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APPLICATIONS OF DIGITAL COMPUTERS IN THE QUALITY CONTROL OF CONCRETE

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PAPER SP 16-2 Digital computers are used for control of concrete quality during its manufacture and for analysis and interpretation of concrete test results. Statistical parameters normally used for quality control are defined and their use is explained with the help of job control charts. The three case histories presented show: (1) the use of computers in establishing the degree of reliance of the strength of accelerated-cured test cylinders; (2) the multiple correlation analyses of 1176 compressive strength test results and several independent variables; and (3) a computer method for the trial and error fitting of a hyperbolic curve to compressive strength test results.

Applications of Digital Computers in the Quality Control of Concrete

By V. M. MALHOTRA

THE ADVANTAGES OF USING statistical methods both in the presentation of field test data and the interpretation of test results are now widely recognized by concrete technologists. Concrete control engineers are well aware that their test results are always subject to certain errors due to the inherent variability of the concrete-making materials. It is most essential that sufficient allowance be made for these variables during the manufacture of concrete and the analysis of the test results. The statistical methods are the only means of doing so since otherwise it is difficult to distinguish between chance causes and real effects of any particular factor, unless the latter are obviously very large when compared with the former causes.

Repetition is a key word in the statistical analysis of test data. When data are limited and a problem has no repetitive aspects, one can carry out the calculations much more quickly by manual methods than one can program it for a computer. However, when data run into hundreds of test results, as is often the case on big hydroelectric projects and paving jobs and in the operation of large ready-mixed concrete plants, manual statistical computations become very tedious and time-consuming and a resort to electronic computing facilities has to be made. This paper presents the applications of digital computers to the quality control of concrete both during its manufacture and during the analysis and interpretation of test results.

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QUALITY CONTROL DURING MANUFACTURE OF CONCRETE

The uniformity of concrete production at a batch plant is measured by the testing of 6x12-in. concrete cylinders. The strength of standard test cylinders not only shows the potential compressive strength of concrete in a structure but is also used to estimate other structural properties of concrete such as flexural strength and modulus of elasticity.

The degree of production control is measured by the uniformity achieved in the testing of cylinders, uniform strength indicating uniform control. Large variations in the 28-day compressive strength of test cylinders invariably require increased average strength, (resulting in increased cost) to meet the minimum design strength. If large variations are allowed in the control strength of concrete, the danger of low strength concrete being placed in the critical sections of a structure is increased.

The variations in compressive strength of concrete are measured by statistical parameters known as "standard deviation" and "coefficient of variation."

Standard deviation is a measure of the spread of observations about the central value. The standard deviation σ of the population is found by extracting the square root of the average of the squares of deviations of individual test values from their average, i.e.:

$$\sigma = \sqrt{\frac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + \ldots + (X_n - \bar{X})^2}{n}}$$

The calculation of σ by this formula is laborious when the number of test results exceeds 30, which is often the case on large construction jobs. Under such circumstances the calculations can easily be handled by a computer using the following form of the formula:

$$\sigma = \sqrt{\frac{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}{n}}$$

The standard deviation is expressed in the same units as the compressive strength. When the number of observations is less than 30, a minor correction called Bessel's correction has to be applied to the above formula.

PROBLEM IDENT	NO. OF OBSERVATIONS	AVERAGE VALUE	STANDARD DEVIATION	COEFFICIENT OF VARIATION
10000	241	.90904564E+03	.18401188E+03	20.24
20000	35	.11905714E+04	.25041083E+03	21.03
30000	49	.15291836E+04	.23064131E+03	15.08
40000	102	.95274509E+03	.24044992E+03	25.24
50000	119	.10810924E+04	.21627372E+03	20.01
60000	171	.13292397E+04	.32025007E+03	24.09
10000	241	.23007468E+04	.42092754E+03	18.30
20000	35	.31820000E+04	.62913011E+03	19.77
30000	49	.39526530E+04	.40822710E+03	10.33
40000	102	.27748039E+04	.57461790E+03	20.71
50000	119	.29768067E+04	.50922007E+03	17.11
60000	171	.36229239E+04	.61295139E+03	16.92

FIGURE 2-1 COMPUTER PRINTOUT FOR AVERAGE, STANDARD DEVIATION, AND COEFFICIENT OF VARIATION

The coefficient of variation (CV) is simply the standard deviation expressed as a percentage of the arithmetic mean, i.e.:

$$CV = \frac{\text{standard deviation}}{\text{arithmetic mean}} \times 100$$

It is a dimensionless quantity.

Some authorities use standard deviation, whereas others, like the U. S. Bureau of Reclamation, prefer the use of coefficient of variation as the statistical tool to indicate the amount of variation. Calculations for both these parameters together with arithmetic mean can be carried out by a computer in one operation. A sample printout from a computer is shown in Figure 2-1.

Test data can be fed into a computer daily, weekly, and monthly as the job demands. The results obtained bring the quality control engineer up to date about the quality of concrete being placed and enable him to effect changes in the mix proportions, if necessary. The accompanying control charts illustrate the use of standard deviation and coefficient of variation in the quality control of concrete.

The control chart in Figure 2-2 shows the results for structural concrete for which the 28-day minimum specified strength was 3000 psi and the number of failures was not to exceed 10 percent. This control chart based upon the results of the first 25 tests indicates that the job was under excellent control with a CV of only 9.4 percent and that there were no test results falling below the minimum specified strength. (It is, of course, assumed that all samples were taken at random.) Using the information from the control chart, the quality control engineer would be justified in effecting changes in the mix proportions in order to reduce the 28-day average compressive strength from 3970 psi to 3440 psi.[1] This means reduction in the cement content per cu yd of concrete, with resulting savings in cost.

The control chart in Figure 2-3 gives the results for a structural mass concrete for which the 28-day minimum specified strength was 2500 psi. The number of failures allowed was 20 percent. The control chart based upon the first 26 test results indicates that the job was under poor control, with a CV of 20.1 percent and 31 percent of the test results falling below the minimum specified strength. Under such conditions the quality control engineer must insist upon much tighter production controls. However, if the circumstances on the job are such that the tighter control cannot be achieved, then the engineer has no alternative but to reproportion the concrete mix. The reproportioned concrete mix must give 28-day average compressive strength of at least 3040 psi as compared to 2825 psi, so that the number of test results falling below the minimum specified strength does not exceed 20 percent.[2] It is stressed that apart from the increased cost, the latter alternative is a poor solution to the problem and always should be discouraged.

ANALYSIS AND INTERPRETATION OF TEST RESULTS

Engineering problems often require the presentation of data showing the observed relationship between two variables. Before the analysis can be carried out the pairs of data are plotted. If the relationship between the observations is nearly exact, a straight line or curve drawn by eye may be sufficient. Unfortunately in concrete technology the exact relationships are indeed elusive. More often than not, the relationship between two variables is less certain and drawing by eye results in graphs which depend too much upon the judgment of the research worker

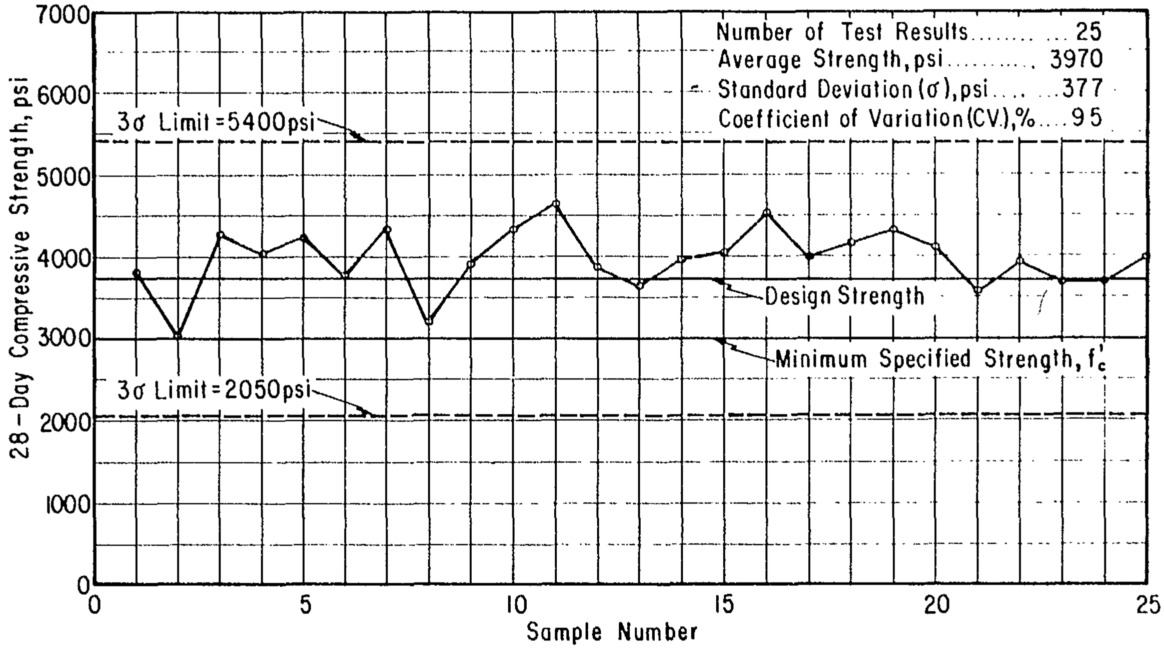

FIGURE 2-2 CONTROL CHART FOR INDIVIDUAL STRENGTH TESTS--STRUCTURAL CONCRETE: CV OF 9.5 PERCENT INDICATES EXCELLENT JOB CONTROL

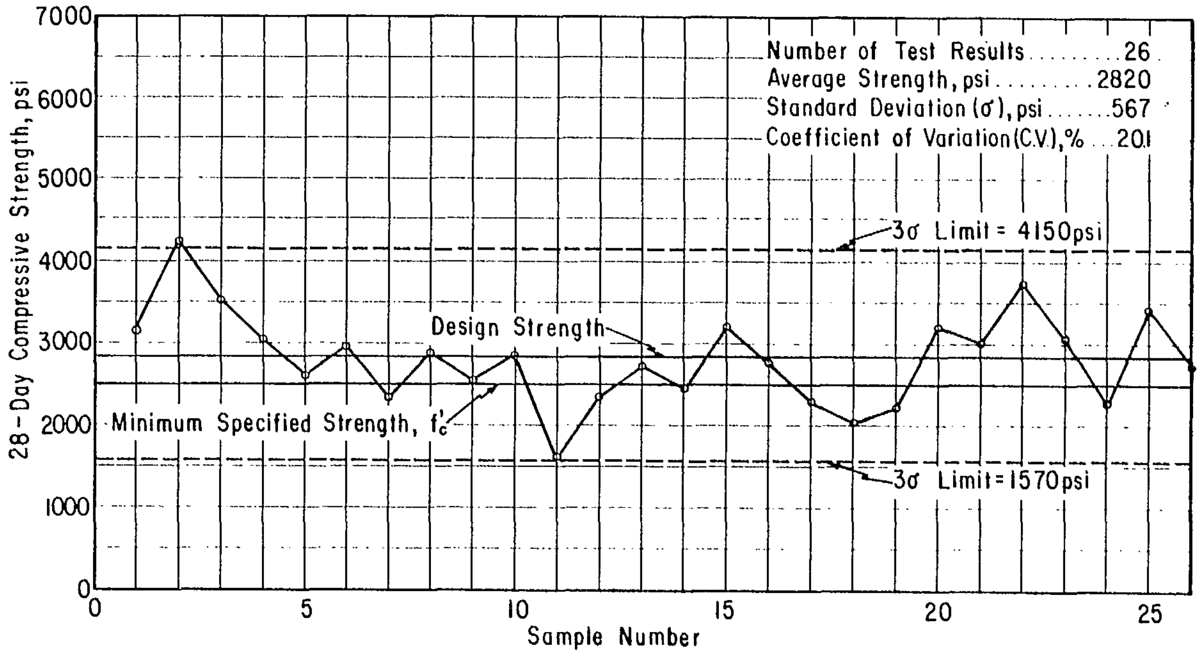

FIGURE 2-3 CONTROL CHART FOR INDIVIDUAL STRENGTH TESTS--STRUCTURAL MASS CONCRETE: CV OF 20.1 PERCENT INDICATES POOR JOB CONTROL

involved. Different investigators observing the same data may produce significantly different graphs. In such cases, fitting a straight line or a curve by eye is not justified and a more precise method of determining the relationship between two variables is needed. In many other types of problems the differences in one variable may be due to several other variables all acting at the same time. The techniques for solving such problems are based upon statistical methods and are known as "linear, multiple linear regression and correlation analyses." Using these techniques one can obtain a mathematical description of the relationship between independent and dependent variables. Such techniques have been used in biological sciences for a long time but their use in concrete technology is of relatively recent origin. The following three case histories illustrate the use of statistical techniques in the analysis of concrete test data. All the necessary computations were carried out using a digital computer.

Case History 1 -- Regression Analysis and Prediction Limits

This case involves the analyses of the results of accelerated 24-hr concrete strengths obtained during the construction of a huge hydroelectric project.[3] The accelerated curing method used consisted of placing the standard 6x12-in. concrete cylinders, 1/2 hr after molding, in a water tank thermostatically controlled at 165 $\pm$ 5F. The cylinders were removed from the tank 21-1/2 hr later and tested at the age of 24 hr. A total of 737 pairs of test results was available. It was necessary to establish the relationship between the accelerated and the 28-day strengths and also to determine the degree of confidence that could be placed in the 24-hr accelerated strengths.

The test results were analyzed using an IBM 1620 computer. Both 90 and 95 percent prediction limits were determined using the following expression:[4]

$$a + bx^{o} \pm t_{\alpha/2;\, n-2} \times S_{y/x}\sqrt{1 + \frac{1}{n} + \frac{(x^{o} - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

where

a = an estimate of the intercept

b = an estimate of the slope

n = number of observations

$t_{\alpha/2;n-2}$ = 100 $\alpha/2$ percentage point of "Student's t" distribution with (n-2) degrees of freedom; values to be obtained from standard statistical tables

```
7           1 717   0 450.       10.      3500.        1.645
7 83   0    1   0 11.00153417E+02 31.47821875E+01                MEAN AND STD ERR
7 83   0    2   0 29.51631799E+02 77.05992288E+01                MEAN AND STD ERR
7 84   0    1   2      .83450                                    CORRELATION CARD
7 85   1    1   1 20.42893417E-01 50.44534364E-03    40.497      .6963 REG COEFF
7 86   1    1   1 29.60906807E+07 29.60906807E+07  1640.020  AN. OF VAR (REG)
7 86   1    2 715 12.90867099E+07 18.05408531E+04            AN. OF VAR (ERR)
7 86   1    3 716 42.51773907E+07                            AN. OF VAR (TOT)
7 87   1    0   0 70.41356256E+01 42.49009921E+01     0.000      .6963   MISC.
```

FIGURE 2-4 COMPUTER PRINTOUT FOR THE REGRESSION ANALYSIS

XO	LOWER LIMIT	UPPER LIMIT	Yo
450.00	921.9107	2324.9645	1623.4376
460.00	942.4029	2345.3301	1643.8665
470.00	962.8943	2365.6967	1664.2955
480.00	983.3845	2386.0643	1684.7244
490.00	1003.8738	2406.4328	1705.1533
500.00	1024.3622	2426.8024	1725.5823
510.00	1044.8496	2447.1728	1746.0112
520.00	1065.3360	2467.5442	1766.4401
530.00	1085.8215	2487.9167	1786.8691
540.00	1106.3059	2508.2901	1807.2980

FIGURE 2-5 COMPUTER PRINTOUT FOR THE 90 PERCENT PREDICTION LIMITS

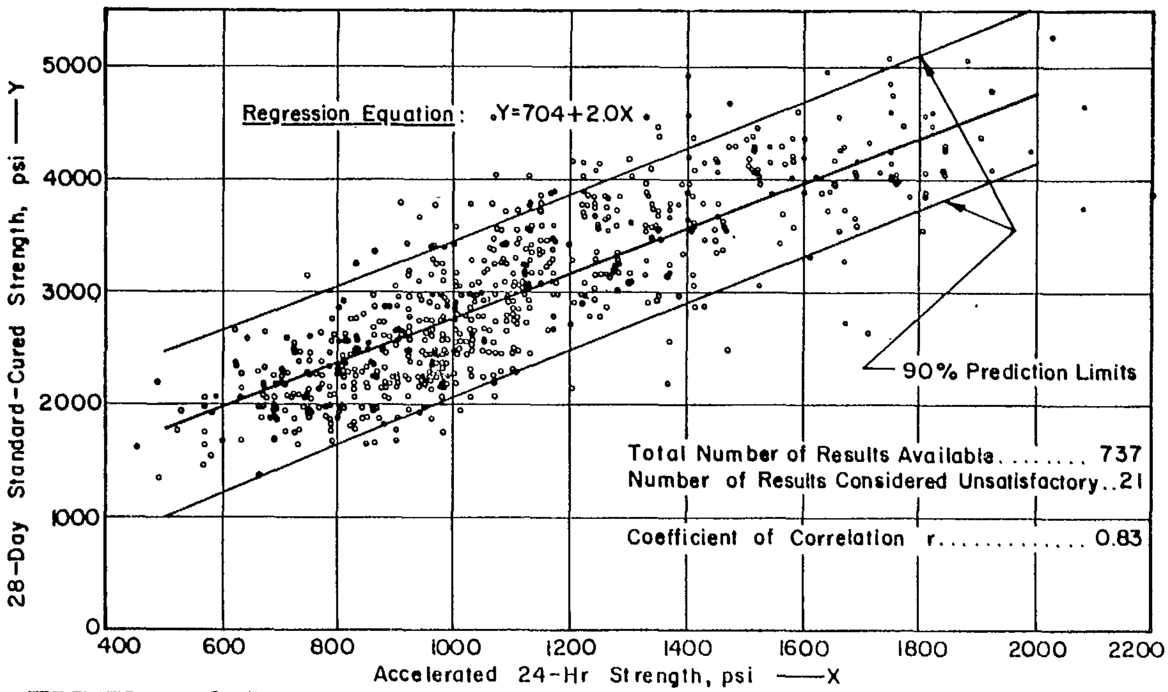

FIGURE 2-6 RELATIONSHIP OF 24-HR ACCELERATED TO 28-DAY STRENGTH; COMBINED DATA FOR ALL TYPES OF CONCRETE

x_i = accelerated 24-hr strength

$\overline{x}$ = mean of accelerated 24-hr strength

x^o = some future observation of accelerated 24-hr strength

$S_{y/x}$ = an estimate of the variability about the line; to be calculated using standard statistical methods

A sample of computer printout in its original form is shown in Figures 2-4 and 2-5. A plot of the test results is shown in Figure 2-6, together with "the line of best fit" and the regression equation. Also shown in this figure are the 90 percent prediction limits. Figure 2-7 compares the above relationship for the same type of concrete but having different water-cement ratios (by weight). The general conclusions drawn from these analyses are:

> The 28-day standard-cured strength can be predicted from the accelerated 24-hr strength with a reasonable degree of accuracy.
>
> At low strengths the relationship between the accelerated 24-hr strength and the 28-day standard-cured strength is dependent upon the water-cement ratio of the concrete mix. This conclusion is rather significant since it is not generally practical to carry out this large number of computations manually.

Case History 2 -- Multiple Correlation Analysis

In concrete quality control it is very desirable to predict the strength development trends of concrete on the basis of the available field and laboratory test data. This invariably involves the selection of a minimum number of test results, which at the same time is large enough to encompass all the possible variables so that the compressive strength at the selected age will be predicted with reasonable accuracy. This case history is one in which statistical studies using a digital computer were performed to correlate the different variables and the 28-day compressive strength, and to select the minimum number of test results required for the analysis of the entire test data.

A total of 1176 compressive strength test results on 6x12-in. concrete cylinders was available from the construction of a concrete-arch dam.[5] Since the compressive strength of concrete at any age is known to be dependent not only upon the water-cement ratio but also upon such factors as percentage of entrained air and aggregate-cement ratio, it was important to establish the relationship between the 28-day compressive strength and the following variables:

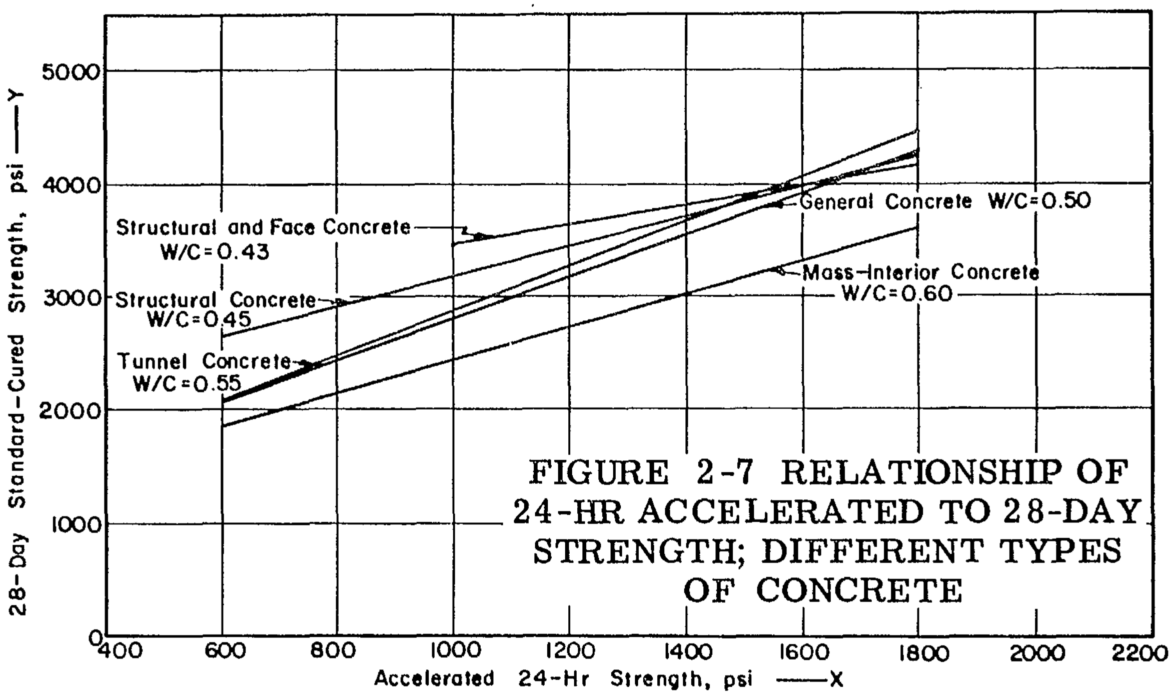

FIGURE 2-7 RELATIONSHIP OF 24-HR ACCELERATED TO 28-DAY STRENGTH; DIFFERENT TYPES OF CONCRETE

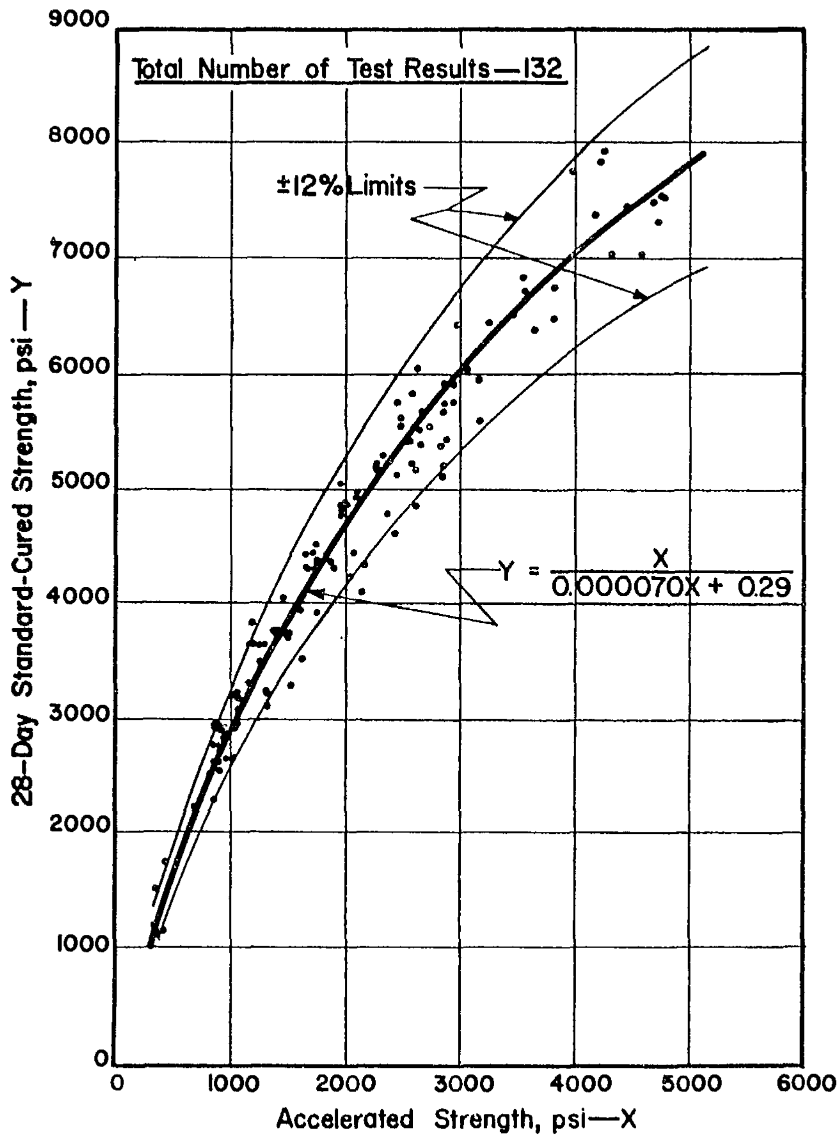

FIGURE 2-8 28-1/2-HR ACCELERATED VERSUS 28-DAY STRENGTH

1. Log water-cement ratio
2. Air content, percent
3. Density of 6x12-in. concrete cylinders 24-hr after casting, lb per cu ft
4. Slump, in.

It should be emphasized that the effects of variables other than the water-cement ratio on the compressive strength of concrete are relatively small, but to be able to determine their effect quantitatively a large quantity of test data is needed. An electronic computer to process these data is necessary to avoid the tedious and time-consuming manual computation.

Correlation coefficient for the entire test data and for 50 and 200 test results selected at random from the test data were computed. This was followed by the formation of the three multiple linear regression equations, one for each of the three sets under analysis.[6] The regression equations so formed are given in Table 2-1.

TABLE 2-1 SUMMARY OF MULTIPLE LINEAR REGRESSION EQUATIONS

Number	Number of Test Results Under Analysis	Equations*
1	50	$Y = (10,805)X_1 - (111.8)X_2 + (13.9)X_3 - (179.1)X_4 - 1,175$
2	200	$Y = (11,636)X_1 - (38.1)X_2 + (79.7)X_3 - (37.4)X_4 - 12,026$
3	1176	$Y = (11,584)X_1 - (128.3)X_2 + (37.3)X_3 - (71.4)X_4 - 5,122$

*Where: Y = compressive strength at 28 days, psi; X_1 = log (to the base 10) of water-cement ratio, by weight; X_2 = air content, percent; X_3 = density of 6x12-in. concrete cylinders at 24 hr after casting, lb per cu ft; and X_4 = slump, in.

The multiple correlation analysis indicated that the independent variables selected predict the 28-day compressive strength within acceptable limits of accuracy, with the added advantage that these predicted values are available within 24 hr of placing the concrete. Furthermore, the analysis showed that the degree of accuracy in forecasting 28-day compressive strengths based upon 50 test results selected at random from the data is of the same order as that based upon 200 results (selected at random) or 1176 test results.

In order to be able to appreciate the complex nature of the analyses of the above type a similar problem has been worked out manually and is shown in Tables 2-2 and 2-3. In this problem a set of 31 compressive strength test results at 7, 28, 90, and 365 days was available. The data were analyzed so that, in

TABLE 2-2 COMPUTATION OF SUMS OF SQUARES AND PRODUCTS

No.	Compressive Strength at Various Ages, psi 7-Day X_1	28-Day X_2	90-Day X_3	365-Day Y	$\Sigma X_i + Y$			X_1	X_2	X_3	Y
1	450	960	1,830	2,220	5,460	$SP_{x_1x_i}$			79,302,200	123,854,700	144,771,500
2	630	1,420	2,280	3,250	7,580	$S_{x_1} \cdot S_{x_i}/n$			67,159,890	107,750,367	128,439,003
3	570	1,260	2,380	3,120	7,330	$SPD_{x_1x_i}$			12,142,310	16,104,333	16,332,497
4	1,480	3,580	5,180	5,680	15,920						
5	840	2,000	3,560	4,080	10,480						
6	1,790	3,910	5,870	6,300	17,870						
7	1,530	3,490	5,650	5,740	16,410	$SP_{x_2x_i}$				264,443,800	308,435,400
8	1,060	2,390	4,050	5,270	12,770	$S_{x_2} \cdot S_{x_i}/n$				229,421,793	273,471,980
9	1,260	2,880	4,560	5,080	13,780	$SPD_{x_2x_i}$				35,022,007	34,963,420
10	2,270	4,650	6,940	7,560	21,420						
11	1,560	2,670	4,800	5,500	14,530						
12	660	1,780	3,700	4,000	10,140						
13	950	2,230	3,750	4,800	11,730	$SP_{x_3x_i}$					488,332,800
14	960	2,240	3,650	4,300	11,150	$S_{x_3} \cdot S_{x_i}/n$					438,754,535
15	830	1,930	3,270	3,880	9,910	$SPD_{x_3x_i}$					49,578,265
16	710	1,690	2,870	3,820	9,090						
17	1,040	1,900	3,190	3,850	9,980						
18	1,760	3,360	4,580	5,880	15,580						
19	500	1,040	1,860	2,680	6,080						
20	820	1,450	2,570	3,220	8,060						
21	930	1,660	2,630	3,400	8,620					CHECK	
22	730	1,390	2,630	3,060	7,810					SSD_{x_i+y}	133,472,040
23	840	1,610	1,910	2,600	6,960					$SPD_{x_1x_i} \times 2$	89,158,280
24	1,510	3,000	4,140	4,980	13,630					$SPD_{x_2x_i} \times 2$	139,970,854
25	1,260	2,820	3,590	4,360	12,030					$SPD_{x_3x_i} \times 2$	99,156,530
26	1,030	2,320	3,230	3,900	10,480						461,757,704
27	800	1,520	2,460	3,160	7,940						
28	550	1,130	1,820	2,630	6,130						
29	400	950	1,770	2,180	5,300						
30	720	1,530	3,080	3,580	8,910						
31	830	1,820	3,020	3,250	8,920						
S	31,270	66,580	106,820	127,330	332,000						
SS	37,559,100	169,149,400	418,764,800	573,616,500	4,017,370,600						
S^2/n	31,542,351	142,996,658	368,081,045	522,997,706	3,555,612,903						
SSD	6,016,749	26,152,742	50,683,755	50,618,794	461,757,697						
S/n	1008.7	2147.7	3445.8	4107.4							

TABLE 2-3 LINEAR REGRESSION ANALYSIS WITH THREE INDEPENDENT VARIABLES : SOLUTION OF NORMAL EQUATIONS

Explanation of Computations	b_1	b_2	b_3	$SPD_{x.y}$	
(1)	$a_{11} = 6,016,749$	$a_{12} = 12,142,310$	$a_{13} = 16,104,333$	$a_{10} = 16,332,497$	
(2)		$a_{22} = 26,152,742$	$a_{23} = 35,022,007$	$a_{20} = 34,963,420$	
(3)			$a_{33} = 50,683,755$	$a_{30} = 49,578,265$	
(4) = (1) x $\left(-\frac{a_{12}}{a_{11}}\right)$		$-\frac{(a_{12})^2}{a_{11}} = -24,504,152$	$\frac{-a_{12} \times a_{13}}{a_{11}} = -32,499,832$	$\frac{-a_{12} \times a_{10}}{a_{11}} = -32,960,285$	
(5) = (2) + (4)		$a_{22.1} = +1,648,590$	$a_{23.1} = +2,522,175$	$a_{20.1} = +2,003,135$	
(6) = (1) x $\left(-\frac{a_{13}}{a_{11}}\right)$			$\frac{(-a_{13})^2}{a_{11}} = -43,104,535$	$\frac{-a_{13} \times a_{10}}{a_{11}} = -43,715,235$	
(7) = (5) x $\left(-\frac{a_{23.1}}{a_{22.1}}\right)$			$\frac{(a_{23.1})^2}{a_{22.1}} = -3,858,675$	$\frac{-a_{23.1} \times a_{20.1}}{a_{22.1}} = -3,064,596$	
(8) = (3) + (6) + (7)			$a_{33.12} = +3,720,545$	$a_{30.12} = +2,798,434$	
			$b_3 = 0.75216$		$b_3 = \frac{a_{30.12}}{a_{33.12}}$
		$b_2 = 0.06433$			$b_2 = \frac{\left[(a_{20.1}) - (a_{23.1} \times b_3)\right]}{a_{22.1}}$
	$b_1 = 0.57146$				$b_1 = \frac{\left[a_{10} - (a_{13} \times b_3 + a_{12} \times b_2)\right]}{a_{11}}$

Regression Equation:

$$Y = \bar{Y} + b_1 (X_1 - \bar{X}_1) + b_2 (X_2 - \bar{X}_2) + b_3 (X_3 - \bar{X}_3)$$

CHECK = $a_{13} \times b_1 + a_{23} \times b_2 + a_{33} \times b_3 = a_{30}$

or $$Y_{365\text{-day}} = 0.60\, X_{7\text{-day}} + 0.06\, X_{28\text{-day}} + 0.75\, X_{90\text{-day}} + 797$$

future, the compressive strengths at one year could be predicted from those at 7, 28, and 90 days. A quick glance at Tables 2-2 and 2-3 should convince even the most skeptical engineer that in these types of analyses the use of a computer becomes mandatory when the number of test results exceed 20. It should be emphasized that apart from the speed at which the calculations are carried out by a computer, the probability of making an error is minimal whereas in manual computations the chances of making errors and repeating them are always present.

Case History 3 -- Fitting of a Hyperbolic Curve

This analysis problem originated during the research work being carried out by the Construction Materials Section on the development of a satisfactory accelerated test for determining the 28-day compressive strength of concrete.[7] This test procedure is different from the one discussed earlier; in this method 6x12-in. concrete cylinders are moist-cured 24 hr in a standard manner and are then boiled for 3-1/2 hr. The cylinders are then removed from the boiling water tank and are tested 1 hr later.[8]

A total of 792 6x12-in. cylinders was tested in compression at 7 and 28 days, an average of three test cylinders at each age comprising one test result. The nature of the data necessitated the fitting of a curvilinear function to the plot of the accelerated versus 28-day standard-cured compressive strengths. The data were fed into an IBM 1620 computer, and various types of curves were tried using a trial and error method. The best-fitting curve was of the type:

$$Y = \frac{X}{AX + B}$$

where X = accelerated-cured strength, psi; Y = 28-day standard-cured strength, psi; and A and B are constants.

A plot of the test results is shown in Figure 2-8, together with the hyperbolic curve of the above form.

It was concluded from this investigation that the 28-day standard-cured strengths could be predicted from the accelerated strength with an accuracy of about ± 12 percent (Figure 2-8).

CONCLUSIONS

Digital computers can be used with advantage in the quality control of concrete. Furthermore, the case histories discussed effectively demonstrate that when a large number of test results are available for interpretation and analysis it becomes very necessary to make use of the electronic computing facilities.

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