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CANADA

REVIEW AND EVALUATION OF METHODS OF PARTICLE SIZE ANALYSIS.

PART I: THE DEFINITION OF TERMS AND CLASSIFICATION OF SIZING METHODS; PART II: SIEVE ANALYSIS.

by

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MINES BRANCH
INFORMATION CIRCULAR

IC 106

PRICE 25 CENTS

NOVEMBER 3, 1958

PREFACE

During the past few years an increasing interest has been shown by industrial operators and research organizations in particle sizing information and in methods of size analysis. Accordingly, the present studies were undertaken at the Mines Branch to provide both an up-to-date and comprehensive review of different sizing methods and an evaluation of these methods. It is hoped that this basic information will make possible an intelligent selection of the sizing method best suited to any particular problem and will result in greater confidence being placed in sizing information.

Part I of this Information Circular was previously issued on a limited circulation as Research Report No. MD 200 of the Division of Mineral Dressing and Process Metallurgy, April 11, 1956. Because of the wide demand for that report, it has been reorganized and is being re-issued, along with Part II, in the present Information Circular IC 106.

John Convey,
Director, Mines Branch.

Mines Branch Information Circular IC 106

REVIEW AND EVALUATION OF METHODS OF PARTICLE SIZE ANALYSIS. Part I: The Definition of Terms and Classification of Sizing Methods; Part II: Sieve Analysis.

by

R. F. Pilgrim*

ABSTRACT

Part I is intended as a general introduction to the study of methods of particle size analysis. Included are definitions of the terms "particle size" and "particle shape", methods of measuring shape factors and correlation factors (i. e. factors relating the size of particles measured by different methods), and a classification of sizing methods to provide an orderly framework for this study.

Part II, on sieve analysis, is divided into three chapters. The first is concerned with the structural characteristics of sieves, the various sieve series and standard specifications, and methods of certifying sieves dimensionally. The necessarily wide tolerances on sieving cloth permitted by standard specifications provide a logical basis for the detailed study of sieve standardization methods in the second chapter. The third chapter deals with the mechanics of sieving, i. e. the way in which sieves are used in size analysis and the various factors which affect the accuracy of sieve analyses. Sieving is shown to be a statistical process, the probability of a particle passing through a sieve depending on several factors. Each of these is discussed in some detail to show, for example, the advantages of using wet sieving and a rate-defined sieving end-point.

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CONTENTS

PART I - THE DEFINITION OF TERMS AND CLASSIFICATION OF SIZING METHODS

(Continued) -

CONTENTS (Continued)

PART II - SIEVE ANALYSIS

(Concluded) -

CONTENTS (Concluded)

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(88 pages, 11 tables)

GENERAL INTRODUCTION

The methods of measuring the fineness of a particulate material may be divided in a general way into two classes: those that characterize the material by a single figure such as the specific surface (surface area per unit weight) or average particle size, and those that measure directly or indirectly the size distribution of the material. During the past few years there has been a growing activity and interest in the field of fineness determination. Evidence of this is the enormous volume of literature which has been published on methods of particle sizing and related topics. A recent bibliography published by the British Whiting Federation(1) lists 550 references to methods of measuring particle size distribution and specific surface of granular materials; the list is admittedly incomplete.

In view of the large number of methods, and variations of them, which have been described, it has become almost imperative that a complete survey and a critical assessment of the available methods be undertaken(2). This is the purpose of the present Information Circular. Owing to the extent of the field, these studies will be concerned only with the second class of fineness measuring methods, namely, those by which particle size distributions are determined.

A number of reviews have appeared on methods of measuring particle size distribution, by Herdan and Smith(3), Schweyer and Work(4), Heywood(5), Work(6), Berg(7), Roller(8), Taggart(9b), Cadle(10) and DallaValle(2), but they have all lacked completeness and to some extent impartiality.

The increasing importance of particle sizing may be judged by the amount of current literature devoted to the subject and by the wide variety of industries which now rely on size analysis for control. Many of the applications of particle size analysis to research and industry have been listed by DallaValle(2), Hawksley(11) and Heywood(12). No list could be complete, however, since new uses for sizing information are continually arising as technological advances are made.

Particle size analysis is used in almost every phase of mineral dressing. Crushing and grinding, classification, thickening, filtration, flotation,and all the various concentration and extraction processes are regulated and controlled,to a greater or lesser extent, by the information obtained from particle size analysis. While sieve analysis has always been the principal sizing method used in milling operations, the trend toward finer grinding and the treatment of low-grade finely-ground ores has heightened the interest in and the use of many methods of measuring particle size distribution in the sub-sieve range.

Particle sizing methods have also been applied as control and research techniques in many other widely differing fields. They are used in the paint and pigment industry; in powder metallurgy; in the study of various sedimentary products such as gravel, silt, and clay; in the cosmetic industry; in the study and control of atmospheric, industrial and mine dust and smoke, and boiler and furnace grits and scale; in the preparation of grains and cereals, starches, flour, and cocoa; in the grading and manufacture of cement, ceramic clays, and moulding sands; in the preparation of colloidal precipitates; in adsorption studies; and in various phases of biological research.

This list, although incomplete, indicates the importance and wide use of particle sizing methods and equipment, both in controlling and in developing various industrial processes.

PART I - THE DEFINITION OF TERMS AND CLASSIFICATION OF SIZING METHODS

CHAPTER 1

DEFINITIONS OF THE TERM "PARTICLE SIZE"

A common fallacy in interpreting the results of particle size analyses is that the measured or indicated "size" of a particle is the same regardless of the sizing method used. Only in the case of sizing spherical particles is this true. The misconception arises, no doubt, from the fact that many of the methods for sizing irregularly-shaped particles are based on theories which are derived for spherical particles.

While the size of a spherical particle is completely defined by its diameter, the definition of the size of an irregular particle depends on the property of the particle being considered. Different methods of particle size analysis assess different size-dependent properties, such as volume, surface area, or resistance to motion in a fluid. Since many of these methods are based on theories which are derived for spheres, as stated above, the size of an irregular particle is conveniently expressed as the diameter of the sphere having the same volume or surface, etc.

By considering the three basic properties of a particle to be its volume, its surface, and its resistance to motion in a fluid, Hawksley(13) defined three basic sizes: the volume diameter, δ, i.e. the diameter of a sphere of the same volume; the surface diameter,

$\triangle$, i.e. the diameter of a sphere of the same surface area; and the drag diameter, d_d, i.e. the diameter of a sphere having the same resistance to motion in a fluid. Hawksley suggested that, for many sizing methods, the measured "size" could be related to one or more of these basic sizes.

In sieve testing, the particle size is defined as the side of a square aperture through which the particle just passes. Herdan(3) has stated that this is equivalent to the diameter of the sphere which just passes through the square aperture. In his study of size testing by sieving, Heywood(14) noted that the size separation was independent of the largest dimension of a particle. He stated that particles of the same smallest cross section (a function of breadth x thickness) were considered to be equal in size. Andreasen(15) defined the particle size for sieve testing as the side of the cube k equal in volume to the particle, and measured it by counting and weighing particles which just passed through or were just retained by a particular square sieve opening.

In particle size analysis by microscope, many different definitions are used. In general each of these is an approximation to δ, the volume diameter, defined as the diameter of a sphere of equivalent volume(13) or to k, defined by Andreasen(15) as the side of the cube of equal volume. Both δ and k have been measured by counting and weighing particles in closely sized fractions(15, 16).

However, by direct microscopic measurement, where particles are generally viewed in their most stable positions, the estimate of δ is based on the two largest dimensions of the particle. Since the shortest dimension (particle thickness) perpendicular to the focal plane is normally neglected or is difficult to measure, the size estimates are necessarily high. Even when measurements of the three dimensions of the particle are made, the size based on estimated volume can only be approximate, due to particle shape and surface configuration.

In the cases where microscopic estimations were made of the length L, breadth B and thickness T of particles, Work(6) suggested two definitions of particle size. These were given as $\sqrt[3]{L.B.T.}$ and $1/3\ (L + B + T)$. The harmonic mean, equal to $3L.B.T/(LB + LT + BT)$, was suggested by Green(17). Roller(8, 18) showed that the harmonic mean was equivalent to $6V/S$ where V is the volume and S is the surface area of the particle. In terms of the basic sizes defined by Hawksley(13), this size is equal to $\delta^3/\triangle^2$.

In general, speed and ease of measurement are main considerations in conducting microscopic size analyses. For these reasons primarily, the most popular definitions for microscopically measured particle size are based on some dimension or dimensions of the projected outline of the particle.

The mean projected diameter is defined by Heywood(5) as the diameter of the circle having the same projected area as the particle lying in its most stable position. This definition of particle size has been widely used and studied. Wadell(16) called this size the "nominal section diameter"; Beirne and Hutcheon(19) called it the "stable area diameter"; Hawksley(13) called it merely the "area diameter" and stated that the mean area diameter for all particle orientations was equivalent to the surface diameter $\triangle$.

Similar to this size designation is Perrott and Kinney's diameter(20), defined as the side of a square having the same area as the projected area of the particle.

By measuring by microscope a single linear dimension of the particles always in the same direction, one obtains a so-called statistical diameter. Examples of this type are statistical diameters defined by Feret(21) and Martin(22). The diameter defined by Feret is given by the distance between two parallel tangents to the viewed outline of the particle in a fixed direction. According to Martin's designation the diameter is defined as the length of the line bisecting the projected area of a particle in a fixed direction for all particles.

Weigel(23) defined the microscopic size simply as the arithmetic average of the two visible particle dimensions, i.e. $\frac{L + B}{2}$. Work(6) considered the thickness of particles, implicitly at least, by defining the size as the shorter of the two visible dimensions, i.e. B.

Many of these definitions have been listed and described by Schweyer(24).

Elutriation and sedimention methods of particle size analysis are based on Stokes' law which states that the velocity of a sphere of diameter d and density ρ settling in an infinite fluid of density ρ_o and viscosity μ is given by $\frac{1}{18} \frac{(\rho - \rho_o)}{\mu} g d^2$ where g is the gravitational constant. Each non-spherical particle has a characteristic settling velocity in a fluid of known density and viscosity. Hence its size is conveniently defined as the diameter of the sphere of equivalent density and settling velocity calculated by Stokes' law.

The size defined in this manner is thus independent of particle shape. It is often referred to as the sedimentation or the Stokes' diameter, but it might be more properly termed the Stokes' equivalent spherical diameter. In terms of the basic sizes defined by Hawksley(13), the Stokes' equivalent spherical diameter is equal to $\sqrt{\delta^3/d_d}$, where, as before, δ is the volume diameter and d_d is the drag diameter.

Andreasen(15) and Berg(7) extended Andreasen's definition of particle size used for sieving and microscopic sizing(15, 25) to sedimentation and elutriation by relating the size, k, equal to the cube root of the particle volume, to the Stokes' equivalent spherical diameter, d_s, by the equation $k = \sqrt[3]{\pi/6}\ d_s$.

CHAPTER 2

DEFINITIONS AND MEASUREMENTS OF PARTICLE SHAPE FACTORS

The shape of particles has a pronounced effect on the behaviour of material when being sized. The passage of particles through a sieve, the estimation of particle sizes microscopically, and the settling velocity of particles in a fluid are all dependent on particle shape. However, because of the conventions used in defining the term "particle size" for different sizing methods, it is not necessary to quantify particle shape in particle size analysis. Nevertheless, shape factors have been evaluated to determine the relationships between the various "particle sizes". Also, determinations of particle shape factors have been used in studies of various size-dependent properties, such as the covering power of pigment particles, and the packing and sintering of metal powders.

The shape of an irregular particle may be defined in terms of either its geometrical shape, i.e. the closeness with which the particle approaches the shape of a sphere, cube or other regular geometrical solid; or the particle proportions, i.e. length, breadth, and thickness(12). Some shape factors are directly applicable in relating particle size to the volume or surface area of the particle; other factors are merely descriptive.

2.1 Heywood's Shape Factors

The most fundamental approach to the quantifying of particle shape is that due to Heywood(5, 12, 26). His analysis, based on the microscopically determined projected area diameter, d_p, is used primarily to calculate factors for correlating the different particle sizes(12, 27).

Heywood first rigorously defines the dimensions of an irregular particle. Assuming the particle to be resting on a plane in the position of greatest stability, the breadth, B, is defined as the minimum distance between two parallel tangents to the viewed particle profile, and the length, L, is the distance between two parallel lines tangent to the profile and perpendicular to the lines defining the breadth. The thickness, T, is defined as the maximum height of the particle above, and perpendicular to, the plane of greatest stability.

The volume and surface shape factors, α_v and α_s, are defined by the equations

$$\alpha_v = V/d_p^3 \quad \text{and} \quad \alpha_s = S/d_p^2,$$

where V and S are the particle volume and surface area, respectively, and d_p is the projected area diameter.

To evaluate α_v in terms of L, B and T, Heywood defines two expressions describing the geometrical shape of a particle. These are the projected area ratio, α_p, and the prismoidal ratio, P_r, given by

$$a_p = \frac{\text{projected area of particle}}{L \times B} = \frac{\pi\, d_p^{\,2}}{4\, L\, B}$$

$$P_r = \frac{\text{volume of particle}}{T \times \text{projected area of particle}} = \frac{a_v\, d_p^{\,3}}{\frac{\pi d_p^{\,2}}{4}\; T} = \frac{4\, a_v\, d_p}{\pi\, T}$$

By eliminating d_p from these two equation, an expression for a_v is obtained:

$$a_v = \frac{\pi\sqrt{\pi}}{8} \cdot \frac{P_r}{\sqrt{a_p}} \cdot \frac{T}{\sqrt{L\, B}}$$

Two further ratios define the proportions of a particle. These are the flakiness, $m = B/T$, and the elongation, $n = L/B$. The factor $T/\sqrt{L\ B}$ in the equation for a_v is then $\frac{1}{m\sqrt{n}}$.

Heywood states that, if it is imagined that the proportions of a particle are changed without altering the geometrical shape, so that $m = n = 1$, the volume shape factor for this equidimensional particle is given by

$$a_o = \frac{\pi\sqrt{\pi}}{8} \cdot \frac{P_r}{\sqrt{a_p}}$$

and $$a_v = \frac{a_o}{m\sqrt{n}}.$$

Hence, the effect of particle proportions can be determined independently of the geometric shape. A large number of measurements of P_r and a_p for particles of various shapes give values of a_o which vary between fairly narrow limits, 0.38-0.54. By using an average value of a_o and estimating m and n, determinations of the volume shape factor of sufficient accuracy may be obtained.

Heywood's expression for the surface shape factor is

$$\alpha_s = \frac{\pi}{2} + C \left(\frac{\alpha_o}{m}\right)^{4/3} \left(\frac{n+1}{n}\right)$$

$$= \frac{\pi}{2} + C \cdot \alpha_v^{4/3} \left(\frac{n+1}{n^{1/3}}\right)$$

The derivation of this equation(28) is based on an analysis of the effect of varying particle proportions on the surface area of different geometrical solids. The factor C, like α_o, depends solely on geometrical shape. Values of C determined for large particles also vary between narrow limits, 2.1 to 3.3. The surface shape factor, α_s, can be determined by assuming an average value of C along with an estimate of particle proportions.

Heywood's volume and surface shape factors have been criticized by Schweyer(24) and Hawksley(13) on the grounds that determinations of α_o and C were made, of necessity, on large particles and may not apply equally to fine sizes of the same material. Although considerable difference of opinion is evident in the literature regarding the variation of shape with size, such investigators as Andreasen(15), Martin and Bowes(29), and Bond(30) have found shape to be independent of particle size.

2.2 Average Volume and Surface Shape Factors

Heywood's analysis, described in the previous section, provides a means of determining shape factors for individual particles. However, to characterize the non-sphericity of a sample of particles, average volume and surface shape factors are determined, based on the particle of average size, volume and surface. These factors are defined by the equations

$$\bar{v} = a_v d_v^{\,3}$$

and $$\bar{s} = a_s d_s^{\,2} ,$$

where $\bar{v}$ and $\bar{s}$ are the average volume and average surface area of a sample of particles and d_v and d_s are the mean volume and mean surface diameters respectively. d_v and d_s are given by

$$d_v = \sqrt[3]{\frac{\sum(nd^3)}{\sum n}} \quad \text{and} \quad d_s = \sqrt{\frac{\sum(nd^2)}{\sum n}} ,$$

where n is the number of particles of size d. Since a particle count is required, d is measured by sieve analysis for coarse samples and by microscopic size analysis, using one of the many definitions of particle size, for sub-sieve samples.

In the determination of a_v, the mean volume diameter d_v of the sample is first obtained. The mean volume $\bar{v}$ is determined from the number of particles per gram, N, and the density, ρ, of the material by means of the equation

$$\bar{v} = \frac{1}{\rho N} ,$$

from which $$a_v = \frac{1}{\rho N d_v^{\,3}} .$$

This general method was used by Hatch and Choate(31) and by Martin, Blyth and Tongue(22).

The surface shape factor α_s is determined from measurements of the specific surface, S_w, which is the surface area per unit weight, and is given by the equation

$$S_w = \frac{\bar{s}}{\rho \bar{v}} = \frac{\alpha_s \, d_s^2}{\rho \, \alpha_v \, d_v^3}$$

$$\text{or} \quad \alpha_s = \rho \, \alpha_v \, S_w \, \frac{d_v^3}{d_s^2}$$

Since the methods of measuring specific surface give widely different results, the values of α_s vary with the method used. The method described by DallaValle(2), wherein the surface area of quartz particles is determined by solution in dilute hydrofluoric acid, is limited, in applicability, to siliceous materials.

2.3 Descriptive Shape Factors

A number of different methods of defining and measuring factors descriptive of particle shape have been proposed. These are discussed under the heading of the name(s) of the proposing author(s).

(a) Wadell:

Wadell(16) has defined two expressions by which the shape of coarse irregular particles may be characterized. The first is the "degree of true sphericity", defined as $\psi = s/S$, where s is the surface area of a sphere of the same volume as the particle and S is the actual surface area of the particle. Because of the difficulties involved in measuring the surface area and volume of a small particle, Wadell defined a second expression which he called the "degree of true circularity". The circularity, ϕ, is given by c/C, where c is the circumference of a circle of area equal to the projected area of the particle resting in its most stable position, and C is the actual perimeter of the particle outline.

Wadell showed that $\phi = c/C$ was equivalent to d_p/d_c, where d_p is the mean projected diameter defined by Heywood(12) and d_c is the diameter of the smallest circumscribing circle to the particle outline.

More recently, Walton(32) determined geometrically that, when the particle profile had no appreciable re-entrant angles, a measure of the statistical diameter defined by Feret(21) was equivalent to the diameter of a circle having the same perimeter as the particle outline. Hence, he suggested that the degree of true circularity was given by the ratio of the mean projected diameter d_p to Feret's diameter.

Values of both sphericity, ψ, and circularity, ϕ, are 1.0 for spherical particles and < 1.0 for non-spherical shapes.

Besides being descriptive of particle shape, the sphericity and circularity factors were shown to bear definite relationships to the settling properties of particles.

(b) Austen and Gilbert:

To correlate filter performance with particle size and shape, Austen and Gilbert(33) measured Feret's statistical diameter(21) and the coefficient of variation $K = \sqrt{K_1^2 + K_2^2}$, where K_1 is due to scatter in size and K_2 to deviation from spherical shape. K_1 is determined by dividing the standard deviation by the mean of the diameters. K_2 is obtained by measuring the diameter of a number of particles in four orientations spaced at intervals of 45°. The four diameters are divided by their mean and K_2 is then the standard deviation from unity of all quantities obtained in this way. K_2 varies from 0 for particles of circular profile to 0.484 for needle-shaped particles.

(c) Schweyer:

Another descriptive factor is due to Schweyer(24) and is defined as $K = \alpha_s/\alpha_v$. Schweyer favoured the use of K since it bore a direct relationship to specific surface. K is determined by microscopic estimation of the dimensions of representative particles in closely sized fractions. Values of K range from 4 to 14 for particles which are acicular to plate-like in shape.

(d) Beirne and Hutcheon:

In a study of the packing of ground petroleum coke particles and the extent of their orientation in a fluid stream, Beirne and Hutcheon(19) used a descriptive shape factor which is defined by the relation

$$\sigma = \frac{\delta}{d_p} ,$$

where δ = Hawksley's volume diameter(13)

and d_p = Heywood's projected area diameter(12).

These experimenters felt that microscopic estimation of particle size was too prone to personal error. Hence, they contrived to measure three separate factors, namely

$\underline{A}$ = specific projected area = integrated area of the orthogonal projection of particles having a total mass of 1 g in their most stable position without overlap; (square centimetres per gram)

$\underline{V}$ = specific particle volume = integrated volume of particles having a total mass of 1 g, i.e. the reciprocal of the particle density; (cubic centimetres per gram)

$\underline{N}$ = number of particles per gram, i.e. the reciprocal of the mean particle mass.

The particle volume, $\frac{V}{N} = \frac{\pi}{6}\,\delta^3$,

from which $\delta = \left(\frac{6}{\pi}\,\frac{V}{N}\right)^{1/3}$.

Also, the projected area, $\frac{A}{N} = \frac{\pi}{4}\,d_p{}^2$,

from which $d_p = \left(\frac{4}{\pi}\,\frac{A}{N}\right)^{1/2}$.

Therefore, the descriptive shape factor

$$\sigma = \frac{\delta}{d_p} = \frac{6^{1/3}\,\pi^{1/6}}{2}\left(\frac{V^{1/3}\,N^{1/6}}{A^{1/2}}\right)$$

$$= 1.10\,\frac{V^{1/3}\,N^{1/6}}{A^{1/2}}\,.$$

The factor A was determined by a photocell measurement of the light transmitted through a slide on which particles were dispersed, and V was measured by a mercury density method. To measure N, single particles were drawn by a slight vacuum from a dilute suspension through a fine capillary. These particles passed successively beneath the objective of a microscope and intercepted light falling on a photomultiplier mounted in the eyepiece. The counted particles were collected and weighed to give N.

The value of σ is 1.0 for spheres and < 1.0 for non-spherical shapes. Beirne and Hutcheon found that σ for coke was about 0.5, corresponding to particles which are far from spherical in shape.

(e) Steinherz:

A descriptive shape factor, defined as d_p/T_M, is given by Steinherz(34), where d_p = projected area diameter and T_M = mean particle thickness. This factor is determined on sized fractions by first counting and weighing a number of particles to determine the mean particle volume. By microscopic measurements of d_p, the mean projected area of the particles in the same fraction is obtained. Then T_M = mean thickness = ratio of the mean particle volume to the mean projected area.

(f) Bond:

Bond(30) defined a factor f which measured the variation of the effective particle shape from that of the ideal ellipsoid having axes

A, B, and C, where A is the longest dimension of a given particle, B is the longest dimension at right angles to A, and C is the longest dimension at right angles to both A and B. f is defined by the relation

$$f = \frac{w}{\rho Vac}$$

where ρ = density of the material,

w = weight of sample,

V = volume of the sphere of diameter equal to the average particle size,

$$a = \frac{A}{B},$$

and $$c = \frac{C}{B}.$$

In the determination of f, which is made on screen-sized fractions, w is obtained by weighing and counting a representative number of particles. The average particle size is the arithmetic mean of the nominal apertures of the sieve on which the particles are retained and the next larger sieve in the $\sqrt{2}$ series through which the particles pass. The ratio a is determined by Bond's block method. In this method,particles were placed on their flattest side and packed together as closely as possible,without overlapping,in a square block having an area calculated to contain 100 spheres of the same average size. The number of particles actually contained in this area,divided by 100, is equal to a. The ratio c is evaluated by screen analysis of the sample, first on square-aperture sieves in the $\sqrt{2}$ series, and then, after re-mixing the sample, on a series of sieves having slotted openings equal in width to the square openings of the screen scale and having lengths about three times the slot width. Bond plotted both analyses on a log percent passing vs. log size graph and measured the horizontal spacing, n, between the two curves at each square sieve size. Then c was calculated from the relation

$$\log c = -0.1505\, n.$$

Bond's factor f was used to characterize the shape of particles in the studies of the control of particle size and shape in crushing and grinding.

2.4 Summary of Methods for Characterizing Particle Shape

In this section a number of methods for evaluating the shape of irregular particles have been described. Although it cannot be claimed that this list is complete, the methods described represent the approaches that are generally made when a measurement of particle shape is required. Particular emphasis has been placed on Heywood's analysis, for two reasons. The first is that Heywood's is the only method based on fundamental properties of an irregularly-shaped particle. By means of the equations evolved in his analysis, the effect of geometrical shape has been, for the first time, separated from that due to the dimensions of a particle. The second is that, by using Heywood's analysis, a means exists of determining, theoretically, factors which relate the particle sizes defined for different sizing methods.

CHAPTER 3

CORRELATION FACTORS

Methods of size analysis based on different principles, and for which different definitions of particle size are used, give, in general, different results. Correlation factors are defined here as factors relating these differently-defined particle sizes, so that

(1) there will be no break observed in the size distribution curve where two or more methods are used to size a sample in order to cover a wide range of sizes;

(2) size analyses may be compared when two or more methods are used to size a single sample in the same size range.

Only for spherical particles will different methods of size analysis give the same results. Since it is the non-sphericity of particles which causes the results obtained by various methods to differ, factors relating them have been called "shape factors"(3, 13). However, it is suggested that it is preferable to designate these factors as "correlation factors", leaving the term "shape factor" to characterize actual particle shape.

Most correlation factors have been determined empirically by comparing analyses on the same sample by different methods of size analysis. However, as pointed out in the preceding section, Heywood's analysis, by which volume and surface shape factors are determined, may be extended to obtain correlation factors from basic theoretical considerations.

3.1 Derivation of the Factor Relating Microscopic Diameter and Sieve Aperture

The largest particle which will just pass a square sieve opening of side A is one in which $B \simeq T \approx A$, where B and T are the breadth and thickness of a particle as defined by Heywood(26). Hence, to a close approximation

$$A = \sqrt{\frac{B^2 + T^2}{2}} .$$

$$\frac{B}{A} = \sqrt{\frac{2B^2}{B^2 + T^2}} = \sqrt{\frac{2m^2}{m^2 + 1}} ,$$

where m = flakiness = B/T.

Heywood's equation for the projected area ratio is

$$a_p = \frac{\pi d_p^2}{4 L.B} ,$$

where d_p = projected area diameter and L = particle length as defined by Heywood. For angular particles he found $a_p = 0.75$.

$$\therefore \quad d_p = \sqrt{\frac{3 L.B}{\pi}} = \sqrt{\frac{3 nB^2}{\pi}} ,$$

where n = elongation = L/B. It may be written

$$\frac{d_p}{B} = \sqrt{\frac{3n}{\pi}} .$$

$$\therefore \quad \frac{d_p}{A} = \frac{B}{A} \cdot \frac{d_p}{B} = \sqrt{\frac{2m^2}{m^2 + 1} \cdot \frac{3n}{\pi}}$$

$$= \sqrt{\frac{6m^2 n}{\pi(m^2 + 1)}} .$$

Table 1 gives calculated values of this ratio for various values of particle flakiness and elongation(3).

TABLE 1

Values of d_p/A for Particles of Varying Proportions

n ╲ m	1	1.5	2.0	2.5
1	1.00	1.14	1.23	1.28
1.5	1.22	1.39	1.51	1.57
2	1.41	1.61	1.74	1.82
2.5	1.57	1.80	1.95	2.03

The calculated values vary from 1.00 for equidimensional particles to 2.03 for particles of extreme elongation and flakiness.

3.2 Derivation of Factor Relating Microscopic Diameter and the Stokes' Equivalent Spherical Diameter

Robins(27) gave an expression for the ratio d_p/d_s, where d_p is, as before, the projected area diameter and d_s is the Stokes' equivalent spherical diameter. His equation is

$$\frac{d_p}{d_s} = \sqrt{\frac{(\pi \alpha_s)^{1/2}}{6 \alpha_v}} ,$$

where α_v and α_s are Heywood's volume and surface shape factors(12).

The derivation of Robins' equation is as follows:

Hawksley(13) showed that $\psi = (d_s/\delta)^4$, where δ is the diameter of the sphere of equivalent volume and ψ is Wadell's degree of true sphericity(16), equal to the ratio of the surface of a sphere of equal volume to the actual surface of the particle.

By definition, $\psi = \frac{\pi \delta^2}{a_s \Delta^2}$, where Δ = surface diameter as defined by Hawksley(13). Hawksley showed that

$d_p \approx \Delta$,

$$\therefore \psi = \frac{\pi \delta^2}{a_s d_p^2} .$$

Equating the two expressions for ψ ,

$$\left(\frac{d_s}{\delta}\right)^4 = \frac{\pi \delta^2}{a_s d_p^2} .$$

$$\therefore \delta^3 = \sqrt{\frac{a_s}{\pi}} \, d_p d_s^2 .$$

It was shown by Heywood(12) that

$$\frac{\delta}{d_p} = \left(\frac{6 a_v}{\pi}\right)^{1/3} \quad \text{or} \quad \delta^3 = \frac{6 a_v}{\pi} d_p^3 .$$

Equating these two expressions for δ^3,

$$\sqrt{\frac{a_s}{\pi}} \, d_p d_s^2 = \frac{6 a_v}{\pi} d_p^3 ,$$

$$\text{or} \quad \frac{d_p}{d_s} = \sqrt{\frac{(\pi a_s)^{1/2}}{6 a_v}} .$$

The equation for the theoretical relationship between the Stokes' equivalent spherical diameter and the corresponding sieve aperture is obtained by dividing the expression for d_p/A by that for d_p/d_s, i.e. $$\frac{d_s}{A} = \sqrt{\frac{36\,\alpha_V\,m^2 n}{(\pi\alpha_s)^{1/2}\,\pi\,(m^2+1)}}$$

Tables 2 and 3 give values of d_p/d_s and d_s/A as calculated by Robins (27) for particles of various particle dimensions. In making these calculations, Robins selected average values of Heywood's non-dimensional constants, $\alpha_o = 0.47$ and $C = 3.0$.

TABLE 2

Values of d_p/d_s for Particles of Varying Proportions

$n \diagdown m$	1	$\sqrt{2}$	2	$2\sqrt{2}$
1	1.00	1.26	1.39	1.59
$\sqrt{2}$	1.17	1.32	1.50	1.73
2	1.27	1.44	1.61	1.86

TABLE 3

Values of d_s/A for Particles of Varying Proportions

$n \diagdown m$	1	$\sqrt{2}$	2	$2\sqrt{2}$
1	1.00	0.89	0.89	0.82
$\sqrt{2}$	1.02	1.02	0.98	0.89
2	1.11	1.11	1.08	0.99

A number of empirical values of these three correlation factors have been determined by different investigators and are listed in Table 4.

TABLE 4

Empirical Values of Correlation Factors

Material	Mesh Size	d_p/A	d_p/d_s	d_s/A	Ref.
Copper shot	10	1.05	1.00[AAA]		(5)
Galena	325			1.14	(24)
Silica	325			1.09	(24)
Sand	10	1.40	1.39[AAA]		(5)
"	18	1.39			(5)
Sillimanite	10	1.50	1.46[AAA]		(5)
"	100	1.48			(5)
"	300	1.45			(5)
Coal	10	1.48	1.48[AAA]		(5)
"	100	1.46			(5)
"	300	1.40			(5)
"	325			1.09	(24)
"	minus 200		1.56		(27)
Coal	minus 15μ		1.55[AA]		(36)
Glass	minus 15μ		1.49		(36)
Blast furnace slag	10	1.48	1.60[AAA]		(5)
" "	100	1.40			(5)
" "	300	1.41			(5)
Limestone	10	1.56	1.74[AAA]		(5)
Plumbago	10	1.61	1.75[AAA]		(5)
Talc	10	1.76	1.74[AAA]		(5)
"	100	1.60			(5)
"	300	1.65			(5)
Gypsum	10	1.56	1.94[AAA]		(5)
"	100	1.53			(5)
"	300	1.61			(5)
Green slate	325			0.87	(24)
Flake graphite	18	1.69	4.36[AAA]		(5)
Mica	10	1.68	11.60[AAA]		(5)
"	325			0.80	(24)
Petroleum coke	36-52[A]	1.94			(19)
"	52-72[A]	2.02			(19)
"	72-100[A]	2.65			(19)
"	100-120[A]	2.18			(19)
"	120-200[A]	1.87			(19)
"	200-240[A]	1.84			(19)

[A] British Standard Association mesh designations.

[AA] The actual observed value of d_p/d_s was 1.35. However, since the determinations of microscopic size were made on particles in no preferred orientation, the value of d_p was calculated from the relation $\pi \Delta^2 = \alpha_s d_p^2$, where Δ is the surface diameter and α_s = surface shape factor = 2.39 for coal dust.

[AAA] Values of d_p/d_s given by Rose(35) obtained from an empirical relation,

$$d_p/d_s \sim \frac{0.72}{\sqrt{\alpha_v}} ,$$

based on Heywood's settling data(5).

A comparison between the calculated correlation factors (Tables 1, 2 and 3) and the observed factors (Table 4) indicates that there is a considerable measure of agreement. In general it appears that the theoretical factors can be applied with confidence in relating particle sizes defined in different ways.

Factors relating many of the other microscopically determined particle sizes have been given by Schweyer(24), Rose(35), Skinner et al(37), Heywood(26), and Steinherz(34).

It should be emphasized that correlation factors, as such, are not necessary for determining the particle size distribution of a particulate material. However, where a discontinuity occurs in a distribution curve when different sizing methods are used to size different size ranges, or where it is required to assess the differences between size analyses done by different methods on the same sample, it may be necessary to apply such factors as have been described in this section.

CHAPTER 4

CLASSIFICATION OF METHODS OF PARTICLE SIZE ANALYSIS

Particle sizing methods have been classified in various ways by different writers(38, 4, 6). However, for the purpose of this series, the classification suggested by Heywood(12) was selected since it permits the clearest distinction to be drawn between different methods.

Sizing methods are first classified into two groups: those depending on geometrical similarity, i.e. the size and shape of particles alone, and those depending on aerodynamic or hydrodynamic similarity, i.e. the size, shape and density of particles. The first group consists of sieve analysis and microscopic methods, while in the second group are those methods which are based on Stokes' law of settling. Using a similar classification, DallaValle(2) designated the first group as direct, and the second group as indirect methods.

Table 5 gives the complete classification of the methods of particle size analysis to be discussed in future reports of this series.

TABLE 5

Classification of Sizing Methods

Group I - Methods based on geometrical similarity.

- A - Sieve analysis
- B - Microscopic size analysis

Group II - Aerodynamic or hydrodynamic similarity.

- A - Elutriation
  1. Air
  2. Liquid
- B - Sedimentation
  1. Methods giving a size distribution directly (incremental)
  2. Methods giving an accumulation curve from which the size distribution is determined (cumulative)
  3. Methods giving sized fractions

PART II - SIEVE ANALYSIS

CHAPTER 5

SIEVES

5.1 Definition

A sieve or screen is defined by the American Society for Testing Materials (A.S.T.M.) as a plate, sheet or woven cloth, or other device, with regularly spaced apertures of uniform size, mounted in a suitable frame or holder, for use in separating material according to size(39a). The same standard definition states that,unless otherwise specified, the term "sieve" applies to an apparatus in which openings are square, and the term "screen" to an apparatus in which the openings are circular. In general practice, however, the terms "sieve" and "screen" are used interchangeably.

Sieve analysis is applied to the sizing of particles in a wide range, from about 4 in. pieces to particles as fine as 40μ (0.0016 in.).

5.2 Construction of Sieves

In general, sieves for testing purposes are constructed of woven-wire cloth having approximately square openings and are mounted on 8-in. open frames. This standard mounting procedure allows several sieves of different mesh to be nested with a cover and receiver. This is especially convenient where machine sieving is used. In some special cases, punched plates are used instead of the woven-wire cloth. These may have square, circular or slotted openings and can be made to closer tolerances than the woven-wire sieves. Due principally to problems of construction, punched plates are generally available only for sizes in excess of 1/8 in. Punched plates have been used for the sizing of coal.

MacCalman(40b) has described in detail the weaving and construction of woven-wire sieves. Plain weave construction is used for sieve cloth down to 200 mesh (about 74μ aperture). Sieves finer than 200 mesh are manufactured of twill weave cloth.

In general, woven-wire sieves are constructed of brass or of phosphor bronze. For special requirements, stainless steel, monel or mild steel is used. While 8-in. diameter mounting frames are standard for testing sieves, smaller or larger frames ranging from 3 in. to 12 in. are obtainable.

5.3 Standard Sieve Series and Sieve Specifications

So that sieve sizing results may be compared and duplicated, a number of series of sieves have been established. Each of these series consists of sieves having definite nominal apertures and wire diameters.

The history and development of the various sieve series has been described by MacCalman(40a). These series have been established, for the most part, on a regional basis. On the North American continent the Tyler series(41) is widely used and is based on a 200 mesh sieve having an aperture of 0.0029 in., or 74μ. The nominal aperture sizes of successive sieves in the series form an approximate geometric progression, i.e. a constant ratio exists between aperture sizes of successive sieves in the series. In the case of the Tyler series the ratio is $\sqrt{2}$ or, for close sizing, $\sqrt[4]{2}$. The U.S. sieve series(39b) has the same ratio but is based on the 18 mesh sieve having an aperture of 1 mm. Actually, the Tyler and U.S. series are almost completely interchangeable. The tolerances on sieve openings and wire diameters given by the U.S. Standard Specifications are used for both series.

In Britain, the Institution of Mining and Metallurgy (I.M.M.) Standard Laboratory Screen Series, which was set up in 1907, has been almost completely superseded by the British Standards Institution (B. S.) Fine Series(42a). The B. S. series has also a $\sqrt[4]{2}$ inter-sieve ratio and sieves of this series are, in general, interchangeable with Tyler and U.S. standard sieves. The slight variations in aperture sizes are due to differences between British and American wire gauges.

On the European continent, the German Deutsche Industrie-Normen (D.I.N.) series(43) is widely used along with the Tyler series. The German series is similar to the French AssociationFrancaise deNormalisation (A.F.N.O.R.) series(44), which has an approximate inter-sieve ratio of $\sqrt[10]{10} = 1.259$.

Associated with most of these series are standard specifications listing the permissible tolerances on size of openings and wire diameters of sieves for testing purposes. The American, British, German and Canadian standard specifications for fine mesh testing sieves are given in Tables 6-9 respectively. Table 10 lists the comparative mesh designations and nominal apertures of the Tyler, U.S. Standard, I. M. M., B.S., A.F.N.O.R. and D.I.N. sieve series.

TABLE 6

American Standard Specifications for Fine Testing Sieves (39b)

Sieve Designation[A]	Sieve Opening, mm	Permissible Variations		Wire Diameter, mm
		in Average Opening, percent	in Maximum Opening[AA], percent	
3½	5.66	± 3	+10	1.28 to 1.90
4	4.76	3	10	1.14 to 1.68
5	4.00	3	10	1.00 to 1.47
6	3.36	3	10	0.87 to 1.32
7	2.83	3	10	0.80 to 1.20
8	2.38	3	10	0.74 to 1.10
10	2.00	3	10	0.68 to 1.00
12	1.68	3	10	0.62 to 0.90
14	1.41	3	10	0.56 to 0.80
16	1.19	3	10	0.50 to 0.70
18	1.00	5	15	0.43 to 0.62
20	0.84	5	15	0.38 to 0.55
25	0.71	5	15	0.33 to 0.48
30	0.59	5	15	0.29 to 0.42
35	0.50	5	15	0.26 to 0.37
40	0.42	5	25	0.23 to 0.33
45	0.35	5	25	0.20 to 0.29
50	0.297	5	25	0.170 to 0.253
60	0.250	5	25	0.149 to 0.220
70	0.210	5	25	0.130 to 0.187
80	0.177	6	40	0.114 to 0.154
100	0.149	6	40	0.096 to 0.125
120	0.125	6	40	0.079 to 0.103
140	0.105	6	40	0.063 to 0.087
170	0.088	6	40	0.054 to 0.073
200	0.074	7	60	0.045 to 0.061
230	0.062	7	90	0.039 to 0.052
270	0.053	7	90	0.035 to 0.046
325	0.044	7	90	0.031 to 0.040
400	0.037	7	90	0.023 to 0.035

[A] The nominal size of sieve aperture in microns is also used as sieve designation, e.g. No. 18 (1000μ).

[AA] For sieves from No. 18 to No. 400, inclusive, not more than 5% of the openings shall exceed the nominal opening by more than one-half of the permissible variation in maximum opening.

TABLE 7

British Standard Specifications for Fine Mesh Normal and Special Test Sieves (42a)

Mesh Number	Nominal Width of Aperture, microns	Nominal Wire Diameter		Aperture Tolerances, percent					
				Average[AA]		Intermediate[AAA]		Maximum[AAA]	
		microns	S.W.G.[A]	Normal Sieves	Special Sieves	Normal Sieves	Special Sieves	Normal Sieves	Special Sieves
5	3353	1727	15½	3.2	3.0	7	7	11	10
6	2812	1422	17	3.2	3.1	7	7	11	10
7	2411	1219	18	3.3	3.0	7	7	11	10
8	2057	1118	18½	3.3	3.1	7	7	12	10
10	1676	864	20½	3.3	3.0	8	7	12	11
12	1405	711	22	3.4	3.1	8	7	12	11
14	1204	610	23	3.6	3.2	8	7	13	11
16	1003	584	23½	3.5	3.3	9	8	13	12
18	853	559	24	3.9	3.3	9	8	14	13
22	699	457	26	4.0	3.3	9	8	15	13
25	599	417	27	4.2	3.3	10	8	16	14
30	500	345	29	4.4	3.4	11	9	17	15
36	422	284	31½	4.6	3.5	11	10	18	16
44	353	224	34½	4.8	3.6	12	10	19	17
52	295	193	36	5.1	3.7	13	11	22	18
60	251	173	37	5.3	3.8	14	12	23	20
72	211	142	38½	5.5	4.0	16	13	25	22
85	178	122	40	5.9	4.3	17	14	29	24
100	152	102	42	6.2	4.5	18	16	32	27
120	124	86	43½	6.5	4.7	22	18	37	30
150	104	66	45½	7.1	5.1	24	20	41	34
170	89	61	46	7.4	5.4	27	22	49	40
200	76	51	47	8.0	6.0	31	25	53	43
240	64	41	48	8.4	6.8	35	28	64	52
300	53	30	49	9.0	7.1	40	32	71	57
350	44	28	49½	11		48		85	

[A] Maximum permissible deviation from nominal wire diameter is one half gauge.

[AA] Average aperture width shall not be greater or smaller than nominal size by more than average tolerance.

[AAA] No more than 6% of the openings shall be larger than the nominal size, and no more than 6% of the openings shall be smaller than the nominal size by more than the intermediate tolerance.

[AAAA] The maximum positive deviation in aperture width shall not exceed the maximum tolerance.

TABLE 8

German Standard Test Sieve Specifications (43)

Sieve Designation and Nominal Aperture, mm	Previous Designation, mesh per cm	Range of Maximum Tolerances on Aperture Width, percent	Nominal Wire Diameter, mm	Maximum Tolerances on Wire Diameter, percent
6.0		10 to 20	2.5	+8
5.0		10 " 20	2.0	8
4.0		10 " 20	1.6	8
3.0		10 " 20	1.2	8
2.5		10 " 20	1.0	8
2.0		10 " 20	1.0	8
1.5	4	10 " 20	1.0	8
1.2	5	10 " 20	0.8	8
1.0	6	10 " 20	0.65	8
0.75	8	10 " 20	0.5	10
0.6	10	10 " 20	0.4	10
0.5	12	10 " 20	0.34	10
0.43	14	10 " 20	0.28	10
0.4	16	10 " 20	0.24	10
0.3	20	10 " 20	0.20	10
0.25	24	12 " 25	0.17	10
0.20	30	12 " 25	0.13	10
0.15	40	12 " 25	0.10	10
0.12	50	12 " 25	0.08	10
0.100	60	15 " 30	0.065	10
0.090	70	15 " 30	0.055	10
0.075	80	15 " 30	0.050	10
0.060	100	15 " 30	0.040	10

Note: The tolerances on both average apertures and average wire diameters is ± 5% for all sieves in the series. Not more than 6% of the measured apertures shall lie within the range of maximum tolerances.

Not more than 6% of the measured wire diameters shall exceed the tolerance on average diameters.

In determination of the average value of apertures any aperture which is more than 5% below the nominal value shall be disregarded.

TABLE 9

Canadian Government Specifications for Woven Wire Testing Sieves (45)

Designation, microns	Opening, mm	Wire Diameters[A], mm	Tolerances (in each direction), percent[AA]			
			Average Opening		Maximum Opening	
			Standard Sieves	Special Sieves	Standard Sieves	Special Sieves
5660	5.66	1.68	± 3	± 2	+10	+ 5
4760	4.76	1.54	3	2	10	5
4000	4.00	1.37	3	2	10	5
3360	3.36	1.23	3	2	10	5
2830	2.83	1.10	3	2	10	5
2380	2.38	1.00	3	2	10	5
2000	2.00	0.900	3	2	10	5
1680	1.68	0.810	3	2	10	5
1410	1.41	0.725	3	2	10	5
1190	1.19	0.650	3	2	10	5
1000	1.00	0.580	5	3	15	8
840	0.84	0.510	5	3	15	8
710	0.71	0.450	5	3	15	8
590	0.59	0.390	5	3	15	8
500	0.50	0.340	5	3	15	
420	0.42	0.290	5	4	25	10
350	0.35	0.247	5	4	25	10
297	0.297	0.215	5	4	25	10
250	0.250	0.180	5	4	25	10
210	0.210	0.152	5	4	25	10
177	0.177	0.131	6	4	40	20
149	0.149	0.110	6	4	40	20
125	0.125	0.091	6	4	40	20
105	0.105	0.076	6	4	40	20
88	0.088	0.064	6	4	40	20
74	0.074	0.053	7	4	60	20
62	0.063	0.044	7		60	
53	0.053	0.037	7		60	
44	0.044	0.030	7		60	
37	0.037	0.025	7		60	

[A] The average diameter of the warp and weft wires, taken separately, of the cloth of any sieve shall not deviate from the nominal value by more than the following:

Sieves coarser than 590 microns - 5%
Sieves 590 microns to 125 microns - 10%
Sieves finer than 125 microns - 15%

[AA] When the tolerance for maximum opening is 15% or more, not more than 5% of the openings measured shall exceed the nominal width of opening by more than 25% and in any sieve not more than 10% of the openings measured shall exceed the upper limit for size of average opening.

TABLE 10

Comparative Mesh Designations and Nominal Aperture Sizes for Different Standard Sieve Series

Tyler(41)		U.S. Standard(39b)		Inst. of Mining and Metallurgy (42a)		British Standard (42a)		French Standard(44)	German Standard(42a)	
Mesh	mm	Mesh	mm	Mesh	mm	Mesh	mm	mm	Mesh	mm
3½	5.613	3½	5.66							6.0
4	4.699	4	4.76					5.00		5.0
5	3.962	5	4.00					4.00		4.0
6	3.327	6	3.36			5	3.353	3.15		3.0
7	2.794	7	2.83			6	2.812			
8	2.362	8	2.38	5	2.540	7	2.411	2.50		2.5
9	1.981	10	2.00			8	2.057	2.00		2.0
10	1.651	12	1.68	8	1.574	10	1.676	1.60	(4)	1.5
12	1.397	14	1.41			12	1.405			
14	1.168	16	1.19	10	1.270	14	1.204	1.25	(5)	1.2
16	0.991	18	1.00	12	1.056	16	1.003	1.00	(6)	1.0
20	0.833	20	0.84			18	0.853			
				16	0.792			0.80	(8)	0.75
24	0.701	25	0.71			22	0.699			
				20	0.635			0.63		
28	0.589	30	0.59			25	0.599		(10)	0.6
32	0.495	35	0.50			30	0.500	0.50	(12)	0.5
35	0.417	40	0.42	30	0.421	36	0.422		(14)	0.43
								0.40	(16)	0.4
42	0.351	45	0.35			44	0.353			
48	0.295	50	0.297	40	0.317	52	0.295	0.315	(20)	0.3
60	0.246	60	0.250	50	0.254	60	0.251	0.25	(24)	0.25
65	0.208	70	0.210	60	0.211	72	0.211		(30)	0.20
80	0.175	80	0.177	70	0.180	85	0.178			
100	0.147	100	0.149	80	0.157	100	0.152	0.16	(40)	0.15
				90	0.139					
115	0.124	120	0.125	100	0.127	120	0.124	0.125	(50)	0.12
150	0.104	140	0.105	120	0.107	150	0.104	0.10	(60)	0.100
170	0.088	170	0.088	150	0.084	170	0.089		(70)	0.090
200	0.074	200	0.074			200	0.076	0.08	(80)	0.075
250	0.061	230	0.062	200	0.063	240	0.064	0.063	(100)	0.060
270	0.053	270	0.053			300	0.053	0.05		
325	0.043	325	0.044			350	0.044			
400	0.038	400	0.037							

5.4 Methods for Sieve Certification

The A.S.T.M. Standard Specifications for Sieves for Testing Purposes(39b) includes a description of the methods used by the National Bureau of Standards for sieve certification, i.e. for assuring that a sieve conforms dimensionally to specified tolerances. Projection techniques are employed to cast an image of the wire mesh on a ground glass screen, a means being provided to traverse the sieve in both warp and woof directions. It is found that the most reliable results are obtained by measuring five to ten wire diameters, determining the number of wires for a unit length and then computing the size of the average opening. This procedure is followed over the whole sieving surface. At the same time, oversize openings are measured and indications of non-uniformity in the sieving cloth are noted. A similar method is used by the British Standards Institution(42a) to certify that sieves of the B. S. Normal or Special series conform to specifications.

CHAPTER 6

STANDARDIZATION OF SIEVES

6.1 Permissible Tolerances on Wire Mesh Sieves and the Necessity for Standardization

An examination of the standard specifications for sieving cloth given in Tables 6-9 indicates that the tolerances are surprisingly large. However, they are evidently consistent with the difficulties encountered in fine sieve construction. For example the A.S.T.M.

specification(39b) for the U.S. sieve series allows tolerances on the size of the average opening of from ± 5% for the 18 mesh sieve to ± 7% for the 400 mesh sieve. The tolerances on the size of maximum openings vary from +15% for the 18 mesh to +90% for the 400 mesh, with the added condition that not more than 5% of the openings shall exceed the nominal opening by more than half the permissible tolerance on the size of the maximum opening. Accordingly, a certified 400 mesh sieve, having a nominal aperture of 37μ, is permitted to have an average aperture between 34 and 40μ and also to have as many as 5% of its openings between 54 and 70μ.

The British Standards Institution has specified two fine-mesh series for testing sieves -- the "normal" and the "special". Both series have the same aperture sizes and wire diameters, but the tolerances allowed on the special series are considerably smaller than on the normal and hence the special series is used where higher accuracy is required(42a).

To demonstrate the magnitude of the variations in sieve aperture permitted by the American specifications, Table 11 was prepared. Calculations based on specified tolerances were made for selected sieves in the fine mesh range. The other sieve specifications permit variations in aperture size similar to those calculated from the American Standard. It is apparent, then, that certifying that a sieve meets the standard specifications does not ensure that the sieve is structurally perfect.

TABLE 11

Permissible Aperture Sizes for Selected Sieves of the U.S. Standard Sieve Series, According to the American Standard Specifications

Sieve Designation, mesh	Nominal Opening, microns	Average Opening		Not more than 5% of measured openings to lie between limits, and none to exceed upper limit	
		Min microns	Max microns	Lower Limit, microns	Upper Limit, microns
20	840	798	883	903	967
30	590	561	620	634	671
40	420	399	441	473	525
50	297	282	312	334	371
70	210	199	221	236	263
100	149	140	158	179	209
140	105	99	111	126	147
200	74	69	79	96	118
270	53	49	57	77	101
325	44	41	47	64	84
400	37	34	40	54	70

In addition, it has been noted by MacCalman(40c), Pollard(46), Lake Shore Mines(47), the British Standards Institution(42a) and the National Bureau of Standards(48), that sieve certification does not ensure that comparable sieving results will be obtained. Wide variations in results were obtained when different certified sieves of the same nominal apertures were compared, especially at the finer sizes. In view of the wide variations allowed in sieving cloth, as shown in Table 11, such discrepancies in sieving results are not unexpected.

Moreover, most practical sieve sizing is done with uncertified sieves. Hence, the errors in sieving results from these are likely to be even larger than when certified sieves are used.

Some disagreement exists in the sieving literature regarding the relative effect of errors caused by variations in the sieves themselves, as compared with errors caused by other sieving variables, such as sieve loading and duration of sieving. However, it is generally agreed that some method more sensitive than certification for calibrating or standardizing sieves is necessary.

6.2 Methods of Sieve Standardization

Because of imperfections in drawn wire, and in methods for weaving and mounting it, there is a considerable and variable spread in aperture sizes in each particular sieve. As a result, also, of these imperfections, the apertures are not, in general, perfectly square. Obviously, then, the problem of standardizing sieves having such constructional imperfections is not likely to allow of a simple or generally acceptable solution.

The term "standardization" implies that the sieving results, or the sieves themselves, are to be related to some standard. This standard may be either some material for which the size distribution is known and may be assumed or measured, or another sieve with separating characteristics that are known or can be measured.

A number of different methods have been proposed for standardizing testing sieves. Several of these will be briefly described in this section. Although the list is not exhaustive, it includes the principal methods which have been or are being used. In addition, it indicates the varied approaches that different workers in the sieve sizing field have adopted to correct for the unavoidable errors in woven-wire sieves due to the generous tolerances allowed in their manufacture.

In general, sieve standardization methods may be classified into five groups, namely:

1. Those in which the effective separating size of a sieve is determined by measurements of wire diameters and aperture widths.
2. Those in which the effective separating size or size distribution corrections are determined by sieving a standard calibrated sample of spherical particles.
3. Those in which size distribution corrections for sieves are determined by comparison with the sieving results obtained with master sieves.
4. Those in which effective separating sizes of sieves, or size distribution corrections, are determined by using a standard sample having a size distribution which follows some theoretical size distribution law.

5. Those in which the effective separating size of a sieve is determined by size measurements of sieving products.

6.2.1 Standardization by Measurement of Wire Diameters and Aperture Widths:

(a) In a study of the sieving of metal powders, Pollard(46) noted significant variations when different sets of certified sieves were used to size the same sample. He found that these differences were greatly reduced when comparisons were made on the basis of the size of the average opening, as determined by standard certification tests, instead of the nominal opening.

(b) To calibrate testing sieves, Weber and Moran(49) also used a procedure similar to the standard certification tests. Measurements were made on representative groups of five adjacent individual openings along the warp and woof directions, measuring the same number of openings in each direction. Good reproducibility was obtained by 100 measurements on sieves coarser than 200 mesh, and by 200 measurements on 200 mesh sieves or finer. The average opening $\overline{X}$, and the percent standard deviation $S = 100\ \sigma/\overline{X}$, were calculated, σ being the standard deviation of the $\overline{X}$'s. Weber and Moran noted that, when the value of S exceeded 6%, sieve non-uniformity was evident. They concluded that, for sieves having $S < 6\%$, the effective separating size was given adequately by $\overline{X}$. For sieves where $S > 6\%$, indicating a considerable number of oversize openings, the effective size was dependent on the duration of sieving and was then calculated from the following empirical equation:

$$X_t = \overline{X}\left[1 + 0.002t\left(\frac{S - 6}{0.06}\right)^{\frac{1}{2}}\right],$$

where X_t is the effective opening for a sieving time t,

$\overline{X}$ is the average opening measured by the microscopic method described above,

t is the sieving time in minutes, and S is the percent standard deviation.

Weber and Moran stated that these values $\overline{X}$ and X_t were independent of the size distribution of material sieved.

(c) In a thorough investigation of British Standard sieves, MacCalman(40c) used certification techniques to measure dimensional characteristics of a number of sieves, both new and used, from coarse to 200 mesh. Mean aperture sizes, mean wire diameters and the frequency of oversize openings were measured for each sieve. MacCalman attempted to correlate these measurements with the corresponding sieving results on a standard sample, using a rigorous sieving procedure. He found that, although there was an apparent tendency for sieving results to follow the mean aperture size, it appeared impossible to predict sieving results from a knowledge of the dimensional characteristics of the sieves.

6.2.2 Standardization by Using a Calibrated Sample of Spherical Particles:

(a) In his sieving tests to investigate possible correlations between the dimensional characteristics of sieves and the sieving results, MacCalman(40c) used a sample of essentially spherical steel shot. He found that glass beads were unsuitable, apparently because of electrostatic effects in sieving. By using spherical particles, the effect of irregular particle shape on sieving results was eliminated. In addition, by sizing the spheres by another method, i. e. microscopically, it was possible to calibrate the sieves absolutely, rather than relative to one another. The effective separating size of the equivalent "perfect" sieve was thus obtained by interpolation from the calibrated distribution curve, i.e. at the cumulative percentage oversize corresponding to the cumulative percentage retained in the test. He noted, however, that sieve apertures were, in general, rectangular in shape rather than perfectly square. Hence, the effective aperture given by spherical particles was a measure of the lesser width and did not necessarily coincide with the measure of effective aperture in sieving irregularly shaped particles.

(b) Another intense investigation of sieving was conducted at the U.S. National Bureau of Standards, the results being reported by Carpenter and Deitz(50, 51). They used a calibrated sample of spherical glass beads to standardize sieve openings.

Their procedure was as follows: Different lots of glass beads obtained from various manufacturers of highway markings were mixed to produce a large sample having a continuous size distribution in the range from 18 mesh, U.S. Standard (1000μ) to 170 mesh (80μ). Non-spherical beads were separated and rejected. A careful sampling procedure was used to cut an original head sample weighing 56 lb into 256 100-g samples.

The size distribution of one of these sub-samples was measured by microscopic count and size estimation on about 10,000 beads. A curve plotting the cumulative weight percentage finer than various diameters against bead diameters was obtained. For standardization, another 100 g sample was sieved on the sieve in question, using a standard procedure. The effective size corresponding to the measured percentage passing this sieve was obtained from the calibration curve. Conversely, from the calibration curve, the percentage finer than the nominal sieve aperture sizes could be obtained, providing in this way a correction to the percentage passing for each sieve tested.

Carpenter and Deitz showed that the discrepancies between sieve analyses on the same sample using different sieves could be largely eliminated by use of the standard calibration method. When sieving glass beads, they did not find the deleterious electrostatic effects that were noted by MacCalman in his preliminary search for a standard material for sieve calibration.

(c) In the Canadian Government Specifications for laboratory test sieves (45), the use of the glass bead method for routine checking of sieve calibration was suggested. It was also stated that samples of glass beads, already calibrated, may be obtained from the Bone Char Research Project, Charleston, Massachusetts.

(d) Johnson and Newman(52), in a study of the sieving of granulated sugar, used the glass bead method of sieve standardization developed by Carpenter and Deitz and claimed to have obtained good correlation between sieving results. They found that this method provided a means of checking the name-plate designation on a sieve, and also permitted detection of sieves having cracks along the seam by giving inordinately large effective aperture sizes on standardization.

6.2.3 Standardization Using Master Sieves:

(a) As the result of his investigation of sieves, MacCalman(40c) recommended that standardization could best be accomplished by establishing a set of master sieves against which sieves used in routine size analyses could be performance-checked. A sample of material, similar in size range, size distribution and shape characteristics to the material to be sieved, could be used, and a correction factor obtained equal to the difference between the weight percentage retained on the test sieve and that retained by the master. It was preferable that master sieves meet the requirements of standard sieve specifications. MacCalman noted that the correction factors were not strictly applicable to all size ranges, size distributions, and particle shapes. Separate standardizations were generally required for different materials. However, if the test sieves were chosen so that the correction factors were not excessive, it was found that the sensitivity to size frequency variations was reduced.

(b) The extensive work of MacCalman in studying the characteristics of test sieves led directly to the adoption of changes in the British Standard 410:(1931) on Test Sieves. The new standard, B.S. 410:(1943), embodied many of MacCalman's suggestions and included the master-sieve standardization method.

(c) Lake Shore Mines(47) set up 250 and 325 mesh master sieves to standardize size analyses in their investigations of fine grinding. They found that unstandardized sieves were not suitable for determining size analyses in grinding: the magnitude of their errors in many cases masked the true size effect. However, by standardization they found it possible to obtain reliable results on sieve analyses. Because of the nature of their ore, Lake Shore found it necessary to use an involved system of wet and dry sieving in their standardization procedure.

(d) Recently, it was proposed to extend the Lake Shore methods to sieves from 48 to 325 mesh in setting up a set of master screens for the Canadian mining industry, at the Mines Branch in Ottawa. The suggested procedure, described by Djingheuzian(53), was adopted in 1953. Three sets of Tyler sieves were performance-checked, using a number of different ores of differing grinds. These sieves were designated as master, first and second sub-master sieves, the master sieves being those which retained the largest weight percentage. Details of these tests and their results were given by Brannen and Djingheuzian(54). It was suggested that mill operators who wish to have their sieves standardized send them to the Mines Branch, along with a representative sample of the dry mill product to be sized. The sub-master sieves would be used to determine the master correction to be applied to each sieve. It was also suggested that the sieves standardized in this way be retained by the mine as a standard against which sieves in daily use could be checked.

It is important to note that, although the Mines Branch method makes provision for use of the Lake Shore wet and dry sieving procedure, every attempt is made, in standardizing sieves for the mining industry, to arrive at a sieving procedure which is the most economical in the amount of handling and sieving time, consistent with the predetermined sieving end-point.

(e) The A.S.T.M. standard method of test for granular metal powders(39c) specifies the use of certified Tyler or U.S. Standard sieves for size analysis. It is recognized, however, that through use, changes develop in certified sieves. These changes may result in the size of sieve apertures exceeding the standard tolerances. To permit the use of uncertified work sieves for this standard method, it is suggested that a set of certified master sieves be set up, against which sets of work sieves may be performance-rated. The applied correction is equal to the ratio of the weight percentage of a sample passing one sieve and retained on the next finer sieve in the $\sqrt{2}$ series in the master set, to the weight percentage of the corresponding sized fraction in the set of work sieves.

6.2.4 Standardization by Using a Sample Having a Size Distribution which Follows Some Theoretical Distribution Law:

(a) Bond and Maxon(55) used a sample of petroleum coke crushed to 6 mesh in which the size distribution was found to agree closely with Gaudin's theoretical distribution equation(56),

$$w = Cx^m,$$

where w is the weight percentage retained on a sieve of aperture size, x, and passing the next coarser sieve in the $\sqrt{2}$ series, and C and m are constants.

When w was plotted against x on log-log paper, a straight line was obtained in the fine particle range.

A complete set of sieves, including a 200 mesh sieve certified by the National Bureau of Standards (to fix the 200 mesh point), was used to sieve analyze the petroleum coke sample. A straight line was drawn through the plotted points on a log-log graph, thus establishing the "correct" distribution line. The deviations of the plotted sieve analysis results from this line were then taken as the corrections to be applied to each sieve. These correction factors were applied to all analyses with the standardized set of sieves.

After considerable use, wear on the sieves and blinding caused changes in the sieving properties and it was necessary to redetermine the correction factors periodically.

(b) From Gaudin's equation, $w = Cx^m$, Schumann(57) derived the distribution equation,

$$y = 100\left(\frac{x}{k}\right)^m ,$$

where y is the cumulative weight percentage passing the aperture size x, and k and m are constants. A straight line results then when y is plotted against x on a log-log graph.

A comminution product having a size distribution which agreed with this equation was selected by Stairmand(58) to standardize different sets of sieves. Sieving tests were done on each set of sieves, using a standard technique for a fixed sieving period. The best straight line was drawn through the plotted y vs. x points on a log-log graph. Either the effective aperture of each sieve, or the corrected weight percentage passing the nominal aperture, could be determined in this way.

(c) Johnson and Newman(52) used a second method of sieve standardization in their study of sieve analysis of granulated sugar; their use of the glass bead method has already been cited in 6.2.2(d). They sieved a sample of crushed limestone, the size distribution of which followed Schumann's equation in the range below 90% passing. Three sets of sieves covering the size range of the lime rock were used, and the best straight line was drawn through the plotted cumulative weight percent passing against the corresponding sieve apertures on a log-log graph. In this way the sizes of effective apertures were obtained, or a correction was applied to the percentage passing or retained at the nominal sieve aperture sizes. Johnson and Newman claimed that adequate sieve standardization was obtained by either of their two methods.

6.2.5 Standardization by Size Analyses of Sieving Products:

(a) Andreasen(59) proposed a method for measuring the "sorting value" of a sieve. The procedure used was as follows: A sieving test was done on a sample of coarsely ground material, using the sieve to be standardized. The undersize material was removed and the oversize was given a further sieving for a few minutes. The small amount of undersize from this second sieving was collected.

The undersize from the first sieving was then returned to the same sieve and re-sieved for the same length of time as in the first test. The small amount of material retained was collected.

The average particle size of each of these two small fractions was determined by a counting and weighing method. This method was previously suggested by Anderson(60) and described in detail for fine particles by Andreasen(15). In it the average particle size, k, of a sample was defined by the equation

$$k = \sqrt[3]{\frac{W}{\rho N}} ,$$

where W was the weight of a sample of particles, ρ was the particle density and N was the number of particles in the sample.

Andreasen found that the two values of k, determined by this standardization procedure, lay quite close together, sometimes overlapping each other. He designated the mean value of $k = k_m$ as the sorting value of the sieve in question. He found that the ratio of k_m to the nominal aperture size, D, was fairly constant for sieves of regular mesh. For all ordinary ground materials, k_m/D lay in the range 0.8-0.9. For sieves of non-uniform mesh, such as are frequently used in practical sieve sizing, the ratio often exceeded 1 and jagged characteristic distribution curves resulted when the sieving results were plotted against nominal mesh widths. However, by use of sorting values, a smooth distribution curve could be obtained.

(b) In an investigation of the size distribution of ground material, Fagerholt(61) noted that the effective separating size of a sieve was dependent on the duration of sieving. Even after extended periods of sieving, particles capable of passing the sieve were still found in the sieving residue (retained fraction). From simple probability considerations, he showed that the size to which the sieving residue after sieving time t should be referred, i.e. the effective separating size of the sieve, was equal to the value of k of the fraction passing the sieve in the time interval from t to 3t, measured by the counting and weighing method used by Andreasen [see 6.2.5(a)].

(c) A calibration or standardization method was proposed by Hatch(62), based on the size analysis of the material retained on each sieve. He found that the size distribution of material retained on each sieve followed the log-probability law when the particle sizes were measured microscopically, that is, the cumulative weight percentage of particles on a probability scale was directly proportional to the logarithm of the microscopic size. The geometric mean size of each retained product could be obtained from the parameters of the measured size frequency curve. This measurement, he stated, was preferable to measurement of sieve openings or to some arbitrary method of calculating the size of separation.

6.3 Evaluation of Standardization Methods

With the acceptance of the general principle that some method of standardization is required to offset the unavoidable discrepancies in testing sieves resulting from their construction or use, it is then necessary to select a method suitable for any particular operation. In general, any one of the methods described in the preceding section provides a measure of sieve standardization. However, there are advantages and disadvantages to each method which should be considered.

The principal advantage of the first general standardization method, which involves measurement of the average aperture size of a sieve, is that standardization is completely independent of the material to be sized. However, for these measurements, as done by Pollard(46) and Weber and Moran(49), considerable extra laboratory equipment (microscope, projection system, etc.) was required. Also, a great number of time-consuming measurements of wire diameters and aperture widths, in both warp and woof directions, were needed to give useful accuracy. These operations had to be done for each sieve and then repeated at regular intervals to check on effects of wear and blinding.

The second method makes use of a standard sample of spherical particles which are calibrated by microscopic counting and measurement to permit the determination of either a corrected separating size or a corrected weight percentage passing the nominal sieve aperture. The advantage of the method is that it is not necessary to alter standardization corrections for different materials or different size distributions. However, it may be necessary to check sieve standardization from time to time to correct for deterioration of the sieving cloth, due to wear. With glass beads, as used by Carpenter and Deitz(50, 51), there is little likelihood of changes in the size distribution of the calibrating sample during a test due to breakage or wear. The main disadvantage is that a large sample of spherical glass beads is required and a great many of the beads must be counted and sized under the microscope to obtain a reliable and accurate calibrating size analysis. However, it is now possible to obtain calibrated samples of glass beads for sieve standardization(45).

The third general sieve standardization method, suggested by MacCalman(40c), has been widely accepted and used. Sieves are performance-rated against master sieves, using material of similar size range, size distribution and shape characteristics to that for which sieve size analyses are required.

It was recommended that the sieves chosen as master sieves be previously certified to have met standard dimensional specifications. However, as already noted, the permissible tolerances on sieving cloth are large and certification does not ensure that the nominal aperture sizes are correct. Hence, this method essentially provides a means of relating an unknown sieve to a master which may have apertures that vary widely in size from the nominal.

Heywood(14) showed that different master sieve standardization corrections are required, not only for different materials but also for different size distributions of the same material. These differences are noticeable in the results of tests done at the Mines Branch(54) in setting up master sieves.

For the strict application of standardization corrections with the master sieve method, as with each of the other methods which depend on sieving performance, it is important that the same sieving procedure be used for both standardization and for each sieving test to which sieve standardization corrections are to be applied. On this basis the rather involved wet and dry sieving procedure developed by Lake Shore Mines(47) for use with the master sieve method would not be generally attractive to the practical sieve user. However, in the modified Lake Shore procedure used at the Mines Branch(54), the sieving time and washing and drying steps are reduced to a minimum consistent with the sieving end-point for the particular mill products and sieves tested.

As was the case in the other methods, it was necessary to re-standardize, against the master sieves, the sieves which were in regular use, in order to correct for errors caused by wear or blinding.

In the fourth method, sieves are compared performance-wise,using a standard sample of material having a size distribution which is known to follow some theoretical size distribution law. Johnson and Newman(52) used a sample of crushed limestone having a size distribution curve that was essentially a straight line on a log-log plot. They found good agreement between the corrected sieve analyses obtained with this standard sample and with glass beads. The methods used by Bond and Maxon(55) and Stairmand(58) are of this type. They have the advantage that a constant correction is obtained which is independent of the type of material to be sized and of its size distribution. However, unlike the glass bead standard sample, the natural samples tend to wear and deteriorate in sieving. This results in changes in the size distribution calibration of the standard.

The fifth method involves the determination of the sorting value of a sieve by making measurements of sieving products. In the methods used by Andreasen(59) and Fagerholt(61), products are assessed by a counting and weighing procedure. The method proposed by Hatch(62) uses microscopic count and size estimation, and the assumption is made that the size distribution of the sized product follows the log-probability law.

In both cases, the definitions of irregular particle size can be extended to the sub-sieve size range and thus good continuity may be obtained between the size distributions of sieve and sub-sieve size material. However, both the standardization methods are complicated and time-consuming and, in addition, must be done separately for each sieve and for each material to be sized.

With regard to Hatch's method, Fagerholt(61) has indicated that many ground materials do not have size distributions which follow the log-probability law.

In summary, then, it is apparent that there are disadvantages involved in each of these five general methods. However, on the basis of this review, the merits of the glass bead standardization method proposed by Carpenter and Deitz appear to outweigh its demerits, and it is felt that this method should be given serious consideration whenever sieve standardization is required.

CHAPTER 7

THE MECHANICS OF SIEVING

7.1 The Statistical Nature of the Sieving Process

Having provided means for correcting errors in the sieve themselves, it is necessary to examine the mechanics of sieving and the factors which affect the passage of particles through sieve apertures.

In theory, a sample of particulate material is placed on a sieve and the sieve is shaken until the fraction retained contains only those particles having minimum cross-sections which are larger than the essentially square sieve apertures(14). However, in practice, complete sieving is never attained in a finite time. No matter how long sieve shaking is continued, some particles which are capable of passing the sieve remain in the retained sieving residue. Also, if the material which has passed through a sieve in a certain time is returned to the same sieve and sieved for the same period of time, a residue will invariably remain. These observations indicate the statistical nature of the sieving process(3).

The main properties of an individual particle which affect its probability of passing through a sieve are its size and shape. Obviously, a particle with dimensions much smaller than the sieve openings will have a high probability of passing. The shape of such a particle will have no influence on its chance of passing through the sieve. However, for a particle that has dimensions close to those of the sieve aperture, the probability will be low, and the chance that the particle has to pass through an opening will be greatly affected by the particle shape. Of the infinite number of possible orientations with respect to the sieve aperture which an irregularly-shaped particle may present, only a few will permit the particle to pass. In this case sufficient mobility must be given to the particle by shaking the sieve to allow the particle to present at least one of its preferred orientations to the sieve aperture.

In the usual sieving charge, large numbers of particles, having a wide range of individual probabilities of passing, are present. The chance that a particle will pass through a sieve is then influenced by mass effects. The greater the sieving charge, and in particular the greater the weight of near-mesh particles, the more competition there will be for the available sieve openings. This results in a lower passing probability and a slower rate of sieving. The passage of particles much finer than the sieve aperture is associated with a fast sieving rate, and mass effects are negligible. Therefore, it can be seen that it is in the near mesh size range that the principal sizing action occurs and where, as a result, the effect of sieving conditions have the greatest influence.

From the foregoing discussion it is apparent that the process of sieving can be considered to occur in two distinct stages, as suggested by the British Standards Institution(42b). The first is the relatively fast removal of particles much finer than the sieve opening which have a high probability of passing. The second stage is the more critical and slower sorting of near-undersize particles in the remaining charge.

7.2 Sieving Procedures

For practical sieve size analyses, either an exclusively dry sieving procedure or a combination of wet and dry sieving is used.

7.2.1 Dry Sieving:

In dry sieving a weighed sample of dried material is placed on a sieve or on the coarsest of a nest of sieves. A sieve shaking procedure is then followed until the size separation is deemed complete. A full discussion of methods of defining the sieving end-point is given in a subsequent section of this paper.

The shaking of sieves is done either by hand, following a definite manipulatory procedure, or by using a mechanical sieve shaker. In both cases the aim is to impart sufficient mobility to the charge so that each individual particle has an opportunity to present itself in its preferred orientation relative to the sieve aperture and either to pass through or to be retained.

Considerable disagreement exists regarding the relative merits of hand and machine sieving. Hand sieving is specified by several A.S.T.M. standard methods for sieving different materials. Even where machine sieving is specified, some standard methods call for checking by hand-sieving(39d, e, f, g, h). Details of hand sieving procedures are given in many of the standard specifications and by

Gaudin(63) and Taggart(9a). However, a number of A.S.T.M. methods specify machine sieving alone(39i, j, k). Tests by Heywood(14) and by Fahrenwald and Stockdale(64) indicated that more accurate and efficient sieving results were obtained by using a mechanical sieve shaker than with hand-sieving procedures. Gulinck(65) noted that machine sieving not only shortened the time and reduced the labour involved but also had the advantage of eliminating the personal error. Thus,closer comparisons between analyses on diverse samples and materials were obtainable.

The Tyler Ro-Tap Testing Sieve Shaker(41) is widely used for machine sieving, following the general acceptance and use of sieves of the Tyler series. Carpenter and Deitz(50), testing the Ro-Tap machine, found that the most important variable affecting machine sieving was the frequency with which the hammer rapped the sieves during shaking. They stated that the best results were obtained with 115 raps per minute.

Taggart(9a) has described the construction and use of several mechanical sieve shaking devices. A vibratory shaker was designed by Fahrenwald and Stockdale(64) which used high frequency and low amplitude. Their tests showed that the optimum frequency and amplitude were a function of particle size. Mörtsell(66) compared the sieving results obtained by various mechanical shaking devices and found few significant differences.

7.2.2 Wet and Dry Sieving:

The wet and dry sieving procedure takes full advantage of the two-stage nature of the sieving process. By preliminary washing of the sample on a single sieve or on the finest of a nest of sieves, the finest particles in the sieving charge are quickly removed. The washing liquid is generally water or some other non-reacting liquid. The retained fraction is then dried and returned to the sieve in question or to the coarsest sieve in the nest for conventional dry sieving.

Details of conventional wet sieving procedures are given by Taggart(9a) and by the British Standards Institution(42b). A more involved wet and dry sieving procedure was used for the sieve analysis of fine materials by Lake Shore Mines(47) and was adopted in a modified form for sieves up to 48 mesh by the Mines Branch(54). Several A.S.T.M. standard methods(39d, l, m, n, p, q) specify the use of wet sieving or a combination of wet and dry sieving.

Experiments by Heywood(14) have demonstrated that wet sieving produces a marked increase in the speed and efficiency of both the removal of particles much finer than the sieve aperture and the subsequent dry sorting of the coarser material. Wet sieving has the advantages of breaking up aggregates of fine particles which may be formed in drying filtered products, and of washing coarse particles free of dust or slime coatings which normally resist removal by even prolonged dry sieving.

It is evident, then, that preliminary wet sieving is almost obligatory if reliable sieving results are to be obtained for fine mesh sieves. But even for coarse sieves there is the advantage of removing the slime coatings from coarse particles to obtain a product that is more amenable to dry sieve analysis.

7.2.3 The Sieving End-Point:

Because of its statistical nature, sieving never reaches completion and hence it is necessary to adopt some arbitrary definition of the sieving end-point. Heywood(5, 14) has listed three general methods by which the end-point may be defined.

(i) Sieving for a standard period of time

(ii) Sieving until the weight passing in unit time is less than a specified percentage of the weight of the sieving charge

(iii) Sieving until the weight passing in unit time is less than a specified percentage of the weight of the sieving residue.

Heywood(14) and MacCalman(40c) among others have favoured the use of a standard time of sieving as the end-point definition. They felt that sieving results could be obtained with sufficient accuracy in this way. In addition, the standard time method was the simplest in operation, especially where machine sieving was used.

Several A.S.T.M. standard methods for conducting sieving tests have specified this method. For granular roofing materials a 15 min shaking time is specified for a 500-g sample(39j). For molding powders the sieving time is 10 min for a sample ranging in weight from 50 to 200 g (39i). A sieving time of 15 min is called for in specifications for sieve testing calcined magnesia(39 l), aggregates and fillers(39e), and refractory materials(39d).

Bietlot(67) stated that there was considerable danger involved in the arbitrary use of method (i). While this method gave constant sieving conditions, the character of the material might vary, causing the results to be variable.

The British Standards Institution(42b) reserved method (i) for routine tests but stated that a preliminary end-point test based on sieving rate should be done to determine a suitable sieving period under the conditions of operation. The sieving end-point was defined as the time when the amount of material passing the sieve in a 2 min period was less than 0.2% by weight of the original sample.

Methods(ii) and (iii) have also been specified for use in several A.S.T.M. standard methods. For non-granular roofing material a sieving time of 20 min is prescribed for a 100 g sample, with further 10 min sieving intervals until the weight passing in any 10 min period is less than 0.5% of the original sample(39k). Standard methods for hand sieving mineral fillers(39g), powdered coal(39f) and glass spheres(39h) specify sieving until not more than 0.05 g of a 50 g sample passes per minute. Hand-sieving of refractory materials(39d) and machine sieving of powdered coal(39f) are to be done until less than 0.1% of the original sample passes per minute.

For fine and coarse aggregates(39r) and for hand sieving tests on aggregates and filters(39e), specifications call for sieving until less than 1% of the sieving residue passes per minute.

It seems apparent that it is better practice to use one of the methods of defining the sieving end-point based on the actual sieving rate, which is a function of the particular size, grade and type of material, as well as variations in the sieve itself. The main drawback, of course, is the extra handling and weighing required.

7.2.4 The Sieving Rate:

The value of defining the end-point of sieving by means of the sieving rate has been demonstrated in the preceding section. Several investigators have concerned themselves with determining an equation for the sieving rate based on theoretical considerations or on the results of sieving tests.

Bietlot(67) assumed that the number of particles dy passing a sieve during a time dt was proportional to the number of particles capable of passing but remaining in the sieving residue. That is,

$$\frac{dy}{dt} = -at,$$

where a is a constant. Hence $y = y_o e^{-at}$, where y_o is the number of such particles present in the original sample. Fagerholt(61) made a similar assumption in his analysis of sieving behaviour. However, Bietlot stated that the constant a varied with particle size and therefore the equation could not be directly applied to sieve analysis of a product containing a range of particle sizes. He measured the variation in the rate of sieving as a function of time and obtained the equation

$$\frac{dp}{dt} = nt^{m},$$

where dp is the weight passing a sieve in time dt at a time t after the commencement of sieving, m and n being constants. Bietlot found m= -1.72 in the test he conducted. Fagerbolt(61) gave m = -1.5, while tests by Carpenter and Deitz(50) showed that m varied between -1.1 to -1.7. Gulinck's equation(65),

$$p = p_o + b \ln t/t_o ,$$

when differentiated gives

$$\frac{dp}{dt} = Kt^{-1}, \text{ i.e. } m = -1.$$

It can be seen, then, that the form of the sieving equation as stated by Bietlot is in general agreement with the results of experiments by several investigators. Hence, by plotting the sieving rate against the duration of sieving to a log-log scale, a straight line should result. This type of plot, then, should permit the end-point based on the sieving rate to be readily determined. Gulinck(65), in addition, used the values of the constants to calculate the weight of undersize particles which still remained in the retained fraction after any period of sieving.

7.2.5 Sieve Loading:

The probability that particles will pass through a sieve is greatly influenced by the weight of the sieving charge. As indicated previously, the larger the sieving charge, the more competition there will be for the limited number of available apertures in a particular sieve. This causes a slower rate of sieving and, in the case where sieving is done for a fixed time, incomplete sieving results are obtained.

Tests by Shergold(68) showed that a reduction in sieve loading was a more effective method of obtaining accurate sieving results than was an increase in sieving time. He recommended using sample weights as small as were convenient for handling, noting at the same time that, while a small sample shortened the time and reduced the chance of wear on particles, accuracy might be lost in sampling and weighing. His tests also showed that the effect of overloading was greater, the smaller the sieve aperture.

Bietlot(67) explained the effect of loading by considering a sieve B of the same area as sieve A, but having apertures n times smaller. For the same weight of sieving charge, sieve B will have to pass n^3 times more grains than sieve A while presenting fewer than n^2 times more openings (since the wire diameter does not decrease as quickly as n). It follows, then, that the optimum time of sieving increases more quickly than the first power of the size of the mesh openings and, hence, to avoid very long sieving times and the resulting possibility of wear on particles, it is preferable to reduce the sample size when using fine mesh sieves.

While the total weight of the sieving charge has been shown to affect the sieving probability, tests by Porter(69) indicated that particles much coarser than the sieve aperture had no deleterious effect on the sieving rate and often gave improved sieving. He found that the presence of near-oversize and near-undersize particles was the main cause for long sieving times and inefficient sieving results.

Experiments by Warner, cited by Gulinck(65), indicated that the sieving efficiency, defined as the ratio of the weight of particles which pass a sieve to the weight of particles capable of passing, was related to the number of "difficult" grains, i.e. near-mesh particles.

Carpenter and Deitz(50) defined "near-fit" particles as those which passed through the next larger sieve and were retained on the next smaller sieve in the $\sqrt[4]{2}$ series than the sieve being used. They reasoned that, since such near-mesh particles have to fall on the openings a large number of times before it is known whether or not they will pass, the time required to reach some pre-defined sieving end-point should depend on the number of near-fit or near-mesh particles per sieve opening.

The number of near-mesh particles, N, which have a weight, W, and diameter, d, is given by

$$N = k_1 \frac{W}{d^3} ,$$

where k_1 is a constant.

Also, the number of sieve openings, H, in a constant sieve area, having an aperture size d is given by

$$H = \frac{k_2}{d^2} ,$$

where k_2 is another constant. Hence the number of near-mesh particles per sieve opening is given by

$$\frac{N}{H} = \frac{k_1}{k^2} \frac{W}{d} .$$

That is, N/H is proportional to the ratio of the weight of near-mesh particles to the size of the sieve opening and hence, according to the original hypothesis, the sieving time should be proportional to the ratio $\frac{W}{d}$. Carpenter and Deitz found experimentally that this relationship held true for sieves of 35 mesh and finer. For coarser sieves they found that the time was proportional to the weight W of near mesh particles.

To use this relationship in test sieving, Carpenter and Deitz made a preliminary trial sieving run to determine the percentage of near-mesh particles. From this they determined the constants and, using them in the first order equations relating the sieving time T to $\frac{W}{d}$ or W, they were able to determine the 'correct' sieving time.

7.2.6 Particle Wear in Sieving:

The particle wear due to sieving action was studied by Mörtsell(66). For long sieving times he found that the material passing a sieve comprised, in addition to particles which were actually smaller than the sieve openings in the original sample, the following three classes of particles:

(i) Comparatively large fragments detached from corners and edges;

(ii) Worn off, relatively fine particles originating from projecting parts;

(iii) Particles which only just pass through the sieve owing to deformation and reduction in size in the course of sieving.

He stated that these various classes of particles were present to a greater or lesser extent, depending on the brittleness and hardness of the material being sieved and on such other factors as the particle size, the weight of the charge, and the mode of operation of the sieving machine.

Mörtsell's sieving tests on quartz and dolomite indicated that most of the variation due to particle wear was of the second type, that is, there was a noticeable rounding off of particles after long sieving periods and a measurable increase in extreme fines in the passing product.

Gulinck(65) and Carpenter and Deitz(50) concluded that the effect of wear on sieving results was not great compared with other factors involved in sieving. However, they suggested shortened sieving times for soft material.

7.2.7 The Effect on Sieving of the Density and Dampness of the Charge:

Carpenter and Deitz(50) sieved materials in a density range from 1 to 10 and found no significant differences in sieving characteristics. They also studied the effect of the dampness of the sieving charge on dry sieving. For some materials a small percentage moisture could be tolerated without deleterious effect on the results. However, in general it was found that pre-drying was beneficial. Although most "blinding" of sieves is caused by near-oversize particles, Porter(69) noted that excessive dampness of the sample also contributed to this effect. Hence, most dry-sieving procedures specify pre-drying at 100 °C.

CHAPTER 8

SUMMARY AND CONCLUSIONS

The foregoing discussion of sieve testing has been written with the aim of bringing to the attention of those who use sieves, or those who rely on sieve testing information, some of the factors which affect the accuracy of sieve size analyses. In general, considerable confidence is placed in sieving and sieving results. The magnitude of the errors involved in sieving is often overlooked.

It is generally agreed that the principal error in sieving, and the one which is often neglected, is in the sieves themselves. This is due to the difficulties encountered in manufacturing woven-wire sieves and is reflected in the relatively wide tolerances on wire diameters and aperture sizes allowed by the various standard sieve specifications. For this reason this paper has placed considerable emphasis on the methods of standardizing testing sieves. Five general methods were described and evaluated, and it was concluded that the glass-bead standardization method proposed by the U.S. National Bureau of Standards was preferable and to be recommended to sieve users who may wish to standardize sieves.

The remaining errors in sieving arise from the way in which sieve testing is done. The concept of the statistical nature of the sieving process has been developed and explained by showing the effect of the various factors which influence sieving results on the probability of particles passing a sieving aperture.

Following the British Standard method for using fine-mesh test sieves(42b), the sieving process has been considered to consist of two stages. The first is the relatively fast removal of particles much finer than the sieve aperture having a high probability of passing. This step can be accelerated by wet sieving, even on coarse sieves. The second part is the much slower passage of so-called near-mesh particles for which the probability of passing is low. This low probability is due not only to the size of particles as compared with the size of the sieve apertures, but also to the particle shape, the method of shaking sieves, the sieving time, the total weight of the sieving charge, and, in particular, the weight of near-mesh particles.

Each of the above factors has been discussed with detailed reference to the sieving literature. In addition, the various ways of defining the sieving end-point have been outlined. For accurate sieve analyses on different materials, the end-point based on the sieving rate is preferred. The rate of sieving has been shown, empirically, to follow the general equation

$$\frac{dp}{dt} = nt^m,$$

where m lies in the range -1 to -2. The use of this equation makes the rate-determined sieving end-point more amenable to machine sieving.

In summary, then, it is evident from the foregoing that unreliable sieve analyses are probable if the errors inherent in sieves and in the sieving process are neglected. However, improved practice based on the awareness of these errors should result in more accurate sieving and in greater confidence being placed in size analysis done using woven-wire test sieves.

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