CANADA

DEPARTMENT OF ENERGY, MINES AND RESOURCES
MINES BRANCH
OTTAWA

THE ANALYSIS OF THE VISCOUS PROPERTY OF ROCKS FOR CLASSIFICATION

R. C. PARSONS AND D. G. F. HEDLEY

FUELS AND MINING PRACTICE DIVISION

Reprinted from Int. J. Rock Mech. Min. Sci. Vol. 3, pp. 325-335, Pergamon Press Ltd., 1966

Reprint Series RS 36

Price 25 cents

Available by mail from the Queen's Printer, Ottawa,
and at the following Canadian Government bookshops:

OTTAWA
Daly Building, Corner Mackenzie and Rideau

TORONTO
Mackenzie Building, 36 Adelaide St. East

MONTREAL
Aeterna-Vie Building, 1182 St. Catherine St. West

or through your bookseller

A deposit copy of this publication is also available
for reference in public libraries across Canada

Price 25¢ Cat. No. M38-8/36

Price subject to change without notice

ROGER DUHAMEL, F.R.S.C.
Queen's Printer and Controller of Stationery
Ottawa, Canada
1967

Int. J. Rock Mech. Min. Sci. Vol. 3, pp. 325-335. Pergamon Press Ltd. 1966. Printed in Great Britain

THE ANALYSIS OF THE VISCOUS PROPERTY OF ROCKS FOR CLASSIFICATION

R. C. PARSONS* and D. G. F. HEDLEY†

(*Received* 12 *April* 1966)

Abstract—The time-dependent deformation of geological materials can be thought of in terms of classical rheological models. The viscous component though usually present, may not be of sufficient magnitude to measure over the limited time spans available. For rock classification purposes, several specimens from each of twenty different rock types have been tested in uniaxial compression to determine the magnitude of the viscous component. Several methods of evaluating the results to determine the magnitude of the viscous coefficients are discussed and compared. Two basic methods use extrapolation or the shape of the time-deformation curves.

It is found that for the great majority of the rock types tested, the semi-log relationship, log time vs. strain, closely fits the data. Extrapolation to the region of 200 min gives a realistic value of the strain rate. The strain rate of 2 μ in/in/hr appears to be a practical value for the division between elastic and viscous behaviour of the rock substance.

1. INTRODUCTION

MANY rock materials which differ from one another in their geological classification, exhibit similar deformation responses when subjected to stress over a period of time. Such materials should be classified together for the purpose of rock mechanics, where the rheological properties of a rock material are the important parameters. A simple criterion for classification is the prominence of the viscous component of the rock substance. What is actually required from the classification is some indication as to whether, under a given stress, the material will remain rigid, flow a little and stop, or flow continuously. No rock material behaves in either a perfectly elastic or viscous manner and it is necessary to determine which of these properties are predominant. Deformation behaviour is complicated in that a rock material may alter with environmental conditions, or a rock which exhibits elastic properties at relatively low stresses may behave viscously above its yield stress. Some rocks, which are normally elastic, become viscous when subjected to high confining pressures and/or high temperatures, while others, which are normally viscous, become predominantly elastic at sub-zero temperatures. This variation in behaviour does not really affect the problem from a classification point of view, since we are interested in prefracture deformation at normal temperatures and below the yield point and the above complications are seldom encountered in practice. For classification purposes, the time–strain behaviour at 20°C and 50 per cent of the compressive strength is used.

This report describes the various ways of analysing test data in order to determine the predominancy of either the elastic or viscous properties. For design purposes, a compre-

* Scientific Officer, Rock Mechanics Laboratory, Mines Branch, Department of Mines and Technical Surveys, Ottawa, Canada.

† Post Doctorate Research Fellow, Mining Research Laboratory, Department of Mines and Technical Surveys, Elliot Lake, Ontario, Canada.

hensive study would be required with emphasis on the predominant property as exemplified by its position in this classification. The results obtained from twenty different rock materials tested at the Mining Research Laboratories, Ottawa are given, together with a number of methods of analysis.

2. TYPES OF CREEP

The strain which results when a rock substance is subjected to a constant stress may be classified into four parts:

(i) Instantaneous strain
(ii) Primary creep at a decelerating rate of strain
(iii) Secondary creep at an approximately constant rate of strain
(iv) Tertiary creep at an accelerating rate to fracture.

The instantaneous strain is the elastic strain and is independent of time. It need not then be considered in the analysis of viscosity. Since the collection of data for this study was made for a uniaxial stress of 50 per cent of the compressive strength of the substance, the fourth type of flow above is of no direct concern. Primary and secondary creep are the two main types to be considered.

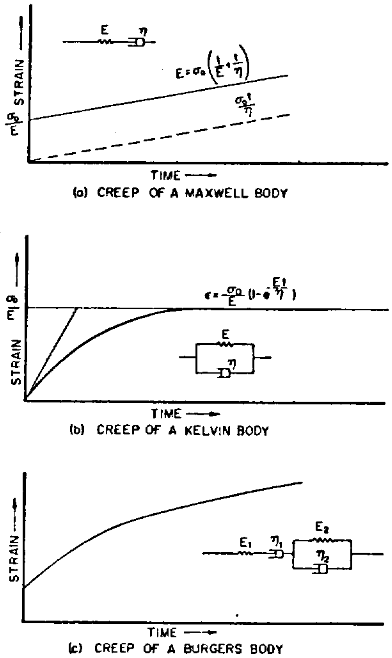

FIG. 1. Rheological models.

The general behaviour of time-dependent materials can be expressed in terms of elements representing elastic and viscous properties. The Maxwell body, a series combination of a Newton and a Hook element, illustrates the property of secondary creep [Fig. 1(a)]:

$$\dot{\epsilon} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta}. \tag{1}$$

Under conditions of constant stress

$$\epsilon = c + \frac{\sigma_0 t}{\eta} \tag{2}$$

where σ_0 = constant stress
η = coefficient of viscosity
E = Young's modulus
c = a constant.

If the application of load is fast in the creep test, the first term of equation (1) is large compared with the second. Then at

$$t = 0,\ \epsilon = \frac{\sigma_0}{E} \text{ and at } t > 0,\ \epsilon = \frac{\sigma_0}{E} + \frac{\sigma_0 t}{\eta}.$$

Neglecting the elastic strain, it is seen that the flow is directly proportional to time:

$$\epsilon \propto t. \tag{3}$$

The creep exhibited by the Maxwell body is non-recoverable.

The Kelvin body, composed of a Newton and Hook element in parallel, exhibits the property of primary creep which is recoverable upon removal of the stress field [Fig. 1(b)]. The recovery however may take long periods of time.

$$\sigma = E\,\epsilon + \eta\,\frac{d\epsilon}{dt}. \tag{4}$$

The time–strain relationship can be shown to be

$$\epsilon = \frac{\sigma_0}{E}\,(1 - e^{-Et/\eta}). \tag{5}$$

The total strain occurring is that allowed by the Hook element of the parallel combination and the attainment of this deformation is retarded by the viscosity element.

Since natural substances rarely exhibit the ideal behaviour of either of these models, a series combination of the two sometimes more closely represents the actual behaviour. Such a combination is known as a Burgers model [Fig. 1(c)]:

$$\epsilon = \frac{\sigma_0}{E} + \frac{\sigma_0}{E_2}\,(1 - e^{-Et/\eta_2}) + \frac{\sigma_0 t}{\eta_1}. \tag{6}$$

Depending on the value of the viscosity elements η_1 and η_2 in the Burgers body, the flow may be either entirely primary and recoverable, or entirely secondary and non-recoverable or a combination of the two in any proportion.

Rheological models are useful in grasping a physical picture of the mechanics of deformation. They are not essential, however, in the analysis of the viscous component of rocks.

The type of flow may be summarized by considering the time–strain to be proportional to the negative n-th power of time:

$$\dot{\epsilon} = At^{-n}. \tag{7}$$

When $n = 0$, $\dot{\epsilon} = A$, $\epsilon = At$, (8)

which is the secondary flow represented by the Maxwell body.

$$\text{When } n = 1,\ \dot{\epsilon} = At^{-1},\ \epsilon = A \log t. \tag{9}$$

The flow represented by equation (9) is analogous to the primary flow of the Kelvin body, the flow rate reducing to zero at an infinite time. Expression (9) above has been used by GRIGGS and PHILLIPS for the primary creep in rocks [1, 2].

Some experiments have used the relationship $\log \epsilon = A \log t$ to represent flow. For values of $n < 0$ in equation (7),

$$\dot{\epsilon} = At^n$$

and $$\epsilon = \frac{At^{n+1}}{n+1}.$$

Then $$\log \epsilon = \log A - \log (n+1) + (n+1) \log t$$

or $$\log \epsilon = b \log t + c. \tag{10}$$

This expression relates to a slower decrease with time than does equation (9) as shown below.

It is observed that the power function (log–log) relationship between strain and time gives higher estimates of the extrapolated strain rates than the exponential function (semi-log). The equations representing these functions are as follows:

Exponential $$\epsilon_1 = B \log t$$

$$\dot{\epsilon}_1 = \frac{d\epsilon_1}{dt} = \frac{B}{t}$$

where ϵ = strain, t = time, B = constant and $\dot{\epsilon}$ = strain rate.

Therefore $$\dot{\epsilon} \, \alpha \, 1/t.$$

Power $$\epsilon = At^n$$

$$\dot{\epsilon}_2 = \frac{d\epsilon_2}{dt} = Ant^{n-1}$$

$$\dot{\epsilon}_2 \, \alpha \, t^{n-1}$$

where ϵ_2 = strain, t = time, A and n are constants and ϵ_2 = strain rate. For a decreasing strain rate with time the value of n can vary $0 > n < 1$: therefore $\dot{\epsilon}_2 \, \alpha \, 1/t^{1-n}$. Consequently, since $1/t^{1-n}$ is a larger value than $1/t$ for values of t greater than 1, the strain rate $\dot{\epsilon}_2$ decreases at a slower rate with time than $\dot{\epsilon}_1$.

It has been observed that in flow tests some rock substances reach a constant value of strain in a finite time. This condition is approximated to when the value of η in the Kelvin body [equation (5)] is small. An alternate equation is [3]

$$\epsilon = \frac{at}{b+t} \tag{11}$$

or $$\frac{a}{\epsilon} = \frac{b}{t} + 1$$

where ϵ = strain, t = time, a and b are constants.

Plotting values of $1/\epsilon$ against $1/t$ should result in a straight line and the intercept on the $1/e$ axis when $1/t = 0$ is equivalent to $1/a$.

3. METHOD OF TESTING

The method of testing is described in Appendix B of the previous paper by COATES and PARSONS [4].

4. RESULTS AND ANALYSIS

The different types of rock tested are as follows:

1.	Limestone 1	11.	Sandstone
2.	Limestone 2	12.	Granite 3
3.	Conglomerate	13.	Calcium Fluoride
4.	Hematite	14.	Siderite
5.	Specularite Magnetite	15.	Chlorite
6.	Granite 1	16.	Blastonite
7.	Granite 2	17.	Peridotite
8.	Potash	18.	Quartzite
9.	Shale	19.	Halite
10.	Diabase	20.	Shale 6

A geological description of each rock type is given in Appendix A of the previous paper by Coates and Parsons [4].

For the purpose of classification, the strain rates obtained at a stress level of approximately 50 per cent of the compressive strength Qu are used. A strain rate of 2 μin/in/hr based on practical considerations, has been used as the dividing rate between viscous and elastic flow.

A number of the rock materials that were tested exhibited minimal time-dependent properties and the deformation reached, or nearly reached, a constant value during the duration of the creep test. These rocks can immediately be classified as predominantly elastic, and are as follows:

Conglomerate	Calcium Fluoride
Specularite Magnetite	Siderite
Granite 1	Peridotite
Diabase	Quartzite
Nodular Fluorite Ore	

The remaining test materials continued to show deformation with time during the 30-min test duration. For these types of rock, the results are fitted to equations (9) and (10), semi-log and log–log functions. Examples of semi-log and log–log plots, for shale and potash specimens respectively, are shown in Figs. 2 and 3. Both relationships produce straight lines from the intially non-linear, time–strain graphs. Straight lines on log–log and semi-log plots relate respectively to power and exponential relationships between strain and time. In Table 1 the extrapolated values of strain rate between 200 and 260 min are given. The relationship which gives the best straight line is used: where there was some doubt both values are given. It has been found that most rocks which show time–strain can be best fitted to the semi-log relationship with time on the log axis.

The estimated strain rates for the granite, blastonite and periodite specimens are finite but below 2 μin/in/hr and these rocks can be classified as predominantly elastic.

The estimated strain rates for the potash, halite and shale specimens are greatly in excess of 2 μin/in/hr and these rocks can be classified as predominantly viscous. The limestone 1 specimens also exhibit strain rates greater than 2 μin/in/hr and are classified as viscous.

The other four rock materials, limestone 2, hematite, sandstone and chlorite, are all borderline cases in that time–strain data fits both relationships for the limited time over which the data is collected.

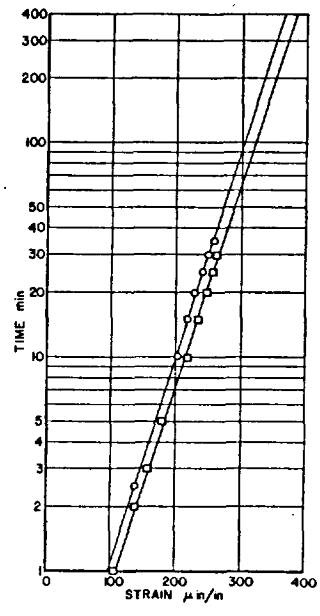

FIG. 2. Shale curves—log time vs. strain.

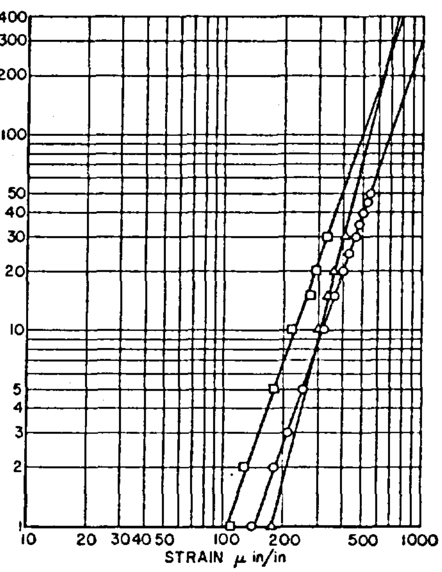

FIG. 3. Potash curves—log time vs. log strain.

As shown in Table 1, the strain rates centre, for most rocks, about 2 μin/in/hr when extrapolated to the region of 200 min.

It is observed that the power function (log–log) relationship between strain and time gives consistently higher estimates of the extrapolated strain rates than the exponential function (semi-log), but that the values are close in magnitude. It is suggested that the use of the log–log relationship can be eliminated entirely.

TABLE 1. STRAIN RATE BETWEEN 200 AND 260 MIN AT 50% Q_u

Rock type	Strain rate (μ in/in/hr) semi-log	log-log
Limestone 1	3·3	4·7
	4·3	11·0
Limestone 2	1·3	3·8
	1·6	5·3
	1·7	4·3
Hematite	1·9	3·0
	2·3	3·5
	2·5	4·5
	2·2	3·3
Granite 2	1·7	
	1·2	
Potash		40
		80
		65
Shale	11·0	
	10·2	
Sandstone	0·8	1·2
	1·8	2·5
Chlorite	1·3	4·2
	1·4	3·1
	1·5	3·2
	1·5	3·3
Blastonite	0·8	
	1·9	
Peridotite	0·5	
	1·2	
Halite	68	
	78	
	85	
	86	
	100	
	125	
Shale	230	

The average values of the other physical properties of the rock materials are given in Table 1 of the previous report [4]. Generally the specimens with high compressive strengths exhibit elastic properties and the materials which are viscous or nearly viscous have relatively low compressive strengths. However, the limestone 1 specimens which displayed appreciable viscous properties have the highest compressive strength measured. The materials which are classed as viscous have a relatively low elastic modulus; however, some other rocks which have a low elastic modulus also behaved elastically.

The effect of specimen direction with regard to bedding planes and foliation on the strain rate is insignificant. The spread of values between specimens in different directions is no greater than the spread from specimens in the same direction.

An alternative method of analysis is to classify the rock materials according to the shape of the creep curves. It would be expected that a material which deforms at a relatively fast rate would be more viscous than a material deforming at a slower rate. The classification in this case would be as follows:

(i) No time-dependent deformation—elastic
(ii) The deformation reaches a constant value at a finite time and is recoverable—visco-elastic
(iii) The strain rate decreases with time exponentially—viscous
(iiia) The strain rate decreases with time as a power function—viscous.

A simple formula which can be used to estimate the constant value of deformation for (ii) above is given in equation (11).

It has already been shown that the power function relates to a slower decrease in strain rate with time than the exponential function. These three types of functions should cover all the cases from a large decrease in strain rate with time [case (ii)] to a slow decrease in strain rate with time [case (iv)]. However, in many creep tests the measured results are not accurate enough to differentiate which function gives the best approximation. An example of this problem is shown in Fig. 4. The measured values of strain for the sandstone specimen fit both the power and exponential functions. The results for the granite specimen fit both the exponential and asymptotic functions [equation (11)] over the measured range, although the curves do diverge at larger values of time. Another difficulty is that the power function is very sensitive to the initial measurements of strain and a relatively small error will produce a non-linear line when the results are plotted on log–log graph paper. Consequently, this method of analysis, although probably theoretically feasible, does not provide an accurate assessment for the measured results.

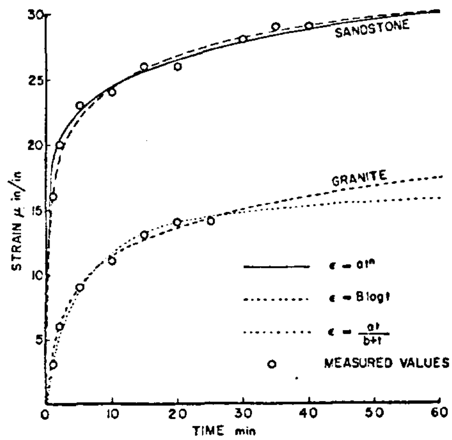

FIG. 4. Comparison of the measured and calculated values of strain.

Another alternative method of analysis is to represent the creep curves by equation (9):

$$\epsilon = B \log t + Ct$$

and

$$\dot{\epsilon} = \frac{d\epsilon}{dt} = \frac{B}{t} + C \tag{12}$$

where ϵ = strain, t = time and B and C are constants.

This equation has the property that as time t becomes large, the term B/t becomes small compared with C and the strain rate tends to approach a constant value equal to C. The term Ct is often referred to as representing the viscous flow and the term $B \log t$ the elastic flow. This type of equation makes extrapolation unnecessary but may require a longer than 30-min time period.

Plotting the strain rate against 1/time should result in a straight line, with a gradient equal to B and an intercept on the strain rate axis equal to C. An example of this type of plot, for four different rock materials, is shown in Fig. 5. The results appear to be in excellent agreement with the equation. The intercepts C are greater than the values previously determined in the first method of analysis, but are still of the same order of magnitude.

TABLE 2

Rock type	Strain rate (μin/in/hr)	
	1st Analysis Equations (9), (10)	3rd Analysis Equation (12)
Potash	80	170
Shale	11	55
Limestone	3·3–4·7	8
Sandstone	1·8–2·5	4

A further example is shown in Fig. 6. In this test, on a rock-salt specimen, the strain was measured over a period of 360 min. It can be observed that the extrapolated intercept from the initial part of the creep curve (i.e. larger values of 1/time) produces an excessively high value for C, 1650 μin/in/hr, compared with a measured rate of 53 μin/in/hr after 360 min. Consequently, this type of equation does not accurately predict the values of constant strain rate from short-term creep tests.

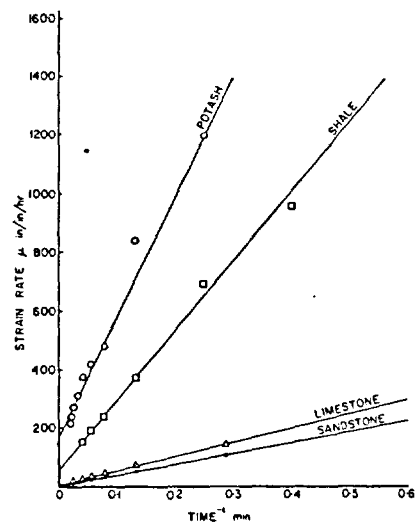

FIG. 5. Strain rate as a function of time^{-1}, potash, shale, limestone and sandstone.

5. CONCLUSIONS

Although none of the methods described is completely satisfactory in determining the viscous component of deformation, the first method, equation (9), has much to recommend

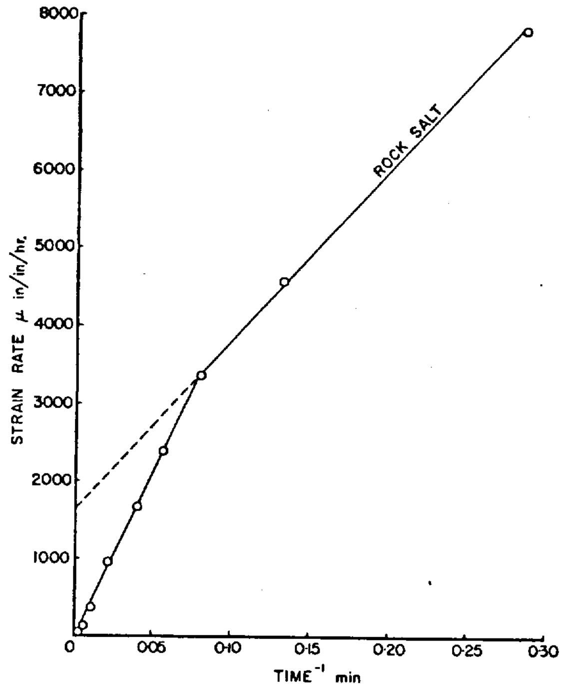

FIG. 6. Strain rate as a function of time^{-1}, rock salt.

it. The extrapolated values lend support to the designation of 2 μin/in/hr as the dividing line between elastic and viscous behaviour. The extrapolation of the log time curves to 200 min gives consistent results for the same rock type as evidenced by the low standard deviation of the results [4].

Whether or not the rate at 200 min is the secondary steady-state flow should not really concern us at the moment. The important thing is that the extrapolation provides a number which is both close to what one would expect in practice and also consistent for a particular rock type. The short time required and the ease of determination are also favourable factors.

In cases where the data fits both log–log and semi-log relationships, a longer measuring time, say 60 min instead of 30 min, will, it is felt, distinguish which is the more representative.

The second method of analysis using equation (10), although independent of the assumptions made in the first analysis, did not accurately relate to the behaviour of the actual rock materials. The third method of analysis was found to adequately cover the short-term creep tests, but the predicted failure behaviour was in serious error. It may be necessary to perform longer creep tests and to compare the short-term behaviour with that at greater times. Only if a reasonable correlation exists will it be possible to rely on the results from the short-term creep tests for absolute values. At the same time, for the purpose of classification an absolute value may not really be necessary.

Acknowledgements—We are indebted to J. SULLIVAN and S. COOK for their help in conducting the tests.

REFERENCES

1. GRIGGS O. T. Creep of rocks, *J. Geol.* **47**, 225–251 (1939).
2. PHILLIPS F. *Phil. Mag.* **9**, 513 (1905).
3. POTTS E. L. J. An investigation into the design of room and pillar workings in rock salt, *Trans. Instn. Min. Engrs* **124**, (49) 6, 10, 11, 13 (1964).
4. COATES D. F. and PARSONS R. Experimental criteria for classification of rock substances, *Int. J. Rock Mech. Min. Sci.* **3**, 181–189 (1966).