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RESEARCH IN IMPROVED METHODS OF ROCK BREAKAGE

L.B. GELLER

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Research in improved methods of rock breakage

L. B. Geller Dipl. Ing., B.Sc.(Eng.), M.A.Sc.
Department of Energy, Mines and Resources, Mines Branch, Ottawa, Canada

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Synopsis

In planning research to improve methods of rock breakage one of the most difficult problems is that of deciding which of the almost endless aspects one should tackle first. In an effort to assist such a decision a critical review is offered of past results in both fundamental research and practical applications. Excavation, as well as crushing and grinding, is covered. Certain areas which seem to represent appropriate points of departure in a continuing search for improved methods of rock breakage are suggested as being worthy of investigation both at the fundamental and applied levels.

The costs to the mining industry of breaking rock, from mining the ore through all stages of processing, are enormous. Potential savings, even by but relatively modest improvements to standard methods, are very substantial. An investment in research is therefore justifiable even in an area where a failure of conventional procedures is not in question, provided that there seems to be a reasonable chance of developing more economical methods. The problem is to determine where to start with work aimed at finding improved methods of rock breakage.

A study of rock breakage and comminution involves a very large variety of dissimilar disciplines. Very many studies have already been published on relevant research and developmental efforts; for example, both the practical and the theoretical aspects of crushing and grinding were covered in detail by Mular in Canada,[1] Harris in the U.S.A.,[2] Beke in Hungary,[3] by scientists working for what was formerly the Department of Scientific and Industrial Research in the United Kingdom[4] and by others at the First European Symposium on Size Reduction.[5] Furthermore, drilling and excavation methods were reviewed at length by Antonides,[6] Ledgerwood,[7] Farmer,[8] Epshteyn and colleagues[9] and scientists of the Hughes Tool Company.[10] The very profusion of available articles tends, however, to cloud the issue: this is so because, in the first place, even review articles are confined to such topics as the mechanics of excavation alone, or to those of crushing and grinding, or to some of the fundamental aspects of one or the other basic rock breakage processes. Secondly, there appears to be a serious lack of correlation between the hundreds of papers discussing the vast number of practical and theoretical aspects involved.

The object of the present paper is to survey the entire spectrum of hard rock breakage—from the *in situ* mass to the commercially desired end product. Theoretical as well as practical aspects are included. These are critically analysed in an effort to determine where to commence research aimed at improving rock breakage. These improvements should, if feasible, result in some sort of a continuous mining process. The theory and practice of blasting are therefore not reviewed. No claim is made of even approaching a complete coverage of any of the fields discussed. These have, however, been outlined in their essentials. Also, sufficient publications have been cited in every case to provide a good point of departure, especially in view of the extensive bibliographies contained in many of the references listed.

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Fundamentals of rock breakage

General remarks

In order to improve on existing methods of rock breakage it is of paramount importance to obtain a proper understanding of the physical laws governing (1) the fundamental modes of fracturing; (2) the stress fields produced by specific loading conditions; (3) the specific energy requirements of fragmentation; and (4) the sources of energy dissipation.

These laws are probably unaffected in substance by (*a*) the physical means used to induce the stresses which lead to failure and (*b*) by the size of the rock mass relative to the tool which is used to break it; but since, by using the latter two groupings, it is possible to classify all practical rock breakage methods, they are also used in discussing the fundamentals of rock breakage. These fundamentals have been extensively studied by means of both man-made and natural rock breakage events. The present review covers results achieved by man-made means only.

Fundamentals of rock mass breakage by mechanically induced stresses

Results achieved with uniformly distributed loading

Uniformly distributed uniaxial and triaxial tests have been performed on rock specimens since the early 1900s by a number of scientists, including Kármán, Adams, Böker and others. A concerted effort has recently been made to study the deformation of rock specimens under various combinations of uniform triaxial compressive and torsional loadings, at various temperatures, confining pressures, strain rates and pore pressures.[11-14] These tests confirmed in greater detail the limited results obtained by the earlier experimenters. In particular, it was shown that nearly all rocks acquire ductility at certain confining pressures. The brittle to ductile transition was demonstrated to be a function of temperature, confining pressure and interstitial fluid pressure; it may occur at relatively low confining pressures, in the order of 1000 atm[11] or less,[15] even with dry rocks at room temperature. It has been postulated that the transition between the two failure types occurs when the compressive to tensile rock strengths are in the ratio of about 4 : 1.[16]

Stress–strain curves give a clear indication of whether a rock is in a brittle or in a plastic state. Unfortunately, they give no clue whatever regarding the flow mechanism involved at failure. This has, for example, been found to consist mostly of mechanical twinning and translation gliding in case of Yule marble twisted under confining pressure.[11]

Tests with uniformly distributed triaxial loading also showed that Mohr envelope curves can be fitted to the extreme principal stress circles obtained. Even though these indicate that, for example, most sedimentary rocks will fracture when the ratio of the extreme principal stresses reaches about 3 or 4 (figures modified by temperature, strain rate and pore pressure), no known strength theory could yet be shown to apply to rocks in general. Existing hypotheses, e.g. the Coulomb–Mohr theory, fail even to correlate results for one material subject to different states of stress. Nevertheless, the stress level at failure can be estimated by existing theories. One theory, which has stood the test of time particularly well, was originally enunciated by Griffith[17] and was later augmented by Irwin, Orowan, Murrell, Brace, McClintock and Walsh and others. It depends on considerations of a balance between the loss in stored elastic energy and the gain in surface energy as the fracture proceeds. It takes into account the multitude of microscopic cracks which abound in all natural rocks. This theory has been successfully applied to various types of crack-initiated failures.[18–21]

As outlined, experiments with uniformly loaded cylindrical test specimens have yielded a great deal of valuable information regarding the modes and energy requirements of rock failure. In particular, they led to the conclusion that, provided Griffith's criterion is applicable, only a small fraction of the specific energy which is necessary to bring about rock breakage represents work done in forming new surface energy. For example, in the case of uniaxial tension the ratio of elastic energy (required to produce failure) to new surface energy is $2\pi S/H$, where S is one-half the length of the longest internal crack and H is the height of cylindrical specimen.[20] The inference drawn from this fact is that rock fragmentation is an extremely inefficient operation. This conclusion would, however, only be true if all energy expended could be used for creating new free rock surfaces alone. Manifestly, this is an impossible proposition, since other energy requirements must also be met. These include elastic strain energy in a much larger volume of rock than is represented by the broken-out chips alone. This energy is largely wasted after failure, e.g. by inducing stress waves which arise as a consequence of very rapid crack extension.[5]

Results achieved with concentrated loading

The basic modes of failure, as well as the various energy requirements involved therein, have previously been described on the basis of results achieved with carefully machined test pieces under strictly controlled and uniformly distributed loading conditions. Moreover, the rock mass was relatively small compared with the loading tool. In this section test procedures approach *in situ* rock breakage conditions more closely, insofar as concentrated loads act on a theoretically semi-infinite mass of rock.

In the case of concentrated loadings, laboratory studies most commonly involved wedge-shaped chisels with various included angles and of varying dullness. These were loaded either statically or dynamically. Both mathematical and experimental procedures were used. The rock mass was considered to be either predominantly brittle or ductile. Major objects of these studies were to establish: (1) the modes of rock failure; (2) the stress fields induced at the point of breakage; and (3) force–tool penetration relationships.

The modes of failure, in the case of brittle behaviour during tool penetration, were found to involve four major phases, of which the following are among the salient features.

(*a*) The material in the immediate vicinity of the tool–medium interface is crushed and compacted, while subjected to a state of high triaxial compression: the force between the tool and rock builds up relatively slowly.

(*b*) The forces on the tool rise rapidly until initial failure occurs in the rock: usually, this consists of a large crack directly beneath the tool tip and directed along the tool's axis of symmetry.

(*c*) The tool forces the two quarter spaces apart so that these now act virtually independently of each other. The forces on the tool rise less rapidly than in phase (*b*). Secondary failures occur in the quarter spaces, including sideways fracturing and conchoidal flaking; this results in more rapid tool penetration and a drop of tool forces.

(*d*) If additional energy remains available, the forces on the tool may again rise to a second peak, while more rock is broken away.

The spatial location of the modes of failure described above, as well as the induced stress fields responsible for their development, have been carefully analysed in terms of mathematical equations.[22,23] They describe stresses and strains in satisfactory agreement with experimental results.[20,22] Recently, a theoretical analysis utilizing recurrence relations established a numerical method for calculating not only the force and energy requirements but also the crater geometries which obtain during the successive stages as the tool advances into a brittle material, either at a constant rate or under a constant load.[24] Numerical methods too have been successfully employed to determine the stress distribution.[25] The latter methods can be used to allow for additional forces besides the tool load, e.g. for overburden and drilling fluid pressures.

Mathematical analyses of stress distributions have been augmented by the use of photoelastic methods.[22,23,26,27] These were further refined by using high-speed photography.[28] The most often used experimental methods, however, involved instrumented drop testers. The principal parameters measured with these were applied bit force versus penetration and energy of loading versus crater volume. In essence, the results showed that mechanical power input into the rock is the primary parameter determining rate of penetration.[29–32] The expressions establishing relationships between penetration characteristics and input energy have been summarized by, among others, Maurer.[33]

While most of the experimental studies were conducted on natural rocks, exhaustive tests have also been undertaken to determine the penetration of coal by wedge-shaped tools, both by indentation and by ploughing.[34–39]

The studies summarized above were primarily concerned with brittle failures occurring at atmospheric pressure. Similar work has also been performed at a range of elevated confining pressures.[40] Experimental studies yielded linear and continuous force–displacement curves indicative of the absence of distinct chip generation (at any rate during bit penetration). They also demonstrated a drop in crater volume, per unit energy input, with increasing confining pressures. Analytical studies of plastic rock deformation were based on two-dimensional plane strain conditions, with the rock failing in accordance with the linear Coulomb–Mohr yield criterion, assuming either a perfectly smooth or a perfectly rough tooth–rock interface.[15,41,42] Calculated and experimental results were in satisfactory quantitative agreement. Both the theoretical and experimental work has lately been extended to cover ductile yielding of rocks in case of parabolic (rather than straight-line) yield envelopes,[43] and in the case of indexed (rather than single) indentations.[44,45]

Experimental results were obtained under both static and dynamic loading conditions. These results may be correlated since it is known that at loading velocities achieved with present-day mining equipment static analyses can replace dynamic ones. The situation is different, however, at loading rates achieved with explosive shock waves. At rates of this order dynamic rock strengths are many times greater than static ones.[46]

One important object of these studies was to obtain a better understanding of tool force and tool geometry versus rock penetration relationships, and of the basic principles involved in the interaction of the fundamental variables during rock breakage processes. Another was to establish some universal measure of drillability, such as the specific energy concept for example.[32,47] Unfortunately, no such single parameter could be established. Instead, a measure of 'drillability' (like that of 'grindability' and 'spallability') has to be based on tests on natural rocks, performed with well defined loading tools under specified methods of energy application and strictly controlled test conditions. Several types of these tools have been used, e.g. microbit roller cutters,[48,49] diamond drills,[32] simple rotary drag bits,[50] and others.[51]

Results achieved by studying 'extraneous' conditions

The fundamentals of rock breakage can perhaps best be investigated by a minute examination of conditions arising within the rock mass proper during, and immediately preceding, breakage. Much can be learned, however, by studying 'extraneous' conditions as well. These are inherent in the mechanics of currently used methods, evolved as a result of long years of practical experience. These 'extraneous' parameters (e.g. rate, level and characteristics of loading, indexing and debris clearance) are intimately related with specific tool designs. They are therefore referred to in the sections that deal with practical methods, except for debris clearance and flushing agents which are briefly described now.

The effect of debris clearance has been studied, both experimentally and theoretically, by several investigators.[33,52,53] In practice, this has led to 'reverse flushing' methods, to tests with various nozzle geometries and cleaning agents, and to the use of exceptionally large amounts of flushing air.

With regard to flushing agents, it was found that their consistency (such as viscosity, density, filtration rate and gel strength) is an important parameter, especially in the case of deep-hole drilling. As an example, their kinematic viscosity must be kept as low as possible to ensure favourable drilling rates. Furthermore, penetration rates can be improved by using certain electrolytes in them.[53] These are commonly referred to as 'hardness reducers'.

Fundamentals of rock particle breakage by mechanically induced stresses

Mechanism of size reduction

Much work has been done in an effort to clarify the basic physical events occurring during crushing and grinding processes. In particular, tests have been devised to study events which occur in single particles by, among others, Schönert and Rumpf in Germany[3] and by Bergstrom and his colleagues in the U.S.A.[2,54–57] They found that substantial differences exist in the way final failure occurs in different types of rock particles, and also that the specific energy required to fracture spherical specimens of several natural rock types is inversely proportional to the Schuhmann size modulus of the fragments thus created. Moreover, the kinetic energy (amounting to about 45 per cent of the strain energy required to initiate fracturing) causes considerable secondary fragmentation.

Where very many particles are involved, three basic processes are said to be operative during comminution: these are referred to as impact, chipping and abrasion events. The characteristics of the feed and of the particle sizes therein are mainly responsible for determining which of these three events is dominant.[1,58–62]

The actual mechanism of fracturing is believed to be based upon the formation of microscopic cracks. One school of thought, represented for example by Poncelet and Bond, stipulates that these cracks do not necessarily preexist before the material is loaded.[1,3] Instead, they are created by the strain energy impressed upon the material, at a point well below the theoretical breaking stress. The other school assumes that, in practice, all solids contain a mass of randomly orientated microscopic cracks or flaws and that fracturing originates from these preexistent points of weakness. This theory, originally established by Griffith,[17] has been amply substantiated and extended.[63,64] In either case all the problems of fracture formation and propagation, as well as of their mathematical development, are intimately involved. These have been examined on several occasions.[65–67] Moreover, numerous papers have been published about them, for example, by Scheidegger,[68] Barenblatt,[69] Holland[70] and many others. One handicap in developing mathematical theories proved to be a lack of physical test data—a shortcoming painfully evident in many other related fields as well.[4] No less serious is the lack of precise terminology, even in such relatively basic cases as surface stress, surface energy[1,4] and shear strength.[71]

The mechanism involved in size reduction has been approached from other sides as well—for example, by rate theories and by the matrix method. The former include kinetic theories and such concepts as nucleation and complex radioactive decay processes.[1,4] The latter is based upon an original hypothesis of Epstein, who postulated that comminution depends upon two basic functions. These are referred to as 'selection' and 'breakage' functions. His concepts have been further developed by, among others, Broadbent and Callcott, Brown, Berenbaum and Meloy.[1,3]

Particle size analysis

Particle size analysis *per se* has little direct connexion with the fundamental problems of rock breakage. But particle size distribution has; and before the latter can be studied a meaningful method for determining relative proportions of comminuted material within certain ranges must be available—either on a number or on a weight basis.

Practical methods of particle size analysis comprise, first and foremost, sieving. It provides a means for determining size distribution by weight (down to a lower limit of about 76 μm) by using standardized sets of sieves.

Smaller particles, down to sizes of 1μm (with visible white light) or even 0·1 μm (with ultraviolet light in air) can be classified by the use of optical microscopes. For even smaller fractions, down to about 0·01 μm, electron microscopy can be used. In either case distribution can only be determined by numbers.[3,4]

Another procedure for analysing dimensions below sieve size involves elutriation and sedimentation. Particle sizes down to about 2 μm can be determined by this means.

Finally, centrifugal sedimentation has been developed to settle out particles below about 5 μm within a reasonable length of time, despite the delaying effect of the Brownian movement. It can be used on particles down to about 0·1 μm, suspended either in air or in water.

One of the above mentioned methods must be used if comminution is to produce commercial end products within a specified size range. If, however, it is only necessary to produce an end product with a high specific surface, it is possible to side-step size analysis altogether. Instead, only a direct indication of the surface is needed, obtained by methods based on measurement of permeability, of molecular adsorption[4] or on sedimentation and on light absorption (or turbidimetric) tests.[3]

Particle size distribution

The particle size distribution, obtainable by crushing and grinding, has been carefully analysed through the years. Certain characteristic features and regularities of the distribution pattern have transpired. For example, it was noted that during the early stages of comminution the distribution curves may reflect characteristics of both feed and grinding machinery. As grinding proceeds, finer particle sizes abound and a steady state distribution is reached. Additional grinding further reduces the scatter of size distribution, but only at an inordinate cost—due to an increased tendency of agglomeration and to a lack of particles with preexistent flaws. Eventually, distribution curves reflect the characteristics of (*a*) the material ground; (*b*) the dominant size reduction mechanism involved; and (*c*) the properties of the suspending fluid used.

How to formulate mathematical expressions describing these distribution patterns has been of great interest to many eminent scientists and mathematicians. Various theories have been presented, resting on a set of assumptions regarding the liability of a particle to fracture and using probability theory to derive the distribution functions for a set of events.[72–74]

In the case of single-event impact fracture distribution functions two widely used ones have been derived by Gilvarry[73] and by Gaudin and Meloy.[2,74] Gilvarry's theory is based on the assumption that fracture is due to activation of the Griffith flaws distributed along an edge.[2,63] Work by Gilvarry and Bergstrom[75] confirmed the validity of the relevant function, provided that exoclastic particles are removed from the distribution spectrum.

Another widely accepted function has been derived by Schuhmann.[2–4] His expression states that

$$y = (x/k)^{\alpha}$$

where y = cumulative weight fraction finer than size x for a material of uniform density
k = size modulus (constant) for a given size distribution
α = distribution modulus

The shape of single-event chipping and abrasion distribution functions has also been obtained, but only experimentally and not from theoretical models.

In the case of complex size reduction processes within practical grinding devices the distribution of fragments is determined by the relative frequency with which any of the three basic fracture mechanisms occurs. Which will be dominant depends to a large extent upon specific machine design features. Apart from these, size distribution is influenced mostly by characteristics of the feed (e.g. by its shape, cleavage, hardness and size) and by those of the suspending fluid.[1,58,59,76]

That complex comminution is an averaging process comprising the three basic single-event mechanisms has been shown, for example, by grinding quartz and limestone in ball-mills either separately or as parts of a mixture; the distribution modulus was the same in either case.[76–78] Therefore the Gaudin–Schuhmann and Gaudin–Meloy distribution functions can be successfully applied in this case too, even if certain limitations do apply. As an example, the Gaudin–Meloy equation is only valid for feed sizes smaller than a critical size or for grinding times longer than a critical period.[60] The critical particle size is believed to establish the point of transition between predominantly impact and predominantly attrition type of grinding.

In European practice and research the most commonly used distribution function is the empirical formula established by Rosin and Rammler. They stipulated that size and surface distribution within the range produced by grinding follows the exponential law[1–3]

$$y = 1 - e^{-(bx^m)} \text{ or } R = 100\, e^{-(bx^m)}\ \%$$

where y = cumulative weight fraction finer than size x for a material of uniform density

b and m = constants, depending on the type of material ground and on the type of grinding process used

R = percentage of material retained on a sieve of mesh size x μm

This expression has been slightly modified by Bennett to obtain the probability curve for residue R on an x mesh sieve.[3] Particle size distribution functions are obtained from these 'fractional residue greater than' formulae by differentiation. Tables have been published, for example by Meloy, to assist in the computation of the values of the relevant functions.[79]

Another function, describing particle size distribution obtained by complex comminution processes, is known as the log-normal one. Here a function is sought which fits a straight line in a system wherein the abscissa is usually the particle size on a logarithmic scale and the ordinate an error integral scale of cumulative weight fractions.[1] Unfortunately, when plotting distribution by weight on this basis, the resultant graph usually exhibits a slight upward bend at the coarse end of the particle size fractions. Additional mathematical procedures, resulting in a 'renormalized log-normal distribution' have been developed to rectify this error.[3]

Energy–size reduction relationships

Comminution is a physical process involving the rupture of adhesive bonds and the creation of fresh surface areas. Among its most basic problems are the fracture mechanisms involved, the obtainable size distributions and the maximum obtainable efficiencies. The two former have been discussed; a brief review of the latter follows.

One of the earliest attempts to relate expended energy with size reduction is Rittinger's theory, expounded in 1867. It is based upon the concept of cleavage and postulates that the expended energy is proportional to the new surface area created, i.e. to the reduction ratio.[1,3] The energy of the freshly cleaved surface is therefore a fundamental parameter. Recently, it has been measured directly, e.g. for quartz,[4] orthoclase[80] and glossy polymers.[81]

On this basis efficiencies in the order of 1·7–26·5 per cent were obtained by crushing single quartz crystals, and up to 60 per cent by impact loading them. In the case of multiple particles, however, these values did not exceed 1·5 and 3 per cent respectively. While Rittinger's law has been upheld by many, it has also been severely criticized on the basis that (*a*) only small reduction ratios were used in evaluating tests; (*b*) the precrushing history of the materials was usually ignored; and (*c*) the methods used to obtain surface areas were of doubtful validity.

The second well known energy–size reduction theory is usually referred to as Kick's volume theory, proposed in 1885. According to his hypothesis, input energy is expended in straining the volume beyond its elastic limit, i.e. input energy is proportional to volume of fractured material.[3] Kick's formula thus covers energy requirements, but neglects to specify the reduction ratio to be obtained by expending them.

A newer, but equally well known, theory was proposed in 1952 by Bond.[1–4] His theory regards the energy requirements as being proportional to the square root of the reduction ratio. He postulated that the feed size represents a certain amount of energy content, which must be considered. Accordingly, the energy required to reduce feed size x_1 to product size x_2 is the difference between the two work inputs characterizing each of these sizes. Bond's concepts have had a profound influence on comminution calculations since their inception.

Other scientists have attempted to devise a formula based on real, rather than uniform, particle size distributions in the feed and product.[4] Such work has been done, for example, by Walker and co-workers, Charles, Holmes and Svenson and Murkes.[3,4,76] Schuhmann too derived an energy–size reduction relationship, basically similar to the type found by the above mentioned workers, by looking upon each fragment of every particle crushed during a complex comminution operation as if it had been crushed by a single impact event. Each comminution event is thus assumed to produce fragments characterized by Schuhmann's size distribution function, even when different materials are ground together.[3] The size reduction of mixtures has also been analysed by Fuerstenau and his colleagues[76–78,82] A recent, and somewhat different, physical theory was presented by Tartaron.[83]

As is clear from even this brief review of published energy–size reduction relationships, there is no shortage of contending theories. Some of these contain embarrassing contradictions. The disputes about their relevant merits, especially in the case of Rittinger versus Kick, have been both protracted and acrimonious. Further efforts to prove or disprove the absolute validity of any one theory would appear to hinder rather than promote comminution research, especially since different theories are probably equally valid during different stages of the comminution process and in different sections of the particle size range.[84] A sensible compromise seems to have been established by hypotheses which stipulate that input energy is used partly to create the new surface of fragmented products, partly for producing new inner weaknesses and mostly to elastically strain the material. It is also noteworthy that the comminution history of the feed has now been involved.

Another group of scientists chose a different way out of the above mentioned difficulties. They suggested the use of a thermodynamic method for determining how fracture initiation and propagation, input and surface energies, as well as heat, are interrelated. In this case the useful input energy is considered to be the difference between net energy input and sensible heat developed during the grinding operation. The net energy input is assumed to be totally absorbed in creating new surface energy and in producing heat.[1] This approach was adopted by Djingheuzian and others to analyse complex breakage processes.[85]

Fundamentals of rock mass breakage by thermally induced stresses

Rocks can be broken by two basic methods—by mechanically induced stresses and by thermally induced ones. The fundamentals of the former method have been studied far more extensively than those of the latter; but research work in the thermal field has also been performed. It covers analytical studies of temperature, stress and strain distributions,[86–93] as well as experimental test work.[94] As a result, it was shown that the stress distribution at any point in the rock mass, prior to spall formation, is radically different from the one obtained by mechanical means. It is such that one principal stress, normal to the free rock surface, is zero and that equal compressive stresses exist in any two directions at right angles to each other and parallel to the free surface. The stresses do not depend on the temperature gradient into the rock mass but only on the temperature attained. The heat flux into the solid determines the temperature gradient, and thus the rate at which a given temperature level will be attained. In consequence, the temperature distribution into the solid also represents the stress distribution. The actual temperature distribution depends on the conditions of heating; it is not critical so long as the heated layer is relatively thin.

While the temperature distribution, and thus the induced stress field, can be theoretically determined under specific conditions of thermal shock loading, the exact mechanism of spall formation has not yet been established. A tentative picture is that of shear failure causing a ring crack to form first in the surface, which delineates the spall. The moment this crack forms (at less than 45° to the surface), both the shear stress which caused it as well as the end confinements are relieved. A new stress pattern is now formed which enables the ring crack to propagate inward from its base, possibly due to shear or tension.

Even though the exact mechanism of spall formation is not known, it is recognized that spalling represents the most efficient method of purely thermal rock decrepitation. Moreover, it is known that some of the hardest and most abrasive rocks, which are the most difficult ones to break by mechanical means, are the best spalling ones.[86,95] Tests have further disclosed that when spalling does occur, characteristics of the heating medium influence the rate of spalling and the size of the spalls.

'Spallability' has been expressed by various formulae.[86] Soviet scientists, for example, have published the following parameter[96]

$$P = \beta E / \sigma_t C v$$

where β = coefficient of linear thermal expansion
E = Young's modulus
σ_t = tensile strength
C = volumetric heat capacity
v = coefficient of thermal plasticity and
P = a measure of the rock's capacity to spall during thermal shock loading

As with the much sought after 'drillability' and 'grindability' indices, it seems most unlikely that a single mathematical formula can be developed to conveniently and reliably express the spalling tendency of crystalline aggregates as complex as rocks. This would be so even if the known temperature dependence of relevant parameters (e.g. of thermal diffusivity and expansion) were to be disregarded. All that can be said with certainty is that the situation is so intricate in the case of thermal rock breakage methods that spalling can only be achieved so long as all conditions are exactly right; otherwise fissuring, melting and vaporization occur.

Fundamentals of rock particle breakage by thermally induced stresses

The breakage mechanism of commercial interest, when exposing a small area of a relatively semi-infinite rock mass to thermal shock loading, is spalling. It consists of a violent dislodgement of small, flaky particles from the rock's surface. Cracking, fissuring, melting and volatilizing is to be avoided because they hinder the economical breakage process.

The situation is different when relatively small, individual rock particles are to be loaded by thermally induced stresses. In this case cracking, fracturing or particle liberation by thermal means are of equal interest to spalling. In this field much of the past work has been done on ceramic and refractory materials; lists of relevant references may be found elsewhere.[86,97] The object of these analyses was to discover how to inhibit the very conditions sought when thermally breaking rocks; however, how to promote breakage has also been studied.[98] Thus it was shown that by heating rock prior to crushing its friability was improved, and that the size distribution of the crushed product was considerably changed. Both of these effects were of the kind to be expected when intergranular fractures occur due to heating.

Recently, thin circular rock discs were subjected to both heating and cooling shocks along their periphery.[97] A new criterion of failure was thus established, referred to as the 'average stress theory', to supplement theories which attribute failure (*a*) to some 'fracture stress' at a given point which exceeds the material's tensile strength (critical stress theory) or (*b*) to an excessively high general stress level (Weibull's statistical theory). The present theory considers both the volume within which elastic energy is stored at time of fracture, and also the corresponding stress distribution. It postulates that the time at which elastic energy attains a maximum can also be taken as the time when the product

$$\text{(volume under tension)} \times \text{(average stress over this volume)}$$

reaches its maximum. Should failure occur under these conditions then the corresponding maximum tensile stress in the body is taken to represent the thermal shock strength of the material. Experimentally established stress values are in reasonably good agreement with values calculated by using the average stress theory.

Practical methods of rock breakage

General remarks

The list of presently used equipment for breaking rock masses by mechanically induced stresses is very extensive. No attempt is made here to cover this list in its entirety, especially since several recent publications have done so.[6–10] Instead, these machines are briefly reviewed on the basis (*a*) of how they transmit input energy to the rock mass (tool bit design) and (*b*) of how they produce and transmit input energy to the tool bit. Tool bit design determines the amount of available energy which the rock can be made to accept, besides the shape of the induced stress and strain fields. Energy production and transmission methods determine loading times and levels. Other constructional details are not considered. Insofar as they might affect production, maintenance and operating costs, however, they must not be neglected in the final analysis.

The length of the section describing equipment used for breaking individual rock particles by mechanically induced stresses again bears no relation to the diversity of available crushing and grinding machinery. Excellent reviews of the subject have been published elsewhere[4,5] and therefore only an outline is offered of this extensive subject, classified according to (*a*) the medium in which comminution occurs (wet and dry grinding), (*b*) the medium used for effecting comminution (autogenous grinding or otherwise, fluid energy mills, etc.), and (*c*) according to the mechanical characteristics of specific designs (speed ranges, ball-, rod- or hammer-mills, etc.).

The practical methods of rock breakage by thermally induced stresses are relatively few and novel compared with their mechanical counterparts. Ancient methods of no practical importance (e.g. open fires and water dousing) are disregarded.

Practical methods of rock mass breakage by mechanically induced stresses

Standard methods

Standard methods are characterized by rigid mechanical connexions between solid tool bit and motive power. The rock is broken by indentation, i.e. first by crushing and then by chipping and fracturing. The three principal standard methods are known as (*a*) 'percussive', (*b*) 'rotary' and (*c*) 'rotary–percussive' ones.

Percussive methods are those in which input energy is transmitted to the rock as an impulse-type loading, usually normal to the free rock surface. Moreover, only a small amount of static loading is used (usually insufficient to seat the tool at the instant it is struck), no force is transmitted to the rock during tool rotation (which is for indexing purposes alone), and relatively low tool wear is achieved at economic penetration rates. Today, as 2000 years ago in China,[10] percussive drilling is performed almost exclusively with wedge-shaped chisels. Of course, they are now engineered better, mounted in groups (e.g. wing bits) as well as singly, and are of incomparably superior materials (e.g. with sintered tungsten carbide inserts). Nevertheless, exhaustive analyses, based on single as well as indexed blows,[45] merely confirmed the sound theoretical basis of their protracted and widespread use.[22,23,26,28,29,31,34,42,44,99,100] Only recently have other shapes also been marketed, e.g. hemispherical indenter equipped percussive tool bits.

While tool bits have hardly changed, methods of energy production and transmission have. As regards generation of input energy, mechanical means have been supplemented by pneumatic, hydraulic and electrical ones. Moreover, supply air pressures have been increased from 100 to 500 lb/in^2 gauge,[29,101,102] and blow rates from a few per minute (with simple churn drills) up to 3800/min (with modern air hammers) and even several thousand per minute (with vibration drills).

As for energy transmission, exhaustive studies have been made (both theoretically and experimentally) of the relevant fundamental problems.[103–112] Apart from consequent design improvements of drill-rods, connectors, etc., perhaps the most fundamental advance was made by moving the energy source from above ground (as with drifter-type hammer drills) to a position immediately on top of the tool bit ('down-the-hole' designs). Among the advantages of the latter design are high energy transfer efficiencies and relief from drill-rod failures. Disadvantages include severely limited motor dimensions. 'Down-the-hole' hammer drills are almost exclusively air-operated in the U.S.A.[7,10] while in the U.S.S.R. hydraulically powered designs (with some concomitant advantages) are said to also operate satisfactorily.[9]

Rotary methods are those in which all input energy is used to rotate the tool bit with no impact to augment tool loading. Moreover, a very high static loading constantly forces the tool bit into the rock. Tool bits are solid indenters, mostly in the shape of wedges or hemispheres. These are usually mounted on the lateral surface of conical bodies. Lately, cylindrical roller cutters too have been tried, either with continuous cutting ridges ('bobbin'-type cutters) or individual teeth.[113] Another experimental roller-type tool bit carries the teeth on its end face and is of stepped annular design. The teeth are either wedge-shaped or rotating bevelled discs.

In rotary drilling, as with percussive drilling, indenters are forced into the rock almost at right angles to its free surface, except in the case of drag bits, ploughs and diamond drills. Ploughing is of little interest in hard rocks; drag bits, however, have lately been improved to handle these.[114] Diamond drills, whether core drills, plug drills or diamond-faced cutter wheels,[7] are different insofar as they work mostly by attrition with a relatively low static loading. The fundamentals of rotary drilling have been studied at length.[35,36,115–117]

Rotary drilling machines are mostly powered from above ground. As with percussive methods, however, and for similar reasons, efforts have been made to perfect 'down-the-hole' designs. The latter are powered either by electric motors (electrodrills) or by air- or drilling fluid operated turbines (turbodrills) and motors.[7] In theory, these offer considerable advantages, owing to increased power and speed at the tool bit. In practice, technical difficulties, mostly with bearings and electrical connexions, have seriously curtailed their use.

Rotary–percussive methods are, as yet, more experimental than either the purely rotary or purely percussive ones: they combine certain advantages of the latter two—notably the high penetration rates of rotary methods and the low tool wear of percussive ones. Input energy is expended both in rotating and impact loading the tool bit. No basically new tool bit has yet been developed for rotary–percussive drilling. Instead, vibration has been superimposed on conventional ones, including wing and drag bits, cone-type rotary cutters and even ploughs.[118] While these orthodox tools wear less and drill faster when used in this way rather than as they were originally intended,[8,9,119] they can only be so used in rocks no harder than a given limiting value—otherwise excessive bearing wear and insert chipping result. Although no special rotary–percussive tool bits have yet been marketed, a few special machines, primarily pneumatic ones, have been designed for this purpose. They incorporate more flexibility, rather than basic new concepts, by utilizing such means as independent rotating and striking motors,[111,119,120] different working media for each of these motors[8] and rotary drill mounted air hammers.[121] A basically different machine, often referred to as a vibratory drill, was also built. It was powered by a magnetostrictive core material. The fundamental principles and practical aspects of rotary–percussive rock breakage were extensively analysed with this tool.[112,123] Unfortunately, technical difficulties eventually led to the abandonment of this study, despite successes achieved with the prototype machine.[7] Nonetheless, it can be said that high-frequency (about 100–1000 cycles/sec) rotary–percussive drilling methods possess a great potential in all types of rocks, despite present-day difficulties in other than soft formations.[8,9] This favourable forecast is strengthened by the successes achieved in fields other than rock breakage, e.g. in ultrasonic machining and forming of glass, ceramics and metals.[124]

Experimental methods

Experimental methods are understood to be those which mechanically load the rock surface, either with non-solid 'tool bits' or else with solid ones which are not, however, rigidly connected to the motive power. The principal methods are known as (*a*) erosion by water jets, (*b*) erosion by steel pellets, (*c*) implosion, and (*d*) shock wave produced breakage achieved by either explosive or (*e*) by electrohydraulic means. None of these is likely to attain commercial importance in the foreseeable future, except in the case of specialized applications.

Fracturing with water jets has so far been used satisfactorily only in soft rocks, sand, clay and coal. The method is advantageous insofar as its 'tool bit' (i.e. the ultra high pressure water jets) can enter minute cracks as these are created and because water can both break and transport the rock in a single, continuous cycle. Drawbacks include the excessive power requirements of hard rocks,[125] and the expense of water as a commodity, especially at high pressures and quantities. Its removal and, on occasions, its freeze-up, also involve a host of problems. The system is therefore likely to be limited to coal mining, where it is of practical interest.[10] Nonetheless, the method is being studied, e.g. with the aid of high-speed photography.[126] Specific technical problems, such as the development of intensifiers in the 1000–1700 atm range, are also being actively investigated.[8,9,127]

Fracturing with solid particles usually involves high-pressure fluid jets, loaded with solid particles. The latter are steel pellets in the case of impact drills. The object is to propel these solid spherical 'indenters' against the rock surface at great speed, the advantage being that worn indenters can be replaced without round tripping. Unfortunately, the operation is very inefficient in practice because, within the limited space available, the pellets cannot be accelerated to significant velocities and therefore high blow energy, the *sine qua non* of efficient rock breakage, is lacking. Moreover, rock chips have to be reduced to unnecessarily small sizes to enable their removal; this is another source of inefficiency. Studies of relevant fundamental conditions, namely of rock breakage by high-velocity solid particles, have been conducted with projectiles accelerated to 2000 ft/sec,[128] and even close to 24 000 ft/sec.[129] Another tool, operating on the same basic principle, is known as an abrasive jet drill.[7] Steel pellets are replaced by smaller solids (e.g. sand) which are not recirculated and must therefore be constantly replenished.

The Calyx drill also achieves rock breakage by means of small solid particles, e.g. with steel shot, but these are not propelled at high velocities. Instead, they are fed between the rock surface and the face of a blunt rotating cylindrical tool.[7]

Fracturing by implosion is based on producing a succession of compressive and tensile loads on the rock's surface, e.g. by propelling hollow glass spheres against it at high velocities. The method has been tried and dismissed as being impractical.

Fracturing by explosive charges on a large scale means blasting, first and foremost, either by chemical or nuclear explosives. The unique advantages of this method reside in its ability to affect an exceptionally large rock surface in tension, and in the very large amounts of specific energy involved. Blasting, however, also involves the tremendous expenses of blast-hole drilling and downtimes. Improved rock breakage methods are therefore sought which can eliminate blasting and still reduce overall costs.

'Explosion tools' break rock with explosions on a relatively small scale. In the U.S.A. early experiments in this category involved shaped charges.[7] The principal drawback is the time required after each shot for round tripping to reload, and for reaming out holes to the required gauge. A recent technique endeavours to overcome these problems by pumping 4–5 ft long plastic capsules down a pipe. Alternate capsules contain 'shaped' and 'gauging' charges.[130] In the U.S.S.R. a rapid series of small explosions is produced by liquid charges, mixed only on the rock face proper.[9] The force of the explosion removes most of the rock chips. The remainder is blown out by compressed air. The process can break both hard and soft rocks, but only down to relatively shallow depths of about 330 ft. With greater depths, down to about 5000 ft, spherical explosive charges are detonated on the rock face while chips are continuously flushed out by liquid.[9] Serious technical difficulties persist with explosion tools but, even so, their potential is highly rated since they utilize a very concentrated source of energy and 'tool bits' which do not wear.

Fundamentals involved in explosive rock fracturing were exhaustively studied both on models and in the field.[9,130,131]

Fracturing by electrohydraulic means is based on shock wave generated mechanical stresses. The waves are produced by underwater electrical discharges. To date, this method has had some success in experimental crushing of rock particles and deep-well drilling. The future of this type of rock breakage seems promising within limited areas of application. Much depends, however, on the successful solution of technical problems at commercially acceptable costs. A particular need exists for low inductance and resistance capacitors, capable of many millions of continual discharges at high repetition rates and voltages. The possibility of storing energy magnetically in air core superconducting inductor coils appears to offer some promise.[132]

Practical methods of rock particle breakage by mechanically induced stresses

Depending on the medium in which comminution occurs, two types of grinding are distinguished—wet and dry. Wet grinding mills are known as cascade, ball-, tube- and rod-mills. They are used at relatively low speeds—of the order of 26–32 rev/min. Advantages include low maintenance and capital costs, insensitivity to foreign matter, reliability over extended periods of continuous operation and quick response to load changes. Drawbacks include high power consumption per unit output, particularly at partial loads, and large floor space requirements.

All the above mentioned mill types can also be used for dry grinding at low speeds, provided that they are equipped with appropriate feeding, discharge or air classifying mechanisms. Modern cascade mills include the type known as 'aerofall mills'. Operation of autogenous grinding has lately been studied at supercritical speeds (in the order of 125 per cent of critical); a remarkable increase in output and a decrease in specific power requirements were noted at this speed.[5] Dry grinding at medium speeds is performed mainly in roller-type mills, such as the Raymond, Loesche or Lopulco, Berz and B. and W. E-type mills. Speeds are in the order of 70–220 rev/min. Advantages include low maintenance costs, high availability, economy in power consumption, relatively low floor space requirements and the ability to economically handle moist feed; in addition, the Berz and E. mills can operate under both pressure and suction. Disadvantages include the exclusion of certain types of small-size mills from the range of economically feasible ones. Dry grinding can also be performed in high-speed mills, equipped with hammers, pinned discs or other types of disintegrators. Modern equipment is available with single and double entries and also in single- and two-stage construction. Grinding is achieved by a combination of impact and attrition. Advantages include compact size, relatively low capital cost, simple design, reasonable efficiency at small mill sizes and high operating temperatures, whenever these are required for handling damp and sticky materials. Drawbacks include high power requirements when grinding to fine sizes, heavy maintenance costs and the difficulty of maintaining product fineness throughout the life of the wearing parts. Special high-speed designs are available for imparting single blows to single particles.[5] In particular, one design is notable in that it provides a measure of the particle's impact strength and also establishes its comminution history.[133]

Other types of grinding mills include fluid energy ones. In these comminution depends upon a kind of collision grinding, whether the particles are propelled against each other or against a fixed target. Ultrafine sizes can be achieved with very little or no contamination.[5] A comparatively recent process is based on the principle of forced vibrations in vibratory mills. The object is to propagate preexistent flaws by impressing upon the feed an oscillatory movement as close as possible to its natural frequency. This method is said to achieve fine grinding at a relatively modest input energy level.[3,5] Another recent process is known as electrohydraulic crushing and grinding. It is based on comminution by shock waves, generated by underwater electrical discharges. Technical difficulties persist in limiting its usefulness. Recently, however, a segmented crusher has been patented which operates on this principle. The history of this technique in the U.S.S.R., Britain and the U.S.A. has been thoroughly reviewed elsewhere.[134]

Another method of comminution, often referred to as explosive pulverization or explosion shattering, relies on sudden pressure reduction around the material which had previously been subjected to a rise in pressure (and possibly also in temperature). The system can be operated with both air or steam.[4]

Practical methods of rock mass breakage by thermally induced stresses

Practical methods of thermal rock mass breakage involve heat sources which primarily affect the rock either on its surface or in volume at depth. The former include internal combustion jet burners, diffusion-type burners, plasma jets, oxygen lances, lasers, electron beams, electric arcs and hot drill 'shoes'. The latter comprise direct electric and magnetic heating methods and embedded heat sources. Only one or two of these methods have so far been accepted by industry.

In practice, thermal rock breakage by internal combustion jet burners, often referred to as 'jet piercing', is the method most widely used. It is based on the advantages inherent in the use of a supersonic velocity jet flame, such as its ability to 'chamber' any part of a borehole,[135] to remove rock debris, and to operate against considerable back pressures. Moreover, exceptionally high heat transfer rates are obtainable with this burner; also, it can penetrate certain types of hard rock faster and more economically than any comparable mechanical tool so far developed. Disadvantages include the fact that jet flames can only penetrate a limited range of rocks and that, even in those, spalling is often superseded by uneconomical melting and fissuring. The method's high costs are also a drawback. Recent steps taken to reduce these include a cutback in cooling water consumption[95] and substitution of compressed air for pure oxygen.[86,136–138] 'Jet piercing' has been extensively studied both in theory[87–93] and in practice. It is particularly widely used in certain iron ore areas of the U.S.A.[135,139,140] and of the U.S.S.R.[141–161] The process has been carefully studied in Canada too, where it is being used by the mining industry.[86,95,136,162,163] Underground experiments have been reported from the U.S.S.R.;[9] elsewhere, the method is limited to operations in the open.

Thermal rock breakage with diffusion-type burners is not practised on a large scale today. Theoretically, there is little, if any, difference between it and the previously described method involving internal combustion jet burners. Substantial practical differences do, however, exist. These are inherent in the diverse characteristics of the two flame types. The method has been carefully studied and successfully applied, as an example, in a Central European gold mine.[164]

Rock breakage with plasma jets,[8,165] oxygen lances, electron beams,[10] electric arcs,[7] hot drill 'shoes'[166,167] and lasers[10,168,169] is still in the experimental stage. These tools tend to produce excessive densities, or amounts, of energy so that instead of merely thermally spalling rocks they are likely to melt or even vaporize them. It therefore seems unlikely that any of these methods will achieve practical importance within the foreseeable future, except in the case of specialized tasks or whenever penetration speeds rather than costs count. In any event, practical acceptance is at present curtailed by difficulties such as high production and operating costs, bulky equipment and insufficient energy levels.

Direct electric and magnetic heating methods seek to produce volume heating of the rock material without mechanical or thermal means of energy transmission to the rock's surface. Some of these methods depend on inductive heating by high-frequency magnetic fields through hysteresis and eddy current losses; others depend on heating by means of high- or low-frequency electric fields through dielectric and resistive losses. In practice, this is achieved by such means as loop coils[170,171] or capacitor plates and electrodes of various shapes.[10,172,173] A practical solution is known as 'electrical disintegrating drilling', wherein current is established between teeth of a specially designed tool bit and a ground wire.[174] Results therefore depend to a large extent on the electrical properties of the rock mass. Generally speaking, direct electric and magnetic heating methods are only efficient when breaking hard igneous rocks and with secondary, rather than primary, rock breakage. In common with other thermal methods, the rock may often only be weakened by heating; complete breakage is then best achieved by additional mechanical means.

Certain thermal methods of rock mass breakage are characterized by embedding the heat source in the rock mass before activation. On a small scale, mostly for secondary rock breakage, this has been done with thermite cartridges.[9] On a large scale nuclear cores have been suggested.[10]

Practical methods of rock particle breakage by thermally induced stresses

Few, if any, crushing or grinding processes have yet been developed purely on the basis of thermal-type particle breakage. References to particle liberation and comminution by purely thermal means can, however, be found in the technical literature.[5,97] Generally speaking, these processes are limited to very special cases only, e.g. heating of barite and fluorite. While technically feasible, they are economically impractical. Other experiments in the field of thermal rock particle breakage appear to be of wider interest,[85,175,176] but in these cases heat was only used to assist standard mechanical means of grinding.

Other practical methods of rock breakage

Practically all present-day methods of rock breakage are based on stresses induced in the rock by purely mechanical or purely thermal means. The few that do not fit into this classification are those which combine thermal and mechanical rock breakage, and chemical methods.

In crushing and grinding heat has so far been used only to assist otherwise standard mechanical procedures.[85,175,176] Overall efficiencies were low, probably because studies have not been completed as to the most advantageous methods for adding and preserving heat in the grinding cycle.

In rock mass breakage most of the combined methods do not induce thermal and mechanical stresses simultaneously, or else they produce only very localized heating. One such method is based on high tool tip temperatures, achieved by friction between tool and rock.[8] In practice, this is done by high static tool loading in the case of drill bits,[9] or else with plain metal discs of low thermal conductivity and special design features. Another such method is the ancient one of heating and cooling before mechanical loading. The modern equivalent might involve hydrocarbon fuel burners or direct heating methods in conjunction with steel balls and mechanical shovels. A more efficient thermal/mechanical method envisages a proper combination of an internal combustion jet burner and appropriately loaded mechanical tool bits.[160] Possibly an even better type of combination is represented by a design embodying an air–fuel jet burner, the flame of which carries an abrasive suspension, e.g. blasting sand.[138]

Chemical rock breakage methods have also been used on an experimental basis. One concept involves the use of high-velocity chemical jets which react with the rock surface by molecular exchange.[7] Rock debris consists of volatile products which can be blown away by the jet streams. One drawback, in the case of borehole drilling, is the time lost in frequent round tripping necessary for recharging. Another method involving chemistry is based on materials which affect the rock's hardness by adsorption or reaction,[7] but this method does not destroy the rock—it merely assists mechanical rock breakage.

Improved methods of rock breakage

General remarks

So far, an effort has been made to highlight important facets of the complex problem summarily referred to as rock breakage. It now remains to critically analyse the mass of available data, with the aim of localizing the most promising points of departure in a search for improved hard rock breakage methods. Continuous mining would be particularly welcome. Improved methods of blast-hole drilling are not to be overlooked, however, for they would lead to a fuller exploitation of the considerable advantages inherent in the use of explosives. Economics are to be judged on an overall cost basis, including all stages from breaking of the *in situ* rock mass to crushing and grinding.

It is suggested that the search in question should depart from a study of those events within the rock proper which occur at the moment of final failure, and which immediately precede it. Once these phenomena are clearly understood it should be a relatively simple task to devise a tool which will establish critical conditions most economically. Nonetheless, it is recognized that the mining industry cannot content itself with fundamental research alone, whose step by step advances may be difficult to translate into meaningful practical terms. Technical improvements such as tungsten carbide inserts, 'down-the-hole' designs, jet piercing, aerofall mills and a host of others have, after all, greatly advanced specific rock breakage techniques, even without an unequivocal explanation of all inherent theoretical intricacies. Consequently, close scrutiny of a few practical rock breakage methods is also proposed.

Proposals for fundamental research

Fundamental studies of rock breakage can be grouped into three major classes, according to whether they are undertaken on the basis (*a*) of very large scale natural phenomena, such as rockbursts,[177] (*b*) of laboratory scale studies with rock masses of relatively semi-infinite extent with respect to the loading tool or (*c*) of laboratory scale studies with rock masses of relatively small size compared with the loading equipment. In each case breakage is basically predicated upon the same fundamental set of circumstances. This is recognized, for example, by methods such as those of Hansági and co-workers[177,178] which correlate *in situ* rock mass strength with that of rock samples. Numerous further examples can be cited to illustrate the point. These include the following observations, established with both 'semi-infinite' and relatively small rock samples.

(1) Slow loading can be more efficient than the impact type.[4,123]

(2) The basic theories, as formulated by Griffith[17] and confirmed and extended to comminution by others,[63,64] apply.

(3) Some analogies from metal cutting are applicable.[179]

(4) Indices such as 'grindability' and 'drillability' can only be established by using standardized tools under strictly defined and controlled test conditions.

(5) A large percentage of the input energy is expended in elastic deformation and subsequently 'wasted'; over-grinding must therefore be avoided ('reverse flushing' drills, 'free crushing' mills) to maintain efficiency.[4,180]

(6) Adsorption of electrolytes and of surface-active agents from solution can affect the economics of both grinding and drilling.[5,53]

When determining the most promising points of departure for studies into the fundamentals of rock breakage it is therefore relatively unimportant whether research is based on grinding mills, drop testers or *in situ* field tests. Results of crushing tests on single particles may be applicable to grinding, excavation or even rockbursts. Energy requirements and costs in grinding are influenced by methods of excavation. All aspects of rock breakage are thus intimately interrelated. Proper correlation and evaluation of all results probably matters more than the specific methods used to obtain them.

Generally speaking, the object of fundamental rock breakage studies should be to determine the steps required (*a*) to reduce the critical volume of elastically strained material at breakage and to determine the spatial distribution of the theoretical minimum required; (*b*) to assure that the maximum possible amount of input energy is used to raise the stress level to critical levels at the incipient points of catastrophic failure propagation; and (*c*) to facilitate the completion of those cracks which will produce the largest possible rock fragments.

More specifically, fundamental studies should be devised to study the effects of properly superimposed stress fields. These should either involve two mechanically induced stress fields or else a mechanically and a thermally induced one.

Studying, by means of the standard line load formula, spatial stress distributions generated by indenter-type tools, it appears that principal stresses decrease in direct proportion to the radial distance from the point of indentation and to the angular distance from the line of loading. Experimental evidence also indicates that prior to catastrophic failure a number of secondary fractures radiate a little distance out from the tool–rock contact zone and that the first minute crack does not necessarily appear in immediate contact with the tool.[23,24] With this picture in mind it seems that an inordinate amount of the input energy, required to push the tool down to chip forming levels, is perhaps used to increase elastic straining of material in the immediate vicinity of the tool and to proliferate minute cracking there rather than for completing an already initiated crack. Consequently, it is proposed that if it were possible to assist the completion of partially formed cracks, instead of producing additional ones, energy requirements could be lowered. It is for this reason that it is proposed to study the effect of superimposing a second stress field, at the right time and place, by the 'secondary' tool which is independent of the 'primary' one. As already mentioned, both of these superimposed stress fields could be mechanically induced, or else one only would be mechanically generated and the other thermally. The fundamentals of rock breakage with solid indenters have been fairly well explored; those of purely thermal fracturing have been relatively neglected and should be studied in greater detail.

In practice, some of the advantages inherent in the principle of superposition are already made use of, e.g. in the case of rotary–percussive methods, stepped tool designs and indexing. All of these, however, involve mechanically induced stress fields only; moreover, rotary cum percussive loading lacks flexibility insofar as the two loading types act upon the same tool bit; stepped tool bits are only being loaded in rotation, thus the different tool levels induce stresses simultaneously at all times; and although indexing ensures that in case of consecutive chisel blows the effect of the first one benefits the second, it entails unloading of the rock between blows.

As for the apparatus required to study the effects of superimposed stress fields, it is proposed that either a static or dynamic loading arrangement can be used with various tool bit geometries, including stepped contour ones. The surface of the rock sample should be subjected to secondary stresses; these should be either compressive or tensile, induced either mechanically or thermally. In addition to single indentation studies, involving observation of both surface and subsurface damage, repeated blows should also be investigated. Consecutive blows should be aimed either at identical or at properly displaced spots to ensure that more specific energy is not expended in reforming favourable crater formations than had been gained by producing them in the first place.[20]

Thermal/mechanical stress superpositions should also be studied with standard laboratory grinding equipment, properly insulated against heat losses. The relatively slow application of heat (or cold) would seem to offer the best hope of obtaining a separation of mineral grains with a predictable degradation in size. Particle size distribution at various combinations of the mechanical and thermal energy level should be studied. The heat balance of the entire cycle is of particular importance. Feed could be heated, and then cooled by ground material upstream of the grinding mill. This measure, and similar ones, would help to reduce excessive costs by preventing unnecessary losses caused, for example, by drawing off heated grounds. Carefully instrumented experiments of this kind might lead to a clearer understanding of exactly how and why heating assists grinding, and how the thermal and mechanical energy inputs must be combined to achieve maximum economies.

Proposals for applied research

Applied research must be founded on results obtained by basic studies of rock breakage phenomena; exceedingly heavy costs may otherwise be incurred in building machines which will not fulfil their sponsor's expectations for fundamental reasons (e.g. lack of sufficient energy at the right time in the right place) or for technical or economic ones (bearing troubles, inordinate tool wear, energy transmission problems, excessive times for round tripping and the like). Unfortunately, far too little is known as yet about exactly why, when and how different rocks break under different circumstances to ensure the best possible tool design for every given set of conditions. Nonetheless, three possible points of departure are proposed here for research in applied methods of rock breakage. They are based on results obtained by past theoretical and practical work.

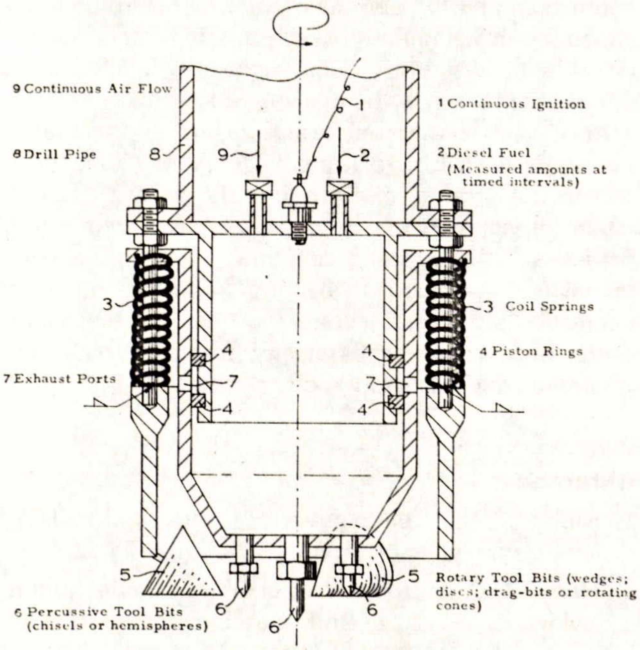

Fig. 1 Schematic arrangement of proposed combustion-powered rotary–percussive tool bit

First, it is suggested that tools should be developed which are capable of superimposing two independent stress fields upon each other. The hoped for advantages were outlined earlier when establishing proposed criteria for fundamental research. In particular, tools should be developed which can superimpose two independent mechanically induced stress fields. One way would be generally on the lines of the experimental drilling machine which produces a pilot hole ahead of the full gauge one, with completely independently loaded rotary and percussive tool bits.[181] It is claimed that total electric power consumption per unit length of tool penetration averages 25 per cent less with this tool than with a conventional tricone bit. Another similar design could again involve independent rotary and percussive tool bits at different levels, with a third independent tool mounted inside the pilot hole to load the wall of the latter radially outward, in the manner of a boring tool. A third experimental design is sketched in Fig. 1. Here, too, two independent tool bits are provided—one loaded in pure percussion, the other in pure rotation. The frequency and load level of the percussive section can be varied at will; so can the speed of rotation and static loading of the rotary section. The tool can operate purely in percussion, purely in rotation, or with any desired combination of the two loading types. The blows are generated by the principle upon which internal combustion engines are based. Thus it is hoped to utilize the advantages inherent in chemical explosions (such as extremely high values of specific energy release at explosive rates) without the drawbacks of present-day explosion drills (such as their inability to establish a sufficiently rapid series of explosions at satisfactory energy levels).

Second, tools could be developed which combine thermally and mechanically induced stresses within the rock mass. Actually, the hot exhaust gases leaving the tool outlined in Fig. 1 may enlarge the hole diameter in the case of spallable rocks, or weaken the walls in the case of non-spalling ones. Thermal mechanical arrangements in the true meaning of the term are designs wherein the thermal and mechanical components assist each other's progress at all times—not only when one or the other is incapable of progressing on its own. A combination of this general type, incorporating jet burners in conjunction with solid tool bits, has been described in a Soviet publication.[160] It is not clear how this design can maintain substantial mechanical and thermal rock loading simultaneously for any length of time. It is proposed that a modification be tested wherein an air–fuel jet burner flame is placed upstream of the leading mechanical tool bit and downstream of another set of mechanical tools. The flame is to point radially outward, or even slightly upward, instead of axially forward. Its principal role is to enlarge the pilot hole and to assist the tool bits at the second level of the stepped hole contour.

Another thermal/mechanical arrangement of some promise incorporates a solid particle laden jet burner flame.[138] It is suggested that this concept be extended by testing other types of solid suspensions than have previously been used, as well as different methods of solid introduction into the flame. In particular, the particles in suspension might include some capable of exothermic reactions and also rock debris. The latter could then participate in breaking up rock while itself being reduced to the desired final fineness. These burners could be used for blast-hole drilling. Alternatively, located within a comminution cycle, they would provide the thermal means for a combined thermal/mechanical grinding process. The latter could, and should, also be investigated by more orthodox means, on the lines previously discussed under the heading of proposed fundamental research.

Combined thermal/mechanical grinding could perhaps be investigated by radically novel means too, namely by shock waves which pulsate inside a spherical or cylindrical combustion chamber at resonant frequency. The process is known as pulsating combustion. It consists of a self-regulating and self-sustaining series of rapid explosions. In theory, it is capable of releasing, by relatively simple means, extremely large amounts of energy at high temperatures and within a small space.[182] In practice, technical difficulties have so far prevented the application of this attractive principle to other than a few special cases of heating and locomotion.

Third, it is suggested that the possibility of focusing the input energy be investigated so that only the desired amount of rock mass is strained at predetermined locations. Possibly a very high frequency and short wave source of energy could do this and thus achieve a weakening of the rock at selected depths, perhaps immediately ahead of a mechanical tool bit such as a rolling disc. In this connexion it is interesting to read that Soviet scientists have experimented with focused electromagnetic energy by means of special directional antennae and contact electrodes.[5]

These three proposals involve criteria for building improved rock breakage tools which could be applied either on novel tunnelling and blast-hole drilling machines or within crushing and grinding cycles. In conclusion it may be worth while to consider the possibility of using some of these tools—or even more standard ones, such as diffusion-type burners—over large areas and to shallow depths rather than for drilling relatively few and deep holes. In quickly passing over the selected areas these 'primary' tools could, under favourable circumstances, produce a vast number of small chips; otherwise they could at least weaken the rock surface. 'Secondary' mechanical tools would follow to dislodge chips and weakened rock layers, which could then be swept directly into a nearby mobile crushing plant.

Summary and conclusions

The volume of broken rock depends primarily upon the amount of specific energy which it can be forced to accept. Better rock breakage methods can therefore be devised by suitable technical improvements to standard energy production and transmission media—improvements which ensure that ever increasing loads are available at the rock face under economically acceptable conditions. Such improvements can be as numerous as the designs and machinery components involved. It is not considered within the scope of this article to discuss any of these technicalities in detail. It is fully recognized, however, that success or failure of even the best theoretical solution depends on them.

Improved rock breakage tools have been proposed in general terms, mainly on the principle of superposition. Specifically, it is recommended that two independently induced, yet complementary, stress fields be superimposed. These are to be generated by two mechanical methods or by a mechanical and a thermal one. The additional flexibility provided by such an arrangement is of considerable importance in either case. It is, however, particularly necessary in the case of thermal tools. It seems unlikely that, on an economically viable basis, purely thermal rock breakage methods can be greatly expanded beyond their specialized fields of acceptance, because thermal spalling can only be achieved when conditions are exactly right.

If blasting is to be avoided, the improved rock breakage tools should be used for tunnelling or for rapid rock chipping over large areas and to shallow depths. The considerable advantages of blasting, however, could also be better utilized if, on implementing suggested changes, blast-hole drilling costs were found to be reduced.

Suggestions have also been outlined for fundamental studies designed to determine the most economical loading rates and levels, as well as the best tool bit geometries and their points of attack. These studies are aimed at investigating basic rock breakage mechanisms in the case of two superimposed stress fields and in the case of purely thermally induced stresses.

An attempt has been made at reviewing and evaluating the entire gamut of rock breakage events from the *in situ* to the ground, end-product stage. This entails a rather unwieldy mass of material, some of which (e.g. particle size analysis and distribution functions) seems to have little immediate bearing upon the events of interest. Nonetheless, a correlation of all these data is useful because advances at any one stage may considerably affect research at other ones.

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