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# THE USE OF A PULSE-HEIGHT ANALYSER AS A CURVE PLOTTER

by

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## SYNOPSIS

A method of employing a hundred-channel analyser to produce, graphically, a family of error function complement curves is described. By matching these curves with experimental results obtained in diffusion studies the diffusion coefficient may be rapidly ascertained.

## RÉSUMÉ

Le présent bulletin décrit un procédé qui implique l'emploi d'un analyseur à cent bandes pour produire graphiquement une famille de courbes de fonctions d'erreurs complémentaires. En superposant le graphique des courbes susmentionnées et celui que l'on a obtenu lors des études de diffusion, on peut en tirer rapidement le coefficient de diffusion.

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## INTRODUCTION

The determination of diffusion coefficients, even with the aid of a computer, is a rather tedious process in those cases where the boundary conditions result in error function complement curves and when the initial surface concentration $C_o$ is not known. It is the purpose of this report to draw attention to a simple analogue method of plotting a series of error function complement curves from which, in a particular experiment, both $C_o$ and the diffusion coefficient D can be obtained by curve matching. Results can be obtained to any desired accuracy by producing a sufficiently large number of the curves and may be used to provide starting values for computer programming.

The solution of the diffusion equation for extended initial distributions is given by

$$C(x,t) = \frac{C_o}{2} \operatorname{erfc} \frac{x}{\sqrt{4Dt}}, \qquad \text{(Eq 1)}$$

where C (x, t) is the concentration at penetration x and anneal time t, $C_o$ is the initial concentration for $x < 0$, and D is the diffusion coefficient (1). The concentration-penetration relation may be displayed in a number of ways, but for convenience we have chosen to plot $\log_{10} C$ against $x^2$. This is common practice in many laboratories, because of the linear relationship that exists between these variables when diffusion takes place from an instantaneous plane source.

The series of curves has been produced employing a hundred-channel analyser* in conjunction with an X-Y recorder**. Convenient values of $\frac{x^2}{4Dt}$, in the range 0.1 to 4.8, were chosen, for which the corresponding ratio $^C/C_o$ was obtained from Equation 1. The logarithm (to base ten) of this ratio was adjusted in powers of ten and entered into the memory of each channel corresponding to a particular value of $\frac{x^2}{4Dt}$. A sufficient fraction of the hundred channels available was loaded so that a smooth curve could be drawn through the points printed out on the X-Y recorder. The Y-amplifier of the recorder was adjusted so that the counts contained in Channel 1 corresponded to the correct ratio $^C/C_o$, i.e., 0.5 for $\frac{x^2}{4Dt} \approx 0$, when plotted on the appropriate graph paper. The values employed to load the channels are given in Table 1. To produce a series of such curves corresponding to different values of Dt, it is only necessary to vary the writing speed of the recorder pen in the X-direction. The family of curves has been drawn on 3-cycle semi-logarithmic paper***, this number of decades being sufficient for most cases.

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*Computing Devices of Canada Ltd., Type AEP 2230.

**F.L. Moseley Co. Model 2D.

***Keuffel and Esser Co. No. 359-71LG.

TABLE 1

Numerical Values Determined from Equation 1, for Loading into the Analyser Channels

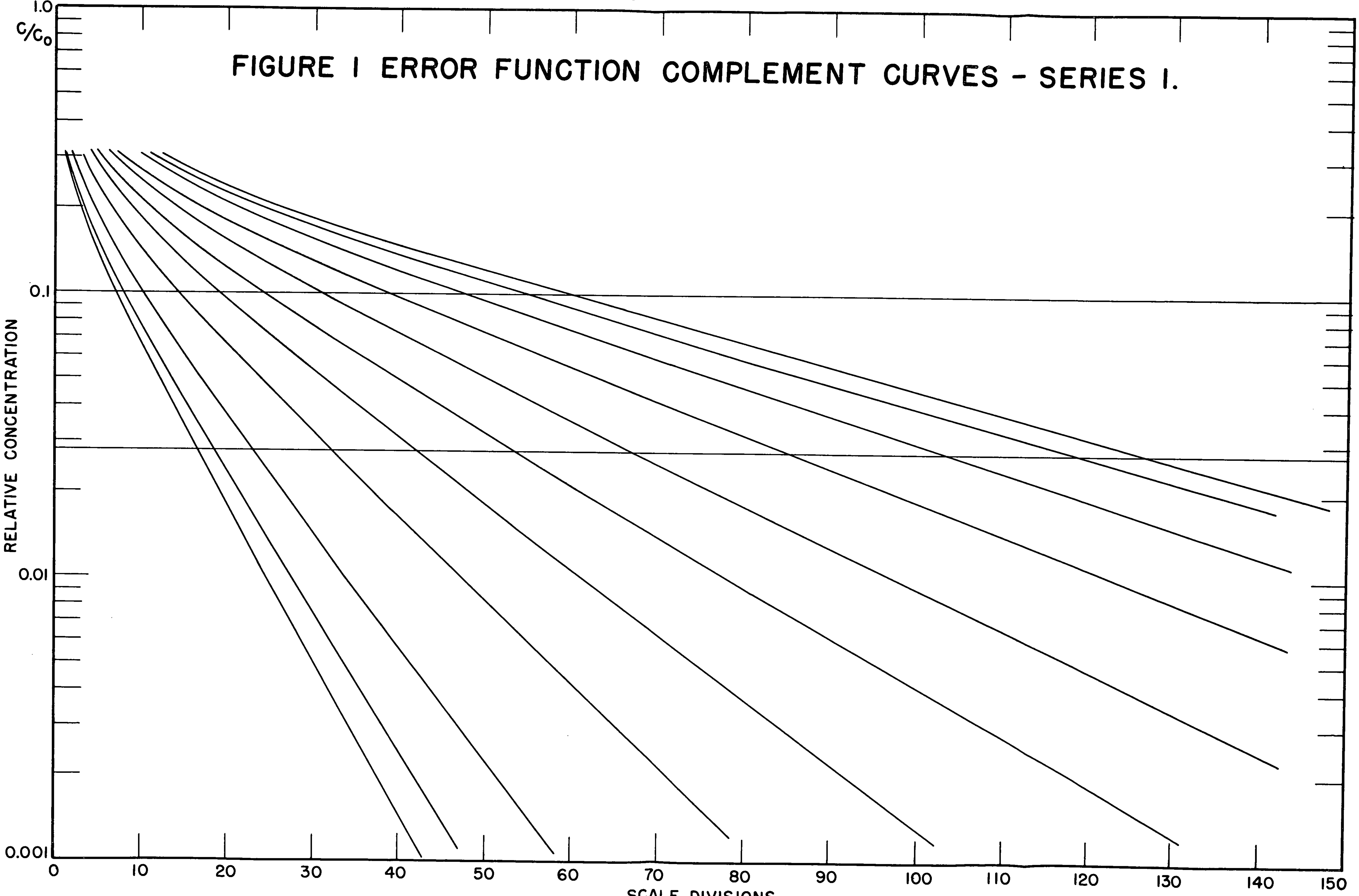
| $\frac{x^2}{4Dt}$ | $\frac{x}{\sqrt{4Dt}}$ | erfc $\frac{x}{\sqrt{4Dt}}$ x $10^3$ | $C/C_o$ x $10^3$ | $\log_{10} C/C_o$ x $10^2$ | Channel No. |
|---|---|---|---|---|---|
| 0 | 0 | 1000 | 500 | 270 | 1 |
| 0.1 | 0.3162 | 654.7 | 327.4 | 252 | 2 |
| 0.15 | 0.3873 | 583.9 | 292.0 | 247 | 3 |
| 0.2 | 0.4472 | 527.1 | 263.6 | 242 | 4 |
| 0.25 | 0.5000 | 479.5 | 239.8 | 238 | 5 |
| 0.3 | 0.5477 | 438.6 | 219.3 | 234 | 6 |
| 0.35 | 0.5916 | 402.8 | 201.4 | 230 | 7 |
| 0.4 | 0.6325 | 371.1 | 185.6 | 227 | 8 |
| 0.45 | 0.6708 | 342.8 | 171.4 | 223 | 9 |
| 0.5 | 0.7071 | 317.3 | 158.7 | 220 | 10 |
| 0.55 | 0.7416 | 294.3 | 147.2 | 217 | 11 |
| 0.6 | 0.7746 | 273.3 | 136.7 | 214 | 12 |
| 0.65 | 0.8062 | 254.2 | 127.1 | 210 | 13 |
| 0.7 | 0.8367 | 236.7 | 118.4 | 207 | 14 |
| 0.75 | 0.8660 | 220.7 | 110.4 | 204 | 15 |
| 0.8 | 0.8944 | 205.9 | 103.0 | 201 | 16 |
| 0.85 | 0.9220 | 192.3 | 96.2 | 198 | 17 |
| 0.9 | 0.9487 | 179.7 | 89.9 | 195 | 18 |
| 0.95 | 0.9747 | 168.1 | 84.1 | 192 | 19 |
| 1.0 | 1.0000 | 157.3 | 78.7 | 190 | 20 |
| 1.2 | 1.095 | 121.5 | 60.8 | 178 | 24 |
| 1.4 | 1.183 | 94.3 | 47.2 | 167 | 28 |
| 1.6 | 1.265 | 73.6 | 36.8 | 157 | 32 |
| 1.8 | 1.342 | 57.7 | 28.9 | 146 | 36 |
| 2.0 | 1.414 | 45.5 | 22.8 | 136 | 40 |
| 2.2 | 1.483 | 36.0 | 18.0 | 126 | 44 |
| 2.4 | 1.549 | 28.5 | 14.3 | 116 | 48 |
| 2.6 | 1.613 | 22.5 | 11.3 | 105 | 52 |
| 2.8 | 1.673 | 18.0 | 9.0 | 95 | 56 |
| 3.0 | 1.732 | 14.3 | 7.2 | 86 | 60 |
| 3.2 | 1.789 | 11.4 | 5.7 | 76 | 64 |
| 3.4 | 1.844 | 9.1 | 4.6 | 66 | 68 |
| 3.6 | 1.897 | 7.3 | 3.7 | 57 | 72 |
| 3.8 | 1.949 | 5.8 | 2.9 | 46 | 76 |
| 4.0 | 2.000 | 4.7 | 2.4 | 38 | 80 |
| 4.2 | 2.049 | 3.8 | 1.9 | 28 | 84 |
| 4.4 | 2.098 | 3.0 | 1.5 | 18 | 88 |
| 4.6 | 2.145 | 2.4 | 1.2 | 8 | 92 |
| 4.8 | 2.191 | 1.9 | 1.0 | 0 | 96 |

In order to facilitate calculation of Dt, two lines parallel to the x-axis were drawn through the family of curves so that the separation between lines corresponded to $\frac{x^2}{4Dt} = 1$. For any given curve, the value of the intercept on the x-axis between these two lines is equal to 4 Dt. In practice, the curves have been drawn on transparent graph paper or tracing paper, and the tracing paper is placed above the experimental points with the ordinate axes superimposed. The tracing paper may then be moved up or down until the points fall on one of the plotted curves. The projection of the intercept between the horizontal lines on the family is read on the abscissa of the experimental graph, which gives the value of the corresponding 4 Dt. With the anneal time t known, D is readily calculated. The value of $C_o$ may be obtained from y-intercept on the experimental graph.

There are obviously an infinite number of curves in this family and it is not practical to attempt to reproduce more than an adequate fraction. We have found that twenty is a sufficient number of curves to permit an estimate of D and $C_o$ within 10%, and in most cases within 5%. In this particular application, once a family of curves has been obtained, there is no further need of the hundred-channel analyser. However, there is no reason why a similar process cannot be adapted to any curve-fitting problem, when the family to which a particular curve belongs is known.

The set of curves employed in this laboratory for determining D and $C_o$ is shown in Figure 1 and Figure 2. For convenience, the set has been divided into series 1 and series 2. The experimental curve to be matched is compared with the curves of the appropriate series.

FIGURE I ERROR FUNCTION COMPLEMENT CURVES - SERIES I.



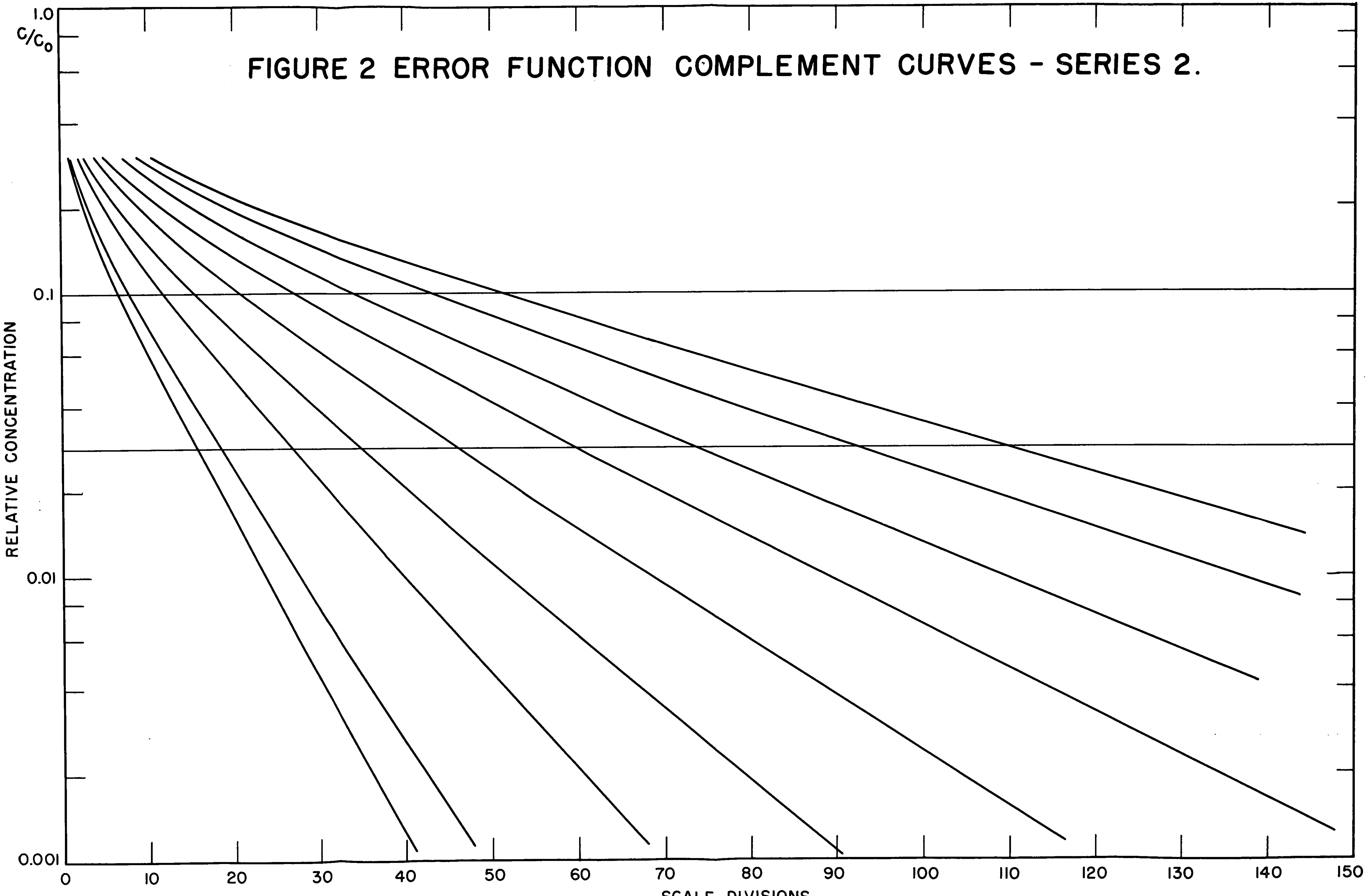


FIGURE 2 ERROR FUNCTION COMPLEMENT CURVES - SERIES 2.
1.0
C/C0
0.1
0.01
0.001
RELATIVE CONCENTRATION
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
SCALE DIVISIONS

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REFERENCE

1. J. Crank, "The Mathematics of Diffusion", Oxford University Press, London, p. 12 (1956).

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