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A COMPUTER PROGRAM FOR CALCULATING PRINCIPAL STRESSES IN PHOTOELASTICITY

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A COMPUTER PROGRAM FOR CALCULATING PRINCIPAL STRESSES IN PHOTOELASTICITY

by

M. Gyenge[x]

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ABSTRACT

Slope stability research is being conducted to increase our knowledge of the mechanics of rock slopes -- a subject of great importance in open pit mining. Stress analysis problems, where the major active force is gravity, give rise to several special requirements when photoelastic models are being used.

The use of a computer program for the separation and computation of the principal stresses means that the stress distribution within the entire model can be obtained quickly. It saves enormous amounts of time in comparison with manual computation, and therefore the efficiency of the model testing procedure is greatly increased.

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Direction des mines

Bulletin technique TB 88

UN PROGRAMME D'ORDINATEUR POUR CALCULER LES CONTRAINTES PRINCIPALES OBTENUES PAR PHOTOÉLASTICIMÉTRIE

par

M. Gyenge*

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RÉSUMÉ

L'auteur poursuit des recherches sur la solidité des pentes afin d'accroître nos connaissances de la mécanique des pentes rocheuses. C'est un sujet d'importance considérable pour les exploitations à ciel ouvert.

Les problèmes d'analyse des contraintes, là où la force active la plus importante est la gravité, donnent lieu à plusieurs exigences spéciales lorsque des modèles photoélastiques sont utilisés.

L'usage d'un programme d'ordinateur pour la séparation et le calcul des contraintes principales signifie que la répartition des contraintes dans l'ensemble du modèle peut être obtenue rapidement. Il permet d'épargner beaucoup de temps en comparaison du calcul ordinaire et d'accroître ainsi de beaucoup l'efficacité de la méthode d'essai des modèles.

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CONTENTS

INTRODUCTION

A slope stability research project was initiated to study the mechanics of rock slopes. As the first step it was necessary to study the stress distribution in the slopes of homogeneous and isotropic materials. From among the several experimental stress analysis methods available, that of photoelastic model testing was selected, for the following reasons:

(a) With this method it is possible to obtain an overall picture of the shear-strain distribution throughout the body.

(b) It is possible to determine the stresses with the same order of accuracy in all shapes, even in irregular ones.

(c) The minimum and maximum stresses can be located qualitatively.

Within the photoelastic model test program, several simple slopes (of various geometry) are being investigated. Before regular model testing could start, however, several problems had to be resolved in detail, by means of trial test runs. The results of these trial tests have been given in Mines Branch Divisional Report FMP 67/8-MRL, issued in 1967 [1].

Because for a complete analysis it is essential to know the stress conditions on the whole area of the model, the magnitude of principal stresses must be determined at a large number of points. These calculations would involve such a large amount of computing if done manually that it was considered justifiable to write a computer program in order to increase the efficiency of the model testing procedure.

SEPARATION OF PRINCIPAL STRESSES

The photoelastic method of stress analysis furnishes us directly with two essential pieces of information, namely: (1) the difference between the principal stresses (p-q), and (2) the angle of inclination (θ) of the principal stresses to the selected axis. The method most commonly used to separate the principal stresses at any desired point within the model is the "Shear Difference Method"[2] . This method, which is based upon differential equations of equilibrium in conjunction with the data provided by the photoelastic technique, was applied in our case.

The basic forms of the equations of equilibrium, for the problem under investigation, are:

$$\frac{\partial \sigma_x}{\partial_x} + \frac{\partial \tau_{xy}}{\partial_y} = 0 \qquad \text{Eq. 1}$$

and

$$\frac{\partial \tau_{xy}}{\partial_x} + \frac{\partial \sigma_y}{\partial_y} = -\rho g \qquad \text{Eq. 2}$$

Across arbitrary straight sections, the normal stresses σ_x and σ_y can be determined by using the above equations. From Equation 1 it follows that

$$(\sigma_x)_B = (\sigma_x)_A - \int_A^B \frac{\partial \tau_{xy}}{\partial_y} dx, \qquad \text{Eq. 3}$$

where $(\sigma\ )_B$ and $(\sigma\ )_B$ denote, respectively, the stresses at points B and A.

When actually applying the "Shear Difference Method", in order to obtain the horizontal normal stresses along a straight line parallel to the x-axis, two auxiliary lines are chosen parallel to, on opposite sides of, and at a distance of $\Delta y/2$ from, the line of stress computation. The horizontal shear stresses, τ_{xy}, are first determined along each of these auxiliary lines; also, their difference, $\Delta\tau_{xy}/\Delta y$, is calculated at the intermediate points at Δx distance apart.

Provided that σ_x at point A is known, the horizontal normal stress at point B can be calculated by summation of the $\frac{\Delta\tau_{xy}}{\Delta y}\Delta x$ quantities between these two points, thus:

$$(\sigma_x)_B = (\sigma_x)_A - \sum_{n=1}^{n=i} \frac{(\Delta\tau_{xy})_i}{(\Delta y)_i} (\Delta x)_i \qquad \text{Eq. 4}$$

At points along the unloaded boundary of the model, the difference between the two principal stresses completely defines the stress condition. Here the principal stress perpendicular to the boundary is zero, while the other one acts in the direction of the boundary, its magnitude being equal to the difference (p-q). With the principal stresses known, the horizontal normal stress, σ_x, can also be calculated for the point on the boundary. Therefore, it is usual to choose a point on the free boundary, where the stress conditions are known, as the starting point of integration.

The shear stresses for every point of the model are calculated from the photoelastic data by using the equation:

$$\tau_{xy} = \frac{p - q}{2} \sin 2\theta \qquad \text{Eq. 5}$$

τxy may be positive or negative, depending on the sign of θ, as the (p-q) quantities are always positive.

Knowing the horizontal normal (σ_x) and the shear (τ_{xy}) stresses, as well as the direction of the principal stress (θ). we can calculate the vertical normal stress (σ_y) and then the major (p) and minor (q) principal stresses. All these calculations are based on the following relationships between the stresses, as represented by Mohr's circle:

$$\sigma_y = \sigma_x - (p - q) \cos 2\theta \qquad \text{Eq. 6}$$

and

$$\left.\begin{aligned} p &= \frac{\sigma_x + \sigma_y}{2} + \tau_{max} \\ q &= \frac{\sigma_x + \sigma_y}{2} - \tau_{max} \end{aligned}\right\} \qquad \text{Eq. 7}$$

Obviously, the integration could be performed along either of the straight lines parallel to the y-axis. Similar sets of equations would then be obtained from Equation 2. However, in our case it was more convenient to integrate in a horizontal direction.

THE COMPUTER PROGRAM

The program was written for the CDC 3100 system computer, and for the attached Calcomp 750 magnetic tape plotter. The programming was done in the most general form possible, since the program will be used over and over again to separate the principal stresses obtained in a number of different types of photoelastic models. Subroutine programs were also used to increase the flexibility of the program.

The entire model area was evenly divided by L horizontal and M vertical lines. The grid points were taken at the points of intersection of these lines. The computer was supplied with input data for these grid points. These input data -- namely the difference of the principal stresses (p - q) and the direction of the principal stresses (θ)-- were obtained by means of the photoelastic tests. The computer program was then executed. The computer presented the results in both written and plotted forms, to allow for further studies.

The equations used for separating the principal stresses had to be rearranged into forms which were better suited for use in the computer. Following were the necessary steps:

Another form for Equation 4, the expression for the horizontal normal stress, is:

$$\sigma_x = \sigma_{xo} - \Sigma\left(\frac{\Delta x}{\Delta y}\right)\Delta\tau_{xy} \qquad \text{Eq. 4a}$$

Since the integration is always started at the left-hand boundary of the model (where the value of σ_{xo} is zero), it follows that:

$$\sigma_x = -\Sigma\left(\frac{\Delta x}{\Delta y}\right)\Delta\tau_{xy} \qquad \text{Eq. 4b}$$

or,

$$\sigma_x = -\left(\frac{\Delta x}{\Delta y}\right)\Sigma\,\Delta\tau_{xy} \qquad \text{Eq. 4c}$$

As the sign of Δy is negative in the chosen coordinate system, the ratio of $\Delta x/\Delta y$ is always a negative quantity; it is also a constant within the model. Therefore,

$$\sigma_x = c\,\Sigma\,\Delta\tau_{xy} \qquad \text{Eq. 4d}$$

The vertical normal stress is calculated from the equation:

$$\sigma_y = \sigma_x - (p - q)\cos 2\theta \qquad \text{Eq. 6}$$

or, on substitution,

$$\sigma_y = c\,\Sigma\,\Delta\tau_{xy} - (p - q)\cos 2\theta \qquad \text{Eq. 6a}$$

The maximum and minimum principal stresses, respectively, are given by the equation:

$$\left.\begin{matrix} p \\ q \end{matrix}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \tau_{max} \qquad \text{Eq. 7}$$

or, on substitution,

$$\left.\begin{matrix} p \\ q \end{matrix}\right\} = \frac{c\,\Sigma\Delta\tau_{xy} + c\,\Sigma\Delta\tau_{xy}}{2} - \frac{(p - q)\cos 2\theta}{2} \pm \frac{p - q}{2} \qquad \text{Eq. 7a}$$

Also,

$$\left.\begin{matrix} p \\ q \end{matrix}\right\} = c\Sigma\Delta\tau_{xy} - \frac{p-q}{2}\cos 2\theta \pm \frac{p-q}{2} \qquad \text{Eq. 7b}$$

Using these expressions, the values of the principal stresses at a grid point (in the j^{th} column and i^{th} row) are:

$$\left.\begin{matrix} p_{i,j} \\ q_{i,j} \end{matrix}\right\} = c \sum_{1}^{j-i} \Delta\tau_{xy} + \left[(\tau_{xy})_{i-1,j-i} - (\tau_{xy})_{i+1,i-1}\right] - \left(\frac{p-q}{2}\cos 2\theta\right)_{i,j} \pm \left(\frac{p-q}{2}\right)_{i,j}$$

Eq. 8

where

$$(\tau_{xy})_{i-1,j-1} = \left(\frac{p-q}{2}\sin 2\theta\right)_{i-1,j-1}$$

and

$$(\tau_{xy})_{i+1,j-1} = \left(\frac{p-q}{2}\sin 2\theta\right)_{i+1,j-1} \qquad \text{Eq. 9}$$

Let in Equation 7b

$$R = c\Sigma\Delta\tau_{xy} - \left(\frac{p-q}{2}\cos 2\theta\right) \qquad \text{Eq. 10}$$

The expressions for the maximum and minimum principal stresses, respectively, are

$$p = R + \frac{p-q}{2}$$

and

$$q = R - \frac{p-q}{2} \qquad \text{Eq. 11}$$

The computer program is separated into four parts, namely, a main program and the input, result and plotr subroutines.

(a) Main Program

```
                                        3200  FORTRAN   (2.1)    16/01/67
      PROGRAM D45000
      COMMON PMQ(25,60),THETA(25,60)
      DIMENSION GAMMA(25)
      READ(60,1)L,M,CONST1,CONST2
      READ(60,2)(GAMMA(I),I=1,L)
      DO 10 I=1,L
      DO 10 J=1,M
      PMQ(I,J)=0.0
   10 THETA(I,J)=0.0
      X=1.0
      CALL INPUT(PMQ,L,M,X)
      X=0.03490658
      CALL INPUT(THETA,L,M,X)
      CALL RESULT(L,M,CONST1)
      CALL PLOTR(GAMMA,D,L,M,CONST1,CONST2)
      STOP
C
    1 FORMAT(2I5,2F10.5)
    2 FORMAT(8F10.3)
      END

        3200 FORTRAN DIAGNOSTIC RESULTS - FOR    D45000

NO ERRORS
```

This part of the program sets up two two-dimensional arrays of input data for (p-q) and for θ. It reads the value of the initial stress field (GAMMA) for each row, the maximum number of rows (L) and columns (M), the value of $\Delta x/\Delta y$ ratio (CONST 1), and the fringe value of the model (CONST 2). Finally, it calls for the subroutine programs.

By means of the Input subroutine program the photoelastic model test data -- namely the (p-q) and θ values at the grid points -- are stored in those two-dimensional arrays which have been reserved by the Main Program. By its structure this subroutine provides assurance on the correct storage of the input data. The program will not be executed if by error (1) the input data cards contain higher row and/or column numbers than are specified in the Main Program, (2) the input data cards are not in the right order, (3) the row and/or column number is duplicated.

(b) Subroutine Program, Input

```
                                    3200   FORTRAN    (2.1)    16/01/67
      SUBROUTINE INPUT (A,L,M,X)
      DIMENSION A(25,60),T(10)
    5 READ(60,1)I,J,T
      WRITE(61,7)I,J,T
    7 FORMAT(1H ,2I5,5X,10F10.2)
      IF(I)100,100,10
   10 IF(L-I)90,20,20
   20 IF(M-J)95,30,30
   30 S=0.0
      KM=J+9
      IF(M-KM)35,38,38
   35 KM=M
   38 CONTINUE
      DO 40 K=J,KM
   40 S=S+ABSF(A(I,K))
      IF(S)99,50,99
   50 KM=M-J+1
      IF(KM/10)60,60,55
   55 KM=10
   60 DO 70 K=1,KM
      A(I,J)=X*T(K)
   70 J=J+1
      GO TO 5
   90 WRITE(61,2)I
      GO TO 110
   95 WRITE(61,3)J
      GO TO 110
   99 WRITE(61,4)I,J,S
      GO TO 110
  100 RETURN
  110 STOP
C
    1 FORMAT(2I5,10F5.0)
    2 FORMAT(4H0I = I5)
    3 FORMAT(4H0J = I5)
    4 FORMAT(7H0AT I = I5,5X,7HAND J = I5,5X,5HSUM = F10.5)
      END

        3200 FORTRAN DIAGNOSTIC RESULTS - FOR    INPUT

NO ERRORS
```

(c) Subroutine Program, Result

```
                                  3200   FORTRAN     (2.1)            16/01/67

      SUBROUTINE RESULT(L.M.CONST)
      COMMON A(25,60),B(25,60)
      L1=L-1
      M1=M-1
      DO 10 I=2.L1,2
      S=0.0
      DO 10 J=2,M1,2
      DELT=0.5*(A(I-1,J-1)*SINF(B(I-1,J-1))-A(I+1.J-1)*SINF(B(I+1,J-1)))
      S=S+DELT
   10 B(I.J)=CONST*S-0.5*A(I.J)*COSF(B(I,J))
      RETURN
      END

        3200 FORTRAN DIAGNOSTIC RESULTS - FOR   RESULT
```

Using the input information, this subroutine computes only the R values (as given by Equation 10) at the grid points that bear even subscript numbers, because the rows with odd numbers are regarded as the auxiliary lines required by the "Shear Difference Method".

(d) Subroutine Program, Plotr

```
                                  3200   FORTRAN     (2.1)            16/01/67

      SUBROUTINE PLOTR(C,D,L,M,C1,C2)
      COMMON A(25,60),B(25,60)
      DIMENSION S1(3),S2(3).C(25),D(60)
      L1=L-1
      M1=M-1
      YL=L
      YL=0.5*YL
      XM=M
      XM=0.5*XM
      XRED=C1*XM
      S1(1)=1.0
      S1(2)=1.0
      S2(1)=0.0
      S2(2)=1.0
      S2(3)=0.0
      SC=-0.5
      DO 30 N=1.2
      DO 20 KK =1,3
      CALL AXISXY(01,XRED.YL,0.5,XM,YL.0.0,0.0,0.0,0.0,0.5)
      DO 40 I=1,L
```

```
      AI=I*0.5
      CALL PLOTXY(-1.0,AI,0,0)
      ENCODE(3,1,FF) I
      CALL LABEL(3,1,0,FF)
   40 CONTINUE
      DO 50 I=1,M
      AI=I*0.5-0.3
      CALL PLOTXY(AI,-0.5,0,0)
      ENCODE(3,1,FF) I
      CALL LABEL(3,1,0,FF)
   50 CONTINUE
      DO 15 I=2,L1,2
      Y=I
      Y=0.5*Y
      S1(3)=1.0/C(I)
      DO 10 J=2,M1,2
      X=J
      X=0.5*X
      FELRAK=0.0
      IF(A(I,J))60,70,60
   60 FELRAK=C2*(S1(KK)*(B(I,J)+SC*A(I,J)))+S2(KK)*C(I)
   70 D(J)=FELRAK
      CALL PLOTXY(X,Y,0,12)
      ENCODE(7,2,FF) FELRAK
      CALL PLOTXY(X,Y,0,0)
   10 CALL LABEL(7,1,3,FF)
      WRITE(61,3)
   15 WRITE(61,4) I,(D(K),K=2,M1,2)
   20 CALL ENDPLOT(01)
      SC=-SC
   30 CONTINUE
      RETURN
C
    1 FORMAT(I3)
    2 FORMAT(F7.3)
    3 FORMAT(1H0,17X,1H2,9X,1H4,9X,1H6,9X,1H8,8X,2H10,8X,2H12,8X,2H14,
     18X,2H16,8X,2H18,8X,2H20)
    4 FORMAT(1H0,I5,5X,10F10.3/(1H0,10X,10F10.3))
      END
```

By the use of the appropriate constant values (S1, S2 and SC), this part of the program writes out, as well as plots, the computed values of (1) minor principal excavation stresses, (2) minor principal resultant stresses, (3) ratio of minor principal excavation stresses over initial stresses, (4) major principal excavation stresses, (5) major principal resultant stresses, and (6) ratio of major principal excavation stresses over initial stresses.

The program is being used successfully for calculating the principal stresses in open-pit photoelastic models. For example, in Table 1 the major principal excavation stresses (p), for part of a model, are given in print-out form, while in Figure 1 the same results are shown, for one-half of a model, as they are plotted by the computer.

REFERENCES

1. Gyenge, M., and Coates D.F., "Development of Stress Analysis by Photoelasticity for Slopes", Divisional Report FMP 67/8-MRL, Mines Branch, Ottawa (1967).

2. Frocht, M.M., "Photoelasticity", Vol. 1, John Wiley and Sons, Inc., New York, 1941.

MG:(PES)DV.

TABLE 1

Major Principal Excavation Stresses, in Print-Out Form

	2	4	6	8	10	12	14	16	18	20
12	.001	.005	.016	.032	.049	.066	.079	.107	.126	.151
	.176	.201	.231	.269	.317	.364	.408	.476	.567	.661
	.816	1.062	1.382	1.909	2.902	3.728				
	2	4	6	8	10	12	14	16	18	20
14	.000	-0.002	-0.004	-0.001	.007	.024	.044	.070	.091	.111
	.133	.155	.180	.223	.269	.310	.362	.417	.497	.640
	.721	.798	.812	.683	0	0				
	2	4	6	8	10	12	14	16	18	20
16	.000	.002	.001	-0.002	-0.006	-0.006	-0.000	.007	.021	.036
	.053	.081	.095	.108	.114	.142	.133	.129	.086	.041
	-0.094	-0.345	-1.218	0	0	0				
	2	4	6	8	10	12	14	16	18	20
18	.000	-0.003	-0.009	-0.017	-0.023	-0.024	-0.020	-0.009	.010	.031
	.055	.077	.082	.091	.090	.071	.038	-0.009	-0.096	-0.237
	-0.494	-1.112	0	0	0	0				
	2	4	6	8	10	12	14	16	18	20
20	.000	-0.005	-0.011	-0.021	-0.035	-0.054	-0.068	-0.077	-0.086	-0.085
	-0.092	-0.087	-0.085	-0.099	-0.116	-0.150	-0.178	-0.212	-0.293	-0.483
	-0.928	0	0	0	0	0				

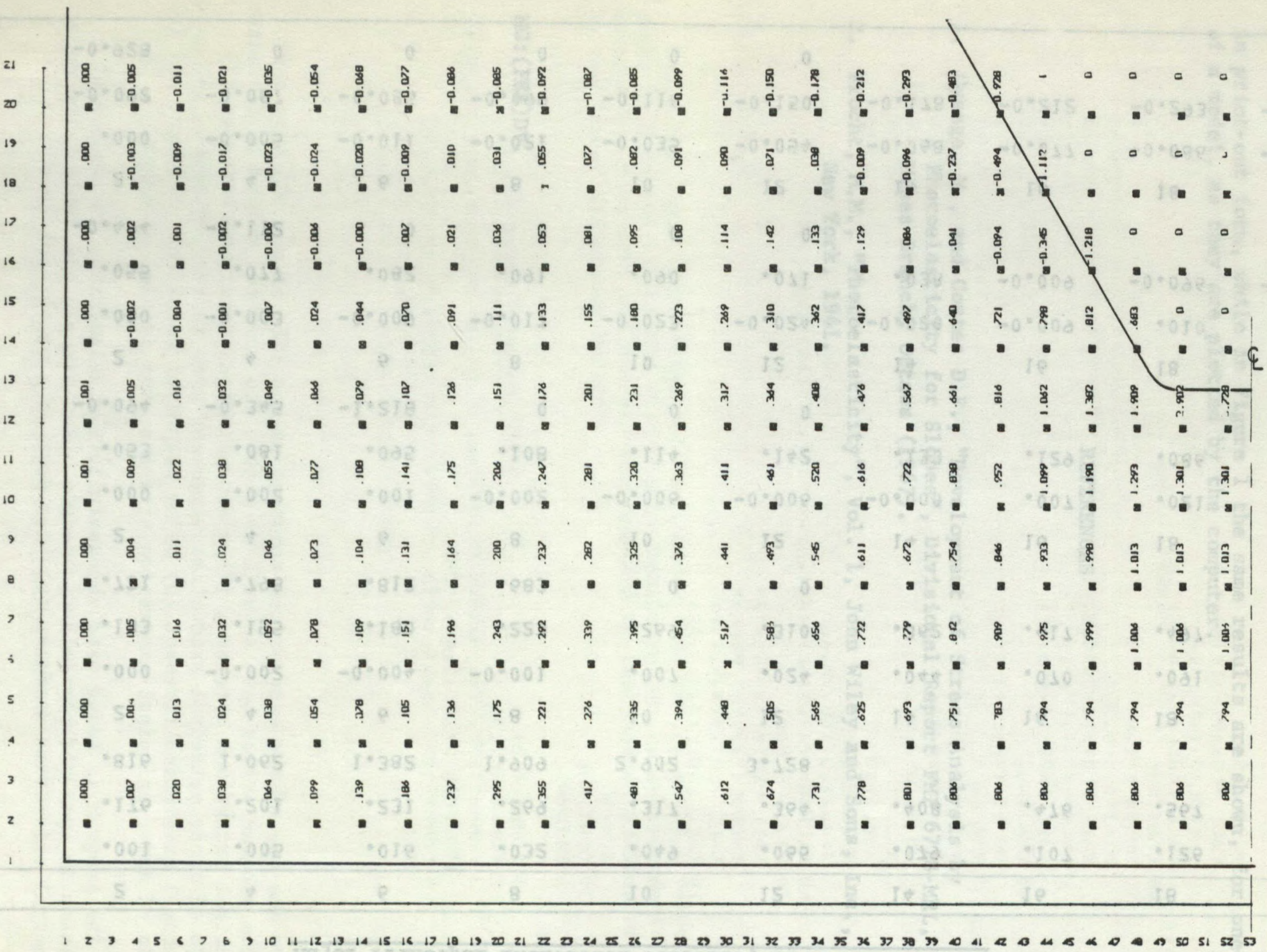

Figure I - Major Principal Excavation Stresses (p), Plotted by Computer