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ANALYSIS OF GRADING EFFECTS ON HYDRAULIC AND CONSOLIDATED FILL

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MINE FILL SYSTEM DESIGN BASED ON OPTIMIZATION

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Analysis of Grading Effects on Hydraulic and Consolidated Fill

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SUMMARY

The Mining Association of Canada has suggested that research should be conducted on fill. It is known that by changing the size distribution of fill its physical properties can be modified. Consequently, following a literature survey and the drafting of tentative specifications for percolation testing, some engineering studies have been made to identify the modifications that have the greatest probability of cost savings.

The in-place density of most fills could be increased by modifying the grain size distribution. By increasing the density, the volume of stope filled by a ton of fill is decreased, the stiffness of the fill is increased, the percolation rate is decreased and the amount of Portland cement required for consolidation may be decreased. These preliminary studies indicate that the minimum increased cost of producing significantly modified fill from typical mill tailings would be 6 cents per ton of fill or 4.5 cents per ton of ore. Whereas the stiffness would be increased two to three times, it would still be highly compressible compared to ore pillars and consequently would not likely have much effect on ground control costs. The percolation rate would be unsatisfactorily cut approximately in half. In consolidated fill, whereas strength would be increased, it is unlikely that the cement could be reduced below a 1/30 ratio without eliminating significant cementing action; consequently, only where higher cement contents are being used could savings of possibly 50 cents per ton of fill result.

It has been established that approximate percolation rates can be predicted from the grading, which could be useful for comparative studies.

The grading requirements for a filter to permit water to exit freely from a granular mass but to prevent the fine particles from being carried away are well established. It is possible that the use of filters at bulkheads or to receive decanted water might be less costly than cleaning the slimes out of the haulageways.

Whereas the equivalent viscosity of a fill pulp can be changed many-fold by changing the gradation of the fill, it is shown that for typical pulps the effect on the friction factor in pipes and boreholes is relatively small, so that little advantage is indicated for any attempts to optimize this property.

In conclusion, it seems that current practices produce gradings of typical mill tailings fill that are not far from the optimum under present economic conditions.

INTRODUCTION

AS A RESULT OF A SURVEY OF ITS MEMBERS a few years ago, the Mining Association of Canada concluded that one of the areas in which mining research should be concentrated was that of mine fill. Accordingly, a literature survey was conducted at the Mines Branch on current international practices[1], and tentative standard specifications for the percolation test were compiled[2]. Then, as it has been suggested in the past that the adjustment of the grain size distribution of hydraulic fill might be an important consideration, as many of the fill properties can be varied in this way[3], an engineering study was made of this variable by examining the properties of density, permeability, filter characteristics and pipeline friction to locate the areas of greatest possible cost savings.

This subject of gradation has been of interest for many years in the fields of concrete and soil mechanics as well as more recently in mining. Consequently, there is a considerable amount of information available from research in these various fields. The present analyses have been based on existing technical information together with cost estimates on technical improvements that might be made. In general, the indication is that current practices produce gradings that are not likely to be changed under present economic incentives.

However, in the process of making these preliminary studies, pertinent information has been brought together which conceivably could be of some use to mining engineering departments at operating properties. For this reason, the known relations between mechanical properties and grain-size distribution, together with some indication of their utilization, are described below.

STRUCTURAL EFFECTS OF BULK DENSITY CHANGES

Optimum Gradings

By varying the grain-size distribution of a fill, its in-place density can be changed. By increasing the density, several dependent properties are altered: (a) the volume of stope filled by a ton of fill is decreased; (b) the stiffness, or modulus of deformation, is increased; (c) the percolation rate, or coefficient of permeability, is decreased; and (d) the amount of Portland cement required for consolidation, or cementing, may be decreased.

In concrete research, the grading requirements for maximum density of aggregate have been the object of many studies. Although several criteria have been suggested, none have been completely valid, owing to the significant variations that can occur in particle shape and angularity. As a first approximation, guidance can be obtained from Talbot's equation:

$$p = 100(D/D_m)^{0.5} \qquad \ldots\ldots\text{(Eq. 1.)}$$

where p is the per cent passing the sieve size, represented by D, and D_m is the maximum size of particle[4]. *Figure 1* shows a grading curve according to this equation. On the St. Lawrence Seaway Project, to economize on cement, the quarried concrete aggregate was processed to follow the above equation except that the index was changed from 0.5 to 0.4, which gave a better result for the angular, elongated particles.

An alternate approach has been achieved through measuring the uniformity of the grain-size distribution using the following equation:

$$C_u = D_{60}/D_{10} \qquad \ldots\ldots\text{(Eq. 2.)}$$

where C_u is the uniformity coefficient, D_{60} is the particle size that is greater than 60 per cent of the material (i.e., 60 per cent passing size) and D_{10} is the size greater than 10 per cent of the material[5]. It is known that generally an increase in C_u will cause an increase in fill density.

The application of these equations can be examined with respect to typical mill tailings with an original grading as follows:

Size	Per cent Retained	Cumulative Per cent Passing
No. 100: 149μ	1.9	98.1
No. 150: 100μ	7.7	90.4
No. 200: 74μ	9.6	80.8
No. 325: 44μ	20.8	60.0
20η	33.2	26.8
10μ	12.2	14.6

Fill utilizing 45 per cent of the above tailings, costing about 12 cents per ton of dry fill and having a reported percolation rate of 5 to 6 ins./hr, had the size-distribution curve shown in *Figure 1* with the following gradation:

Size	Per cent Retained	Cumulative Per cent Retained	Cumulative Per cent Passing
No. 65: 230μ	0.4	0.4	99.6
No. 100: 149μ	2.5	2.9	97.1
No. 150: 100μ	14.2	17.1	82.9
No. 200: 74μ	24.5	41.6	58.4
40μ	35.5	77.1	22.9
20μ	19.9	97.0	3.0
10μ	0.6	97.6	2.4

By using Talbot's equation (Eq. 1), the grading for maximum density can be determined. It is assumed that the D_{10} size, 0.029 mm, must remain constant. The maximum particle size, D_m, can then be calculated as follows:

$$0.10 = (0.029/D_m)^{0.5}$$

$D_m = 2.9$ mm (approximately No. 7 sieve).

As can be seen, with the maximum size in the tailings being equivalent to approximately the No. 65 sieve, a large amount of sand would have to be added to achieve the Talbot grading.

As a compromise, following the experience of others where it has been found that a coefficient of uniformity, C_u, of about 5 gives fairly high in-place density[6], the D_{60} size can be calculated from Eq. 2 as follows:

$$5 = D_{60}/0.029$$

$D_{60} = 0.145$ mm (equivalent to No. 100 sieve).

The grading for this specification would then be approximately as follows:

Size	Per cent Retained	Cumulative Per cent Passing
No. 65: 230μ	20	80
No. 100: 149μ	20	60
No. 200: 74μ	29	31
40μ	15	16
20μ	13	3

This would require, for every 100 lbs of the mill-tailings fill, the removal of approximately 15 lbs of the minus-200-mesh material and the addition of about 50 lbs of plus-100-mesh material — yielding 135 lbs of modified fill.

The average in-place density of the above typical mill tailings fill would be about 90 pcf*. With the fill modified to a C_u of 5, the average in-place density would be approximately 105 pcf. A material with the Talbot grading would have an in-place density of 115 pcf or more (glacial till fulfils the Talbot grading naturally and can have a dry density of as much as 145 pcf and a total density, including moisture, of 155 pcf).

The cost of producing this modified fill would depend substantially on the cost of the added sand, which might vary from $0.25/T if located immediately adjacent to the mill to $1.00/T if trucking for a few miles is required. If it is assumed that the total processing cost of the mill tailings is substantially the same, regardless of the amount that is used for fill, the unit cost of producing the modified fill (requiring 100 lbs of tailing fill and 50 lbs of sand for 135 lbs of modified fill) would be as follows:

Cost of modified fill
(sand at $0.25/T) = 0.12 x 100/135 + 0.25 x 50/135
= $0.182/T of fill

Cost of modified fill
(sand at $1.00/T) = 0.12 x 100/135 + 1.00 x 50/135
= $0.459/T of fill

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*pcf = lb per cu.ft.

It can be seen that even at the minimum cost of pit sand, the cost of producing the fill is increased by 50 per cent, although in absolute figures an increase of $0.06/T on the total of approximately $0.80/T in-place would be acceptable if the resulting benefits more than paid for this extra cost. At the same time, with fill of increased coarseness, pipeline wear, as discussed below, would be increased.

Effect on Volume

With a density of 90 pcf, the mill tailings fill would have a specific volume of approximately 22 cf/T, whereas the modified fill at a density of 105 pcf would have a specific volume of 19 cf/T. Assuming that the specific volume of the ore is 11 cf/T, the cost of the fill can be restated as follows:

Cost of tailings fill = 0.12 x 11/12 = $0.06/T of ore
Cost of modified fill
(sand at $0.25/T) = 0.181 x 11/19 = $0.105/T of ore
Cost of modified fill
(sand at $1.00/T) = 0.459 x 11/19 = $0.266/T of ore

This shows that besides the extra cost of processing the modified fill, the mining cost of ore would be increased by $0.04 to $0.21/T due to the requirement for more fill in the stopes (i.e., more weight for the same volume).

Effect on Stiffness

The modulus of deformation of the typical mill tailings fill would be of the order of 12,500 psi[7]. In a steeply dipping orebody, fill exerts an active pressure on the walls through the force of gravity acting down-

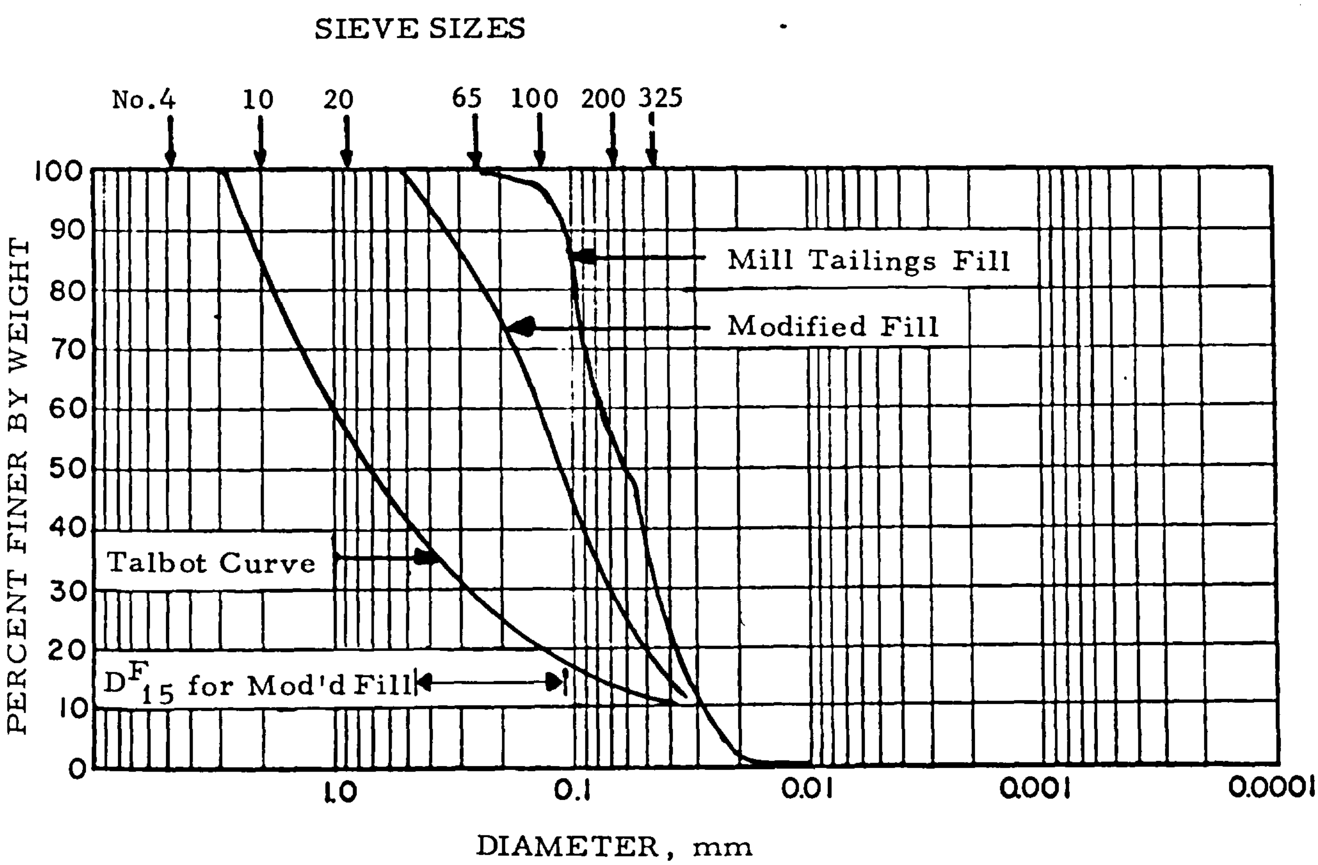

Figure 1.—Grain-Size Distributions.

ward and tending to push the fill out sideways. This horizontal pressure would be of the order of half the vertical pressure. However, closure of the walls will compress the fill, which will then act like a spring to give the greatest back pressures. In a stope 10 ft wide from wall to wall, to generate an increase in back pressure of 100 psi would require a closure of approximately 1 inch. An extreme case would occur in a stope 100 ft wide; here, to generate an increase in back pressure of 1000 psi would require a closure of 8 ft. These figures indicate that, compared to the pillars, fill is a spongy material.

The above modified fill could be expected to be stiffer and have a modulus of deformation of approximately 33,000 psi. With this stiffness, to generate an increase in back pressure of 100 psi in a 10-ft stope would require closure of 0.4 in.; to generate a back pressure of 1000 psi in a 100-ft stope would require closure of 3 ft. Whereas the modified fill is stiffer, it is still a spongy material compared to the ore pillars. Consequently, it is questionable whether the increased stiffness of the modified fill would, in most mines, have any effect on costs; e.g., the cost of rehabilitating service drifts in the walls that break down as a result of closure or the cost of rehabilitating areas subjected to rockbursts. Possibly the increased stiffness would be helpful in soft-rock mines or in rockbursting mines where even the small difference in back pressure of the modified fill would reduce bursting frequency. These are only speculations, however, as no definite information exists to permit analysis.

Effect on Cemented Fill

Several mines are now using Portland cement in hydraulic fill to produce consolidated, or cemented, fill to obtain the advantages of a material that will stand by itself for heights of 20 to 30 ft. Portland cement consists of complex compounds, primarily of calcium, silicon, aluminum and oxygen, which hydrate to form a cemented mass. The amount of combined water is approximately one-quarter by volume of the amount of the cement. The extra water that is used for transportation dilutes this optimum mixture and reduces potential strength. As produced by manufacturers, the particle sizes of Portland cement commonly vary bebetween 1 and 80μ.

In concrete work, it has been found that maximum mixing time with continuous agitation, without affecting the ultimate strength, is about 6 hrs (although normal specifications call for a few minutes of active mixing and a maximum of 2 or 3 hrs of gentle agitation). Also, it has been found in concrete work that leaching water dissolves some of the readily soluble calcium compounds, leading to a reduction in strength. Solidification occurs by coating the inert, or aggregate, particles with cement that bonds these particles together by forming a continuous crystalline cement compound. With high cement contents, cementing can also be achieved by filling the voids with the cement mortar, thus forming a continuous matrix within which the inert aggregate particles are held.

As the modified fill described above has a lower void ratio and a higher modulus of deformation than the typical mill-tailings fill, it should either give a stronger consolidated fill, if the cement content is maintained constant, or it should require less cement for the same strength. A rough calculation can be made of the specific surface (the total surface area per unit weight of fill) of the two fills and the amount of cement that would be used up if all the fill particles were coated. Assuming that the specific surface is substantially represented by the fraction of the fill smaller than D_{10} and assuming that the coating of cement would be equivalent to a thickness of 10μ, it turns out that in a 1/30 (cement to fill ratio by weight) mixture, the total volume of the Portland cement would be used in coating all the particles (if no segregation occurred during pouring). Consequently, the cementing mechanism of having the voids filled with a mortar would not be effective unless, in some areas, a higher concentration occurred as the result of segregation. Consequently, with the reduction in void ratio that occurs in the modified fill, it cannot be argued that this would logically reduce the amount of Portland cement needed to achieve the same strength.

Alternatively, it could be argued that because the modulus of deformation of the modified fill is increased by 164 per cent, then the compressive strength of the consolidated fill should be increased by a similar amount; e.g., 20 psi to 53 psi. Consequently, if a compressive strength of 20 psi is good enough, less cement in the modified fill should be satisfactory. However, with a cement to fill ratio of 1/30, the material can be so weak under mining conditions that it is barely cemented, and it is possible that any further reduction in Portland cement content would produce no cementing at all. In other words, with an increase in density of the fill an increase in strength of the consolidated fill could be expected, but no decrease in cement could be considered under field conditions without abruptly returning to the condition of simple hydraulic fill. At the same time, the possibility of finding a cheap admixture to prevent segregation might make this factor worth reconsidering.

In cases where higher cement contents are used, this procedure of modifying fill for optimum density might be more feasible. Consider the effect on costs of reducing the cement content from 1/20 to 1/30. If the cost of Portland cement to the collar of the transportation system is $30 per ton of cement, it follows that:

Cost of cement in 1/20 fill = $30 x 1/20 = $1.50/T of fill

Cost of cement in 1/30 fill = $30 x 1/30 = $1.00/T of fill

Clearly, this is an area with incentive for some research, although some research might be directed toward the increased use of Portland cement for the additional benefits that could result.

Alternatively, recognizing that the addition of Portland cement to the fill decreases its percolation rate (more than would be expected simply from the small increase in slimes content as the chemical reaction initially produces a jelly-like mortar), possibly the initial operation of processing the typical mill tailings should be reconsidered. If the minus-10μ fraction were reduced to 10 per cent instead of being substantially eliminated (owing to the acceptance of lower percolation rates in consolidated fill), a C_u of 5 would still be obtained with a 95 per cent recovery of fill from the tailings. Assuming the production cost to remain constant, the unit cost would be reduced from $0.12/T of fill in the ratio of utilizations (45%/95%) to $0.06/T of fill, which indicates that there might be some cost reduction in this action if no penalties resulted from the increased slimes content (e.g., this factor might nullify the consolidation resulting from the Portland cement content as well as produce unacceptable bulkhead pressures).

GRADING EFFECTS ON PERMEABILITY

Bulk Density Effects

With an increase in density, there will be a decrease in the volume of voids, which means that the cross-sectional area of the passages for water will be decreased. Consequently, the permeability of the fill will be decreased according to the following equation:

$$k_1/k_2 = e_1^2/e_2^2 \qquad \text{.......(Eq. 3.)}$$

where k is the coefficient of permeability and e is the void ratio or volume of voids to volume of solids [$e = (Gw/\gamma) - 1$, where G is the specific gravity of the solid particles, w is the density of water and γ is the dry mass density of the fill][5].

Using Eq. 3, the change in percolation rate from the 5 ins. per hour for the above tailings fill to that for the modified fill can be calculated:

$$k_2 = 5(0.58/0.84)^2 = 2.4 \text{ in./hr} \qquad \text{.......(Eq. 4.)}$$

This looks like another unfavourable consequence of increasing the density of the fill, as higher hydrostatic heads would be generated unless the rate of filling were decreased. At the same time, as the percolation rate is also an indirect measure of how quickly the fill will change from a jelly-like suspension to a firm granular material, it is possible that the appropriate percolation rate for this aspect would be lower for an inherently denser fill.

Grain-Size Effect

Studies originally made on filter sand showed that the coefficient of permeability could be related to the grain-size distribution by the following equation:

$$k = D^2_{10} \qquad \text{.......(Eq. 5.)}$$

where k, in cm/sec., is the coefficient of permeability and D_{10}, in mm, is the size of particle for which 10 per cent of the material is smaller[5]. Since then, research work has shown that the same equation is valid for mill-tailings fill with D_{10} sizes down to as low as 28μ[7]. This coefficient of permeability can be converted to a percolation rate in ins./hr by multiplying by 1470. Tests have shown that the actual value might be at least 60 per cent lower or 50 per cent greater than the calculated values; however, even if the equation just gave the order of magnitude, the quantitative relationship could be useful, particularly for comparative studies.

According to Eq. 5, the D_{10} size for a percolation rate of 4 ins./hr should be the same for all fills and could be determined as follows:

$$D_{10} = (4/1470)^{0.5} = 0.052 \text{ mm} \qquad \text{.......(Eq. 6)}$$

However, it is known that many fills with a D_{10} size as low as 0.03 mm give a percolation rate of 4 ins./hr, which emphasizes that the relationship can only be used in preliminary or comparative studies.

Filter Specifications

Where water exits from a fill, it tends to cause seepage erosion. A sand filter can be placed adjacent to the fill or base material that will not impede the exit of water but will hold back the fine particles of the fill or base material. The specification for such a filter is as follows: the D^F_{15} size of the filter material should be greater than $5D^B_{15}$ of the base material (to ensure that the filter is sufficiently more permeable than the base material) and less than $5D^B_{85}$ of the base material (to prevent the fines from being carried out by the water)[5]. This specification can be expressed as follows:

$$5D^B_{85} > D^F_{15} > 5D^B_{15} \qquad \text{.......(Eq. 7.)}$$

In addition, the D^F_{85} size of the filter material should be larger than twice the gap in any pipe or drainage device carrying off the water.

Normally, it can be expected that fine particles will be carried out of the fill until the remaining particles in the exit zone have an effective grain size distribution such that the D_{85} size is twice the size of the holes in the burlap or whatever is being used in an attempt to filter the water. Consequently, a large volume of mud may be deposited in the drainage ditches of the haulage drifts.

The typical mill-tailings fill described above has a D^B_{15} of 0.032 mm and a D^B_{85} of 0.11 mm; consequently, the filter requirements would be as follows:

$$0.55 > D^F_{15} > 0.16 \text{ mm} \qquad \text{.......(Eq. 8.)}$$

From these figures, it can be seen that the tailings fill will not build up its own filter until a very large volume of fines has been removed, as the maximum sizes in the fill just fall within the filter specification for the D^F_{15} size as shown in *Figure 1*, which should then comprise only 15 per cent of the total filter layer.

Ideally, "mouse traps" or drainage diaphragms should be backed with a filter sand that would effectively prevent any slimes from being carried out with the percolation water. Possibly some arrangement could be devised whereby a row of filter sand or a line of bags of filter sand could be laid out in the stope up to the bulkheads in the same way that gardeners use crushed stone or gravel to make a French drain. Although such a channel of filter sand would not be as open as a mouse trap or drainage pipe, the volume of water required to be passed is not great and its permeability will be more than 25 times greater than that of the fill, which means that it would not impede the outflow.

In some situations, it might be feasible to consider replacing concrete bulkheads in horizontal openings connected to a stope with bags of filter sand. It is known from other work that if the length of such a plug can be made equal to or greater than the least width, it is almost impossible to push the plug out, particularly with pressures generated by a granular mass such as fill. In this case, the sand bags would act both as a retaining bulkhead and as a filter. However, the cost could be high; for example, such a bulkhead in a 7- by 8-ft doghole would require about 400 cu.ft of sand in bags, or 40,000 lbs of sand or four hundred 100-lb bags. These could well cost something like $1/ bag in-place, resulting in a bulkhead cost of $400, although experience could possibly lead to a reduction of 50 per cent in the required volume.

Increasingly, practice is directed toward eliminating fill pulp water by decanting rather than percolating. The decanted water also carries slimes (of the order of 4 per cent by weight), which end up in the haulage drifts. Somehow or other, if this water could be passed through filter trays, the slimes could be caught — with a saving of possibly a thousand dollars a month in cleanup costs. The cost of the filter trays would depend substantially on how frequently the sand would have to be replaced, which, at this stage, could only be determined by experience. It is probable that a 10- by

10-ft tray, 6 ins. deep, containing 2½ tons of sand, would operate satisfactorily until about half as much slimes had been retained, i.e., 1¼ tons of slimes. If the decanted water contained 4 per cent by weight of slimes, then the filter would pass 31 tons of water, or 6,200 Imperial gallons. This could mean that in one pour of 20 ft, in a stope 50 ft wide by 20 ft on strike being filled with a pulp of 60 per cent solids with 15 per cent water retention in the fill, about 140,000 Imperial gallons would have to be passed through the filter. With four trays, the sand would have to be replaced five or six times, which would keep a crew of a couple of men busy during the entire pouring operation. However, such a procedure might, in some cases, give lower costs than current cleanup expenditures.

GRADING EFFECT ON PIPELINE FRICTION

Flow Head Losses

For fluid flow in pipes, the head loss is commonly calculated using the following equation:

$$H_L = f(L/D)V^2/2g \qquad \text{.......(Eq. 9.)}$$

where H_L is the head loss in ft, f is the dimensionless friction factor, L is the length of the pipe, D is the diameter of the pipe, V is the velocity of the fluid in fps and g is the acceleration due to gravity (32.2 fps² for Canadian latitudes). In any hydraulic handbook, the variation of the friction factor, f, with the Reynolds number, R, is shown by a family of curves for water in pipes of varying degrees of roughness. R is equal to the dimensionless product $VD/(\mu/\gamma)$, where μ is the absolute viscosity of the fluid and γ is its density.

It is known from experimental work that the viscosity of a fill pulp can be changed by as much as threefold by modest changes in the minus-325-mesh fraction[9]. Whereas it is known that fill pulps do not behave like ideal liquids, it is interesting to examine the consequences of such a large change in viscosity. It is known that the friction factor, f, for a pulp of 60 per cent solids would be approximately 0.03 for a 3-in. pipe. With an absolute viscosity of 30 centipoise and a velocity of 10 fps, the Reynolds number would be 0.4×10^4. By reducing the minus-325-mesh fraction, the absolute viscosity could be reduced to 10 centipoise, which would increase the Reynolds number to 1.2×10^4. Following the general shape of the curves for water on the f vs R curves, it can be seen that the friction factor would drop or fall to 0.0295 — an insignificant change. If the f vs R curves are not of the same shape for fill pulps, then possibly the change in friction factor would be greater.

In any event, there are few situations in which it is conceivable that spending money on modifying the grain-size distribution to change the friction factor would be more than repaid by savings in other costs. Possibly, in the case of extensive shallow workings, the minimizing of f would permit a significant increase in the distance to which the fill could be transported on the levels by the limited head. Alternatively, in deep workings the maximizing of the friction factor in the borehole or down-pipes might reduce the costs of energy "spoilers" and the pipe at the take-offs on the levels.

Pipe Wear

Some experiments have shown that the loss in weight of a pipe carrying a slurry is related as follows:

$$w = aV^{2.6} \qquad \text{.......(Eq. 10)}$$

where w is the loss in weight per unit time, V is the average velocity and (a) is a parameter that varies, among other factors, with grain size; e.g., the value for minus-16-mesh slurry was found to be 115 times that for a minus-150-mesh slurry and the value for minus-35-mesh slurry was 45 times that for minus-150-mesh slurry (the paper does not describe the actual grain-size distributions)[10]. It would seem that with the modified-tailings fill having a D_{50} size of twice that for the typical mill-tailings fill as shown in *Figure 1*, pipeline wear would be doubled. Whether this would be a significant increase would depend on the pulp density, velocity, mineral type, particle shape and other factors.

The determination of wear rates at the present time is substantially achieved by measurements in the actual system. One step toward predicting these rates might be made by extrapolating measured rates for known conditions to new conditions, e.g., different pipe diameters, different pulp density, different flow rate, etc., through a knowledge of the appropriate functional relations. However, more research is required even to take this step in many cases[11].

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Mine Fill System Design Based on Optimization

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ABSTRACT

An optimization computer program has been developed in order to solve multi-variable problems with non-linear functions. The resultant program can handle substantially any number of variables, and it is readily adaptable for actual conditions.

As a sample multi-variable design problem, the optimization of a simplified mine-fill transportation system was analysed.

From several possible parameters, the following four variables were selected and manipulated during this study: (1) the diameter of the vertical borehole, (2) the diameter of the horizontal steel pipe, (3) the duration of the filling operation in hours per day and (4) the solids concentration in the slurry.

It was found that the optimum cost solution obtained by the analyses resulted in borehole and pipe sizes that are very close to those currently being used in practice for the given conditions.

INTRODUCTION

IN OPTIMIZATION PROBLEMS encountered in the mining industry, the use of linear programming is limited because the functional relationships between the variables are frequently non-linear. Therefore, a computer program was written to use non-linear programming techniques[1]. The resultant program can handle substantially any number of variables.

As a sample multi-variable design problem, the optimization of a simplified mine-fill transportation system was analysed. As the main purpose of this exercise was to test the computer program, the results of this analysis do not necessarily provide the optimum solution of an actual mine-fill transportation system because, with the simplification used, several variables were neglected.

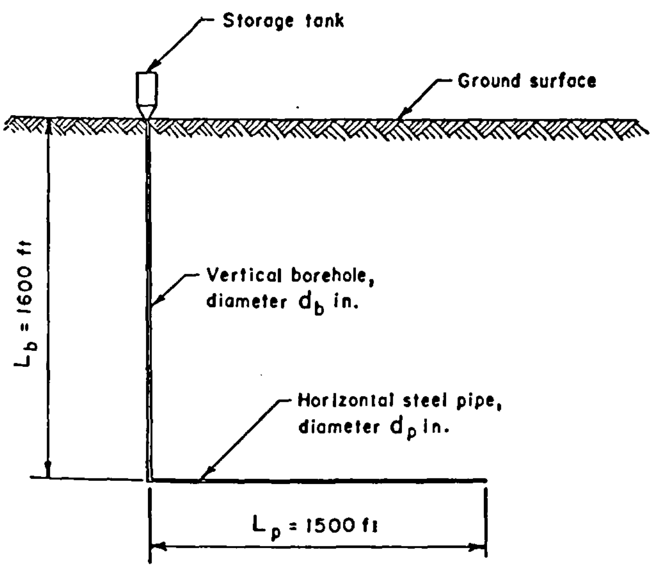

FIGURE 1 — Schematic arrangement of the hydraulic fill system.

This work represents an example of how a multi-variable engineering design problem can be solved explicitly using operational research methods.

ALGORITHM OF THE PROGRAM

In a minimization task, using mathematics, the problem is defined as follows: to find a vector $x^* = (x_1, x_2 \ldots x_n)$, such that the objective function $f = f(x^*) = f(x_1, x_2 \ldots x_n)$ is minimized and such that the constraints of the form $g_i(x^*) = g_i(x_1, x_2 \ldots x_n) \geq 0$, $i = 1, 2, \ldots n$, are satisfied, where both $f(x)$ and $g_i(x)$ may be linear or non-linear.

The algorithm is based on direct search techniques, which work when other methods often fail. The steps of the algorithm are:

1. Start with an initial feasible point
   $x = (x_1, x_2, \ldots, x_n)$,
   define $f_{min} = f(x_1, x_2, \ldots, x_n)$,
   take $k = 1$
2. Form $x_k + \delta$ and $x_k - \delta$, where $\delta \geq 0$, with the other variables held constant.
3. If $g_i\ (x_1, \ldots, x_k + \delta, \ldots, x_n) < 0;\ i = 1, 2, \ldots, n$
   or
   $g_i\ (x_1, \ldots, x_k - \delta, \ldots, x_n) < 0;\ i = 1, 2, \ldots, n$
   form $x_k + w$ and $x_k - w$, where $w < \delta$ such that
   $g_i\ (x_1, \ldots, x_k + w, \ldots, x_n) \geq 0$
   and
   $g_i\ (x_1, \ldots, x_k - w, \ldots, x_n) \geq 0$
4. Choose $f = \min\ [f(x_1, \ldots, x_k + w, \ldots, x_n), f(x_1, \ldots, x_k - w, \ldots, x_n)]$
5. If the procedure described in step 3 has to be invoked, set x_k to the "best" value and go to step 8.
6. Otherwise, increase δ.
7. If $f < f_{min}$, then set $f_{min} = f$ and repeat steps 2 to 6. If $f \geq f_{min}$, set x_k to the "best" value and go to step 8.
8. Repeat steps 2 to 7 sequentially for each variable.
9. Redefine $k = 1$, and repeat steps 2 to 8. The procedure terminates when no further improvement is possible.

The computer program for this algorithm can accommodate equality as well as inequality constraints.

OUTLINE OF THE FILL TRANSPORTATION PROBLEM

A simplified hydraulic fill transportation system is shown in *Figure 1*. The slurry — mixture of sand and water — is stored in the surface storage tank. From this tank, the slurry falls freely through a vertical borehole to the mining level, 1,600 ft below. At this level, a horizontal steel pipe is connected to the borehole. This steel pipe

***Refers to the solution vector corresponding to the minimum value of f**

transports the slurry to the underground mining site where the fill material is placed. The horizontal transportation distance is 1,500 ft.

The diameters of both the vertical borehole and the horizontal steel pipe are at present selected substantially by judgment. In this study, an attempt is being made to design such a fill transportation system by using the tools offered by the methods of operational research.

The problem is a multi-variable one. The following four parameters, from among several, were selected and manipulated in the study:

(1) the diameter (d_b) of the vertical borehole in inches;
(2) the diameter (d_p) of the horizontal steel pipe in inches;
(3) the duration (t_f) of the filling operation in hours per day; and
(4) the ratio (S_w) of weight of solids to weight of slurry.

First, the functional relationships between these variables were established, and then the values of the variables were determined for the given set of conditions to obtain the minimum operating unit cost ($/ton of dry fill) of the filling system. The following three cases were considered:

Case A. Only two variables were used: the diameter of the vertical borehole, d_b, and the diameter of the horizontal steel pipe, d_p. The other two variables were kept constant. The duration of the filling operation, t_f, was assumed to be 11 hours/day, and the solids concentration in the slurry, S_w, was selected to be 0.71.

Case B was similar to Case A, except that the value of t_f was set at 4 hours/day.

Case C. In this case, the optimum values of all four variables — d_b, d_p, t_f and S_w — were determined.

METHODS OF ANALYSES

The derivation of the functional relationships between the selected variables is included in the Appendix.

The total cost per ton of dry solids placed in the hydraulic filling operation, C_T, is the objective cost function. This is obtained by adding the cost of the borehole (C_B), the cost of the pipeline (C_P), the cost of energy (C_E) and the labour cost (C_L). That is:

$$C_T = C_B + C_P + C_E + C_L \text{ (in \$ per ton of dry solids)}$$

The cost elements within this equation have the following general form:

$$C_B = A\, d_b$$
$$C_P = B\, d_p + C/(S_w\, t_f\, d_p)$$
$$C_E = D(1 - S_w)/S_w$$
$$C_L = E\, t_f$$

where d_b = diameter of the borehole, in.
d_p = diameter of the pipeline, in.
S_w = the solids concentration in the slurry, by weight
t_f = duration of the filling operation, hr/day
A, B, C, D and E are constants.

The equations for the cost of the borehole, C_B, and for the cost of labour, C_L, are linear functions. However, the expressions which represent the cost of the pipeline, C_P, and the cost of energy, C_E, are non-linear functions; therefore, the objective cost function also becomes non-linear.

The majority of the constraints are expressed in the following form:

$$K_1 \geq x \geq K_2$$

where x = one of the variables
K_1, K_2 = appropriate constants

However, the constraint which takes into account the flow mechanics and friction losses within the transportation system is a non-linear function of the general form:

$$F = G + f\,(1/d_b^{\alpha}, 1/d_p^{\beta}, 1/S_w^{\gamma}, 1/t_f^{\delta})$$

where F, G = constants
$\gamma, \beta, \alpha, \delta$ = exponents of the variables.

The other constraint which takes into consideration the critical (v_c) and maximum (v_m) velocities is also of a non-linear form, namely:

$$\frac{K_2}{S_w\, t_f} \leq d_p^2 \leq \frac{K_3}{S_w\, t_f}$$

Because the objective cost function and some of the constraints are non-linear functions, linear programming methods are not suitable for the solution of the problem.

The detailed calculations of the constant values are included in the Appendix. Accordingly, for Case A, the final form of the objective cost function is:

$$C_T = 0.01354 d_b + 0.00155 d_p + \frac{0.01214}{d_p} + 0.00319 + 0.091$$

The problem is to find the values of the d_b and d_p variables when C_T is a minimum and when, at the same time, the set of constraint equations is satisfied. The constraints, based on the assumptions detailed in the Appendix, are:

$$2910 - \frac{62{,}800}{d_b^{4.8655}} - \frac{58{,}900}{d_p^{4.8655}} - 635\, d_p^{0.1345} - \frac{199}{d_p^4} \geq 0$$

$$d_p \leq 3.74$$
$$d_p \geq 2.37$$
$$d_b \geq 2.375$$
$$d_b \leq 6.875$$
$$d_p \geq 2$$
$$d_p \leq 8$$

For Case B, the set of simultaneous equations to be solved is:

$$C_T = 0.01354 d_b + 0.00155\, d_p + \frac{0.0333}{d_p} + 0.00319 + 0.0331$$

$$2910 - \frac{405{,}500}{d_b^{4.8655}} - \frac{380{,}000}{d_p^{4.8655}} - 566\, d_p^{0.1345} - \frac{1493}{d_p^4} \geq 0$$

$$d_p \leq 6.19$$
$$d_p \geq 3.92$$
$$d_b \geq 2.375$$
$$d_b \leq 6.875$$
$$d_p \geq 2$$
$$d_p \leq 8$$

The objective cost function in Case C has the following form:

$$C_T = 0.01354 d_b + 0.00155 d_p + \frac{0.0946}{S_w\, t_f\, d_p} + 0.007752\, \frac{1 - S_w}{S_w} + 0.00827\, t_f$$

The constraints are:

$$\frac{4380}{2.74 - 1.74 S_w} - \frac{2{,}845{,}000}{S_w^{1.85}\, t_f^{1.85}\, d_b^{4.8655}} - \frac{2{,}670{,}000}{S_w^{1.85}\, t_f^{1.85}\, d_p^{4.8655}} - \frac{1030 S_w^{1.15} t_f^{0.14} d_p^{0.1345}}{2.74 - 1.74\, S_w} - \frac{18{,}000}{2.74 S_w^2 t_f^2 d_p^4 - 1.74 S_w^3 t_f^2 d_p^4} \geq 0$$

$$d_p \leq \sqrt{\frac{108.6}{S_w\, t_f}}$$

$$d_p \geq \sqrt{\frac{43.5}{S_w\, t_f}}$$

$$d_b \leq 6.875$$
$$d_b \geq 2.375$$
$$d_p \geq 2$$
$$d_p \leq 8$$
$$t_f \geq 0.1$$
$$t_f \leq 24$$
$$S_w \geq 0.5$$
$$S_w \leq 0.739$$

Manual techniques were used for checking the program. Because in Cases A and B the problem is only three-dimensional, it is possible to provide a graphical representation of the manual solution. Consequently, visual inspection is available, which gives a better understanding of the techniques involved. In Case C, as the problem is five-dimensional, the manual effort required to obtain an exact solution is excessive. However, in order to analyse the effect of each variable on the unit cost of the hydraulic fill operation, C_T, the possible ranges are calculated by considering each variable one by one, while all others are kept constant.

RESULTS

Case A

According to the computer solution, the optimum cost is \$0.1352/ton dry fill. The corresponding diameter of the borehole and of the steel pipe is 2.375 and 2.785 in., respectively.

The graphical representation of the manual solution is given in *Figure 2*. The cost function is the equation of a family of parallel curves, the assigned values of C_T being the parameters. From *Figure 2*, it appears that the curve corresponding to C_T = \$0.1350/ton dry fill is the lowest one among those which have common points with the feasible region. Any curve with a lower value of C_T does not intersect the feasible region, therefore it does not yield a solution. On the other hand, the curves which are located above the curve with the parameter of C_T = \$0.135 represent higher costs, besides yielding an infinite number of solutions. Hence the solution of this problem is given by the common point between function C_T and the given constraint. Its coordinates are: d_p = 2.8 in. and d_b = 2.375 in.; moreover, the line with a parameter of C_T = \$0.135 passes through it. The results obtained are in good agreement with the computer solution.

Therefore, 2⅜ in. and 3 in. should be selected for the size of the borehole and for the pipe diameter, respectively.

Case B

The optimum solution obtained by the computer for Case B is: C_T = \$0.0905/ton dry fill, d_b = 2.965 in. and d_p = 5.226 in. The graphical representation of the manual solution is given in *Figure 3*. The values obtained are: C_T = \$0.0908/ton dry fill, d_b = 2.96 in. and d_p = 5.3 in.

A comparison between Case A and Case B reveals that the operating cost is decreased by about 50 per cent (from \$0.135 to \$0.091) as the filling time decreases from 11 hours/day to 4 hours/day.

Case C

The values of the optimum solution obtained by the computer for Case C are: C_T = \$0.0896/ton dry fill, d_b = 3.068 in., d_p = 6.108 in., t_f = 2.734 hours/day and the ratio S_w (weight of solids to weight of slurry) = 0.739.

Within the restrictions of the available sizes of borehole and pipe diameters, and considering the operation requirements originally specified, the fill system shown in *Figure 1* will be at the optimum unit cost if the solids concentration in the slurry is the maximum possible (S_w = 0.739), if the filling operation is about 3 hours per day (t_f = 2.73 hrs), if the diameter of the borehole is 3 inches and if the diameter of the steel pipe is 6 inches.

CONCLUSIONS

An optimization computer program was developed in order to solve multi-variable problems with non-linear functions. The program is readily adaptable for actual conditions.

One of the cases selected to test the computer program was the analysis of a simplified mine-fill transportation system.

It was found that the optimum cost solution obtained by the analysis resulted in borehole and pipe sizes that are very close to those being used in current practice for actual fill systems under similar conditions. However, where the duration of the filling operation per day is also optimized, the pipe diameter required is somewhat larger than is currently used in practice.

The analyses reveal that the unit filling cost is most sensitive to variation in the duration of filling per day. The next variable, in order of importance, is the borehole diameter. The solid concentration in the slurry is the third variable of importance. Finally, cost is least sensitive to variations of the pipe diameter.

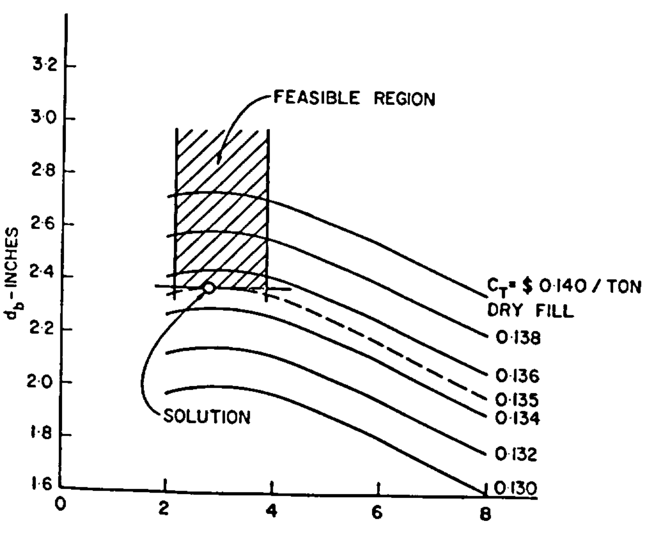

FIGURE 2 — Graphical representation of the manual solution for Case A.

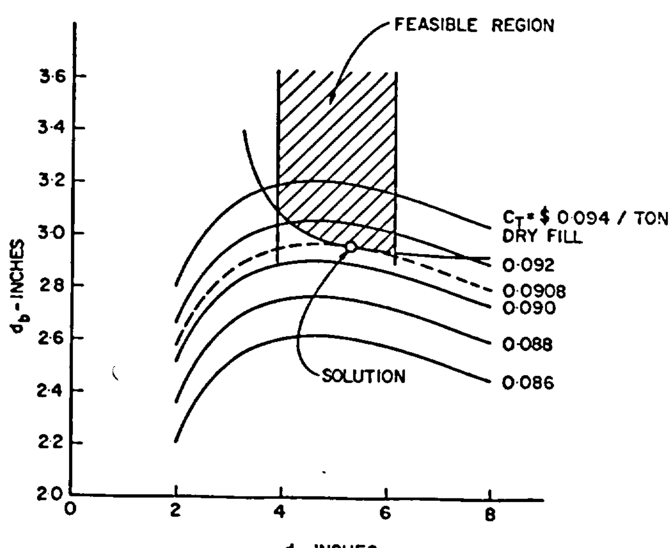

FIGURE 3 — Graphical representation of the manual solution for Case B.

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APPENDIX

Case A

Assume that, for a fill transportation system (as shown in *Figure 1*), the required yearly dry fill in place, Q_F, is 265,000 tons. The mine operates continuously, therefore the number of days per year with filling, D_w, is 365 days. Duration of the filling operation, t_f, is 11 hours/day. The slurry used contains 71 per cent sand by weight, therefore the concentration of solids, S_w, is 0.71. The diameters of the vertical borehole and of the horizontal steel pipe for the given data are to be determined to produce the minimum cost of dry fill.

From the above data, the weight of dry solids transported per hour is:

$$Q_D = \frac{Q_F}{D_w t_f} = \frac{265{,}000}{365 \times 11} = 66 \text{ tons/hour} \quad (1)$$

Friction Loss in Vertical Borehole

Based on experiments conducted on vertical pipes of various diameters with slurries of various sand concentration, Durand[1] concluded that the friction losses for slurries were the same as those for clear water. Therefore, the Williams and Hazen formula can be used to calculate the friction loss. Accordingly:

$$h_w = 0.2083 \left(\frac{100}{C}\right)^{1.85} \frac{q^{1.85}}{d^{4.8655}} \quad (2)$$

where h_w = h_b = head loss (ft water/100 ft borehole) for the slurry in the borehole
d = d_b = diameter (in.) of borehole
C = constant for surface roughness; for design purposes, C = 100 is recommended in the case of concrete pipe[7].
q = q_s = flow (U.S. gal./min.)

The flow is calculated by the equation:

$$q_s = \frac{3.99\, Q_D}{G_s S_w} = \frac{3.99 \times 66}{1.82 \times 0.71} = 205 \text{ U.S. gal/min} \quad (3)$$

In this equation, G_s is the specific gravity of the slurry. It is calculated by using the following equation:

$$G_s = \frac{1}{W_w + \frac{S_w}{G'}} = \frac{1}{0.29 + \frac{0.71}{2.74}} = 1.82 \quad (7)$$

where W_w = ratio of water in the slurry by weight.

Using Eq. (2), after substitution, for the friction loss in the vertical borehole, the following expression is obtained:

$$h_b = 0.2083\left(\frac{100}{100}\right)^{1.85} \frac{205^{1.85}}{d_b^{4.8655}} = \frac{3926}{d_b^{4.8655}} \text{ ft of water/100 ft of borehole} \quad (2a)$$

Friction Loss in Horizontal Steel Pipe

The friction loss in the horizontal steel pipe may be calculated by equation (2):

$$h_p = h_w + S_v h_w \left[\frac{62\, g\, \frac{d_p}{12}(G'-1)}{v_p^2}\; \frac{1}{\sqrt{C'_D}}\right] \quad (5)$$

Or, by substituting the value of gravitational acceleration, g = 32.2 ft/sec.²,

$$h_p = h_w + S_v h_w \left[\frac{166\, d_p(G'-1)}{v_p^2}\; \frac{1}{\sqrt{C'_D}}\right] \quad (5a)$$

where h_p = head loss (ft water/100 ft pipe) for slurry in horizontal steel pipe
h_w = head loss (ft water/100 ft pipe) for water, i.e., the slurry is regarded as water
S_v = the ratio, volume of solids to volume of slurry
d_p = diameter of pipe (in.)
v_p = flow velocity (ft/sec.) in pipe
C'_D = weighted mean drag coefficient
G' = weighted mean specific gravity of solids, here assumed to be 2.74.

With sufficient information, G′ should be determined by equation (2):

$$G' = \frac{P_1G_1 + P_2G_2 + \ldots . P_nG_n}{100} \quad (4)$$

where $P_1, P_2, \ldots P_n$ = percentage of the total for each particle size fraction, established by sieve analysis of the fill material.
$G_1, G_2, \ldots G_n$ = specific gravity of each particle size fraction.

To evaluate the value of h_w, Eq. (2) is used again.

$$h_w = 0.2083\left(\frac{100}{C}\right)^{1.85} \frac{q_s^{1.85}}{d_p^{4.8655}} \quad (2b)$$

C = 100 is taken; this is the value recommended for design purposes when using an old steel pipe[7].

The flow value for the horizontal steel pipe is the same as for the vertical borehole, i.e., q_s = 205 U.S. gal./min.

$$h_w = 0.2083\left(\frac{100}{100}\right)^{1.85} \frac{205^{1.85}}{d_p^{4.8655}} = \frac{3926}{d_p^{4.8655}} \quad (2c)$$

To calculate S_v, the following equation is used:

$$S_v = \frac{G_s - 1}{G' - 1} = \frac{1.82 - 1}{2.74 - 1} = 0.471 \quad (6)$$

The flow velocity in the pipe is determined by the equation:

$$v_p = \frac{1.63 Q_D}{G_s S_w d_p^2} = \frac{1.63 \times 66}{1.82 \times 0.71 \times d_p^2} = \frac{83.8}{d_p^2} \text{ ft/sec.} \quad (8)$$

and

$$v_p^2 = \frac{7030}{d_p^4} \quad (8a)$$

To determine the weighted mean drag coefficient, C'_D, the following equation is used:

$$C'_D = \frac{4}{3}\, \frac{g\, \frac{d'_n}{12}(G'-1)}{v_s'^2} \quad (9)$$

or

$$C'_D = \frac{3{\cdot}57\, d'_n (G'-1)}{v_s'^2} = \frac{3{\cdot}57 \times 0{\cdot}1\,(2{\cdot}74 - 1)}{0{\cdot}45^2} = 3{\cdot}08 \quad (9a)$$

In this expression

$d_n' =$ weighted mean nominal particle diameter (in.), here assumed to be 0.1 in.
$v_s' =$ weighted mean terminal fall velocity (ft/sec.); in this case 0.45 ft/sec. was taken.

It is proposed[2] that d_n' and v' should be determined from the following equations:

$$d_n' = \frac{P_1 d_{n1} + P_2 d_{n2} + \ldots P_n d_{nn}}{100} \qquad (10)$$

and

$$v_s' = \frac{P_1 v_{s1} + P_2 v_{s2} + \ldots P_n v_{sn}}{100} \qquad (11)$$

where $P_1, P_2, \ldots P_n =$ percentage of the total for each particle size fraction, established by sieve analysis of the fill material
$d_{n1}, d_{n2}, \ldots d_{nn} =$ nominal diameter of each particle size fraction
$v_{s1}, v_{s2}, \ldots v_{sn} =$ terminal fall velocity of each particle size fraction.

By substituting the results which are obtained from Equations (2c), (6), (8a) and (9a) into Eq. (5a), the following expression is obtained for the friction loss in the horizontal steel pipe:

$$h_p = \frac{3926}{d_p^{4.8655}} + 0.471 \frac{3926}{d_p^{4.8655}} \left[\frac{166\, d_p\,(2.74 - 1)}{\frac{7030}{d_p^4}} \quad \frac{1}{\sqrt{3.08}} \right]$$

or

$$h_p = \frac{3926}{d_p^{4.8655}} + 43\, d_p^{0.1345} \text{ ft water/100 ft of pipe} \qquad (9b)$$

Constraints

In order to obtain a meaningful solution of the problem, the boundaries of the feasible region or the constraints have to be established. Any solution which falls within the feasible region is acceptable, and solutions outside of the region are rejected.

The first constraint is from the energy balance. The slurry column (L_b) in the borehole has a potential energy which can be expressed in terms of feet of water. That is:

$$G_s L_b = 1.82 \times 1600 = 2910 \text{ ft of water} \qquad (12)$$

While the slurry flows, this potential energy changes to kinetic energy. In the horizontal steel pipe it is:

$$G_s \frac{v_p^2}{2g} = 1.82 \frac{\frac{7030}{d_p^4}}{64.4} = \frac{199}{d_p^4} \text{ ft of water} \qquad (13)$$

However, part of the potential energy is used to overcome the friction resistance in both the vertical borehole and horizontal steel pipe. The energy loss in the vertical borehole is:

$$\frac{L_b h_b}{100} = \frac{1600}{100} \frac{3926}{d_b^{4.8655}} = \frac{62{,}800}{d_b^{4.8655}} \text{ ft of water} \qquad (14)$$

The energy loss at the same time in the horizontal pipe is expressed as follows:

$$\frac{L_p h_p}{100} = \frac{1500}{100}\left(\frac{3926}{d_p^{4.8655}} + 43\, d_p^{0.1345}\right)$$
$$= \frac{58{,}900}{d_p^{4.8655}} + 635\, d_p^{0.1345} \text{ ft of water} \qquad (15)$$

The potential energy and kinetic energy should be equal; therefore

$$2910 - \frac{62{,}800}{d_b^{4.8655}} - \frac{58{,}900}{d_p^{4.8655}} - 635\, d_p^{0.1345}$$
$$= \frac{199}{d_p^4} \text{ ft of water} \qquad (16)$$

The diameters of the borehole, d_b, and of the pipe, d_p, should be selected in a manner such that Eq. (16) is satisfied. The first constraint accordingly is:

$$2910 - \frac{62{,}800}{d_b^{4.8655}} - \frac{58{,}900}{d_p^{4.8655}} - 635\, d_p^{0.1345}$$
$$- \frac{199}{d_p^4} \geq 0 \qquad (16a)$$

The second type of constraint is from velocity restrictions. For the given flow capacity (Q_D) or the corresponding q_s, the diameter of the pipe should be selected in such a way that the flow velocity will fall between v_c and v_m. The critical velocity, v_c, is defined as the lowest possible flow velocity at which no plugging of the line will occur. A critical velocity of 6 ft/sec. was taken in this case. The maximum velocity, v_m, is the velocity at which the pipe wear becomes unacceptable, 15 ft/sec. The foregoing, expressed in equation form, is:

$$v_c = \frac{1.63\, Q_D}{G_s S_w d_p^2} = \frac{1.63 \times 66}{1.82 \times 0.71 \times d_p^2}$$
$$= \frac{83.8}{d_p^2} = 6 \text{ ft/sec.} \qquad (17)$$

and

$$v_m = \frac{83.8}{d_p^2} = 15 \text{ ft/sec.} \qquad (18)$$

From Eq. (17)

$$d_p = \sqrt{\frac{83.8}{6}} = 3.74 \text{ in.} \qquad (17a)$$

and from Eq. (18)

$$d_p = \sqrt{\frac{83.8}{15}} = 2.37 \text{ in.} \qquad (18a)$$

Therefore, the following two constraints are obtained:

$$d_p \leq 3.74 \text{ in.} \qquad (17b)$$
and
$$d_p \geq 2.37 \text{ in.} \qquad (18b)$$

The third type of constraint is given by the available sizes of borehole and steel pipe. The standard borehole sizes are: 2⅜, 3, 3 15/16, 4¾, 5¾ and 6⅞ inches. The inside diameters of commercial steel pipes are: 2, 2½, 3, 4, 5, 6 and 8 inches. The size constraints therefore are:

$$d_b \geq 2.375 \text{ in.} \qquad (19)$$
$$d_b \leq 6.875 \text{ in.} \qquad (20)$$
$$d_p \geq 2 \text{ in.} \qquad (21)$$
$$d_p \leq 8 \text{ in.} \qquad (22)$$

The constraints expressed by the previous equations are plotted in *Figure A-1*. Notice that the feasible region is bounded by the innermost constraints. For example, in Case A the energy constraint, represented by Eq. (16a), is inactive and the size constraint, Eq. (19), is active. Or in other words, if the solution to the condition expressed by Eq. (19) is satisfied, then the condition required by Eq. (16a) is also satisfied. Similar considerations are applied to the other constraints.

Cost Elements

In order to obtain the unit cost directly connected with the mine fill system, the following cost elements are considered: cost of borehole, cost of pipeline, cost of energy and labour cost.

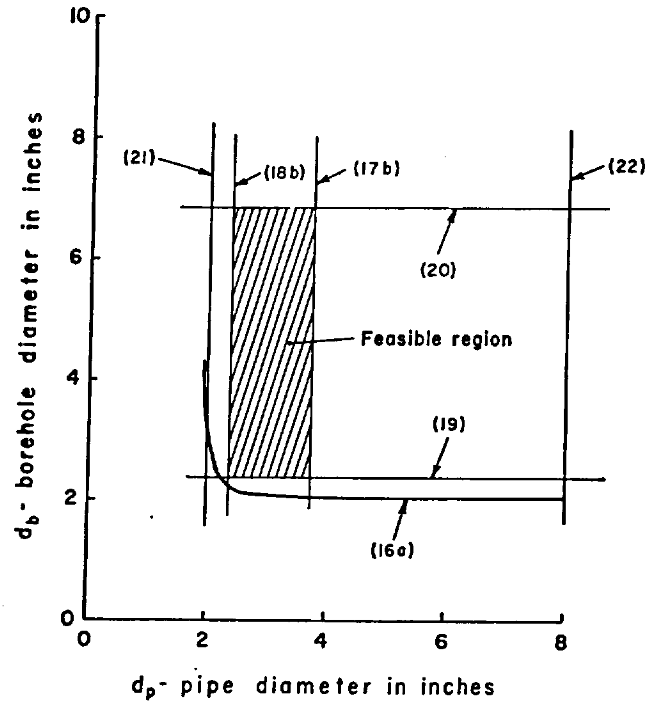

FIGURE A-1 — Constraints and feasible region for Case A.

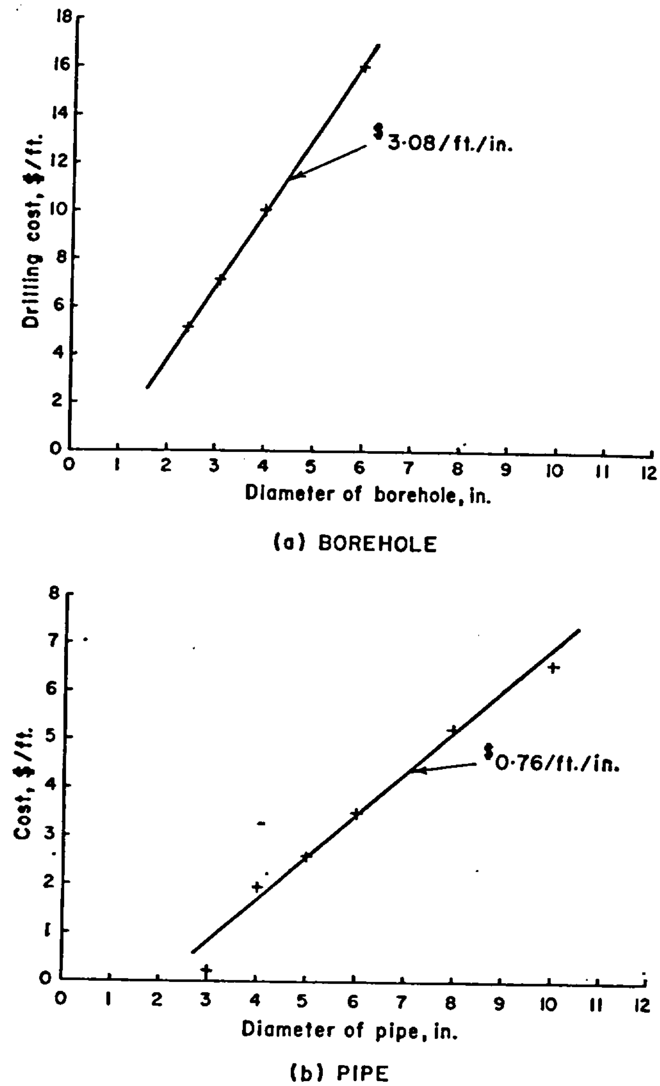

FIGURE A-2 — Unit cost of borehole and pipe.

Cost of Borehole

The total cost of the borehole, C_B, in terms of $/ton of dry fill, is expressed by the following equation:

$$C_B = 2a\,(1 + i_b)\,c_b\,d_b\,L_b/Q_F \quad (23)$$

where a = amortization factor ($ per annum/$ capital)
c_b = unit cost of borehole ($/ft/in.)
i_b = installation cost factor for borehole (dimensionless)
d_b = diameter of borehole (in.)
L_b = length of borehole (ft)
Q_F = dry fill in place (tons/year)

Note that the right-hand side of the equation is multiplied by 2. This reflects the general practice, in that two boreholes are usually drilled in order to ensure a continuous filling operation.

It was assumed that the amortization factor, a, is 0.33 (which is equivalent to a write-off period of 3 years).

The cost of the pipe or hose connections, which are necessary in the intersected drifts at every elevation, and the cost of the connection of the borehole to the horizontal pipe line are covered by the installation cost factor. It was assumed that these costs are 10 per cent of the drilling cost ($i_b = 0.1$).

In *Figure A-2a*, the borehole diameter is plotted against drilling cost. The slope of the straight line thus obtained, i.e., $3.08/ft/in., is the unit cost of the borehole, c_b. By substituting these values into Eq. 23 for the total cost of the borehole, the following function is obtained:

$$C_B = \frac{2 \times 0.33\,(1 + 0.1)\,3.08 \times d_b \times 1600}{265{,}000} = 0.01354\,d_b \quad (23a)$$

Cost of Pipeline

The total cost of the pipeline, C_P, in terms of $/ton of dry fill, can be expressed as follows:

$$C_P = (a + w)\,(1 + i_p)\,c_p\,d_p\,L_p/Q_F \quad (24)$$

where a = amortization factor ($ per annum/$ capital)
w = wear factor ($ per annum/$ capital)
c_p = unit cost of pipe ($/ft/in.)
i_p = installation cost factor for pipeline (dimensionless)
d_p = diameter of pipe (in.)
L_p = length of pipe (ft)
Q_F = dry fill in place (tons/year)

For the write-off period 3 years was again assumed, therefore $a = 0.33$.

The wear factor is defined by the expression:

$$w = \frac{Q_F}{Q_w} \quad (25)$$

where Q_F = dry fill in place (tons/year)
Q_w = tonnage of dry fill passing through which is necessary to wear out the pipe (tons)

From actual pipe wear data[3,4] it was concluded that Q_w is a function of the flow velocity, v_p. In equation form:

$$Q_w v_p = k \quad (26)$$

where k is a constant. Its value depends on the material of the pipe. For steel pipe, $k = 8.68 \times 10^6$ and for black iron pipe $k = 5.76 \times 10^6$ can be used.

From Eq. (1)

$Q_F = Q_D D_w t_f$

and from Eq. (8)

$$Q_D = \frac{v_p G_s S_w d_p^2}{1.63}$$

while from Eq. (26)

$$Q_w = \frac{k}{v_p}$$

Therefore

$$w = \frac{Q_F}{Q_w} = \frac{\dfrac{v_p G_s S_w d_p^2 D_w t_f}{1.63}}{\dfrac{k}{v_p}} = \frac{G_s S_w D_w t_f}{1.63k} v_p^2 d_p^2$$

Substituting the known values, and assuming steel pipe is used, this expression becomes:

$$w = \frac{1.82 \times 0.71 \times 365 \times 11}{1.63 \times 8.68 \times 10^6} v_p^2 d_p^2 = 0.000366 v_p^2 d_p^2$$

Or by substituting Eq. (8a), a final expression is obtained for w:

$$w = 0.000366 \frac{7030}{d_p^4} d_p^2 = \frac{2.572}{d_p^2} \quad \text{(25a)}$$

The unit cost of steel pipe is found from *Figure A-2b*, i.e., $c_p = 0.76$/ft/in. It was assumed that the cost of hardware items amounts to 10 per cent of the pipe cost, therefore $i_p = 0.1$. Now all the values are known and the following function is obtained for the total cost of the pipeline:

$$C_P = \left(0.33 + \frac{2.572}{d_p^2}\right)(1 + 0.1)\, 0.76 \times d_p \times 1500/265{,}000$$

$$= 0.00155 d_p + \frac{0.01214}{d_p} \quad \text{(24a)}$$

Cost of Energy

After the slurry is placed, about 90 per cent of the water is squeezed out and 10 per cent is retained by the fill. The water normally flows by gravity to a sump in the shaft area. From here it is pumped to the surface. The major cost here is the electrical power cost. The power cost depends on volume of water and on the head against which the water must be pumped. The weight of water used for mixing the slurry in tons/day is:

$$W_M = \frac{t_f Q_D (1 - S_w)}{S_w} \quad \text{(27)}$$

90 per cent of this is returned to the surface. The quantity of water which has to be lifted, in terms of lb/min., is:

$$W_P = \frac{2000 \times 0.9}{1440}\left[\frac{t_f Q_D (1 - S_w)}{S_w}\right]$$

$$= 1.25 \frac{t_f Q_D (1 - S_w)}{S_w} \quad \text{(28)}$$

The horsepower requirements then are:

$$HP = \frac{L_b}{33{,}000} 1.25 \frac{t_f Q_D (1 - S_w)}{S_w}$$

$$= 0.000038 \frac{L_b t_f Q_D (1 - S_w)}{S_w} \quad \text{(29)}$$

where L_b = lifting height, which is assumed to be the same as the length of the borehole.

If energy cost, c_e, is quoted in \$/(HP-year) and the efficiency of the pumping system is e, then the total cost of energy in \$/ton dry fill placed is:

$$C_E = 0.000038 \frac{L_b t_f Q_D (1 - S_w)}{S_w e} / Q_F \quad \text{(30)}$$

Assuming that c_e = \$35/(HP-year) and that e = 75 per cent, the total cost of energy is:

$$C_E = 0.000038 \frac{1600 \times 11 \times 66 (1 - 0.71)\, 35}{0.71 \times 0.75 \times 265.000} = 0.00319 \quad \text{(30a)}$$

Labour Cost

The direct labour cost involved is expressed by the following equation:

$$C_L = \frac{n C_l t_f D_w}{Q_F} \quad \text{(31)}$$

where n = number of workers supervising the fill system, assumed to be 2
c_l = hourly salary paid to supervise the pipeline and joints, cleaning up, etc., during the fill time, assumed to be \$3/hr.
t_f = duration of filling operation (hrs/day)
D_w = number of working days in a year
Q_F = dry fill in place (tons/year)

After substitution, the total cost of labour in terms of \$/ton dry fill placed is obtained:

$$C_L = \frac{2 \times 3 \times 11 \times 365}{265{,}000} = 0.091 \quad \text{(31a)}$$

Objective Function

The total cost of the hydraulic fill operation is obtained by adding the cost of the borehole, the cost of the pipeline, the cost of energy and the labour cost. That is:

$$C_T = C_B + C_P + C_E + C_L \quad \text{(32)}$$

or

$$C_T = 0.01354 d_b + 0.00155 d_p + \frac{0.01214}{d_p} + 0.00319 + 0.091 \quad \text{(32a)}$$

Eq. (32a) is the objective function in this minimization problem.

Formulation of the Optimization Problem

Having all the necessary information, the optimization problem can now be formulated in the following manner: minimize the objective function

$$C_T = 0.01354 d_b + 0.00155 d_p + \frac{0.01214}{d_p} + 0.00319 + 0.091 \quad \text{(32a)}$$

subject to the following constraints:

$$2910 - \frac{62{,}800}{d_b^{4.8655}} - \frac{58{,}900}{d_p^{4.8655}} - 635 d_p^{0.1345} - \frac{199}{d_p^4} \geq 0 \quad \text{(16a)}$$

$$d_p \leq 3.74 \quad \text{(17b)}$$
$$d_p \geq 2.37 \quad \text{(18b)}$$
$$d_b \geq 2.375 \quad \text{(19)}$$
$$d_b \leq 6.875 \quad \text{(20)}$$
$$d_p \geq 2 \quad \text{(21)}$$
$$d_p \leq 8 \quad \text{(22)}$$

Case B

The problem here is the same as for Case A — namely to obtain the required and most economical diameter of the borehole and horizontal pipeline of the hydraulic fill system of *Figure 1*. The only difference is that the duration of the filling operation, t_f, is changed from 11 hours/day to 4 hours/day.

Calculated in a manner similar to that given in detail for Case A, the optimization problem is defined: minimize the objective function

$$C_T = 0.01354 d_b + 0.00155 d_p + \frac{0.0333}{d_p} + 0.00319 + 0.0331 \quad \text{(32b)}$$

subject to the following constraints:

$$2910 - \frac{405{,}500}{d_b^{4.8655}} - \frac{380{,}000}{d_p^{4.8655}} - 566d_p^{0.1345} - \frac{1493}{d_p^{4}} \geq 0 \ldots (16b)$$

$$d_p \leq 6.19 \ldots\ldots\ldots (17c)$$
$$d_p \geq 3.92 \ldots\ldots\ldots (18c)$$
$$d_b \geq 2.375 \ldots\ldots\ldots (19)$$
$$d_b \leq 6.875 \ldots\ldots\ldots (20)$$
$$d_p \geq 2 \ldots\ldots\ldots (21)$$
$$d_p \leq 8 \ldots\ldots\ldots (22)$$

The constraints are plotted in *Figure A-3*. Note that the size of the feasible region is increased, shifted toward right and the energy constraint, Eq. (16b), becomes the lower boundary of the region instead of the size constraint, Eq. (19).

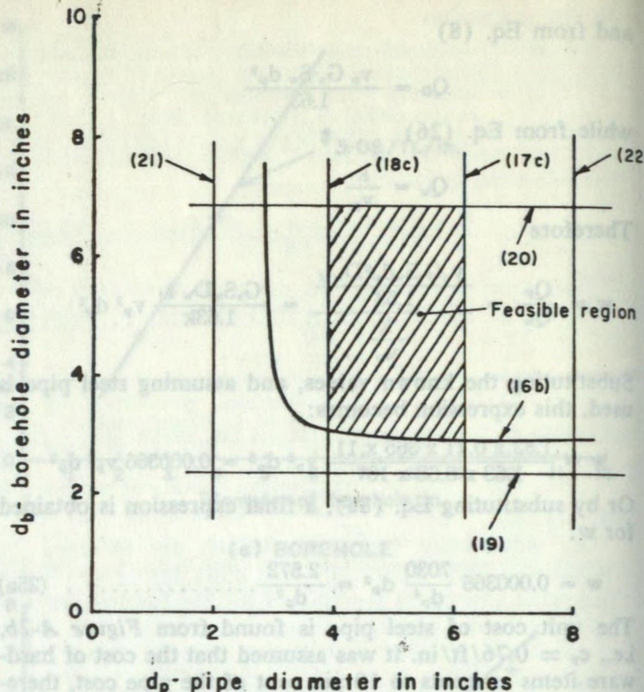

FIGURE A-3 — Constraints and feasible region for Case B.

Case C

For the simplified hydraulic fill transportation system shown in *Figure 1*, the optimization problem consists of four variables (d_b, d_p, t_f and S_w).

The objective cost function in Case C has the following form:

$$C_T = 0.01354d_b + 0.00155d_p + \frac{0.0946}{S_w\, t_f\, d_p} + 0.007752\,\frac{1 - S_w}{S_w} + 0.00827\, t_f$$

The constraints are:

$$\frac{4380}{2.74 - 1.74\, S_w} - \frac{2{,}845{,}000}{S_w^{1.85}\, t_f^{1.85}\, d_b^{4.8655}} - \frac{2{,}670{,}000}{S_w^{1.85}\, t_f^{1.85}\, d_p^{4.8655}}$$

$$- \frac{1030 S_w^{1.15}\, t_f^{0.15}\, d_p^{0.1345}}{2.74 - 1.74\, S_w} - \frac{18{,}000}{2.74 S_w^{2} t_f^{2} d_p^{4} - 1.74 S_w^{3} t_f^{2} d_p^{4}} \geq 0$$

$$d_p \leq \sqrt{\frac{108.6}{S_w\, t_f}}$$

$$d_p \geq \sqrt{\frac{43.5}{S_w\, t_f}}$$

$$d_b \leq 6.875$$
$$d_b \geq 2.375$$
$$d_p \geq 2$$
$$d_p \leq 8$$
$$t_f \geq 0.1$$
$$t_f \leq 24$$
$$S_w \geq 0.5$$
$$S_w \leq 0.739$$

The cost function as well as the majority of the constraints were derived in a similar manner to Case A. The limits given for t_f are self-explanatory. The lower limit of $S_w = 50$ per cent is taken from general practice. To determine the upper limit, the empirical formula, suggested by Stewart *et al.*[6], is used. Accordingly:

$$S_w(\max) = \frac{0.495G'}{0.495G' + 0.505}$$

$$= \frac{0.495 \times 2.74}{0.495 \times 2.74 + 0.505} = 0.739$$