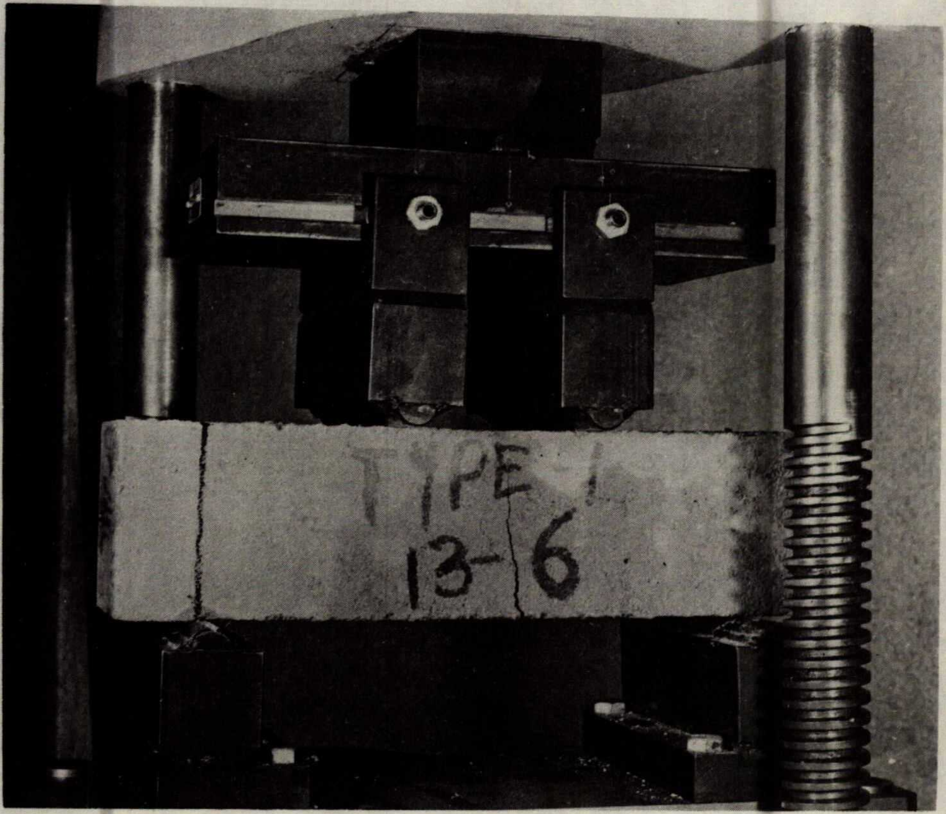


Fig. 10.

Concrete beam being tested by ASTM third-point loading method C 78-64. (After Canada Mines Branch, 1965)

2. Cylinder-Splitting Tension Test

This test was developed in Brazil in 1943 by Carneiro and Barcellos (61, 62) and, independently, in Japan by Akazawa (63) at about the same time. This test is carried out by placing a cylinderical specimen horizontally between the loading surfaces of a testing machine, so that the load is applied to the specimen along the two opposite generatrices. (Figures 11 and 12). The chief advantage of this method is that the same type of specimen and the same testing machine as are used for the compression test can be employed for this test. This perhaps explains some of the popularity this test has gained over the past two decades (64-93).

The ASTM adopted the test as one of its Standards in 1962 (94).

Calculation of Cylinder-Splitting Tensile Strength

The cylinder-splitting tensile test is based on the state of stress developed when a cylindrical specimen is subjected to a compressive force along two opposite generators of its surface. This loading condition produces a biaxial stress distribution within the specimen. Immediately next to the two generators to which the load is applied are small regions of compressive stress; however, an almost constant tensile stress exists over about three-quarters of the vertical plane.

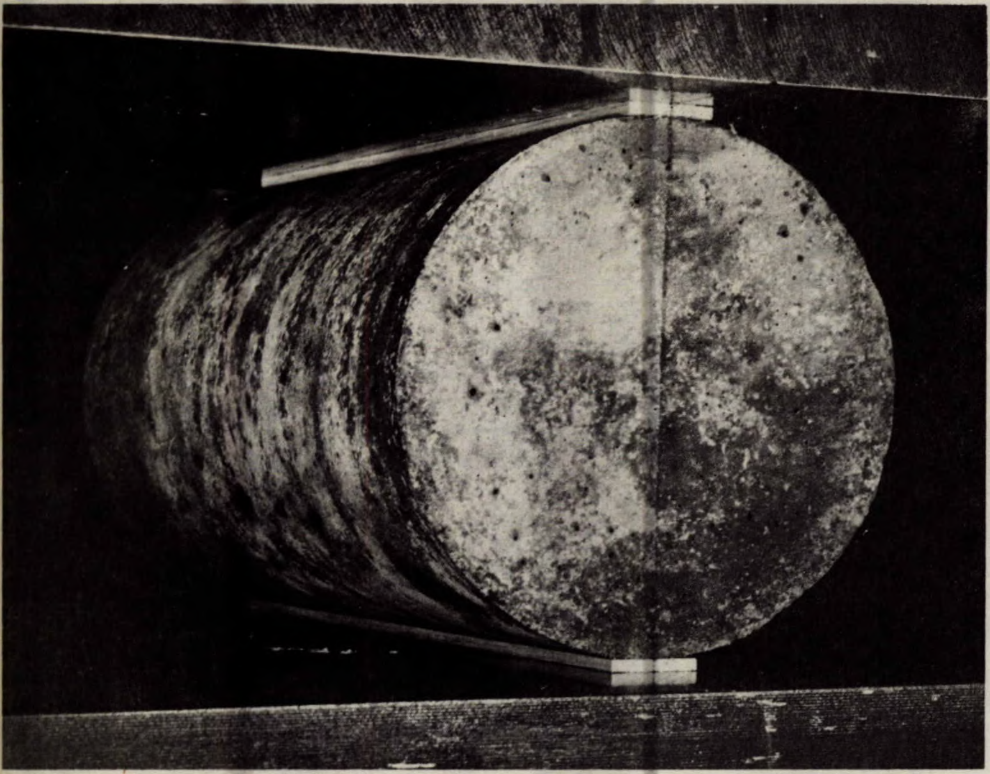
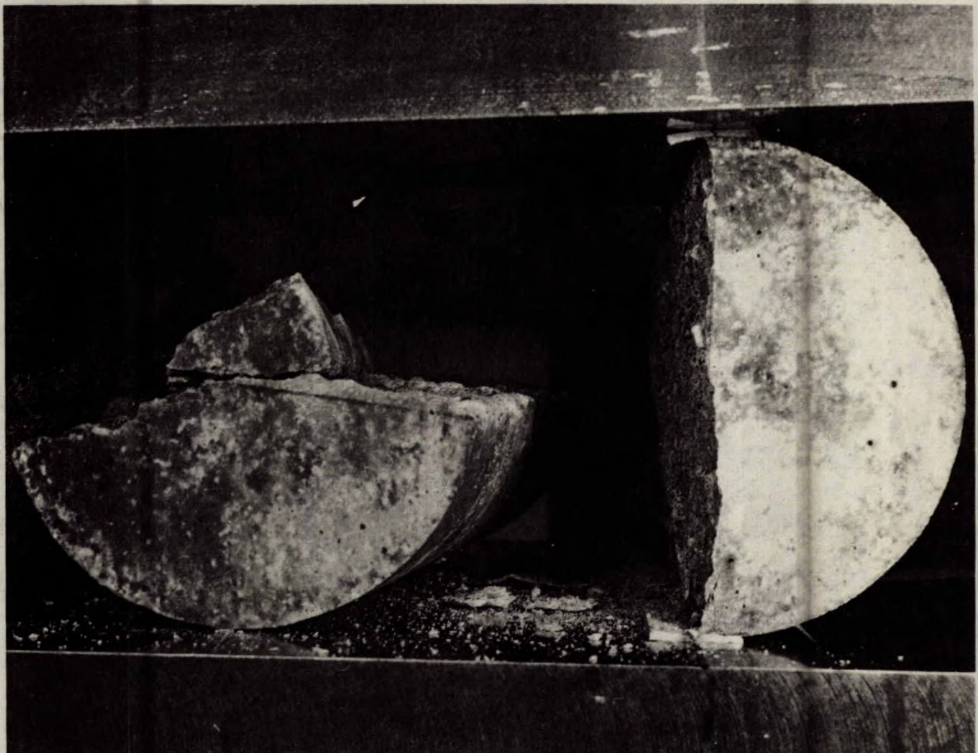


Fig. 11.

Concrete cylinder, 6 x 12 in., ready for splitting tensile test. The packing strips are of 1 x 1/8 in. soft wood, 12 in. long. (After Canada Mines Branch, 1965)

Fig. 12.

Concrete cylinder, 6 x 12 in., immediately after the splitting tensile test. (After Canada Mines Branch, 1965)



The cylinder-splitting tensile strength is calculated from the formula:

$$T = \frac{2P}{\pi LD}, \qquad \text{(Eq. 3)}$$

Where T = cylinder-splitting tensile strength, psi;

P = maximum applied load at failure, lb;

L = length of cylinder, in; and

D = diameter of cylinder, in.

The compressive stresses vary in magnitude along the diameter, from a minimum of $6P/\pi DL$ at the centre to an infinitely high value immediately under the loads (66). It has been shown experimentally that, though high local compressive stresses are developed near the points of application of the load, the failure in a concrete specimen always occurs by separation along the vertical plane. This happens as soon as the tensile strength of concrete is exceeded.

In order to reduce the magnitude of the high compressive stresses near the points of application of the load, narrow packing strips of suitable material are placed between the specimen and the relatively hard loading platens of the testing machine. The packing strips should be soft enough to allow distribution of the load over a reasonable area, yet narrow and thin enough to prevent the contact area from becoming excessive. The width of the packing strips appears to have little effect, but the thickness has a rather unpredictable and irregular effect (66); the ASTM has therefore standardized the packing at 1 in. wide by 1/8 in. thick by 12 in. long (full length of the cylinder).

Reproducibility of Cylinder-Splitting Tensile Strength Test Results

The reproducibility of the cylinder-splitting tensile strength test results, as reported by various researchers, is given in Table 6. The within-batch coefficient of variation ranges from 5.0 to 8.5 per cent, with the exception of one reported value of 2.4 per cent. These values, which are based upon laboratory tests, may be considered relatively high. Few data are available on the reproducibility of field test results.

Relationship Between Cylinder-Splitting Tensile/Compressive Strength Ratio and Compressive Strength

The relationship between the cylinder-splitting tensile strength and compressive strength ratio with the compressive strength as obtained by several investigators is shown in Figure 13. It will be seen that the above ratio decreases with increase in the compressive strength of concrete and reaches a value of less than 10 per cent beyond compressive strengths of 6000 psi. The high value of the ratio at low compressive strengths, as reported by Malhotra and Zoldners, is probably due to the small size aggregate used (3/8 in. max.).

TABLE 6

Reproducibility of Cylinder-Splitting Tensile Strength Test Results *

Name of Researcher	Reference No.	Average Within-Batch Coefficient of Variation, per cent
Wright (England)	66	5.0
Efsen and Glarbo (Denmark)	68	6.3
Rüsch and Vigerust (Germany)	70	6.0
Ramesh and Chopra (India)	76	2.4
Kenis (U.S.A.)	81	8.5
Malhotra and Zoldners (Canada)	109	7.8

* For details of concrete mix design and number of test specimens per test, refer to the footnote at the bottom of Table 3.

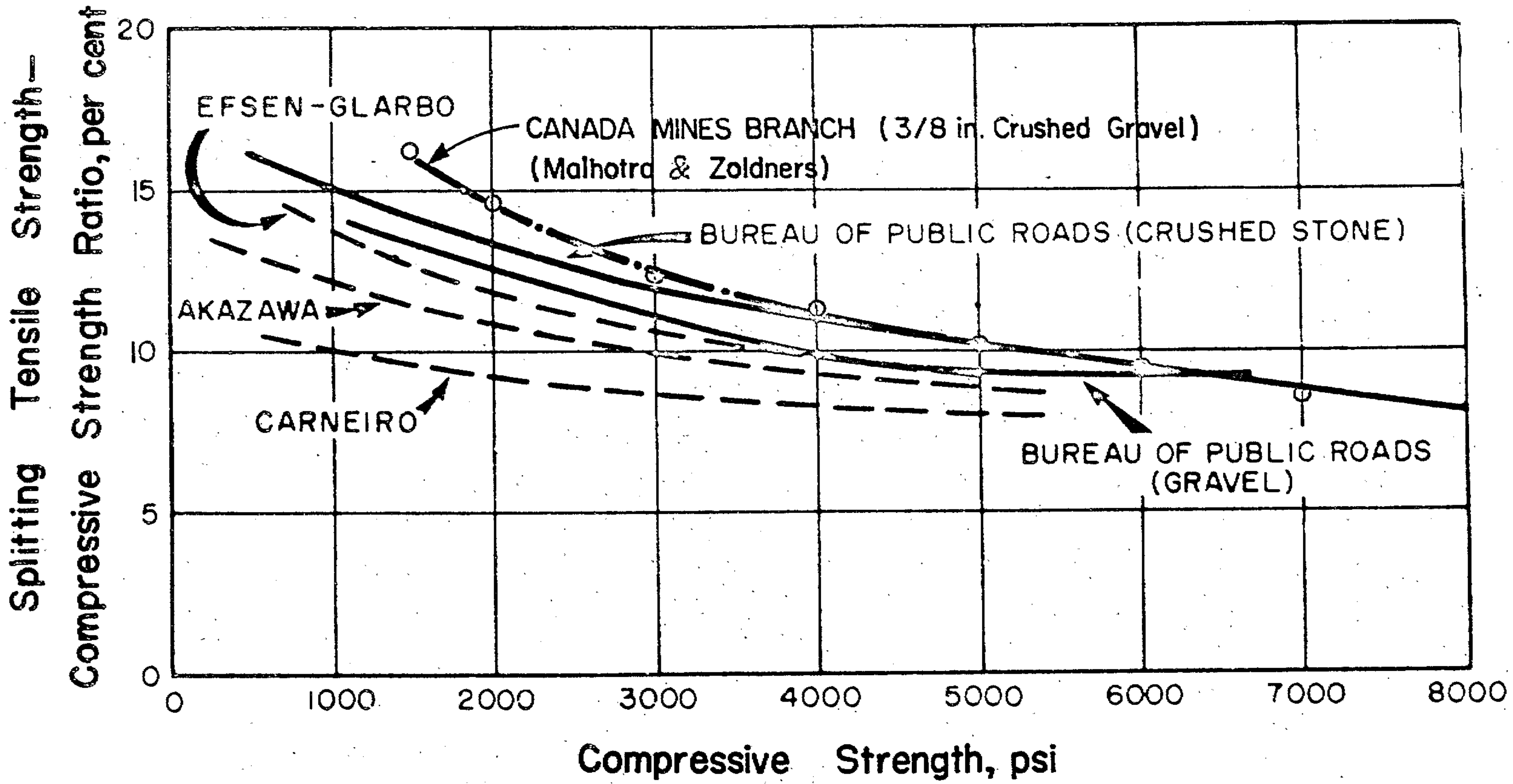


FIGURE 13. COMPARISON OF RELATION BETWEEN SPLITTING TENSILE STRENGTH—COMPRESSIVE STRENGTH RATIO AND COMPRESSIVE STRENGTH

Relationship Between Cylinder-Splitting Tensile Strength and Direct Tensile Strength

In the cylinder-splitting tension test, concrete is under a state of biaxial stress, one of the principal stresses being tensile and the other being compressive. As stated earlier, the principal compressive stress is about three times the tensile stress at the centre of the specimen. Under such stress conditions the strains are additive and therefore theoretically the tensile strength as obtained from the cylinder-splitting tension test should be slightly lower than the "true" value. However, this has not been confirmed by the available experimental data. On the contrary, various research workers (66, 108) have reported cylinder-splitting tensile strengths somewhat higher than the strengths obtained in direct tension tests; the difference depnds upon the type of the direct tension test employed.

The higher values obtained in the cylinder-splitting tension tests are probably due to the following reasons:

(a) Readjustment of stresses in the specimen during testing.

(b) Greater volume of concrete in which failure can take place in the direct tension tests than in the cylinder-splitting tension test.

(c) Stresses due to eccentricity in direct tension tests.

Pincus and Gesund (25) suggest that a correction factor of 0.9 should be applied to the reported cylinder-splitting tensile strength of normal concrete to reduce the reported test data to the "true" tensile strength as obtained from Todd's modified direct tension test. The experimental data of Ledbetter and Thompson (26) suggest that the above correction factor may be as high as 0.75 for lightweight concretes.

A relationship between cylinder-splitting tensile and direct tensile strengths is shown in Table 7. For the data reported by Wright (66), the cylinder-splitting tensile strengths are considerably higher than the direct tensile strengths. This is probably due to the type of the specimens used and the method employed for the direct tension test. For the data of Reference 108, the cylinder-splitting tensile strengths are about 10 per cent above the reported direct tensile strength of concrete.

Limitations of Cylinder-Splitting Tension Test

The major drawbacks of this test are as follows:

(i) The derivation of Equation 3 for the analyses of the test data assumes that concrete is an ideal elastic material (which it is not) and obeys Hooke's Law of Linear Stress-strain Proportionality (which it does not).

(ii) The state of stress in the cylinder under load is essentially biaxial, varying in magnitude from about

TABLE 7

Comparison Between Cylinder-Splitting Tensile and Direct Tensile Strengths

Reference No.	Water/Cement Ratio (by weight)	Compressive Strength at 28 Days, psi		Direct Tensile Strength at 28 Days, psi		Cylinder-Splitting Tensile Strength at 28 Days, psi
		6 x 12 in. cylinders	4 in. cube	6 x 12 in. cylinders	4 x 18 in. cylinders	6 x 12 in. cylinders
66 (Wright)*	0.5	-	5980	-	275	405
109 (Malhotra and Zoldners)**	0.79	2495	-	295	-	345
	0.65	4190	-	440	-	470
	0.47	6235	-	500	-	580

* Coarse aggregate: 3/4 in. gravel; fine aggregate: river sand. Results based on 32 specimens of each type.

** Coarse aggregate: 3/8 in. crushed gravel; fine aggregate: natural sand. Results based on two specimens of each type.

three times the horizontal stress at the centre to a high compressive stress at the loading points (theoretically infinite for line loads). These high stresses are known to have initiated "premature" failure under the packing strips during the testing of lightweight concrete (95).

(iii) The effect of Poisson's ratio is not taken into account in the analysis of the test data. This is strictly not correct when dealing with the biaxial stress conditions in concrete, because strains play a major role in the failure of specimens (71).

(iv) The splitting tensile strengths seem to be 10 to 30 per cent higher than the direct tensile strength of concrete. This difference depends, of course, on the type of direct tension test used and the strength level of the concrete.

(v) The reproducibility of the laboratory test results is of the same order as that of the flexure test.

3. Cube- and Prism-Splitting Tension Tests

In countries where the compressive strength of concrete is determined from cubes rather than from cylinders, the tensile strengths have been approximated using a cube-split or a diagonal-cube-split tension test. This test was first proposed by Rosenhaupt, Van Riel and Wijler (96) in 1957, and since then it has been gaining some acceptance (97, 101). Recently this test has been applied to the broken pieces of prisms tested in flexure (102, 103).

Calculation of Splitting Tensile Strengths

The cube-splitting, diagonal-cube-splitting, and prism-splitting tensile strengths are calculated from the following equation:

$$S = k \frac{P}{s^2}, \qquad \text{(Eq. 4)}$$

where S = splitting tensile strength, psi;
P = maximum applied load, lb;
s = side of a cube or prism; and
k = a constant.

The value of k for different types of splitting tests is as follows:

Cube-splitting tensile test, $k = 0.642$ (Ref. 99)
Diagonal-cube-splitting tensile test, $k = 0.5187$ (Ref. 99)
Prism-splitting tensile test, $k = 0.648$ (Ref. 102)

It is stressed that the values of "k" are relatively approximate. This is especially so for the prism-splitting test because it has been pointed out (Ref. 103) that the value of the constant varies with the ratio of the width of the loading strip to that of the specimen. Values of "k" at the centre of the strip for different ratios of the width of the loading strip to that of the specimen (103) are given below:

Ratio of the Width of the Loading Strip to that of the Specimen	Value of "k" for Prism-Splitting Tension Test
1/16	0.494
1/8	0.484
1/4	0.446
1/2	0.335
1	0.129

Figures 14 to 16 show the cube- and prism-splitting tension test being carried out.

Reproducibility of Splitting Tensile Strength Test Results

There are few data available on the reproducibility of the cube-splitting and diagonal-cube-splitting tension test results. Welch (102) gives a value of 5.6 per cent for the average coefficient of variation for the prism-splitting tensile strengths (Table 8).

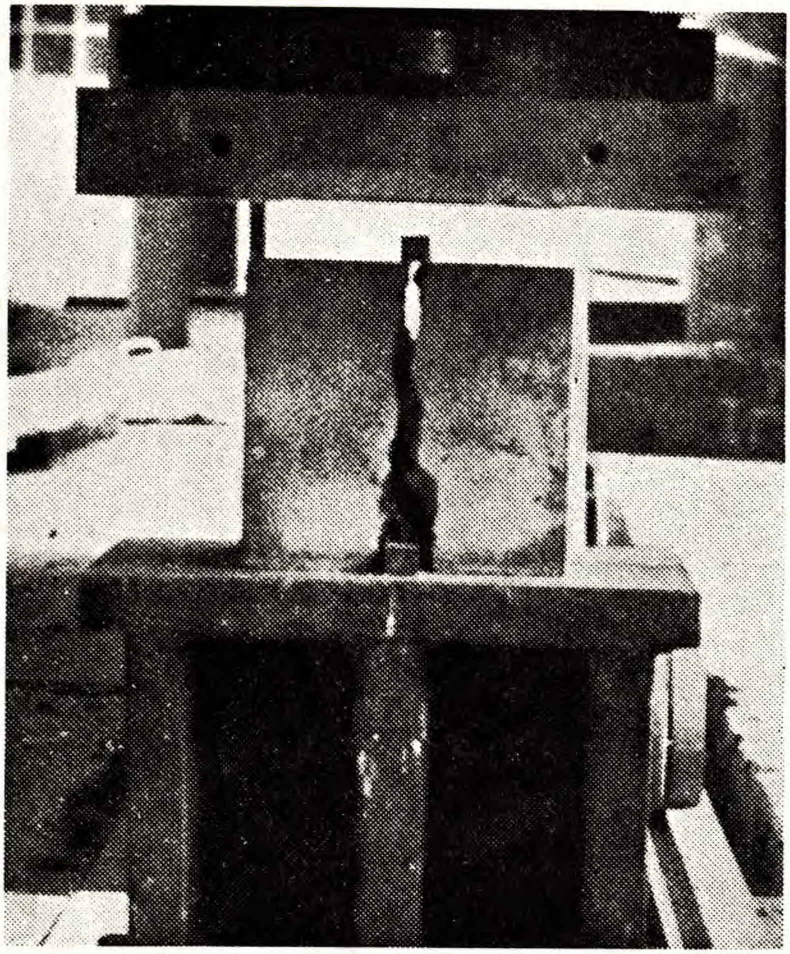
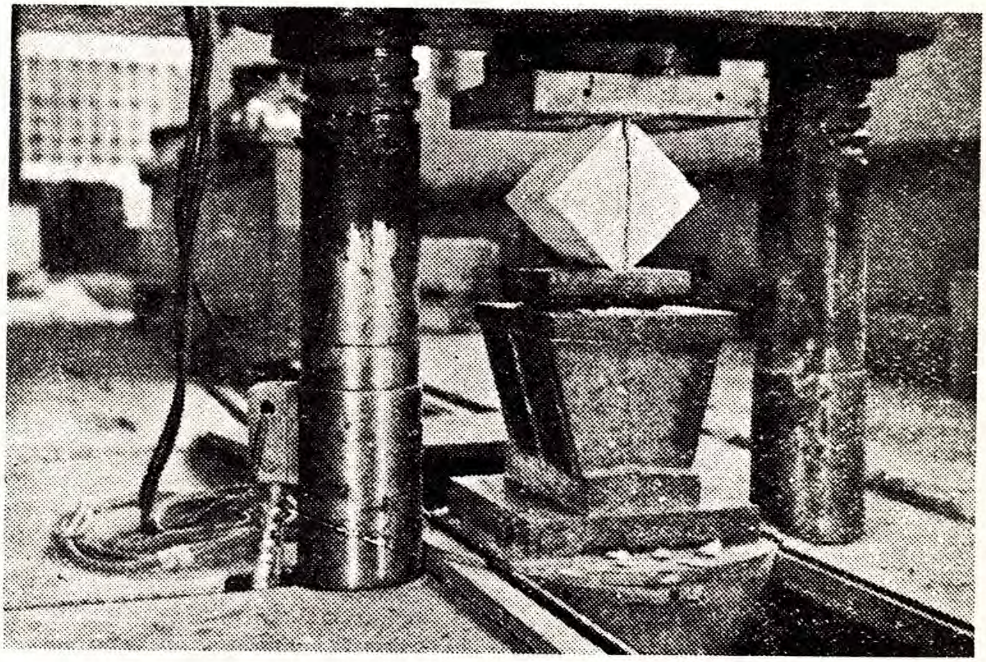


Fig. 14.

A 6 in. cube under splitting tension test. (After Sen and Bharara, 1961, Ref. 97)

Fig. 15.

A 6 in. cube under diagonal-splitting tension test. (After Sen and Desayi, 1962, Ref. 100)



Comments

The above splitting tension tests are esentially the same as the cylinder-splitting tension test described earlier. The advantages and the limitations outlined for the cylinder-splitting tension test apply equally to the above tests.

Reliable data for the relationships between the splitting tensile and compressive strengths and the splitting tensile and direct tensile strengths are not available.

4. Torsion Tension Test

In 1963, Iyengar et al. (104) reported the use of a torsion tension test to obtain the "true" tensile strength of concrete. In this test method, concrete cylinders 4 in. in diameter by 16 in. long are subjected to pure torsion, by means of a specially designed loading frame (Figure 17).

The authors report that the difference between the "true" tensile strength and the tensile strength as obtained in the torsion tension test is of the order of 5 to 8 per cent, with torsion tests giving the lower results; however, this has not been confirmed by other research workers.

There are no data available as to the reproducibility of the test results for this method.

Fig. 16.

A 6 x 6 x 20 in. prism under splitting tension test. (After Sen and Bharara, 1961, Ref. 97)

Fig. 17.

A 4 in. diameter by 16 in. long cylinder under torsion test. (After Iyengar et al., 1963, Ref. 104)

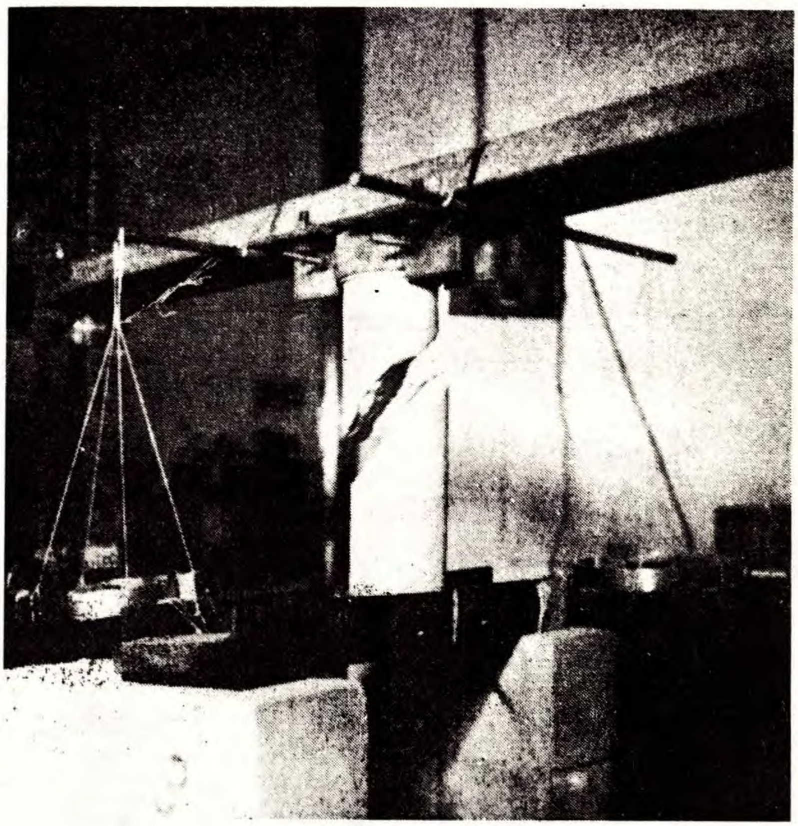


TABLE 8

Reproducibility of Prism-Splitting Tensile Strength Test Results *

Water/Cement Ratio (by Weight)	Size of Specimen, ** in.	Average Prism-Splitting Tensile Strength at 28 Days, *** psi	Coefficient of Variation, per cent
0.35	4 x 4 x 20	675	2.0
		685	5.5
		875	4.4
0.50	4 x 4 x 20	495	10.8
		490	10.2
		665	4.4
0.50	6 x 6 x 28	485	5.8
		420	6.0
		585	3.8
1.00	6 x 6 x 28	170	3.5
		140	8.1
		205	2.4
	Average		5.6

* After Reference 102. Coarse aggregate ranged from rounded gravel to crushed granite. Fine aggregate was from Thames Valley.

** For 4 x 4 x 20 in. beams, max. size aggregate 3/4 in.; for 6 x 6 x 20 in. beam, max. size aggregate 1 1/2 in.

***Each strength figure represents the mean of either eight tests on the 4 in. specimens or four tests on the 6 in. specimens.

5. Ring Tension Test

In 1965, Canada Mines Branch (105-110) reported a new technique, known as the "ring test", for determining the tensile strength of concrete. Briefly, in this new technique hydrostatic pressure is applied radially against the inside periphery of a 6 in. diameter, 1½ in. thick and 1½ in. high concrete ring specimen. The resulting tensile stresses developed in the specimen are determined from the equations for the stress analysis of thick-walled cylinders.

Test Method

Concrete test rings are cast in specially fabricated steel moulds (Figure 18). The testing jig consists of two 11 in. diameter mild steel plates held together by five tie-bolts. The hydrostatic pressure is applied radially from inside through a specially moulded bladder made of 1/8 in. thick nitrile rubber. The bladder is connected to the hydraulic line by means of a valve which is moulded to the centre upper surface of the bladder.

The nuts on the tie-bolts holding the testing jig together are "finger tightened" to ensure that the concrete ring is not restrained and that, when the hydraulic pressure is applied and the tie-bolts are stretched, the test ring actually floats to find its own unrestrained position. Figure 19 shows the testing jig being assembled.

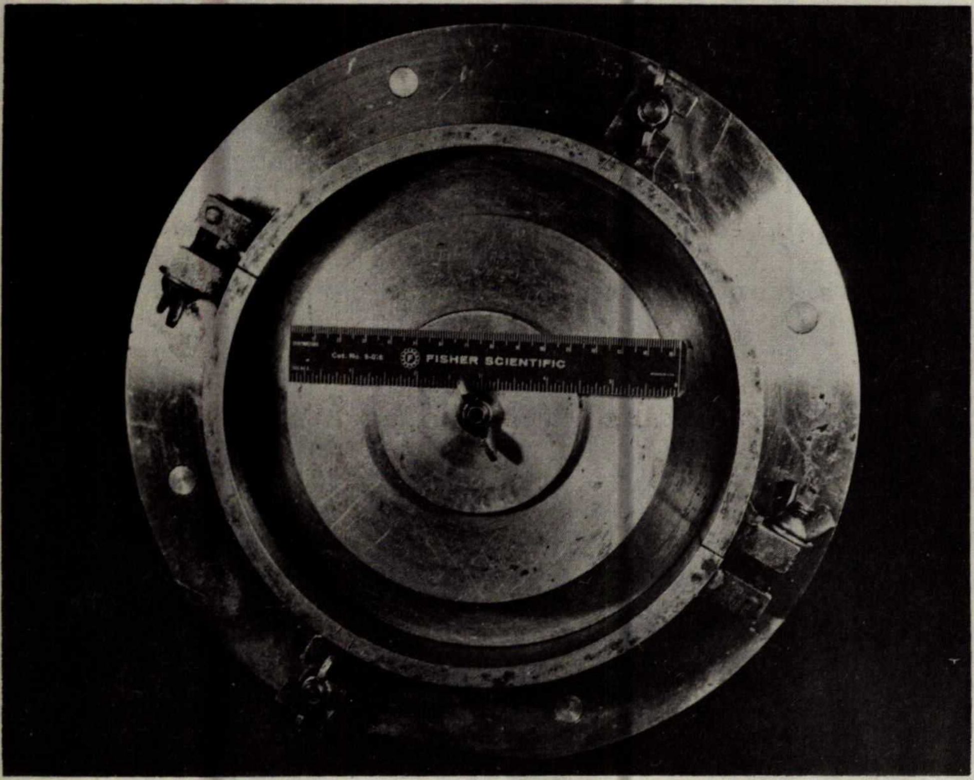


Fig. 18.

Steel mould for casting concrete rings 6 in. diameter by $1\frac{1}{2}$ in. wide by $1\frac{1}{2}$ in. high. (After Canada Mines Branch, 1965, Ref. 109)

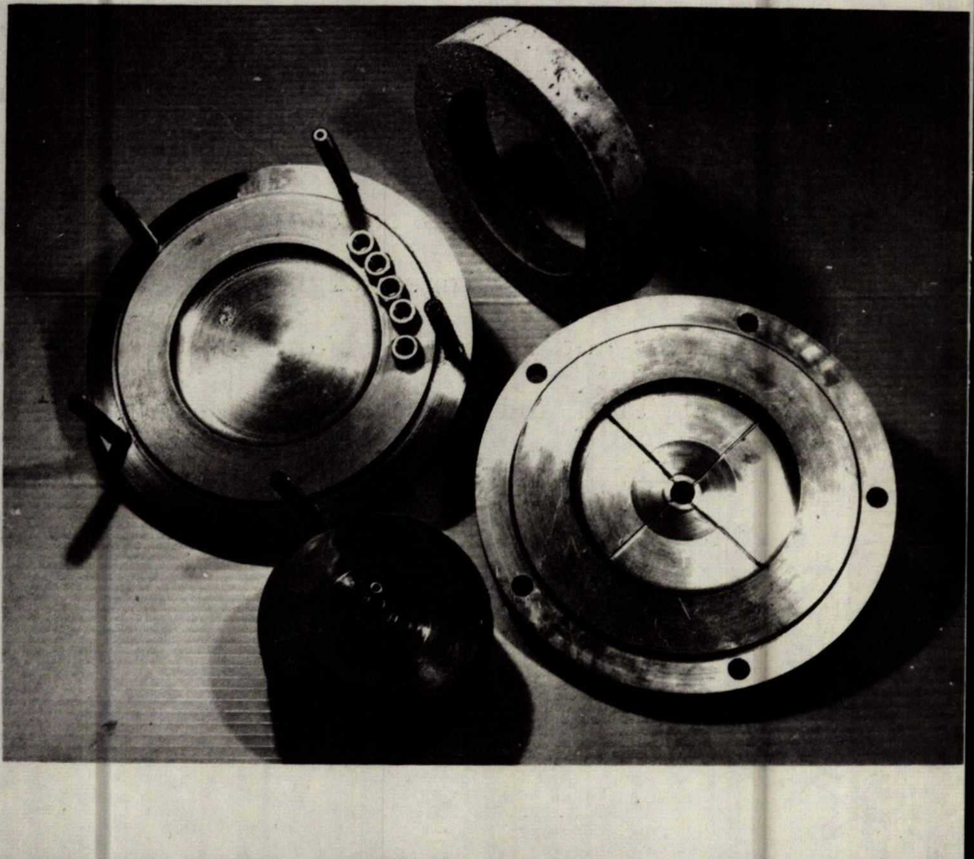


Fig. 19.

A view of a concrete ring, rubber bladder, and top and bottom plates of the testing jig. (After Canada Mines Branch, 1965, Ref. 106)

The hydraulic pressure is obtained by a hand-operated hydraulic jack, using oil as the hydraulic medium. The pressure is indicated on Marsh-type gauges having a range of 0 to 600 psi. The pressure is applied at a rate of 10 psi per second.

All concrete rings are tested in a moist condition. Figure 20 shows the assembled testing jig and Figure 21 a ring immediately after test.

Calculation of Ring Tensile Strength

The ring tensile strength is calculated using the following equation (111):

$$\sigma_t = \frac{P_i r_i^2}{r_o^2 - r_i^2} \left(1 + \frac{r_o^2}{r^2} \right), \qquad \text{(Eq. 5)}$$

where σ_t = tangential (on the inside periphery) tensile stress, psi;
P_i = applied hydrostatic pressure, psi;
r_i = internal radius, in.;
r_o = external radius, in.; and
r = radius at point of failure.

The stresses obtained using the above equation vary from a maximum of 2.6 P_i at the inside periphery of the ring to a minimum of 1.6 P_i at the outside surface. The corresponding compressive stress, calculated using the equation for radial compressive stress (not given above) varies from P_i at the inside periphery and diminishes to zero at the outside periphery.

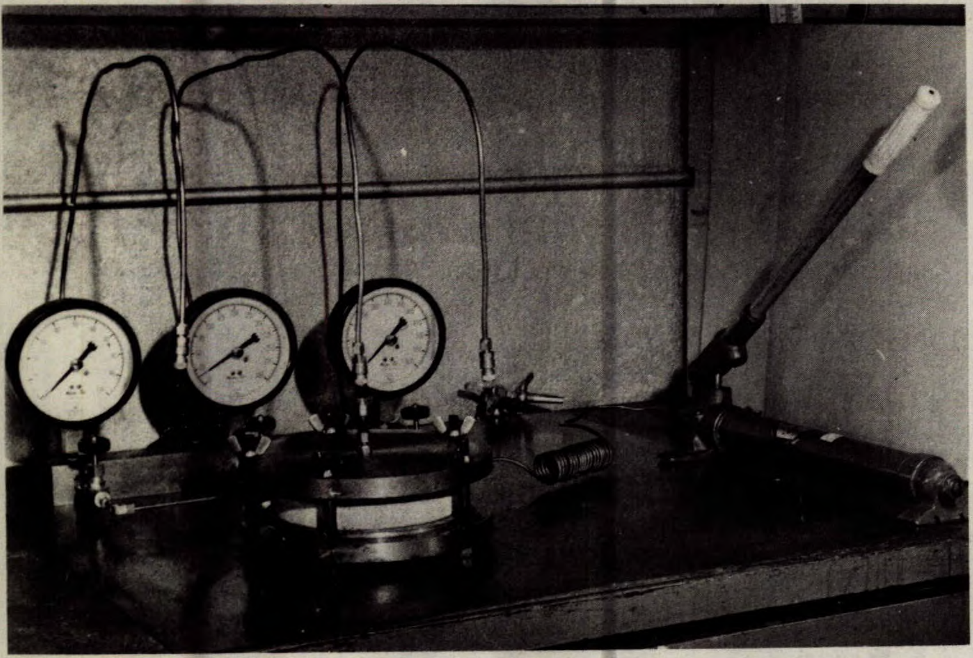
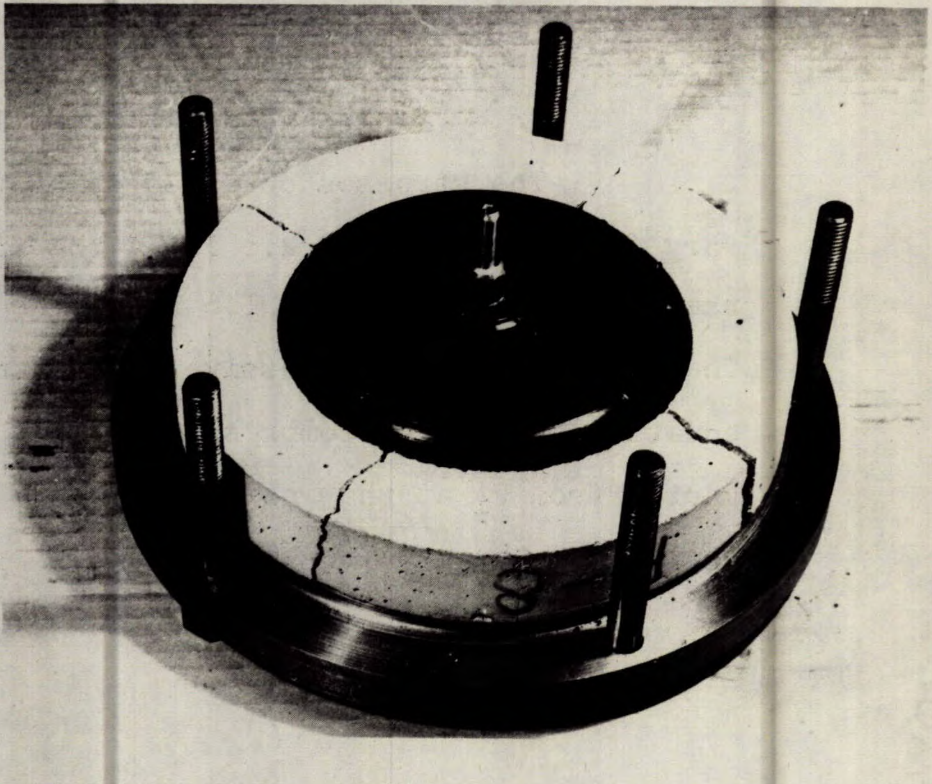


Fig. 20.

An assembled view of the testing jig just before testing, using a hand-operated hydraulic pump. (After Canada Mines Branch, 1965, Ref. 109)

Fig. 21.

Concrete ring after test with four equally spaced broken sections. (After Canada Mines Branch, 1965, Ref. 104)



Reproducibility of Ring Tensile Strength Test Results

The reproducibility of the ring tension test results is given in Table 9. The average within-batch and between-batch coefficients of variation are 4.9 and 3.1 per cent respectively, and these compare very favourably with the reproducibility data for flexure and splitting tension test results.

Relationship Between Ring Tensile Strength and Direct Tensile Strength

A comparison between ring tension and direct tension test results is shown in Table 10. The direct tension tests were carried out using the method in which thick steel plates are glued to the ends of cylinder specimens, which are then pulled in tension. The data in Table 10 are based upon only three mixes, each with a different water/cement ratio and are, therefore, limited in scope.

Relationship Between Compressive Strength and Ring Tensile Strength

The general relationship between compressive strength and ring tensile strengths is shown in Figure 22 (109). Also shown on the figure are the 95% confidence limits for the individual prediction. As with the direct tensile strength, the ring tensile strength/compressive strength ratio decreases with increase in the compressive strength. It varies from about 20 per cent at low strengths levels (1500 psi compressive strength) to about 10 per cent at high strength levels (8000 psi compressive strength).

TABLE 9

Within-Batch and Between-Batch Coefficients of Variation for Ring Tension Test *

Water/Cement Ratio (by weight)	No. of Test Batches	Pooled Average Compressive Strength at 28 Days, psi (6 x 12 in. cylinders)	Coefficient of Variation for Ring Tensile Strengths, per cent (6 in. diameter 1½ in. by 1½ in. rings)	
			Within-Batch Average, **	Between Batch
1.03	4	1610	1.2	1.2
0.92	2	1930	2.7	5.4
0.79	3	2655	9.2	0.7
0.79	3	2845	7.6	3.0
0.65	4	4100	6.2	5.5
0.57	4	4930	6.7	1.8
0.47	3	5940	3.2	3.8
0.47	2	6210	4.0	3.4
0.37	4	6690	3.8	3.3
	Average		4.9	3.1

* From Ref. 109.

** Within-batch average based upon 3 test results per batch.

TABLE 10

Comparison Between Ring Tensile and Direct Tensile Strengths*

Water/Cement Ratio (by weight)	Strengths at 28 Days, psi		
	** Compressive	** Direct Tensile	*** Ring Tensile
	6 x 12 in. cylinder	6 x 12 in. cylinder	6 in. diameter $1\frac{1}{2}$ in. x $1\frac{1}{2}$ in. ring
0.79	2495	295	360
0.65	4190	440	495
0.47	6235	500	685

* From Ref. 109.

** Mean of two test results.
***Mean of three test results.

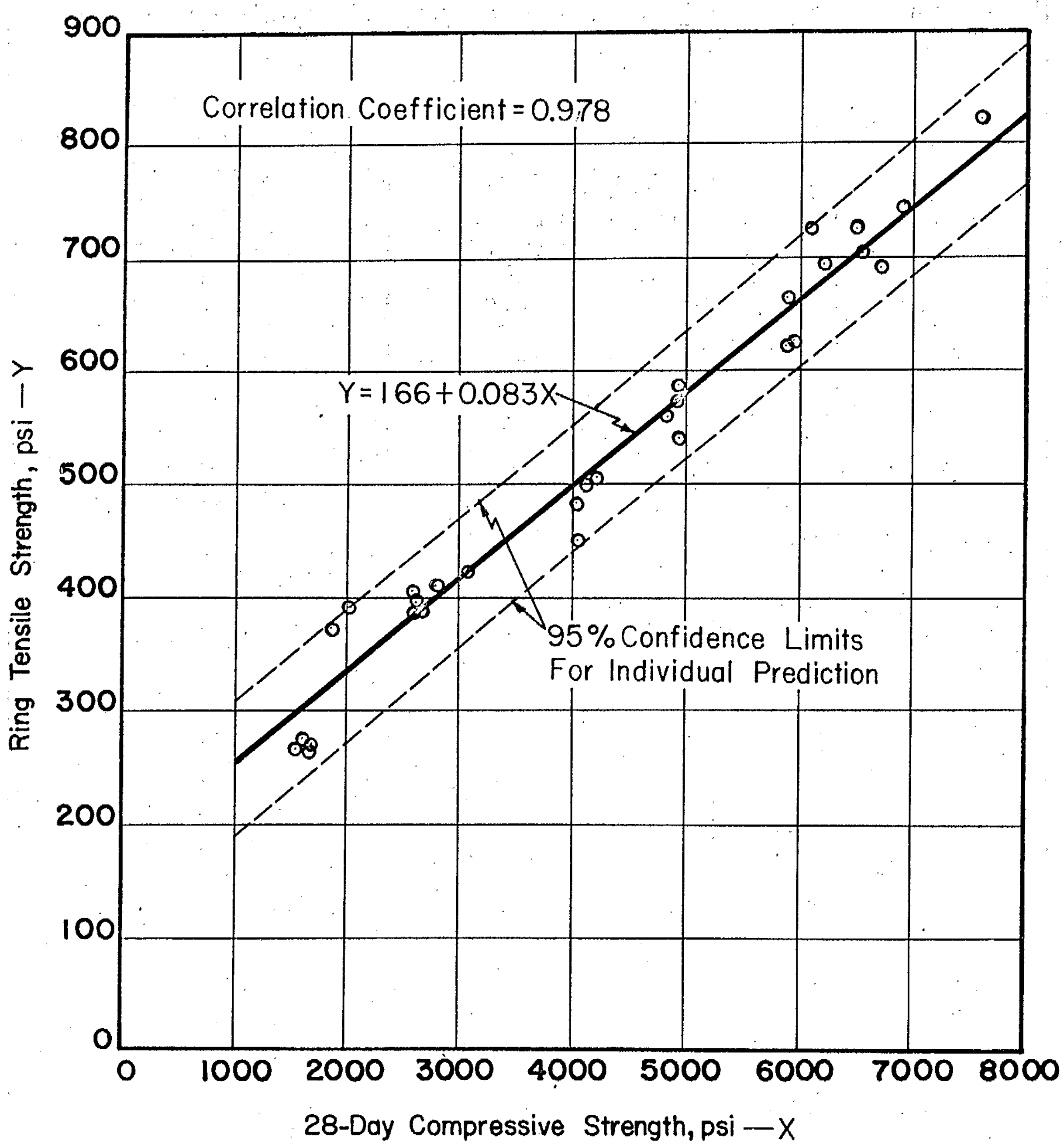


FIGURE 22. RELATIONSHIP BETWEEN COMPRESSIVE STRENGTH AND RING TENSILE STRENGTH

Advantages of Ring Tension Test

The ring test appears to have some distinct advantages over the existing direct and indirect tension tests. These are:

(i) The nature of the load application in the ring test is such that no clamping and misalignment stresses are introduced in the test specimen, a condition difficult to avoid in direct tests.

(ii) The entire volume of the ring specimen is subjected to tensile stresses, with the uniformly distributed maximum stress occurring along the entire internal periphery of the ring. This is never achieved in flexural tests, and even in the cylinder-splitting tension test a compressive load acting on a diametral plane creates a uniform tensile stress over that plane only.

(iii) The magnitude of the radial compressive stress is quite small when compared with the tangential stress. This is a definite advantage over the splitting tension test, in which the minimum compressive stress occurring at the centre line of the splitting plane is about three times the corresponding tensile stress.

(iv) A comparatively high degree of reproducibility appears to be possible in the ring test.

Limitations of Ring Tension Test

The drawbacks of the ring tension test are that, once again, the derivation of the equations used for the stress analyses is based upon Hooke's Law of Linear Stress-strain Proportionality. The ring tensile strengths obtained appear to be somewhat higher than the 'true' tensile strength of concrete; the magnitude of the exact difference has yet to be firmly established.

The data reported to date on the ring tension test relate only to concrete using 3/8 in. max. size aggregate. If the method is to become a standard field test, data on concrete with 3/4 in. max. size aggregate will be required.

CONCLUSIONS

1. The direct tension test methods discussed are relatively complicated and time-consuming, and therefore cannot be recommended for routine laboratory and field use. However, the "true" tensile strength of concrete may only be obtained by the direct test method proposed by Todd.

2. Among the indirect tension test methods, the cylinder-splitting tension test is gaining some acceptance, but the ring tension test appears very promising.

3. The "true" tensile strength of concrete can be estimated by applying correction factors to the test results obtained from various direct and indirect tension tests as discussed in this report. It is stressed that these correction factors are different for each type of test and are also dependent on the type of aggregates used, the type and size of specimen, and the strength level of concrete.

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