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# *THE EFFECT OF STRESS CONCENTRATIONS ON THE STABILITY OF TUNNELS*

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Reprinted from Proceedings of the First Congress of the International Society of Rock Mechanics, Vol. II, 1966, Laboratorio Nacional de Engenharia Civil, Lisbon, Portugal

***Reprint Series RS 35***

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ROGER DUHAMEL, F.R.S.C.
Queen's Printer and Controller of Stationery
Ottawa, Canada
1967

# The effect of stress concentrations on the stability of tunnels

## L'effet des concentrations des contraintes sur la stabilité des tunnels

## Der Einfluss von Spannungskonzentrationen auf die Stabilität von Tunneln

by D. F. Coates

**Summary**

The stress concentrations around an underground opening may theoretically exceed the strength of the rock mass. Deviations of the actual rock properties from those of homogeneity and perfect elasticity can, however, modify the theoretical stress distributions considerably. In addition, the known variation of the strength of rocks with the volume of the rock makes the predictability of failure due to stress concentration questionable. Several experiments have been conducted which show that failure due to compressive stress concentrations can be predicted under favourable circumstances quite accurately. However, considering failure as a stochastic phenomenon, predictions should be in terms of a probability of failure rather than a certainty. As a result of these experiments, it can be seen that the main requirements to improve the correspondence between experimental results and predicted results is to have better methods for determining the mechanical properties of rock masses.

**Résumé**

Les concentrations des contraintes autour d'une gallerie peuvent être théorétiquement plus grandes que la résistance du massif. Quand le massif n'est pas parfaitement homogène et élastique, les concentrations des contraintes d'après les théories peuvent être modifiées considérablement. Aussi, la variation de la résistance des roches qui est bien connue, rend les prédictions difficiles. Quelques expériences ont été entreprises qui montrent que la rupture en compression peut être prédite en circonstance favorable avec pas mal de précision. En même temps, on doit prédire en termes de probabilité de préférence à supposer que nous sommes certains. Les expériences montrent surtout qu'il èst très important d'améliorer les méthodes pour déterminer les propriétés mécaniques du massif.

**Zusammenfassung**

Die Spannungskonzentrationen in der Umgebung einer unterirdischen Öffnung können theoretisch die Festigkeit des Gesteinsmassivs überschreiten. Die Abweichung der gegebenen Gesteinseigenschaften vom homogenen und vollkommen elastischen Zustand haben jedoch eine bedeutende Abänderung der theoretischen Spannungsverteilung zur Folge. Überdies wird jegliche Voraussage der von Spannungskonzentrationen verursachten Brüche recht fragwürdig, infolge der bekannten volumenabhängigen Variation in der Festigkeit des Gesteins. Mehrere Versuche zeigten, dass unter günstigen Umständen die von Druckspannungskonzentrationen verursachten Brüche mit guter Genauigkeit vorauszusagen sind. Da man aber einen Bruch als ein stochastisches Phänomen betrachten kann, so sollten die Voraussagen eher als Wahrscheinlichkeit denn als Gewissheit aufzufassen sein. Aus diesen Versuchen ist zu ersehen, dass die Hauptbedingung für die mögliche Verbesserung der Übereinstimmung zwischen experimentellen Ergebnissen und theoretischen Voraussagen dann vorliegen wird, wenn vollkommenere Methoden zur Bestimmung der mechanischen Eigenschaften des Gesteinsmassivs verfügbar sein werden.

## Introduction

When a hole is cut in a stressed medium, the diffraction of the stress trajectories around the hole produces stress concentrations. The resultant stresses can be many times greater than the field stress and, consequently, may exceed the strength of the material.

The distribution of stress around a circular hole in a stressed medium has been solved in the theory of elasticity, but deviations of material properties from those of homogeneity and perfect elasticity can modify the theoretical stress distributions considerably. For this reason, failure predicted by calculation of stresses may not occur.

The strength of materials, particularly rock, is not easy to predict. Furthermore, as rock masses are not homogeneous strength will vary from point to point and failure must be considered as a stochastic phenomenon. Failure may not occur where predicted as the calculated stress may exceed the average strength but the actual strength at that point may also exceed the average strength.

## Theory

For a circular hole in a biaxial stress field as shown in Figure 1*a*, the theory of elasticity has shown that the tangential stresses, $\sigma_t$, at the critical points on the boundary of the opening can be determined from the following equations:

$$A: \quad \sigma_t = 3\,S_y - S_x$$

$$B: \quad \sigma_t = 3\,S_x - S_y$$

where $A$ is the point equivalent to the wall, or rib, of a tunnel; $B$ is the point equivalent to the crown of a tunnel; $S_y$ is the field, or pre-mining, stress in the $y$-direction (with compressive stresses being positive); and $S_x$ is the field stress in the $x$-direction [1].

By analysing the effects of anisotropy, it has been shown that the maximum boundary compressive stress can be increased as much as 30% or decreased as much as 20% if the ratio of the moduli of deformation at right angles to each other is 4 [2].

Recent work has shown that for a material composed of grains, a more comprehensive solution indicates that the stress concentrations would usually be less than those represented by the above equations but would approach the classical values as the diameter of a tunnel approaches infinity [3]. The differences between the solutions are not considered great enough for the classical equations to be replaced at the present time.

Recent analytical work on the effects of yielding in the material gives the resultant stress concentration factors that can be expected as a function of the yield stress and the moduli of deformation of the material before and after yielding [4]. The effect of yielding, of course, is to reduce the purely elastic stress concentration factor. However, to use this analytically it would be necessary to have a material with a distinct and known yield point together with a knowledge of the plastic properties of the material after yielding.

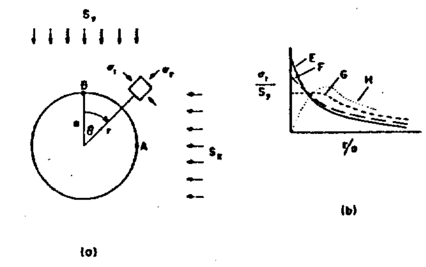


*Fig. 1 — (a) A circular opening in a biaxial stress field; (b) Patterns of stress concentration around an opening;* E *for elastic ground,* F *for non-linear or viscous ground,* G *for yielding ground at constant stress,* H *for partial breakdown and yielding of ground near the surface of the opening*

Figure 1$b$ shows qualitatively the effects of some of the common non-elastic properties of rock. The curve $E$ shows the variation of the stress concentration factor, $\sigma_t/S_y$, with depth into the rock, $r/a$. Curve $F$ shows the modification to the elastic curve that would result from either having a material with a decreasing modulus of deformation with increasing stress or a material that would creep at a rate varying, as it commonly does, with the stress level; the maximum stress concentration at the boundary of the tunnel would be decreased and more load would be shifted into the interior of the mass. Curve $G$ represents an elasto-plastic material which yields and produces a horizontal stress-strain curve. In other words, the concentration of stress could only be equal to the yield point in the material with the result that additional stress would have to be diffracted farther into the mass. Curve $H$ represents the common case in rock where the surface material around a tunnel is partially fractured and can only sustain a very low level of stress, but where the rock is confined this level of stress increases until the material becomes effectively solid and elastic.

For tunnels engulfed by a ground shock wave, photoelastic experiments and theoretical analyses have indicated that the compressive stress concentrations can be predicted with a moderate degree of accuracy by applying the above classical equations to the peak radial and tangential stresses in the shock front [5, 6, 7, 8.] Actually, the compressive stress concentration when the length of the ground shock wave is large with respect to the size of the tunnel is about 10% higher than the static values and the tensile stress concentration seems to be about 15% higher.

On the factors that affect strength, it has been found from uniaxial compression testing that the average strength of a rock sample usually decreases with an increase in volume. It is thought that flaws may have the nature of microscopic, weak grains within the rock, which would have an effect similar to one weak link in a chain. Alternatively, or in addition, flaws may be cracks varying in size from the length of a single mineral grain to the large joints seen in a rock face. It is then visualized that stress concentrations leading to local failure occur around these cracks as described in Griffith's Strength Theory.

Whatever their nature, it is generally assumed that the maximum size of flaw varies with the volume of the rock sample. By assuming a probability density function of the strength of the imperfections, the average fracture strength of samples and the dispersion of the results can be calculated using the statistical theory of extreme values [9, 10]. It can then be shown, assuming that the number of flaws is proportional to the volume, that the relation between the uniaxial strength of the sample and its volume is:

$$Q_b = Q_o\, V^{-a}$$

where $Q_b$ is the compressive strength of a sample of width $b$, $Q_o$ is the compressive strength of a sample with the same shape but with unit width, $V$ is the ratio of the volume of the sample of width $b$ to the volume of the sample of unit width, and $a$ is a parameter that depends on the particular rock.

This equation has been shown to apply quite well, not only to laboratory rock samples, but also to concrete samples up to 1 m in diameter and to brickwork piers up to 0.34 m square [11, 12]. Consequently, it is a factor that must be included in any appraisal of the effects of stress concentrations.

## Previous Experiments

One of the first cases in rock mechanics of comparing stress concentrations to strength was concerned with a study of the failure of mine openings [13]. Models were constructed of a mixture of plaster and silica flour, with a uniaxial compressive strength of 21 ksc. The calculated tangential stress at failure in the walls of openings was found to exceed the strength by 100% and more; also, the loading pressure on the model required to produce failure varied inversely with the dimension of the opening.

The disparity between calculated stress and strength was suggested at that time to be due to the average stress over the finite length of the potential fracture surface being more important than the stress at a point; thus the average stress within a finite distance increases with size of opening, and the larger openings are less stable. Alternatively, it was suggested that the probability of local weaknesses and incipient cracks could be a function of the perimeter of the opening. Thus the larger openings with greater perimeters would have a greater probability of cracking than small openings. The fact that square and circular openings of the same size failed at approximately the same pressures supported this concept.

Another series of experiments was carried out in plates of marble and sandstone in which circular holes had been drilled [14]. One series of tests was conducted using a uniaxial field stress which produced failure in tension at the crown and floor of the holes. A second series of tests was run with a biaxial field stress, $S_v/S_t = 4$, where failure was by crushing in the ribs. The results of these tests are shown in Table 1. The ratios $\sigma_t/T_s$ and $\sigma_t/Q_u$ were obtained from the uniaxial plus biaxial experiments respectively. The models were loaded to failure and the boundary stresses were calculated using the equations cited above.

*Table 1*

**Failure of Circular Openings in Rock [9]**

| Rock | $Q_u$ ksc | $T_s$ ksc | $S_b$ ksc | $\sigma_t/T_s$ | $\sigma_t/Q_u$ |
|---|---|---|---|---|---|
| Marble | 896 | 53 | 129 | 1.75-3.0 | 0.9 -1.3 |
| Sandstone | 1577 | 111 | 231 | 1.8 -2.9 | 0.95-1.2 |

$Q_u$ = uniaxial compressive strength; $T_s$ = uniaxial tensile strength; $S_b$ = modulus of rupture; $\sigma_t$ = boundary stress.

It can be seen in Table 1 that the agreement between theory and experiment was not good for tensile fractures but was very good for compressive failures. It was suggested that the disparity in the case of the tensile fractures might be due to a non-linear stress-strain relation for tensile stresses which would result in the calculated stresses being higher than the actual stresses. Some support was obtained for this explanation by noting that the ratio of the average modulus of rupture to uniaxial tensile strength was almost equal to the average ratio of $\sigma_t/T_s$.

As an extrapolation of this work surveys were conducted underground. It was found that where the major principal field stress could be assumed to be vertical with a biaxial stress condition, any failures were found to be in the side walls. Where field stress conditions were essentially uniaxial, as in a vertical pillar, tension cracks were found in the roof and floor with little evidence of compression failure in the walls. From these observations it was concluded that some information could be obtained on the field stress conditions by observing the location and nature of the failure of rock around underground openings.

In a recent blast trial, vertical holes 0.9 m in diameter were drilled 19.5 m deep at spacings of 13.7 m into basalt [15]. The basalt had a characteristic *P*-wave velocity of 3300 m/s although the shock wave only travelled at 2160 m/s. The density of the rock was 2.66 g/cc, and the uniaxial compressive strength of the substance was 1320 ksc.

Accelerometers and velocity gauges were installed at the bottom of the holes. A charge of 20 tons of nitromethane produced in a hole 27.4 m away a stress, calculated from the peak particle velocity, of 676 ksc. With a positive phase duration of 0.020 for the particle velocity, the length of the shock pulse was 43 m. Based on the stress concentration factor of 1.53 around the end of the hole [20], the peak radial stress in the shock wave was 442 ksc.

From previous work, as mentioned below in the newly reported blast trial experiment, the tangential stress in the shock wave can be assumed to be in tension and equal to about 1/9 of the peak radial stress. Using the static equations cited above, the maximum compressive stress around the hole would have been 1385 ksc giving a $\sigma_t/Q_y$ ratio of 1.05, which is better agreement than is inherent in the analysis. The hole at the range of 13.7 m was completely obliterated and that at the range of 41.1 m sustained very little damage.

## New Experiments

### *Uranium Mine*

In connection with an extensive study of the in situ stresses in the pillars of and in the formation around a uranium mine, some data was obtained on the effects of stress concentrations [16]. A plan of the mine with the location of the stress measuring holes is shown in Figure 2. The rock formation is a Pre-Cambrian quartzite within which is contained the conglomerate ore material.

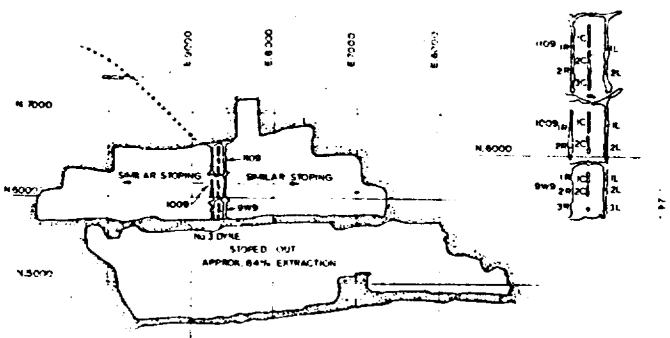



*Fig. 2 — The plan of part of uranium mine showing the location of the stress-measuring holes in the pillars; the depth below the ground surface of these pillars varies from 260 m to 310 m; the average dip of the orebody is 14°*

The quartzite is composed of moderately well-rounded quartz grains firmly cemented by silica. The grains range in size from 0.2 to 1.5 mm. Feldspar occurs in some specimens as small particles between the quartz grains and can be up to as much as 5% of the contents. Sericite alteration is present along many of the quartz grain boundaries. Pyrite is also found but usually forms less than 1% of the total mineral content [17].

The conglomerate consists of quartz pebbles generally in a quartz matrix but with as much as 5% feldspar in its alteration products and as much as 15% pyrite also in the matrix [17].

Table 2 shows the results of this experimental work. The principal stresses in the pillars, $\alpha_1$ and $\alpha_2$, are the average from several readings with some dispersion having been measured around these means.

The average uniaxial compressive strength of the quartzite substance based on 110 tests was found to be 2680 ksc with a coefficient of variation of 15.8%; that of the conglomerate was 2220 ksc with a coefficient of variation of

22.5% [17]. The pre-failure deformation characteristics of both substances are elastic (with strain rates of 0 and 1.0 $\mu$ strain/hr respectively) and failure characteristics are brittle (with plastic strain being 0 and 2% of the failure strain respectively) [17, 18]. The strengths contained in Table 2 are those for the individual holes which also are mean values but with dispersions less than 15.8%. The ratio $\sigma_t/Q_u$ is based on the calculated maximum compressive stress using the classical equations cited above. The flaking areas described in Table 2 consisted of spalls of the order of 3 mm thick.

the stress concentration at this particular zone would be decreased by diffraction into the adjacent zones so that the compressive stress would be decreased below the strength level.

The uniaxial compression tests were conducted on samples 2.5 cm in diameter by 5 cm long. It is possible that the strength of samples this size is representative of the material effectively stressed by the diffraction effects around the 15 cm holes. However, had the openings been of the order of 4 m in diameter (the height of the stopes), it is probable that these strength figures would

*Table 2*

**Wall Failures in Uranium Mine Stress Holes 15 cm in Diameter**

| *Hole No.* | $\sigma_1$ *ksc* | $\sigma_2$ *ksc* | $\theta$ *deg* | $Q_u$ *ksc* | *E* *ksc* | $\sigma_t/Q_u$ | *Size of Flaking Areas* *cm* | *Inclination of Bisector of Flaking Areas* *deg* |
|---|---|---|---|---|---|---|---|---|
| 9-1 R | D | — | — | 2520$^{x}$ | 8.36$^{x}$ × $10^5$ | 1.18$^{y}$+ | 3×13 | 36 |
| 9-2 R | 770 | 320 | 55 | 3160 | 8.02 | 0.63 | 4×10 | 27 |
| 9-3 R | 1000 | 320 | 53 | 2780 | 8.09 | 0.96 | — | — |
| 9-1 L | 740 | 340 | 59 | 2200 | 8.37 | 0.86 | 1×3 | 35 |
| 9-2 L | 675 | 250 | 49 | 2870 | 7.80 | 0.62 | — | — |
| 9-3 L | 1100 | 770 | 54 | 1950 | 7.42 | 1.30 | — | — |
| 9-1 C | 530 | 170 | 53 | 2840 | 8.92 | 0.50 | — | — |
| 9-2 C | D | — | — | 2780$^{x}$ | 7.87$^{x}$ | 1.12$^{y}$+ | 5×13 | 54 |
| 10-1 R | 670 | 250 | 52 | 2090 | 5.73 | 1.32 | 5×8 | 11 |
| | | | | | | | 5×8 | 27 |
| 10-2 R | 610 | 310 | 37 | 2250 | 7.38 | 0.68 | — | — |
| 10-1 L | 600 | 260 | 50 | 2950 | 7.77 | 0.52 | 1×5 | 34 |
| 10-2 L | 560 | 225 | 50 | 2790 | 11.10 | 0.50 | 5×5 | 38 |
| 10-1 C | 985 | 430 | 42 | 3040 | 9.00 | 0.83 | 5×13 | 20 |
| | | | | | | | 5×13 | 44 |
| 10-2 C | 740 | 280 | 51 | 2980 | 8.15 | 0.65 | — | — |
| 11-1 R | D | — | — | 2920$^{x}$ | 8.15$^{x}$ | 0.86$^{y}$+ | — | — |
| 11-2 R | 750 | 680 | 80 | 2050 | 8.44 | 0.78 | — | — |
| 11-1 L | 700 | 330 | 37 | 2960 | 9.04 | 0.60 | — | — |
| 11-2 L | 760 | 330 | 25 | 2840 | 8.15 | 0.67 | — | — |
| 11-1 C | 940 | 310 | 50 | 2970 | 7.80 | 0.85 | 3×3 | 37 |
| 11-2 C | 800 | 280 | 46 | 2980 | 8.02 | 0.71 | — | — |
| 11-3 C | 840 | 390 | 45 | 2270 | 8.58 | 0.94 | — | — |

$\sigma_1$ = average major principal stress in the pillar; $\sigma_2$ average minor principal stress in the pillar; $\theta$ = inclination measured from the vertical to the north of the major principal stress; $Q_u$ = uniaxial compressive strength of rock substance; $E$ = modulus of deformation of rock substance; $\sigma_t$ = calculated maximum ompressive stress at boundary of hole; $D$ = discing in core preventing stress measurements; $x$ = average values from nearest hole on strike; $y$ = assuming $\sigma_1$ = 1100 + ksc and $\sigma_2$ is the same as in the other holes in the same pillar.

It can be seen in Table 2 that in the four cases where the ratio $\sigma_t/Q_u$ is greater than 1, three produced flaking [19]. Of the seventeen cases where the ratio $\sigma_t/Q_u$ is less than 1, eleven had no flaking but six had flaking. In these latter cases it is quite possible that the variation in stress around the mean values would be sufficient to produce actual stresses high enough for a ratio of $\sigma_t/Q_u$ of 1.

If failure occurs around an opening as a result of the compressive stress concentration then, other things being equal, failure should propagate into the wall until the opening collapses. This sequence should occur as the initial failure would theoretically create a shape that would produce a more severe stress concentration and hence additional failure.

In spite of this theoretical concept, the apparent compressive failures in the above holes did not propagate. The explanation might be as follows: the field stress conditions in the rock around the holes varied along their lengths. Failure occurred just at the distances along the holes where these stresses were high enough. These failure zones were of finite length, and consequently, after an initial failure

not be representative of the larger volume of rock being stressed.

It is observed in stopes with geometry starting with an opening 4 m × 4 m and being mined out to 4 m × 20 m that crushing occurs at the corners. In this area the major principal stress is horizontal with a magnitude of about 350 ksc. The maximum compressive stress concentration for these stopes should not be greater than about 5 with a stress of 1750 ksc, which is considerably lower than most of the measured values of strength of the rock substance but still produces failure.

*Blast Trial*

In 1964 a 500 ton blast trial was conducted at Suffield, Canada [20]. As part of the trial, six adits, 15 cm in diameter by 3 m long, were excavated at each of eleven ranges where the ground surface was subjected to overpressure levels varying from 5 ksc to 0.2 ksc (see Figure 3*a*). After the blast, the adits were excavated and the failure conditions observed.

As mentioned above stress concentrations resulting from the engulfment of an underground opening by a shock wave should be only 10 to 15% greater than those calculated using tle classical equations cited above. However, the intensity of failure should be dependent on the duration of the pulse so that for a relatively short duration at most only some slabbing in the sides would occur, as indicated in Figure 3*b*, as opposed to complete collapse of the opening for a long duration pulse of critical magnitude.

Besides the compressive stress concentration due to the diffraction of the *P*-wave, there will be tensile stresses induced in the crown of the adits resulting from the combining of radial compression and tangential tension in the shock wave. This stress would produce radial cracking as shown in Figure 3*b*. At the same time, reflection of the shock wave can produce radial tensile stress and tangential cracking as shown in Figure 3*c*.

Following the *P*-wave, there is an *S*-wave generated by the air blast as shown in Figure 3*c*. The ground stresses in this shock wave being pure shear can be resolved into equal compressive and tensile stresses on planes at 45 degrees to the wave-front. The resultant compressive stress concentrations are less than those from the *P*-wave; however, the tensile stress concentrations can be greater than those from the *P*-wave.

Other effects that might cause failure around the underground openings are the vertical tension that precedes the *R*-wave and the horizontal tension that can follow the moving air blast wave across the surface of the ground. Also, the acceleration downwards induced by this air blast loading may cause a rebound at an acceleration greater than 1 g so that the surface layers actually lift off the underlying ground and leave horizontal «bounce» cracks as shown in Figure 3*d*.

The ground in which the adits were drilled was a cemented silt. The uniaxial compressive strength was found to be 7.18 ksc with a coefficient of variation of 28%. The seismic velocity of the surface layer, which extended down 6 m, was 336 m/s. When broken down the material was a soil of low plasticity with 95% of the particles smaller than the N.° 200 sieve with an average effective angle of internal friction of 27 degrees. The natural moisture content in 1964 was 11% [21]. In situ shear tests showed that the strength of the ground at a depth of 1.2 m was 0.86 of that obtained at depths from 0.6 to 0.9 m.

Table 3 shows the results of this experiment. The maximum compressive tangential stresses, $\sigma_t$, were calculated from the peak overpressures in the air blast at each station taking into account the dispersion with depth which would produce a value of 0.8 of the maximum value at the ground surface at a depth of 1.5 m. In view of this reduction in ground stress magnitude being almost identical with the reduction in strength of the ground, it was considered satisfactory to use average stress values and average strengths. The ratio of the horizontal particle velocity to the vertical particle velocity under superseismic conditions based on previous analyses was considered to be equal to 1/9, and the stresses were assumed to be in the similar ratio. The length of the shock waves varied from 40 m to 111 m.

*Table 3*

**Wall Failures in Blast Trial Adits**

| *Station* | $\sigma_t$ ksc | $\sigma_t/Q_u$ | *Probability of Failure* % | *Frequency of Cracking* % | *Frequency of Sloughing* % |
|---|---|---|---|---|---|
| A | 14.8 | 2.1 | 99+ | 90 | 55 |
| B | 13.8 | 1.9 | 99+ | 68 | 16 |
| C | 12.2 | 1.7 | 99+ | 89 | 11 |
| D | 10.4 | 1.4 | 98 | 70 | 44 |
| E | 8.9 | 1.2 | 90 | 64 | 23 |
| F | 7.7 | 1.1 | 73 | 96 | 26 |
| G | 5.7 | 0.8 | 33 | 53 | 12 |
| H | 3.7 | 0.5 | 8 | 87 | 20 |
| J | 1.8 | 0.3 | 1— | 61 | 0 |
| K | 0.9 | 0.1 | 1— | 42 | 0 |
| L | 0.5 | 0.1 | 1— | 0 | 17 |

$\sigma_t$ = maximum compressive stress in the walls; $Q_u$ = uniaxial compressive strength, 7.18 ksc.

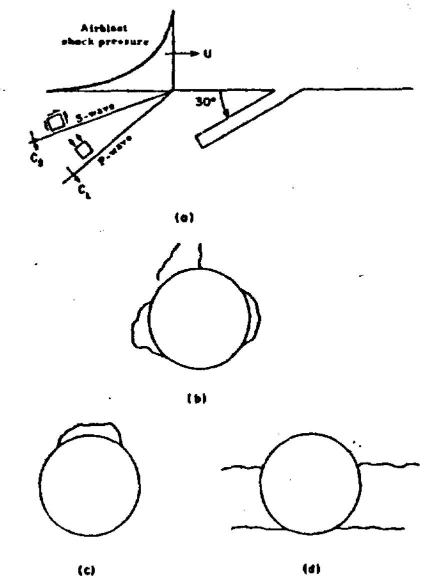
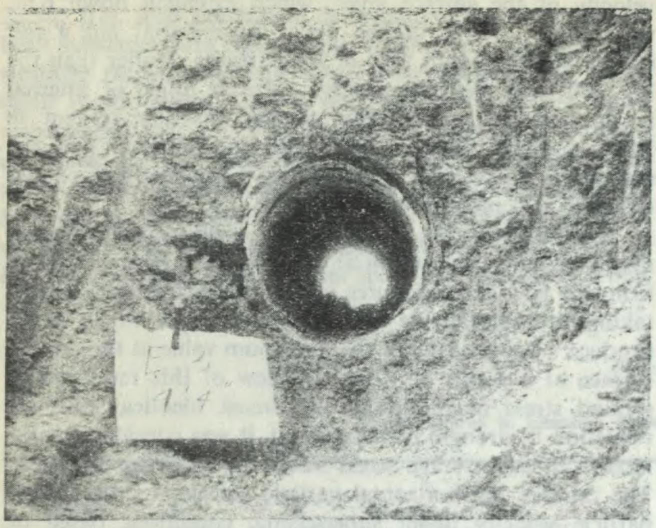


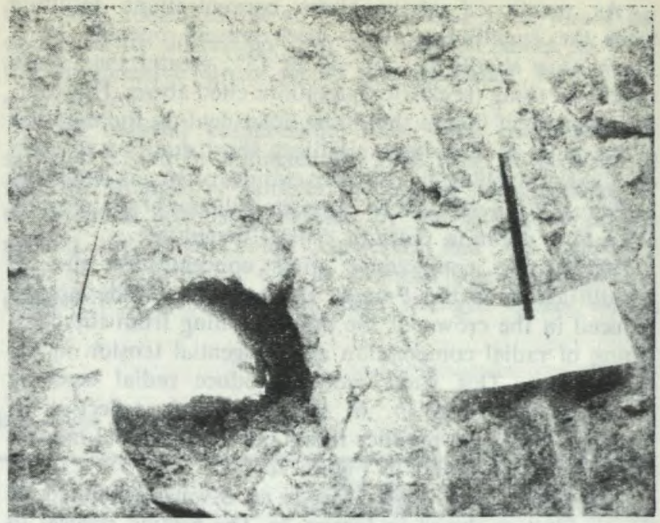

*Fig. 3 — (a) Suffield blast trial: airblast wave travelling at a velocity U initiating P-and S-waves in the ground travelling at velocities of $C_L$ and $C_S$ approaching an inclined adit. (b) Predicted failure patterns from diffraction of P-wave around the adit producing crushing failure in the sides, radial cracks in the roof and secondary tensile cracks in the shoulders as a result of crushing at the sides. (c) Predicted tangential cracking due to reflected P-wave in the roof. (d) Predicted horizontal bounce cracks from rebound of ground after airblast*

Based on the dispersion of strength values that were obtained in the testing program the probabilities of failure were calculated for each station as shown in Table 3. A statistical analysis of the post-shot observations showed the frequency of cracking to vary from station to station but only approximately in the pattern predicted. Similarly, the frequency of sloughing, or where measurements showed the horizontal diameter to be greater than the original diameter, showed a variation from station to station. A similar erratic pattern was found for the frequency of cracking resulting from tensile stress concentrations.

In addition, other types of failure associated with the other dynamic effects as explained above were observed. Figure 4 shows some of these effects on two adits. The possibility of the different effects combining at various ranges may be one reason for the lack of regularity in the frequency of failures together with the spurious effects that can occur as a result of the acceleration or vibration of the entire ground mass containing the adits.



*(a)*



*(b)*

*Fig. 4 — (a) Typical curved slabbing in the sides of the adits subjected to ground shock, resulting from compressive stress concentrations, at a depth of 1.3 m where the peak overpressure in the airblast was 3.16 ksc (45 psi); the effects of a horizontal bounce crack can also be seen to the left of the adit. (b) Extensive cracking, resulting from the combination of radial tensile stress concentrations in the roof, horizontal bounce cracking and possibly some reflected tensile cracking together with some crushing in the sides due to compressive stress concentrations, at a depth of 0.76 m where the peak overpressure in the airblast was 3.16 ksc (45 psi)*

*Sewer Tunnel*

A tunnel 3 m in diameter was excavated, as shown in Figure *5a*, 18 m below the ground surface through a stiff clay. The average density of the clay was 1.67 g/cc; the maximum depth of ground water was 3 m below the ground surface or 13.4 m above the crown of the tunnel.

An extensive testing program before excavation indicated that the average uniaxial compressive strength was 3 ksc for a short-time loading and that this could be expected to decrease by 25% for a long-time loading [22]. This testing program also showed that the deformation characteristics of the material could be represented by a rheological model consisting of a Maxwell element in series with a Kelvin element with the Maxwell spring characteristic being constant at a value of about 630 ksc, the Kelvin dashpot characteristic being constant with a coefficient of viscosity of about $10^6$ poise but with the Kelvin spring characteristic increasing with stress level.

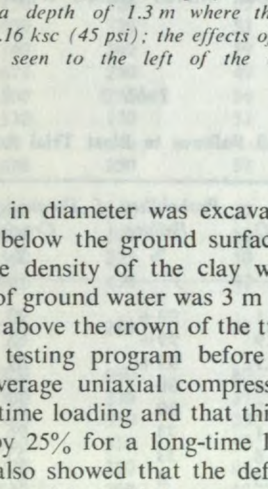



*Fig. 5 — (a) Sewer tunnel, 3 m in diameter at a depth of 18 m with the ground water level 3 m below the ground surface, during excavation with liner plate support within 1.6 m of the face. (b) Failure wedges that were developing during the test period when the compressed air pressure was reduced to zero*

In spite of the conventional stability analysis for this type of excavation giving a safety factor of more than 3, which assumes failure to consist of complete shearing throught to the ground surface [23], it was considered that, in such a relatively stiff soil, stress concentrations might be significant. Consequently, it had been recommended that the excavation should be done under compressed air to obtain the reduction in shear stresses that would be produced by the elevated internal pressure. Based on tests on samples cut from blocks from the face of the excavation, the actual uniaxial compressive strength seemed to vary between 4.7 and 7.2 ksc. Assuming Poisson's ratio for a saturated clay to be 0.5, the maximum compressive stress concentration would produce a stress of 7.68 ksc, which would mean that the ratio $\sigma_t/Q_u$ would be between 1.1 and 1.6.

To determine whether it was essential to excavate the tunnel under compressed air, a test was conducted to determine the reaction of the heading when the air pressure was decreased from 0.35 ksc to zero. Deformation measurements were taken during this experiment, but the significant finding was that on decrease of air pressure, oblique shear surfaces developed in the walls of the tunnel, as indicated in Figure *5b*, in exactly the manner that would be expected from excessive compressive stress [24].

## Conclusions

It has been shown that failure in the walls of tunnels is a stochastic process. In other words, if it is known that the maximum compressive boundary stress is equal to the known average uniaxial compressive strength, then it must be recognized that there is still only a 50% probability of failure. However, it has also been shown that it is possible in some circumstances to determine stresses and strengths sufficiently closely to be able to predict where there is a high probability of failure.

Where analyses of the mechanics satisfactorily explained the observations the volumes of rock subjected to the maximum stresses were of the same order of size as the test specimens used for strength determinations. Consequently, it should not be assumed that one can make equally accurate predictions for larger size openings where, with the increased volume subjected to high stress, the average strength of the rock mass will be less than that of the rock substance.

It has also been observed that stress concentrations resulting from shock waves will produce failures similar to static loading but, in addition, spurious rockfalls result simply from the ground motion and other modes of failure occur due to mechanisms not present under static conditions.

Finally, it must be pointed out that better testing techniques are required to determine the mechanical properties of rock masses before the stability of tunnels can be accurately assessed.

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