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Extreme river flow prediction for river water supply to oil sands mining sites, Athabasca River near Fort McMurray, Alberta

G. Chen and Z. Chen

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Extreme river flow prediction for river water supply to oil sands mining sites, Athabasca River near Fort McMurray, Alberta

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Publications in this series have not been edited; they are released as submitted by the author.

Summary

This report has been prepared at the request of Geological Survey of Canada (GSC). The specific instructions from GSC asked to develop the following deliverables:

1. Computer R source code in digital format;
2. Prediction results of extreme values of annual river flow rates (annual minimum and maximum)
3. A report that describes the methods and application to the Athabasca River near Fort McMurray.
4. The work is to be completed and submitted by January 15, 2011.

This report meets and exceeds the above requirements.

1 Introduction

The Geological Survey of Canada is undertaking a study on future water supply of the Athabasca River in the Fort McMurray region under the Climate Change Science program to investigate impacts of climate change on river flow rate. Although various methods are applied to the predictions of river flow rate, none of the methods have the capacity to estimate the extremes of river flow (particularly the annual low) incorporating different climate change scenarios, which is essential for sustainable water supply under a changing climate. In order to accomplish the objectives set by the ESS Climate Change Science program, external assistance is needed to conduct a study on methodology for prediction of extreme river flow rate considering different climate change scenarios.

In the following sections we address the above issue.

2 Data Description

The Geological Survey of Canada has provided three raw data sets: the monthly flow rates of the Athabasca River from 1957 to 2008; the monthly precipitations at Fort McMurray from 1916 to 2005; and the monthly average temperatures at Fort McMurray from 1908 to 2008. The temperature data set consists of three sub sets, namely, the monthly average temperatures, the monthly average minimum temperatures, and the monthly average maximum temperatures.

We are mainly concerned with the maximum and minimum flow rates. We also want to employ the precipitation and temperature data sets to help predict the future maximum and minimum flow rates. However, due to missing/incomplete data entries in the three raw data sets, the portion of the data that are complete include the maximum and minimum flow rates from 1957 to 2008, the maximum and minimum precipitations from 1924 to 2004, and the maximum and minimum temperatures from 1909 to 2007. These are the data sets to be analyzed in this report. Figure 1 to Figure 3 display these data sets.

3 A Methodology for Modeling Maximum and Minimum Series

The series to be analyzed in this report are annual maximums and annual minimums. We shall use models based on the *generalized extreme value* (GEV) distributions to analyze them. This choice is made because the popular models based on the Gumbel, Fréchet and

Figure 1: Athabasca River annual minimum and maximum precipitations from 1924 to 2004.

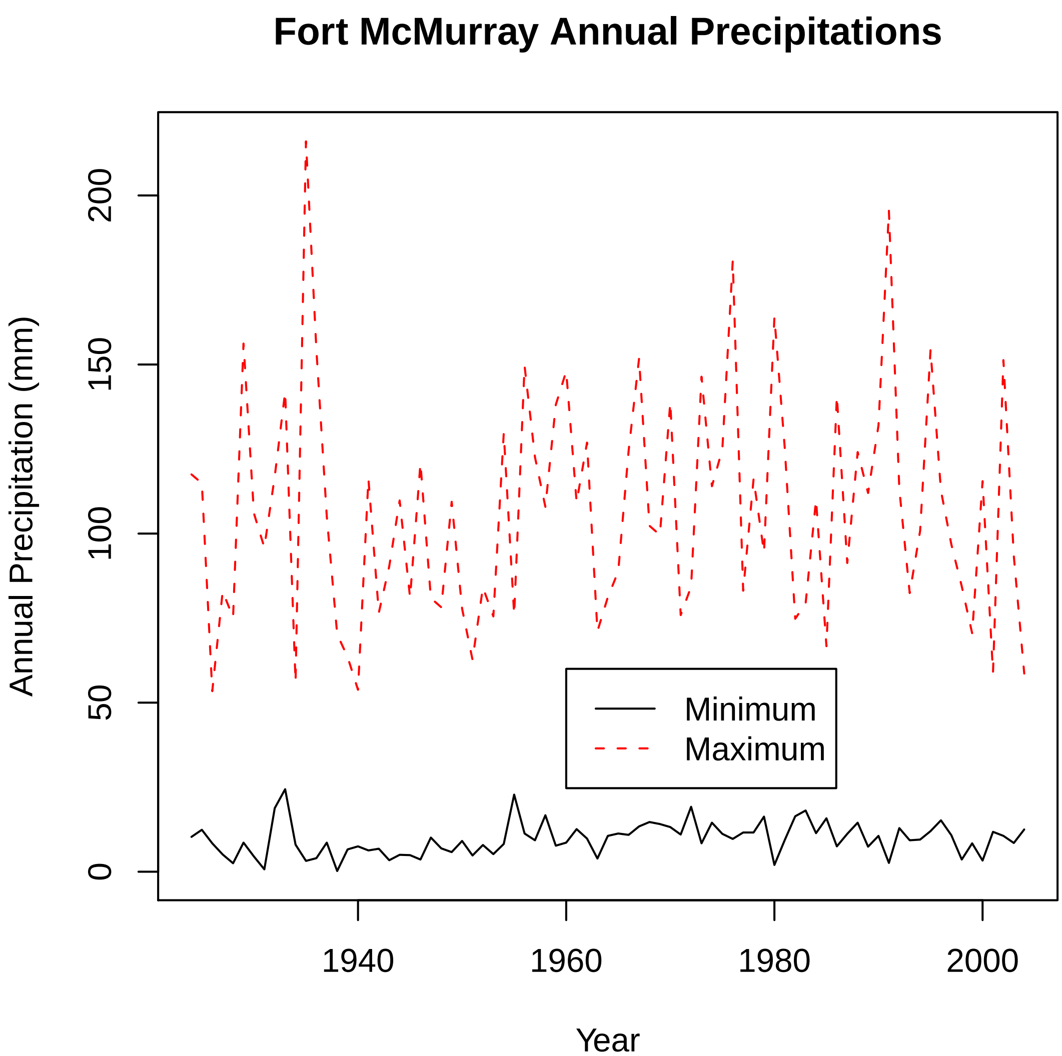


Figure 2: Fort McMurray annual minimum and maximum flow rates from 1957 to 2008.

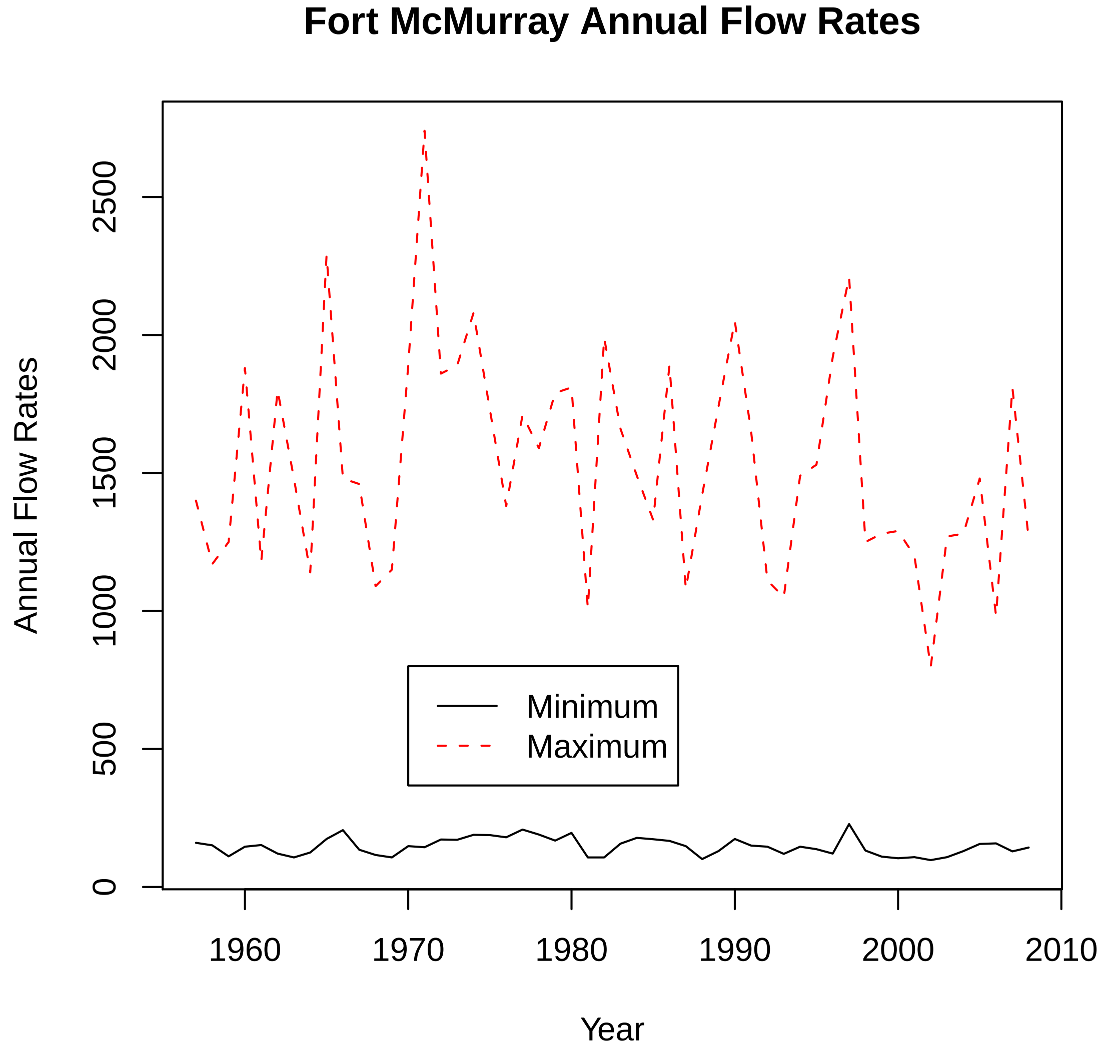
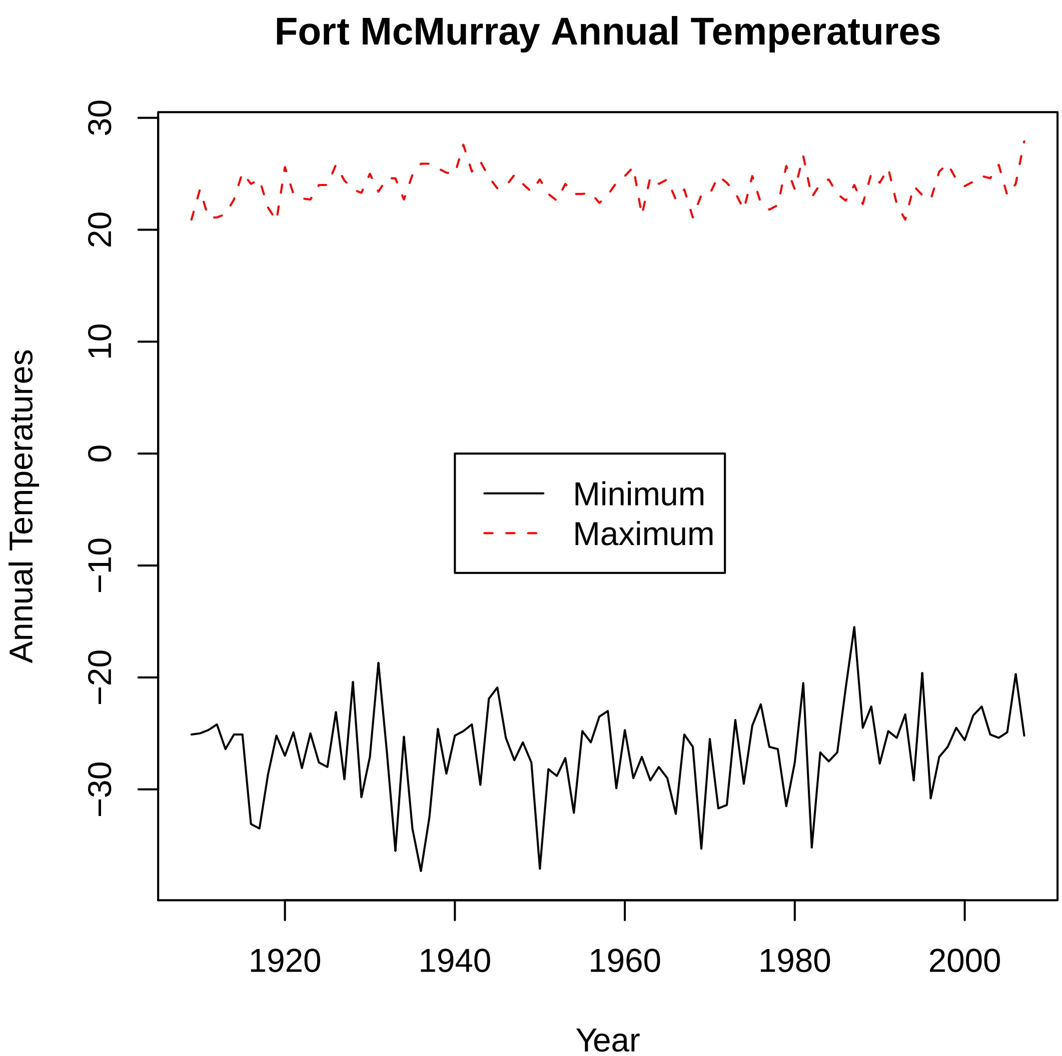


Figure 3: Fort McMurray annual minimum and maximum temperatures from 1909 to 2007.



Weibull distributions are all included in the GEV family of distributions as special cases.

A random variable X has a generalized extreme value distribution for the maximum case, denoted $GEV_{max}(\mu\,\sigma, \xi)$, if the distribution function of X is

$$G(x) = \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\},$$

where $-\infty < \mu < \infty$ is the location parameter, $\sigma > 0$ is the scale parameter, $-\infty < \xi < \infty$ is the shape parameter, and $x \in \{x:\ 1+\xi(x-\mu)/\sigma > 0\}$. The mean of X is given by

$$E(X) = \mu + \frac{\sigma}{\xi}[\Gamma(1-\xi) - 1],$$

where $\Gamma(\cdot)$ is the Gamma function.

Among the various parameter estimation methods (graphical, moment, order statistics, maximum likelihood), we shall use the maximum likelihood method for its all-round utility and adaptability to complex model building. Under the assumption that $x_1, ..., x_n$ is a random sample from $GEV_{max}(\mu, \sigma, \xi)$, the log-likelihood function for μ, σ and ξ when $\xi \neq 0$ is

$$\begin{aligned} l(\mu,\sigma,\xi) &= -n\log(\sigma) - (1+1/\xi)\sum_{i=1}^{n}\log\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right] \\ &\quad -\sum_{i=1}^{n}\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{-1/\xi},\ \text{if } 1+\xi\left(\frac{x_i-\mu}{\sigma}\right) > 0,\ i = 1, ..., n. \end{aligned}$$

When $\xi = 0$, the log-likelihood function is

$$l(\mu,\sigma,0) = -n\log(\sigma) - \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right) - \sum_{i=1}^{n}\exp\left\{-\left(\frac{x_i-\mu}{\sigma}\right)\right\}.$$

If for any i there is

$$1+\xi\left(\frac{x_i-\mu}{\sigma}\right) \leq 0,$$

the log-likelihood function equals $-\infty$ because the value of the density is 0.

Correspondingly, a random variable X has a generalized extreme value distribution for the minimum case, denoted $GEV_{min}(\mu\,\sigma, \xi)$, if the distribution function of X is

$$G(x) = 1 - \exp\left\{-\left[1-\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\},$$

where $-\infty < \mu < \infty$ is the location parameter, $\sigma > 0$ is the scale parameter, $-\infty < \xi < \infty$ is the shape parameter, and $x \in \{x:\ 1-\xi(x-\mu)/\sigma > 0\}$. The mean of X is given by

$$E(X) = \mu + \frac{\sigma}{\xi}[1 - \Gamma(1-\xi)].$$

Under the assumption that $x_1, ..., x_n$ is a random sample from $GEV_{min}(\mu, \sigma, \xi)$, the log-likelihood function for μ, σ and ξ when $\xi \neq 0$ is

$$\begin{aligned} l(\mu, \sigma, \xi) &= -n \log(\sigma) - (1 + 1/\xi) \sum_{i=1}^{n} \log \left[1 - \xi \left(\frac{x_i - \mu}{\sigma} \right) \right] \\ &\quad - \sum_{i=1}^{n} \left[1 - \xi \left(\frac{x_i - \mu}{\sigma} \right) \right]^{-1/\xi}, \text{ if } 1 - \xi \left(\frac{x_i - \mu}{\sigma} \right) > 0, \; i = 1, ..., n. \end{aligned}$$

When $\xi = 0$, the log-likelihood function is

$$l(\mu, \sigma, 0) = -n \log(\sigma) - \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma} \right) - \sum_{i=1}^{n} \exp \left\{ - \left(\frac{x_i - \mu}{\sigma} \right) \right\}.$$

There are no closed-form solutions to finding the maximum likelihood estimators (MLE) of μ, σ and ξ in both the maximum case and the minimum case. Numerical optimization methods will be used in this report.

Once the estimates $\hat{\mu}$, $\hat{\sigma}$ and $\hat{\xi}$ of the parameters in the GEV models are obtained, we would like to conduct some model checking to see whether the models fit the observations well or not. Let $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$ denote the ordered series. For the maximum case, the empirical distribution function evaluated at $x_{(i)}$ gives

$$\tilde{G}(x_{(i)}) = i/(n+1), \quad i = 1, ..., n,$$

and the corresponding model based probability estimates are given by

$$\hat{G}(x_{(i)}) = \exp \left\{ - \left[1 + \hat{\xi} \left(\frac{x - \hat{\mu}}{\hat{\sigma}} \right) \right]^{-1/\hat{\xi}} \right\}, \; i = 1, ..., n.$$

If the GEV model fits the series well, we expect $\tilde{G}(x_{(i)}) \approx \hat{G}(x_{(i)})$, so a probability plot of the points

$$\left\{ \left(\tilde{G}(x_{(i)}), \hat{G}(x_{(i)}) \right), \; i = 1, ..., n \right\}$$

should be close to the unit diagonal line. An alternative plot, the quantile plot, is to plot the model based quantiles

$$\hat{G}^{-1} \left(\frac{i}{n+1} \right) = \hat{\mu} - \frac{\hat{\sigma}}{\hat{\xi}} \left[1 - \left\{ - \log \left(\frac{i}{n+1} \right) \right\}^{-\hat{\xi}} \right], \; i = 1, ..., n$$

against the empirical quantiles $x_{(i)}$, and departures from linearity indicate poor model fit.

From Figures 1 to 3 we see that the maximum and the minimum series to be analyzed do not seem to be identically distributed. We see traces of increasing/decreasing linear trend in the mean levels, or sin/cos types of up and down moves. To quantify these observations, we consider modeling μ, σ and ξ as functions of time t. For example,

$$\begin{aligned}
\mu(t) &= \mu + bt, \\
\mu(t) &= \mu + A\sin\left(\frac{2\pi}{B}t\right), \ B > 0, \\
\mu(t) &= \mu + bt + A\sin\left(\frac{2\pi}{B}t\right), \ B > 0, \\
\sigma(t) &= \sigma + kt, \\
\sigma(t) &= \sigma + C\sin\left(\frac{2\pi}{H}t\right), \ H > 0, \\
\sigma(t) &= \sigma + kt + C\sin\left(\frac{2\pi}{H}t\right), \ H > 0,
\end{aligned}$$

and similar functions for $\xi(t)$. Although it may fit the observed series well, we do not consider quadratic trend because it has poor potential when we extrapolate into the future.

Under the above general setup, we model the maximum series with

$$X_t \sim GEV_{max}(\mu(t), \sigma(t), \xi(t)), \ t = 1, ..., n.$$

The parameters involved are estimated by maximizing the log-likelihood function

$$\begin{aligned}
l(\mu, \sigma, \xi) &= -\sum_{t=1}^{n}\left\{\log\sigma(t) + (1 + 1/\xi(t))\log\left[1 + \xi(t)\left(\frac{x_t - \mu(t)}{\sigma(t)}\right)\right]\right. \\
&\quad \left. + \left[1 + \xi(t)\left(\frac{x_t - \mu(t)}{\sigma(t)}\right)\right]^{-1/\xi(t)}\right\},
\end{aligned}$$

where

$$1 + \xi(t)\left(\frac{x_t - \mu(t)}{\sigma(t)}\right) > 0, \ t = 1, ..., n.$$

To model the minimum series, the likelihood function to be maximized is

$$\begin{aligned}
l(\mu, \sigma, \xi) &= -\sum_{t=1}^{n}\left\{\log\sigma(t) + (1 + 1/\xi(t))\log\left[1 - \xi(t)\left(\frac{x_t - \mu(t)}{\sigma(t)}\right)\right]\right. \\
&\quad \left. + \left[1 - \xi(t)\left(\frac{x_t - \mu(t)}{\sigma(t)}\right)\right]^{-1/\xi(t)}\right\},
\end{aligned}$$

where

$$1 - \xi(t)\left(\frac{x_t - \mu(t)}{\sigma(t)}\right) > 0, \ t = 1, ..., n.$$

For model checking, we work with the standardized random variables

$$Y_t = \frac{1}{\xi(t)} \log\left\{1 + \xi(t)\left(\frac{X_t - \mu(t)}{\sigma(t)}\right)\right\}, \; t = 1, ..., n,$$

for the maximum case and with

$$Y_t = \frac{1}{\xi(t)} \log\left\{1 - \xi(t)\left(\frac{X_t - \mu(t)}{\sigma(t)}\right)\right\}, \; t = 1, ..., n,$$

for the minimum case. In both cases, the standardized random variables Y_t follow the standard Gumbel distribution, that is,

$$P(Y_t \leq y) = \exp\{-\exp(-y)\}, \; \infty < y < \infty, \; t = 1, ..., n.$$

Let $\hat{y}_t$ denote the result of substituting the MLE estimates into Y_t and let $\hat{y}_{(t)}$ denote the ordered $\hat{y}_t$, one can use the probability plot of

$$\exp(-\exp(-\hat{y}_{(t)})) \;\text{ against }\; t/(n+1), \; t = 1, ..., n,$$

or the quantile plot of

$$-\log(-\log(t/(n+1))) \;\text{ against }\; \hat{y}_{(t)}, \; t = 1, ..., n,$$

to check the model fit. If the model fits the series well, both plots should show a linear pattern.

Among the three types of trend to be considered, there is a need to pick the "best" one. This can be accomplished by checking the size of the *deviance statistic* D defined as

$$D = 2\{l_1(M_1) - l_0(M_0)\},$$

where model M_1 contains model M_0 as a sub-model, and $l_1(M_1)$ and $l_0(M_0)$ are the maximized log-likelihoods under M_1 and M_0, respectively. If D is large, model M_1 explains substantially more of the variation in the data than model M_0; if D is small, the larger model M_1 does not bring worthwhile improvement compared with the smaller model M_0. Formally, if the p-value $P(\chi^2_\nu \geq D) \leq \alpha$, we prefer model M_1 to model M_0 at significance level α, where χ^2_ν denotes the chi-square distribution with degrees of freedom ν, and ν is the difference in the dimensionality of M_1 and M_0 (the number of estimated parameters in model M_1 minus the number of estimated parameters in model M_0). Note that χ^2_ν is the asymptotic distribution of D.

4 Modeling Maximum and Minimum Precipitation, Flow Rate and Temperature Series

We first consider the annual maximum precipitation series and restrict our attention to model μ as a function of time t only, namely, we assume $\sigma(t) = \sigma$ and $\xi(t) = \xi$.

Figure 4 displays the results of fitting constant, linear, sin and linear plus sin trends to the maximum precipitation series. The trends plotted in Figure 4 are the means of the four different models. The model fits of these 4 types of trend is assessed in Figure 5. The values of r plotted in the four sub-plots of Figure 5 are the Pearson correlation coefficient for the linear pattern in each sub-plot. We see from Figure 5 that the sin trend model and the linear plus sin trend model fit the data better. Formally, we can compare the four trends using the deviance statistic as below.

Table 1: Comparison of the four types of trend in the GEV model for the maximum precipitation series from 1924 to 2004.

Linear trend vs constant trend	p-value = 0.3761
Sin trend vs constant trend	p-value = 0.0421
Linear plus sin trend vs constant trend	p-value = 0.0964
Linear plus sin trend vs linear trend	p-value = 0.0623
Linear plus sin trend vs sin trend	p-value = 0.9920

For example, for the constant trend model, the maximized log-likelihood is -394.7314 with 3 estimated parameters. For the sin trend model, the maximized log-likelihood is -391.5638 with 5 estimated parameters. So to compare sin trend with constant trend, the deviance statistic is $D = 2(-391.5638 + 394.7314) = 6.3352$, with $\nu = 5 - 3 = 2$, and the p-value $= P(\chi_2^2 \geq 6.3352) = 0.0421$.

At significance level $\alpha = 0.05$, the best fitted trend is the sin trend.

Using the same approach, we model the maximum flow rate series and the maximum temperature series; the results are given in Figure 6 to Figure 9 and Table 2 to Table 3.

At significance level $\alpha = 0.05$, the best fitted trend for the maximum flow rate series is the sin trend, followed by the linear plus sin trend as a borderline trend. For the maximum temperature series, both the sin trend and the linear plus sin trend are significant, with the sin trend slightly more significant.

An entirely parallel analysis is performed to model the minimum precipitation, flow

Figure 4: GEV MLE fits of constant, linear, sin and linear plus sin trends to the maximum precipitation series from 1924 to 2004.

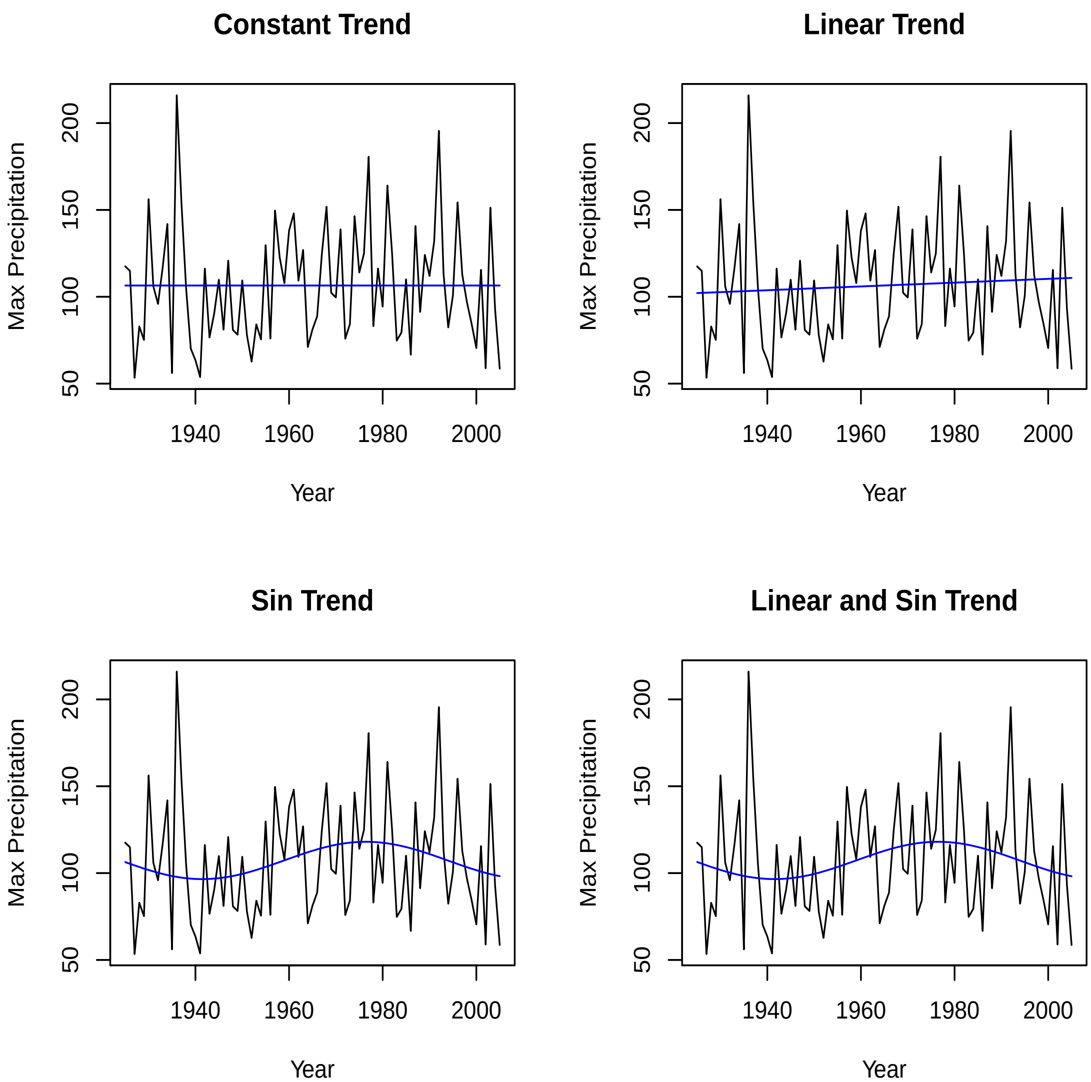


Figure 5: Probability plots of the GEV MLE fits of constant, linear, sin and linear plus sin trends to the maximum precipitation series from 1924 to 2004.

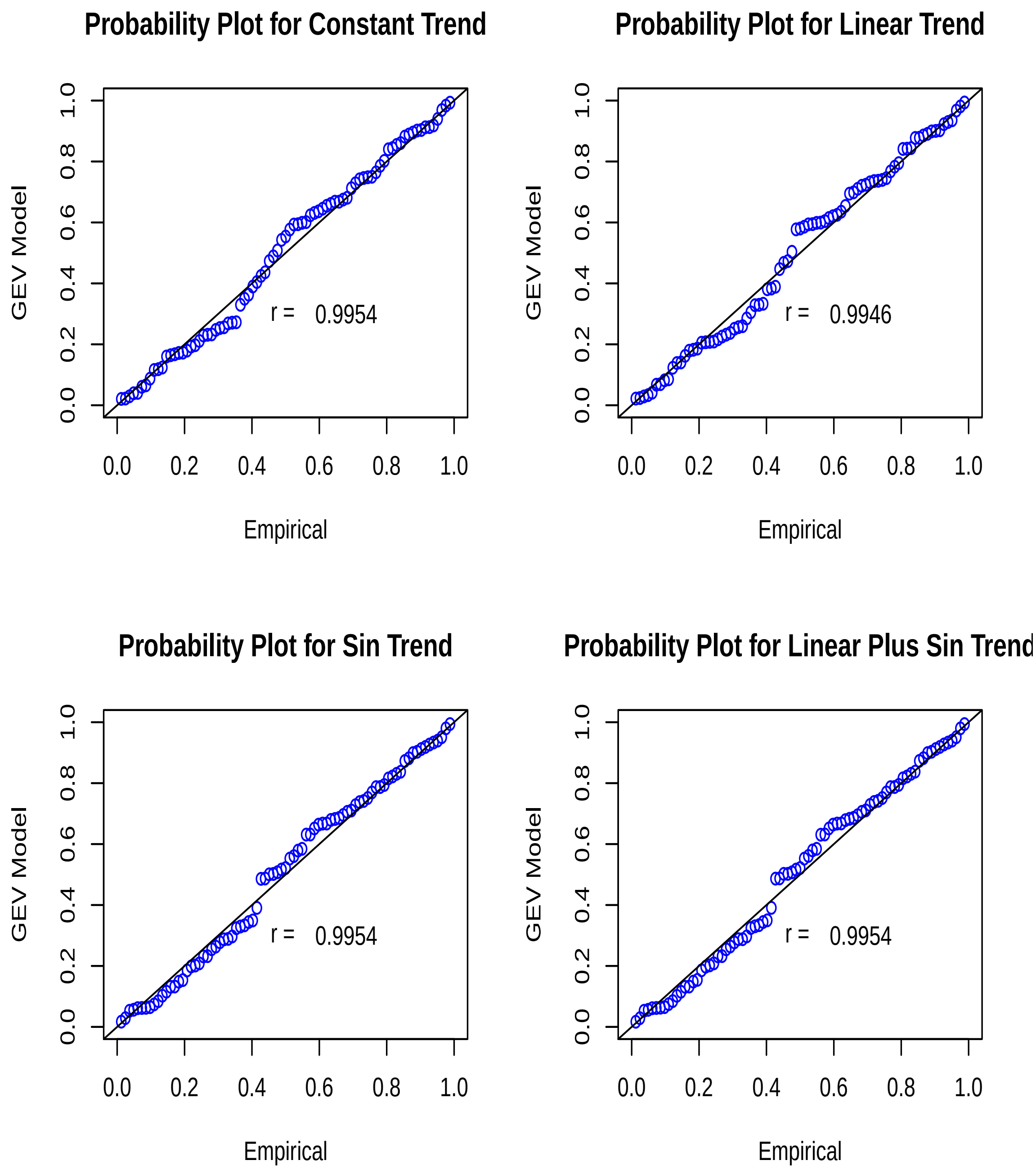


Figure 6: GEV MLE fits of constant, linear, sin and linear plus sin trends to the maximum flow rate series from 1957 to 2008.

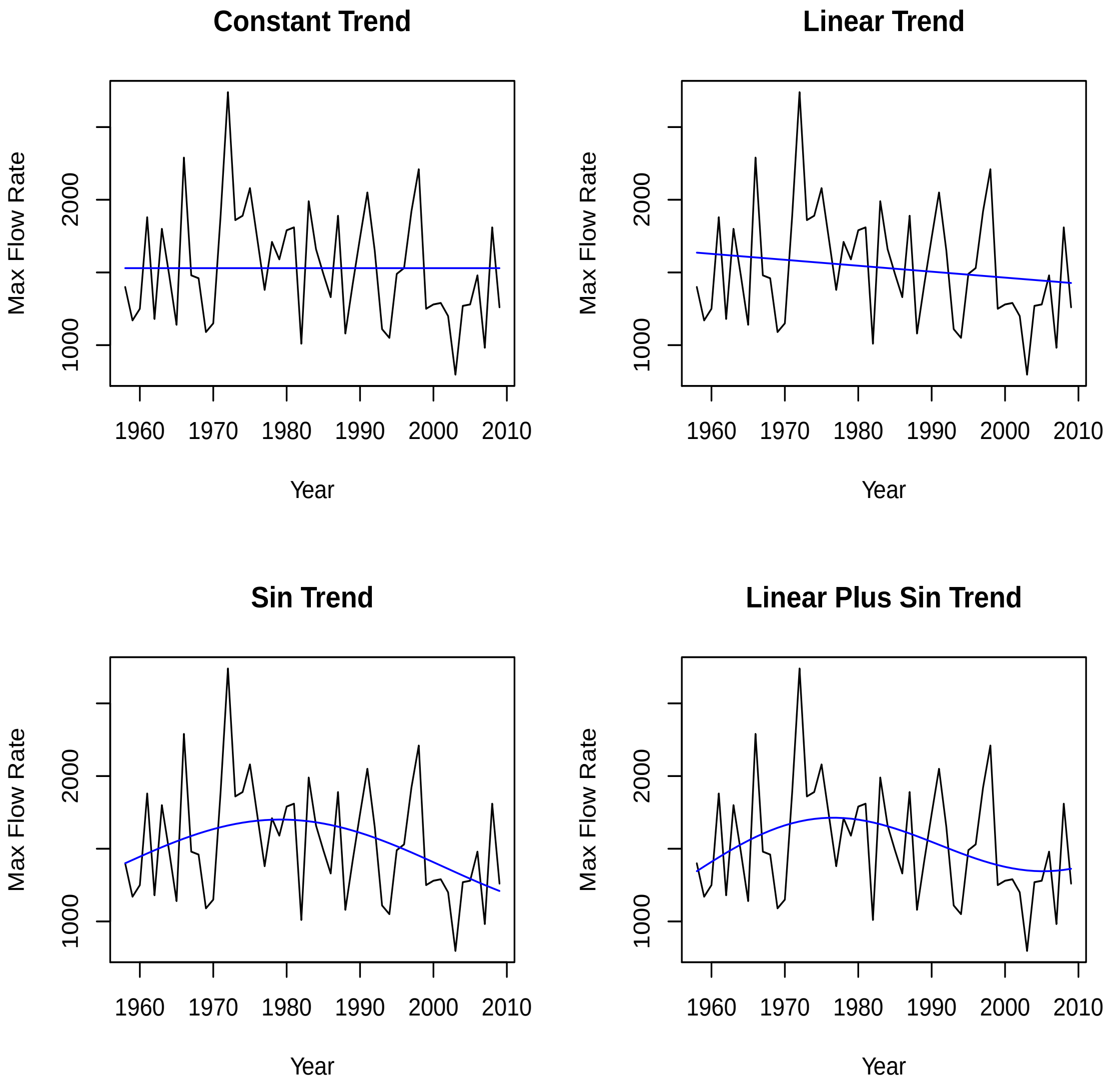


Figure 7: Probability plots of the GEV MLE fits of constant, linear, sin and linear plus sin trends to the maximum flow rate series from 1957 to 2008.

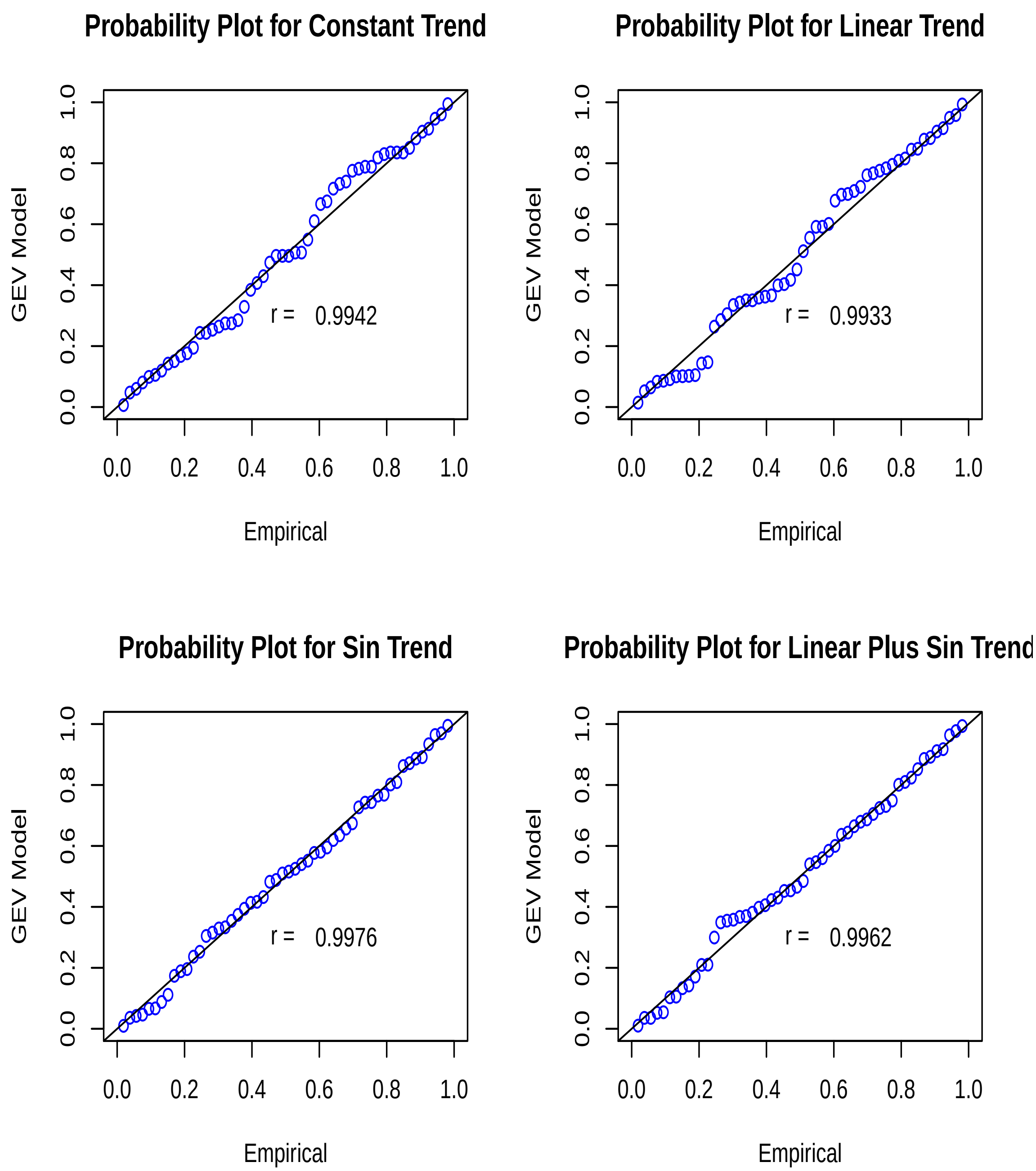


Figure 8: GEV MLE fits of constant, linear, sin and linear plus sin trends to the maximum temperature series from 1909 to 2007.

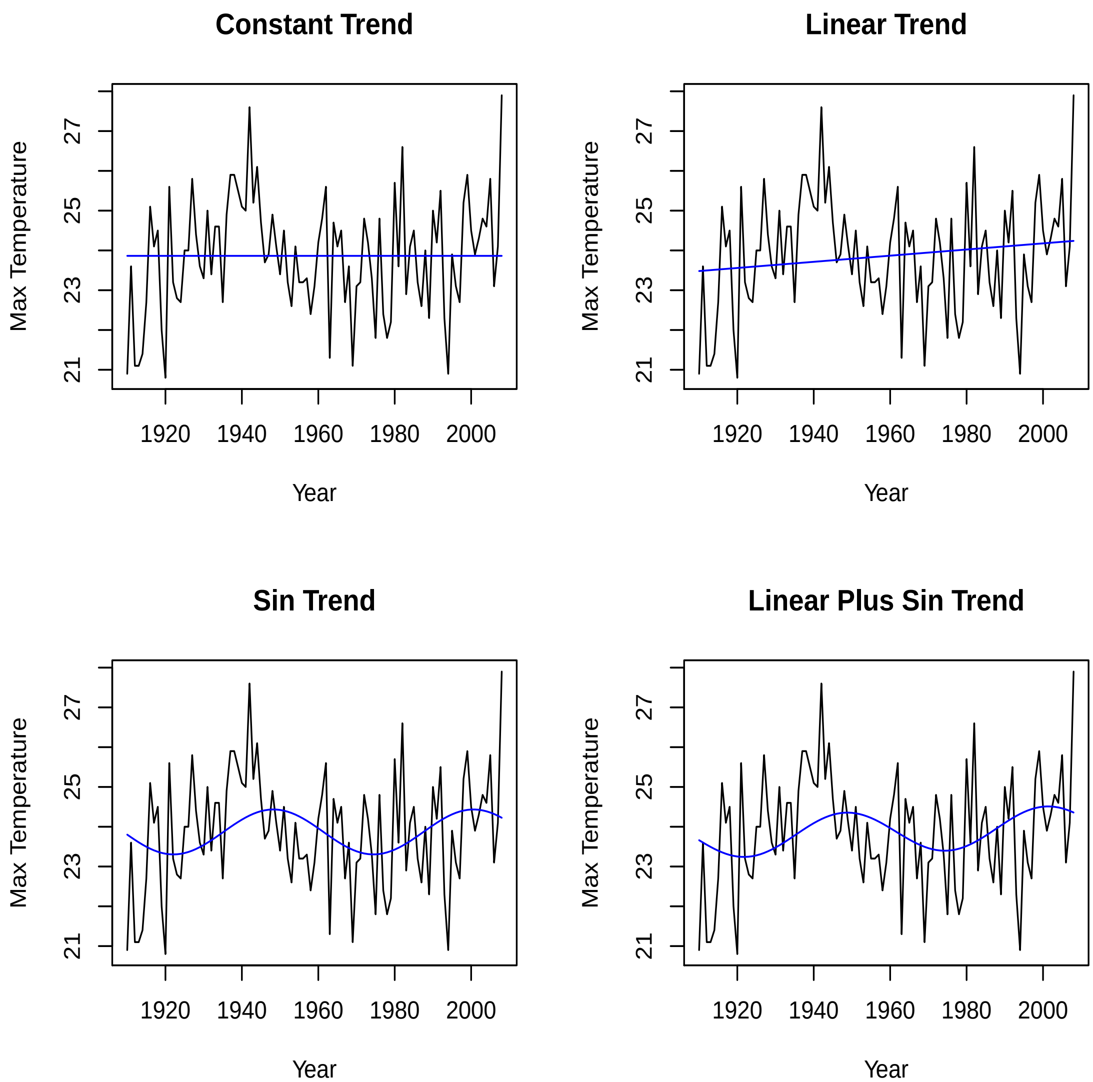


Figure 9: Probability plots of the GEV MLE fits of constant, linear, sin and linear plus sin trends to the maximum temperature series from 1909 to 2007.

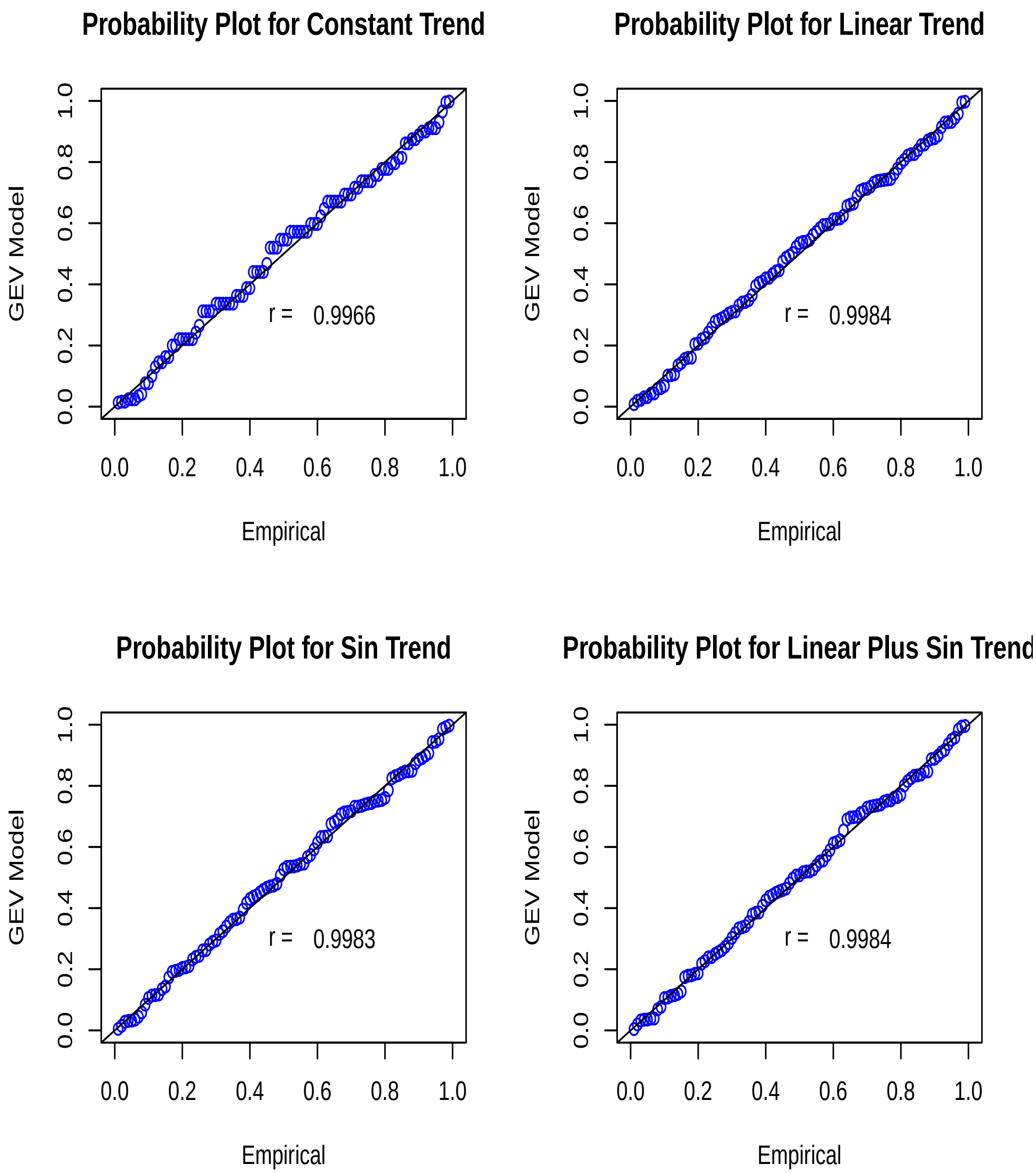


Table 2: Comparison of the four types of trend in the GEV model for the maximum flow rate series from 1957 to 2008.

Linear trend vs constant trend	p-value = 0.1503
Sin trend vs constant trend	p-value = 0.0418
Linear plus sin trend vs constant trend	p-value = 0.0639
Linear plus sin trend vs linear trend	p-value = 0.0744
Linear plus sin trend vs sin trend	p-value = 0.3381

Table 3: Comparison of the four types of trend in the GEV model for the maximum temperature series from 1909 to 2007.

Linear trend vs constant trend	p-value = 0.1143
Sin trend vs constant trend	p-value = 0.0184
Linear plus sin trend vs constant trend	p-value = 0.0395
Linear plus sin trend vs linear trend	p-value = 0.0537
Linear plus sin trend vs sin trend	p-value = 0.5571

rate and temperature series; the results are provided in Figure 10 to Figure 15, and Table 4 to Table 6.

Table 4: Comparison of the four types of trend in the GEV model for the minimum precipitation series from 1924 to 2004.

Linear trend vs constant trend	p-value = 0.0142
Sin trend vs constant trend	p-value = 0.0028
Linear plus sin trend vs constant trend	p-value = 0.0026
Linear plus sin trend vs linear trend	p-value = 0.0163
Linear plus sin trend vs sin trend	p-value = 0.1164

For the minimum precipitation series, the best trend is the sin trend, followed by the linear plus sin trend; for the minimum flow rate series, none of the three investigated trends do better than the constant trend; for the minimum temperature series, the best trend is the linear plus sin trend.

Notice that all of the probability plots show good to very good model fits, but the

Figure 10: GEV MLE fits of constant, linear, sin and linear plus sin trends to the minimum precipitation series from 1924 to 2004.

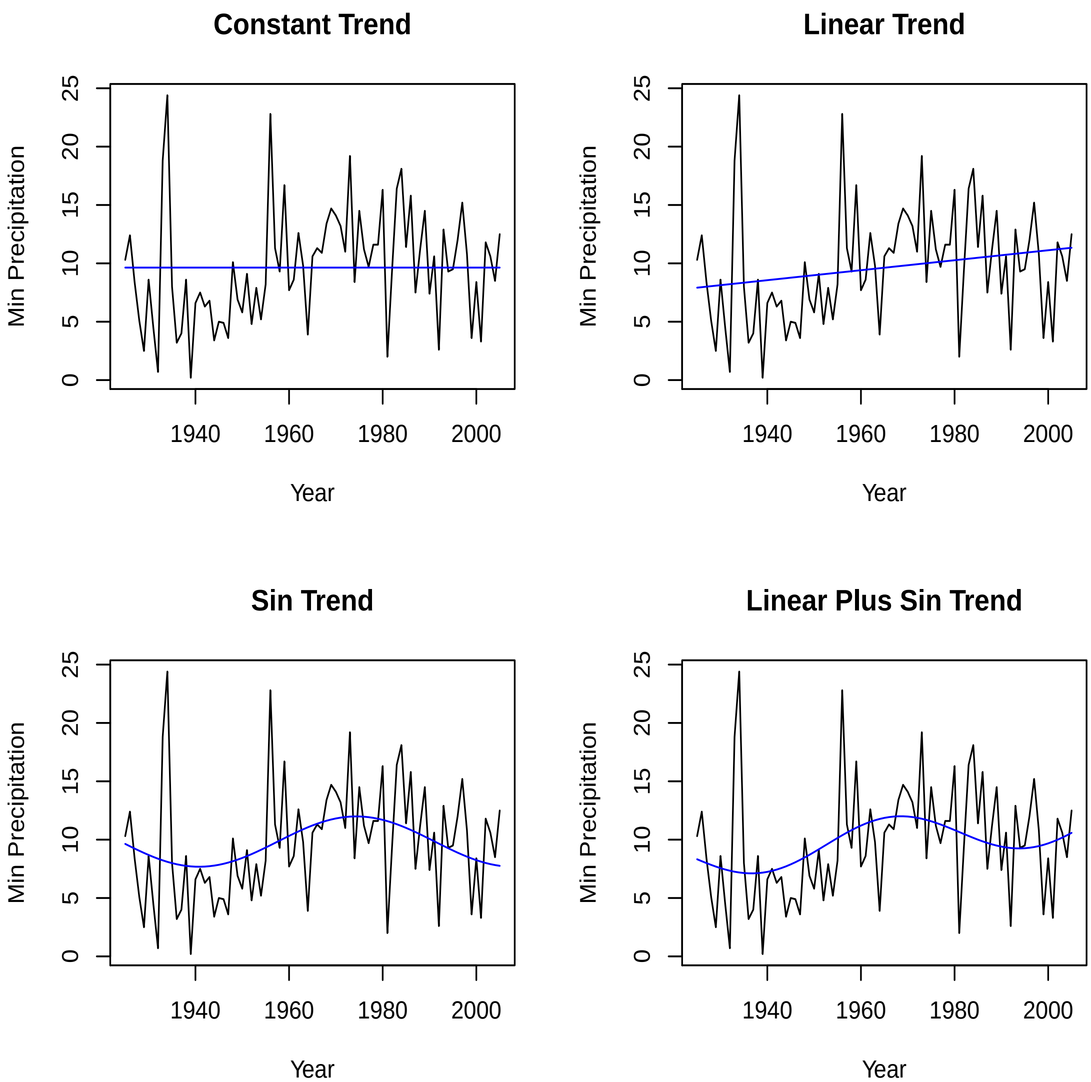


Figure 11: Probability plots of the GEV MLE fits of constant, linear, sin and linear plus sin trends to the minimum precipitation series from 1924 to 2004.

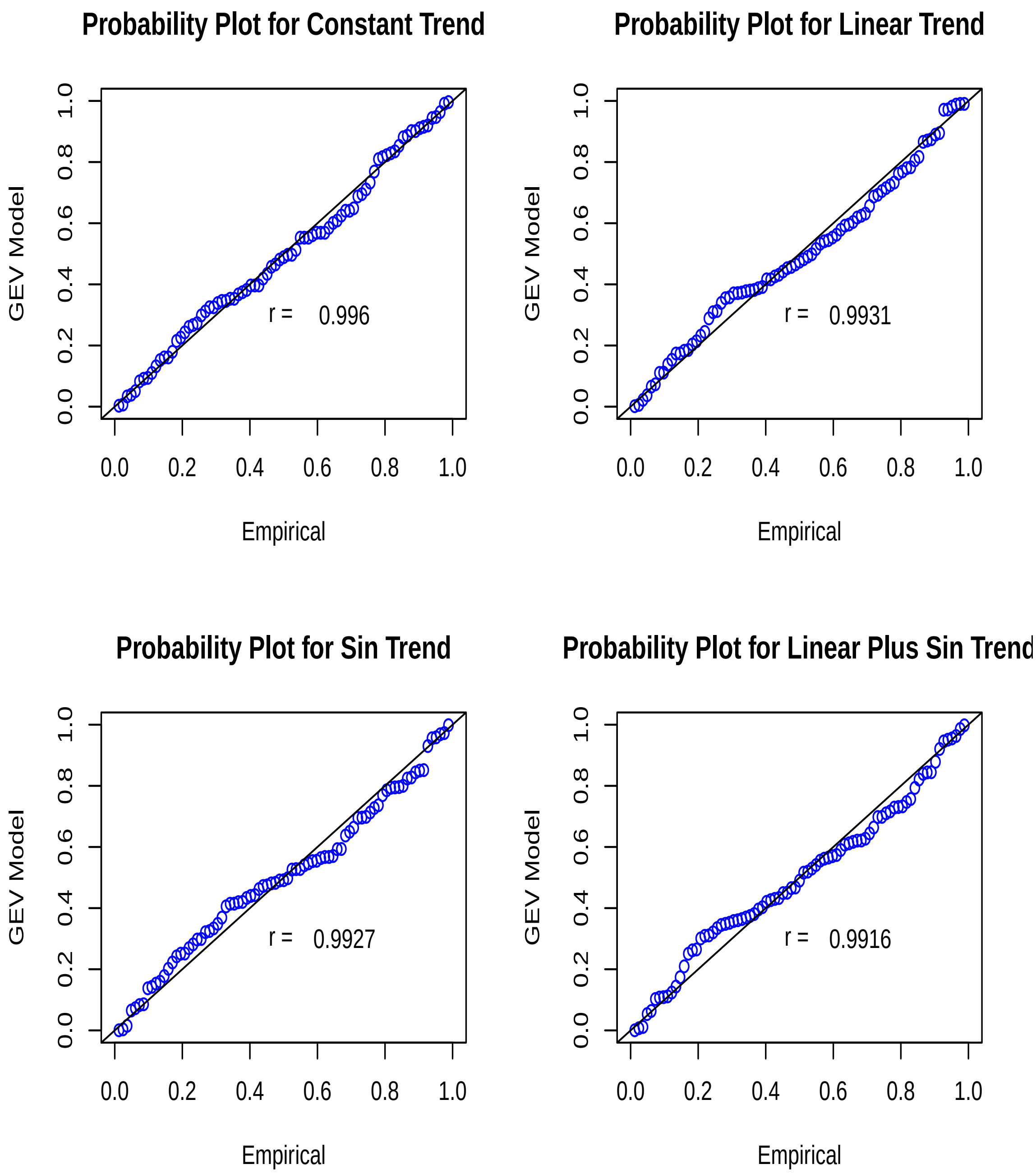


Figure 12: GEV MLE fits of constant, linear, sin and linear plus sin trends to the minimum flow rate series from 1957 to 2008.

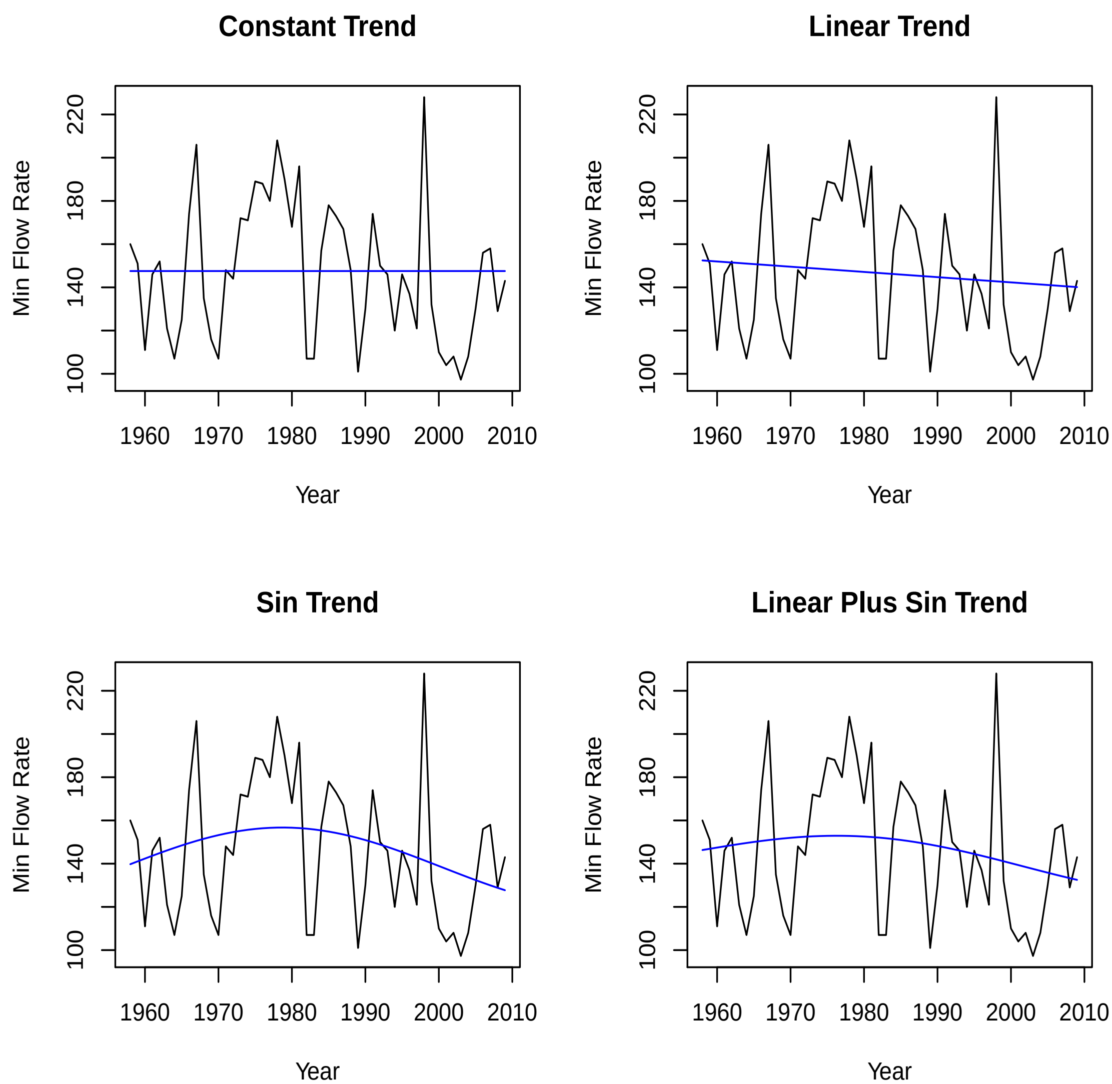


Figure 13: Probability plots of the GEV MLE fits of constant, linear, sin and linear plus sin trends to the minimum flow rate series from 1957 to 2008.

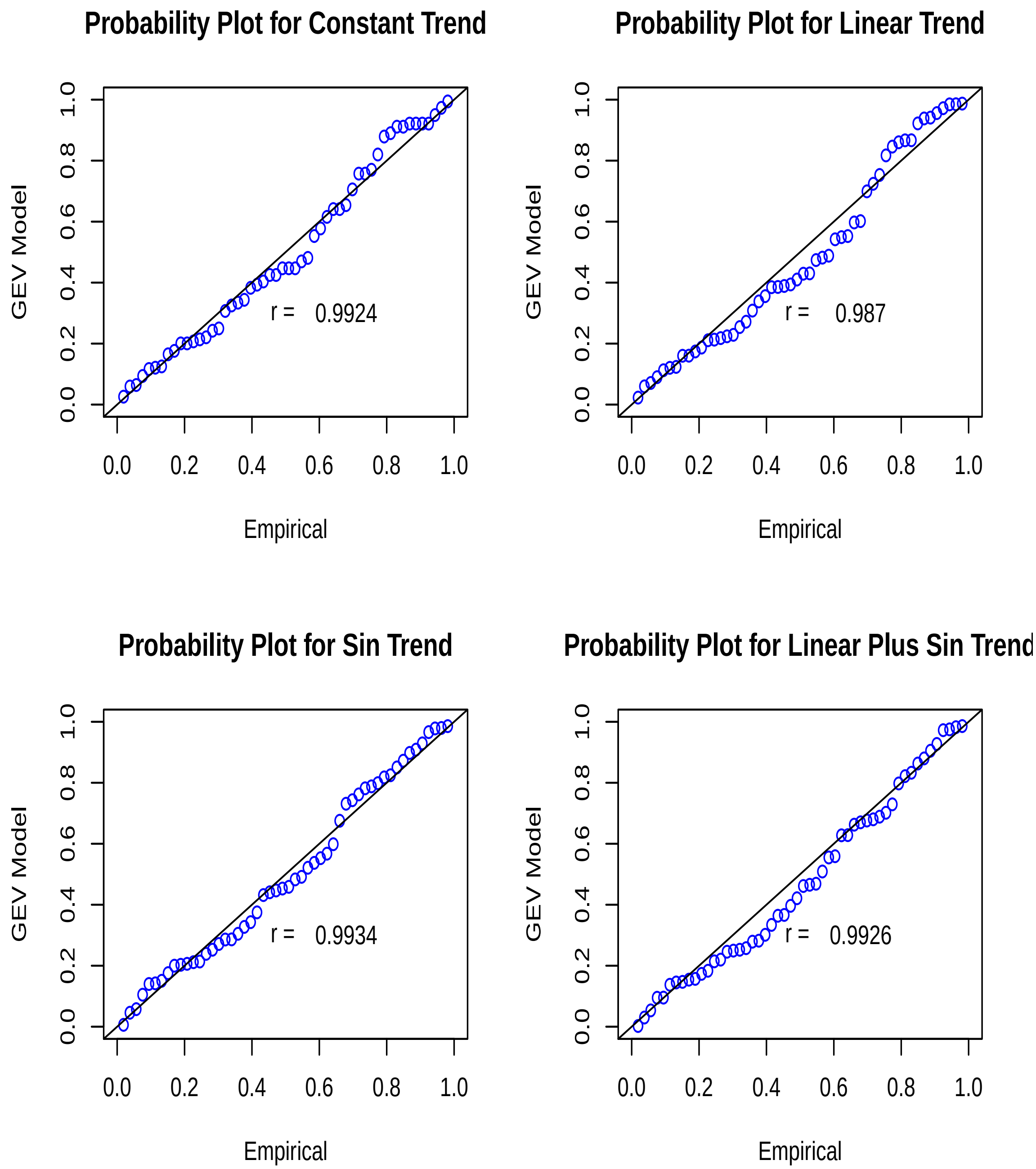


Figure 14: GEV MLE fits of constant, linear, sin and linear plus sin trends to the minimum temperature series from 1909 to 2007.

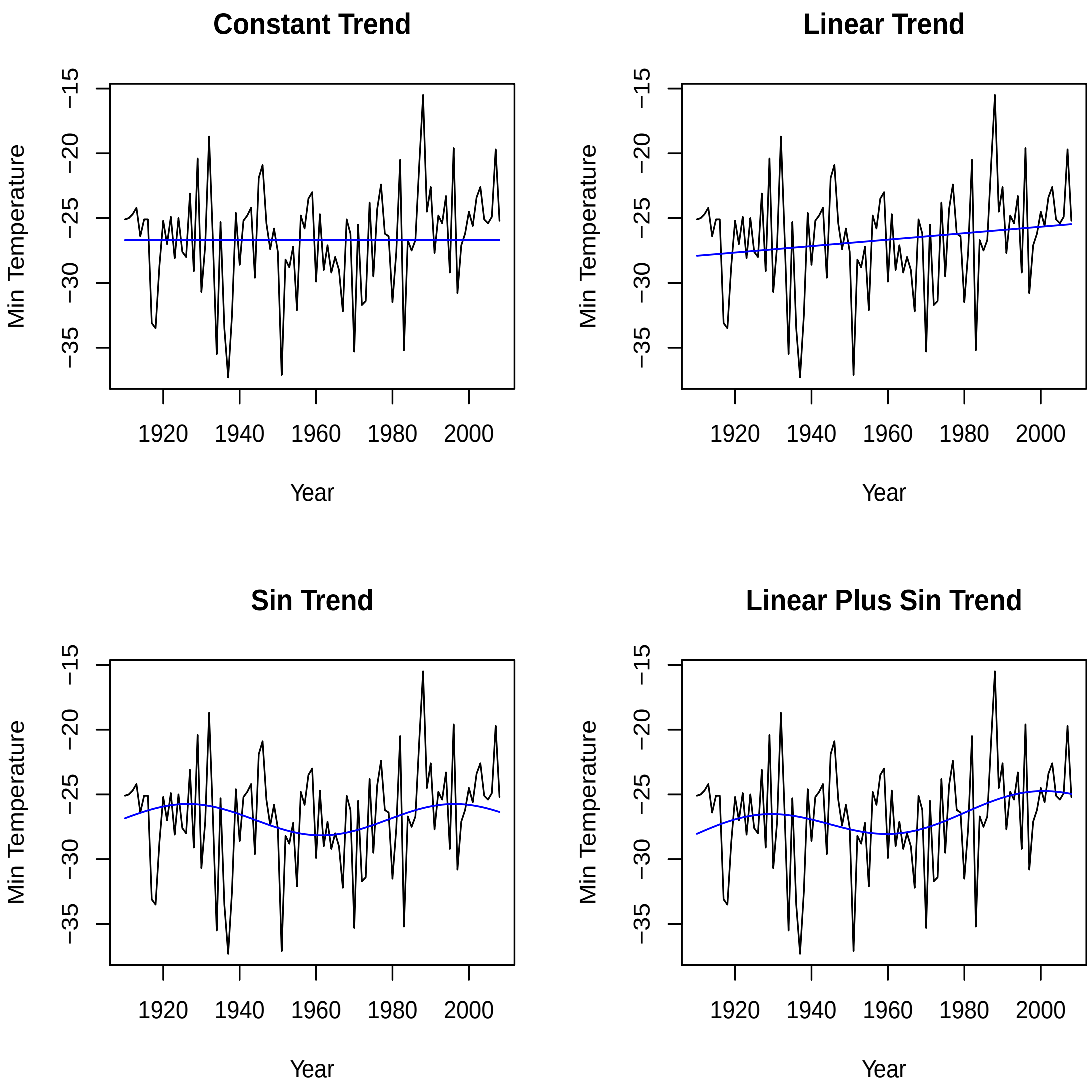


Figure 15: Probability plots of the GEV MLE fits of constant, linear, sin and linear plus sin trends to the minimum temperature series from 1909 to 2007.

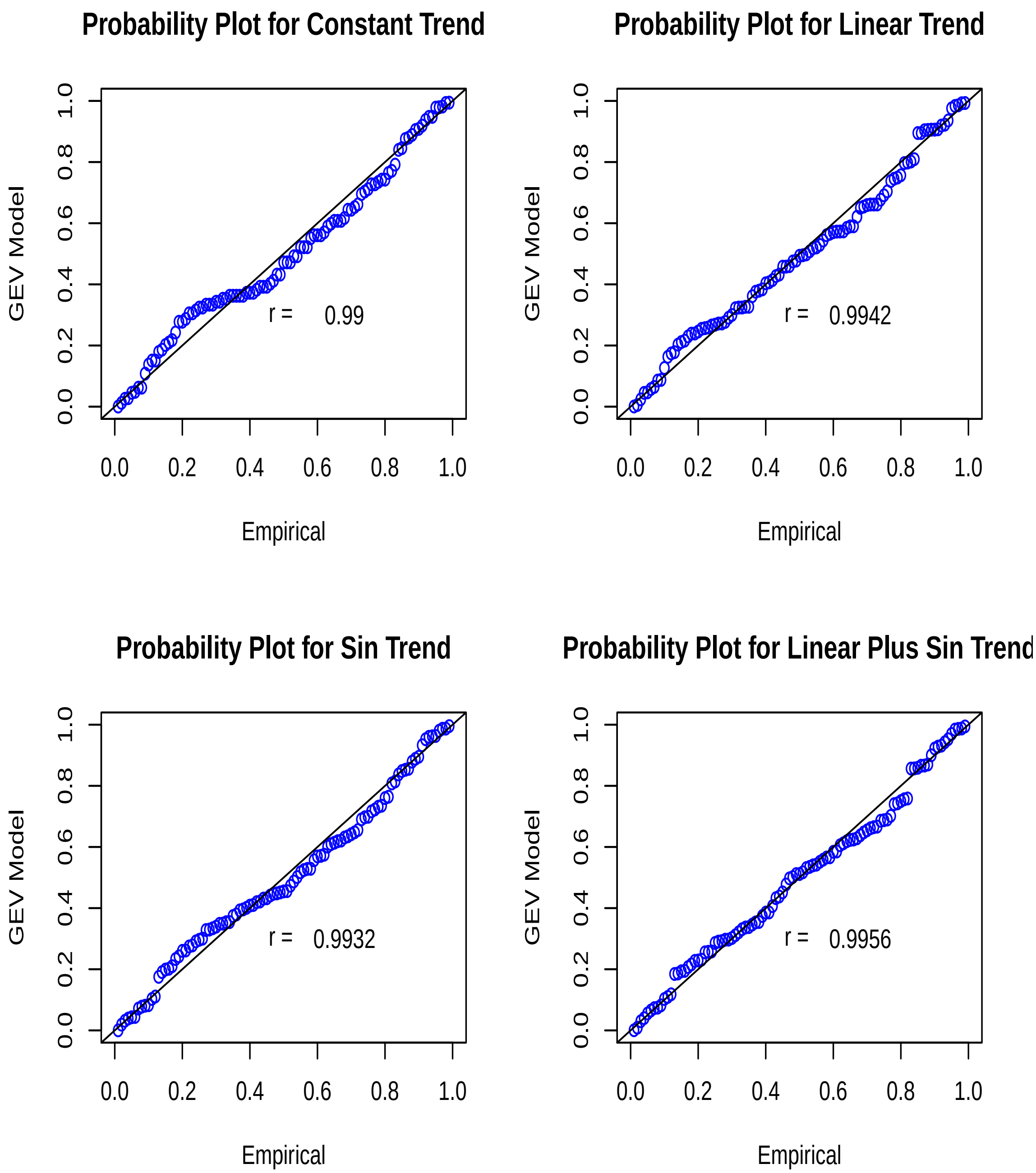


Table 5: Comparison of the four types of trend in the GEV model for the minimum flow rate series from 1957 to 2008.

Linear trend vs constant trend	p-value = 0.1073
Sin trend vs constant trend	p-value = 0.1852
Linear plus sin trend vs constant trend	p-value = 0.3275
Linear plus sin trend vs linear trend	p-value = 0.6522
Linear plus sin trend vs sin trend	p-value = 0.7826

Table 6: Comparison of the four types of trend in the GEV model for the minimum temperature series from 1909 to 2007.

Linear trend vs constant trend	p-value = 0.0579
Sin trend vs constant trend	p-value = 0.1069
Linear plus sin trend vs constant trend	p-value = 0.0445
Linear plus sin trend vs linear trend	p-value = 0.1066
Linear plus sin trend vs sin trend	p-value = 0.0577

likelihood theory based conclusions drawn above (using the deviance statistic) are not always in agreement with the visual impressions that the probability plots display. This is because the measure of "closeness to a straight line" is not the same as the measure of "maximized likelihood". In this report we pay more attention to the likelihood approach.

We have tried to model σ and ξ as functions of time t the same way we model μ as a function of time t. In both the maximum series and the minimum series cases we found no evidence that the scale and the shape of the series change over time.

5 Predicting Maximum and Minimum Flow Rates

We present our prediction results in two situations: (1) using the flow rate series along; (2) using the flow rate series and the precipitation and temperature series.

According to the modeling results of Section 4, for the three maximum series the significant trends are the sin trend (for all three series), followed by the linear plus sin trend for the maximum precipitation and temperature series and (border line) for the maximum flow rate series. For the three minimum series, the significant trends are the sin trend followed by the linear plus sin trend for the minimum precipitation series, the constant trend for the minimum flow rate series and the linear plus sin trend for the minimum temperature series. To help us connect history to the future, we present the above mentioned trends in Figure 16 to Figure 19, going back to 1909 (the first year we have observed data) and extrapolating into the future up to 2070.

We see from Figures 16 and 17 that the sin trend and the linear plus sin trend of the maximum precipitation series have a periodicity of around $\hat{B} = 70$ years, while the periodicities of the sin trend and the linear plus sin trend of the maximum flow rate series are around 90 (Figure 16) and 70 years (Figure 17), respectively. The sin or the linear plus sin trend of the maximum temperature series has a shorter periodicity (around $\hat{B} = 50$) than the precipitation and flow rate series, and has a kind of delayed anti-phase behavior.

From Figures 18 and 19 we see that the minimum temperature series changes periodically and increases slowly. It also has a periodicity similar to that of the precipitation series (around $\hat{H} = 70$) in Figure 18 and has a shorter periodicity (around $\hat{H} = 60$) in Figure 19, together with a kind of anti-phase or delayed anti-phase behavior. The minimum precipitation series changes periodically and may have a slightly upward linear trend (Figure 19). That the minimum flow rate series has a constant trend is understood as either the series is too short to show any other trend, or simply because the annual minimum flow rates all occurred in the cold winter season when the river was frozen, which left very little room for the flow rate to fluctuate.

Next we borrow the information from the precipitation and temperature series to predict the future flow rates. One way to do so is to line up the three maximum seres and the three minimum series from 1957 to 2004 (the range shared by all three series) and model the flow rate series using the precipitation and temperature series as covariates. This is a legitimate approach except that shortening the precipitation series (from 81 values to 48 values) and the temperature series (from 99 values to 48 values) loses valuable information and dramatically changes the features of the two covariate series, so we will not take this approach.

An alternative approach is to use the results of Section 4 to represent the precipitation and temperature series by their respective significant trends. This approach is based on the features extracted from the entire observed precipitation and temperature series, and the values of those significant trends from 1957 to 2008 (the range for the flow rate series) are used as the covariates. There is another advantage of this alternative approach: "future values" of the precipitation and temperature series become available naturally, which is important to predict future flow rates objectively.

For a flow rate series x_t, we consider modeling its mean as a function of the significant trend y_t of the precipitation series and the significant trend z_t of the temperature series,

Figure 16: Group 1: Significant trends for the maximum precipitation, flow rate and temperature series from 1909 to 2070, with the observed series embedded.

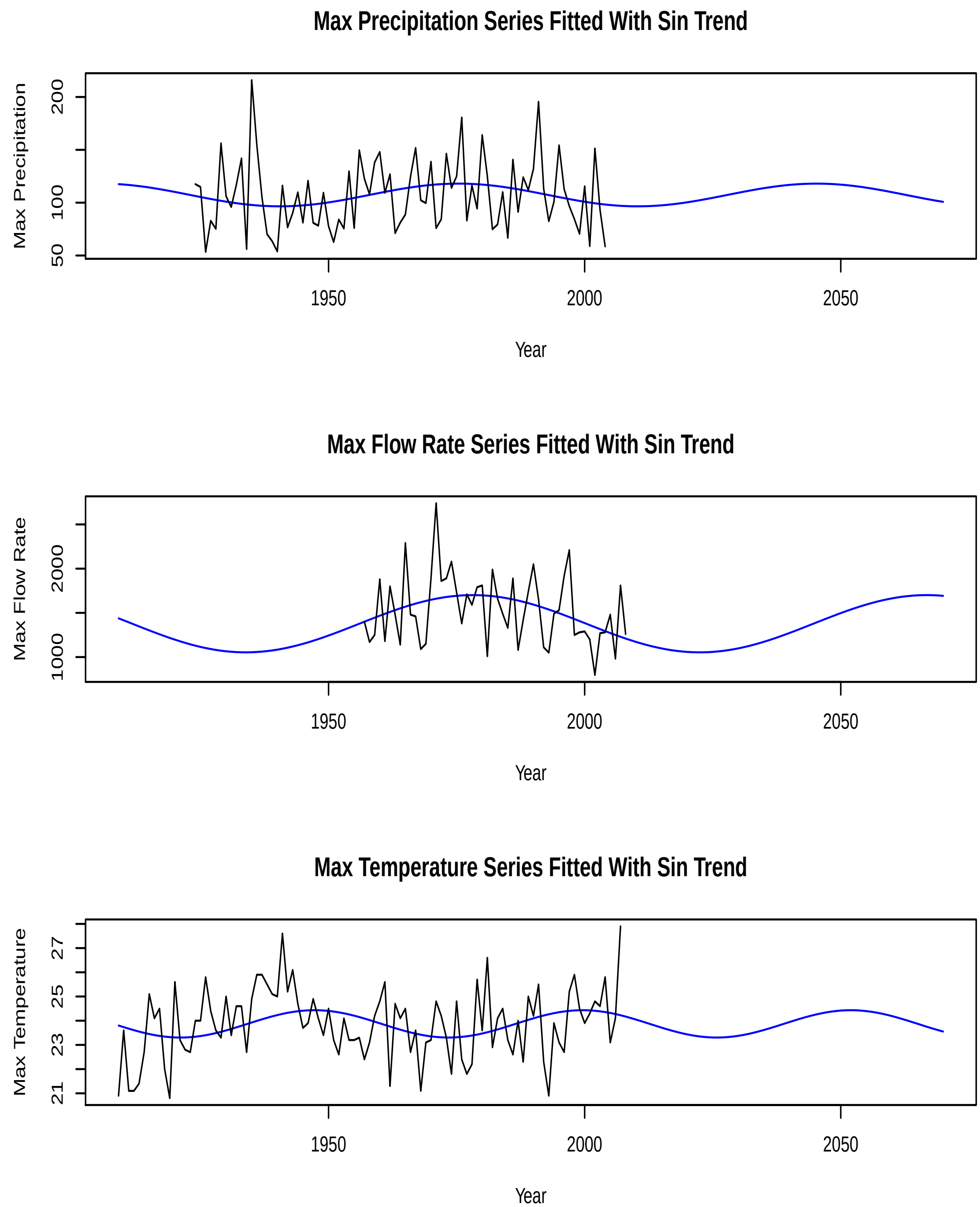


Figure 17: Group 2: Significant trends for the maximum precipitation, flow rate and temperature series from 1909 to 2070, with the observed series embedded.

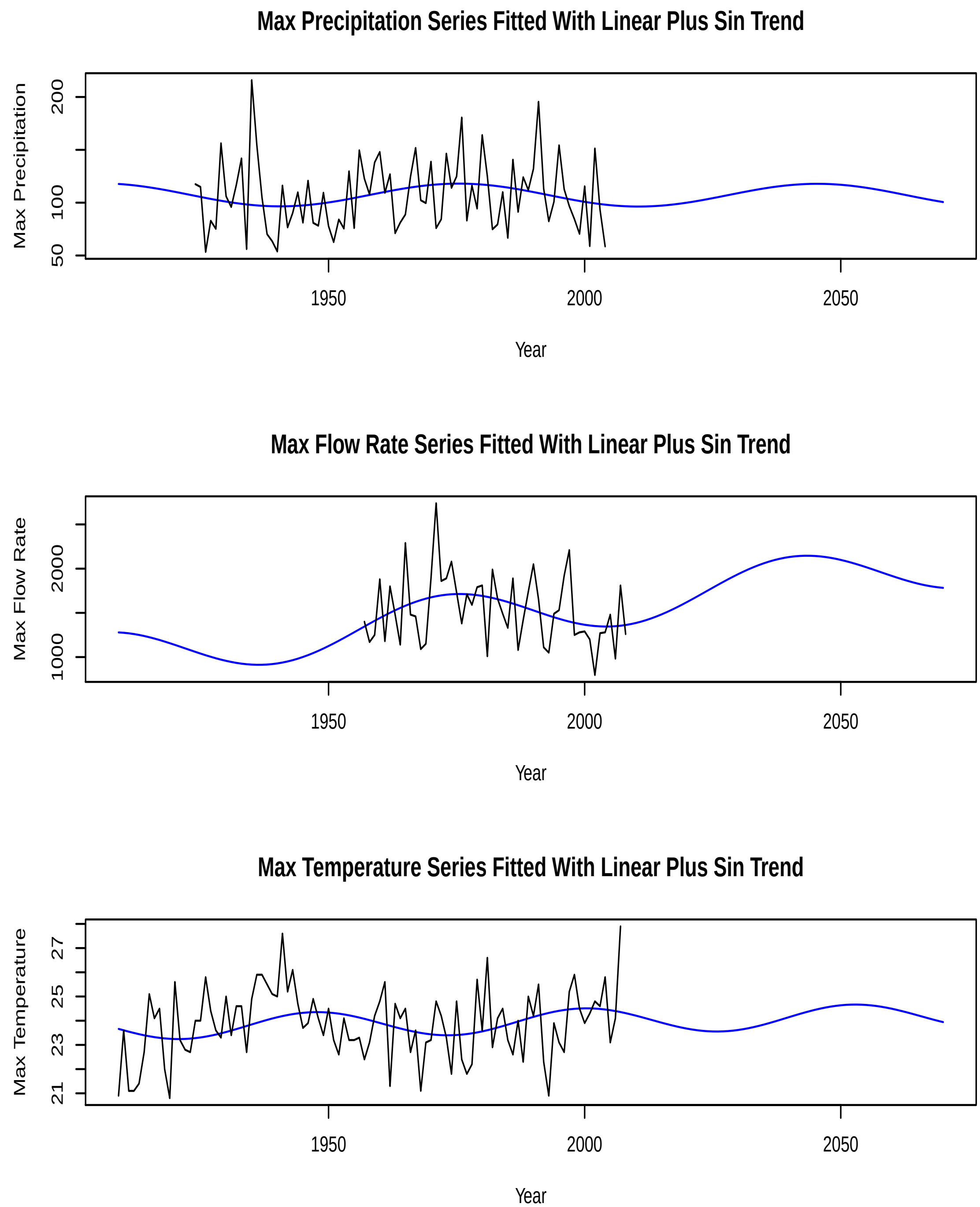
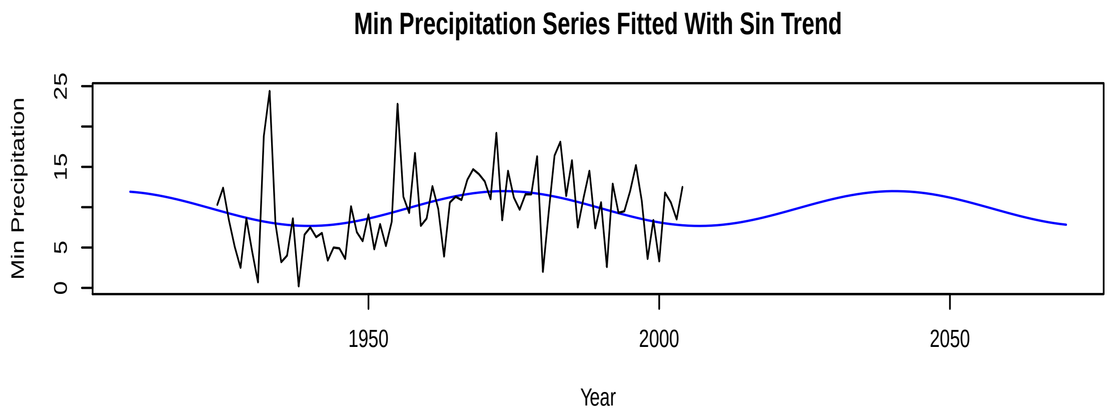
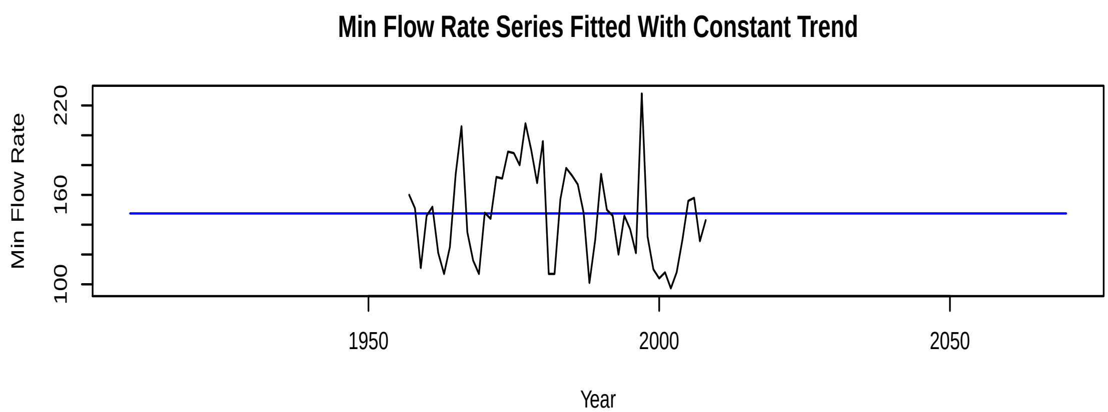


Figure 18: Group 1: Significant trends for the minimum precipitation, flow rate and temperature series from 1909 to 2070, with the observed series embedded.





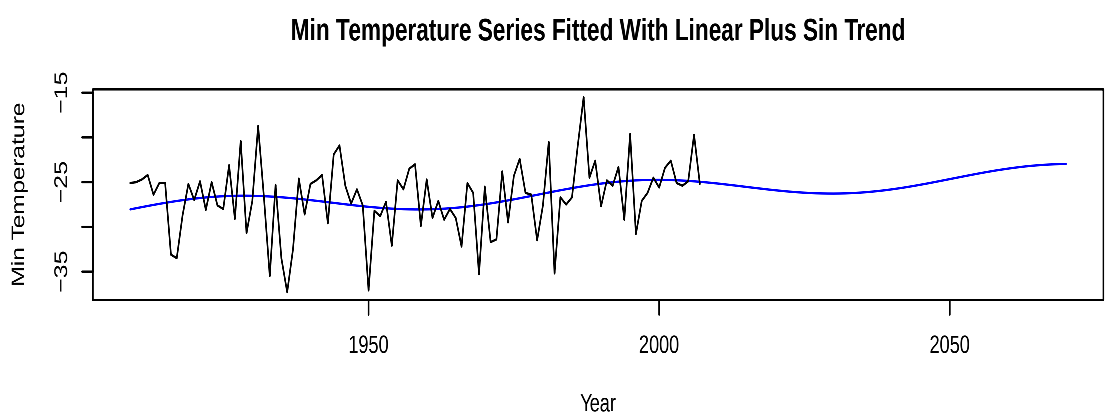
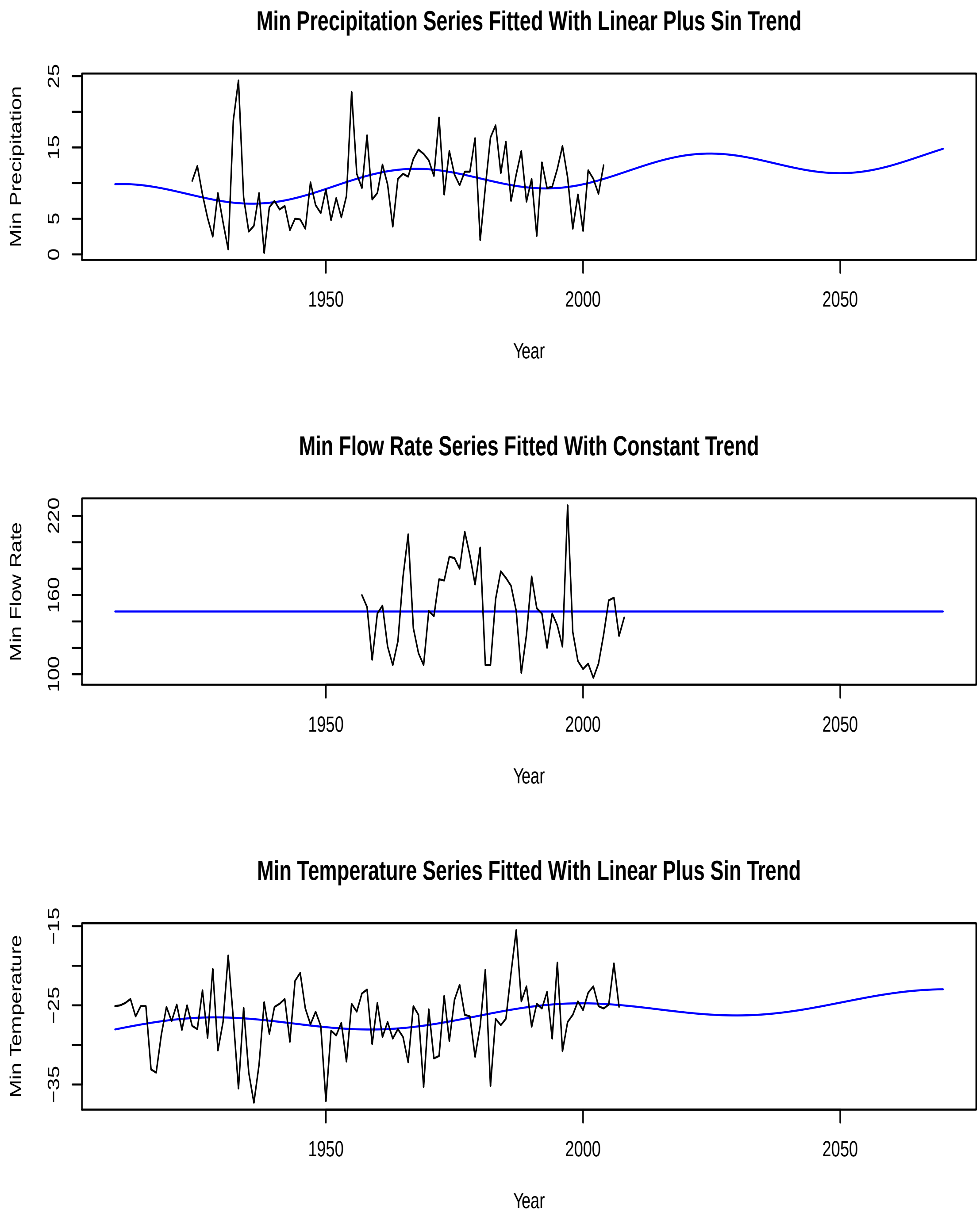


Figure 19: Group 2: Significant trends for the minimum precipitation, flow rate and temperature series from 1909 to 2070, with the observed series embedded.



in addition to the usual linear trend over time t. Specifically, we consider the following six models:

$$\begin{aligned}
\mu(t) &= \mu, \\
\mu(t) &= \mu + bt, \\
\mu(t) &= \mu + Ay_t, \\
\mu(t) &= \mu + Bz_t, \\
\mu(t) &= \mu + Ay_t + Bz_t, \\
\mu(t) &= \mu + bt + Ay_t + Bz_t.
\end{aligned}$$

For the maximum flow rate series, we have the fitted trends, the model checking probability plots and the nested tests using the deviance statistic presented in Figure 20, Figure 21 and Table 7, respectively.

Table 7: Comparison of the six different trends in the GEV model for the maximum flow rate series from 1957 to 2008 using the sin trend for the maximum precipitation and temperature series.

$\mu + bt$	vs	μ	p-value = 0.1503
$\mu + Ay_t$	vs	μ	p-value = 0.0085
$\mu + Bz_t$	vs	μ	p-value = 0.0121
$\mu + Ay_t + Bz_t$	vs	μ	p-value = 0.0306
$\mu + bt + Ay_t + Bz_t$	vs	μ	p-value = 0.0626
$\mu + Ay_t + Bz_t$	vs	$\mu + Ay_t$	p-value = 0.8202
$\mu + Ay_t + Bz_t$	vs	$\mu + Bz_t$	p-value = 0.4106
$\mu + bt + Ay_t + Bz_t$	vs	$\mu + Ay_t + Bz_t$	p-value = 0.5602

From Figure 21 and Table 7 we see that all of the six models fit the maximum flow rate series well, with the

$$\mu(t) = \mu + Ay_t = \mu + A \times \text{Precipitation}_t$$

model achieving the best fit at significance level $\alpha = 0.05$. (The $\mu(t) = \mu + Ay_t$ model, $\mu(t) = \mu + Bz_t$ model and $\mu(t) = \mu + Ay_t + Bz_t$ model all bring in significant improvements over the constant model $\mu(t) = \mu$. However, having Ay_t in the model and adding Bz_t to it or having Bz_t in the model and adding Ay_t to it does not improve the expanded model any more, that is, one of the two models $\mu + Ay_t$ and $\mu + Bz_t$ will suffice. Between

Figure 20: GEV MLE fits of the six different trends to the maximum flow rate series from 1957 to 2008 using the sin trend for the maximum precipitation and temperature series.

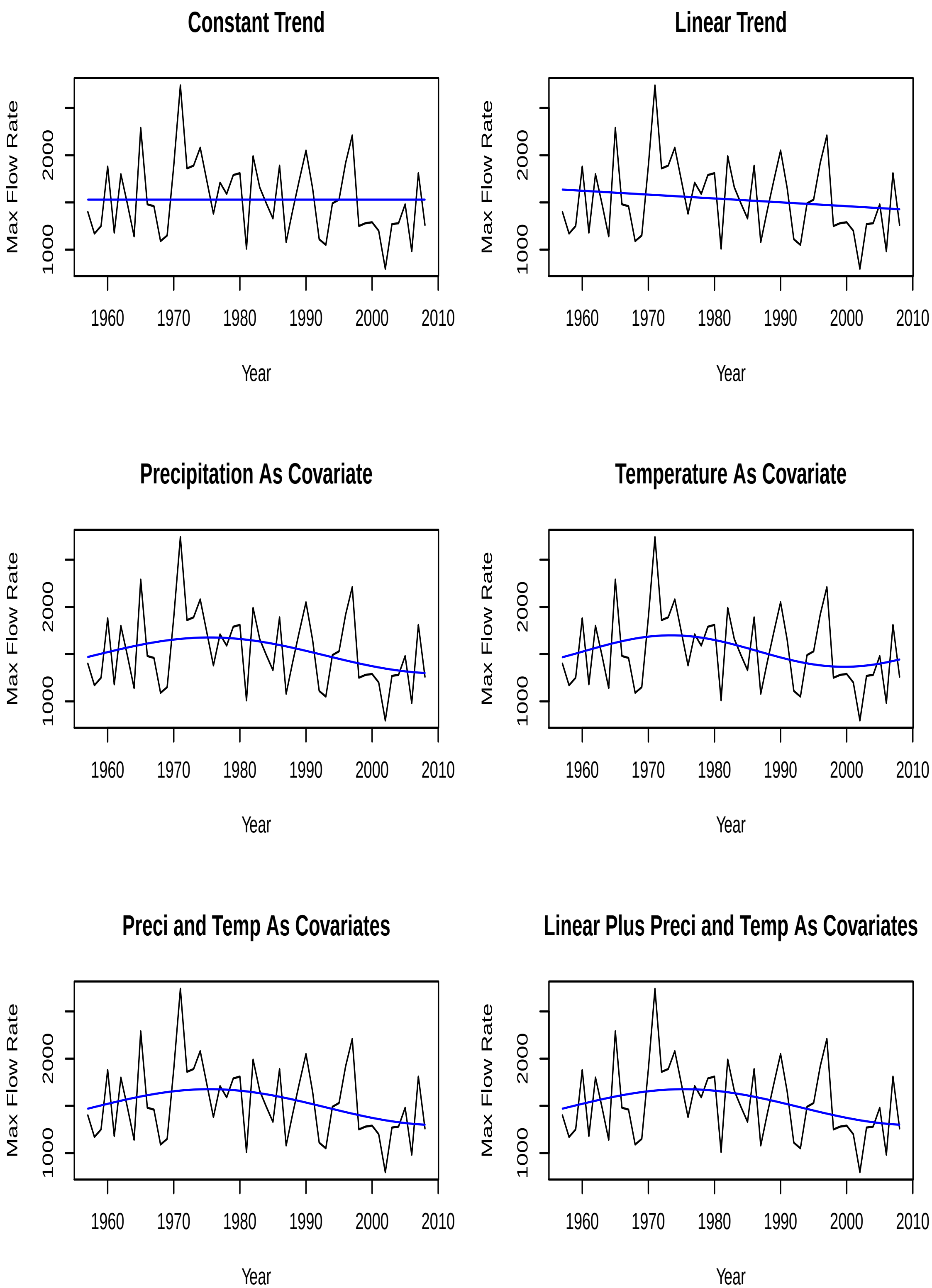
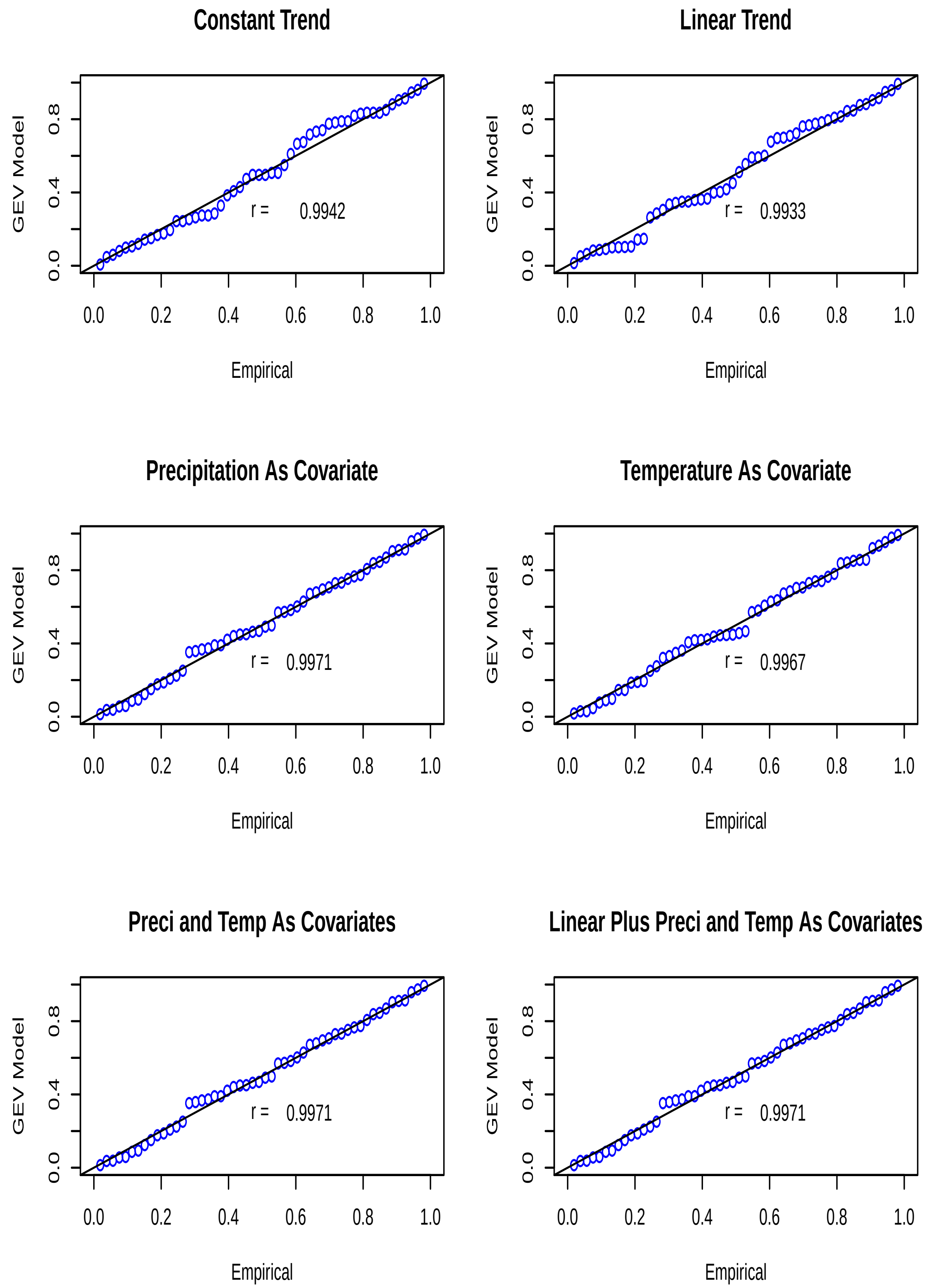


Figure 21: Probability plots of the GEV MLE fits of the six different trends to the maximum flow rate series from 1957 to 2008 using the sin trend for the maximum precipitation and temperature series.



these two models, the $\mu + Ay_t$ model brings in more improvement.) We repeat the above analysis with the linear plus sin trends to represent the maximum precipitation series and the temperature series; the conclusion is the same. The details of the best fitted model are as below:

$$\begin{aligned}
X_t &\sim GEV_{max}(\hat{\mu}(t), \hat{\sigma}, \hat{\xi}), \\
\hat{\mu}(t) &= -564.8465 + 17.7074 y_t, \\
\hat{\sigma} &= 313.2231, \\
\hat{\xi} &= -0.1040, \\
y_t &= \text{the sin trend for the maximum precipitation series} \\
&= 92.2616 - 10.7435 \sin\left(\frac{2\pi}{69.8826} t\right), \ t = 1, \ldots, 48.
\end{aligned} \tag{5.1}$$

For modeling the minimum flow rate series, we summarize the results in Figure 22, Figure 23 and Table 8.

Table 8: Comparison of the six different trends in the GEV model for the minimum flow rate series from 1957 to 2008 using the sin trend for the minimum precipitation series and the linear plus sin trend for the minimum temperature series.

$\mu + bt$	vs	μ	p-value = 0.1073
$\mu + Ay_t$	vs	μ	p-value = 0.0474
$\mu + Bz_t$	vs	μ	p-value = 0.1203
$\mu + Ay_t + Bz_t$	vs	μ	p-value = 0.1250
$\mu + bt + Ay_t + Bz_t$	vs	μ	p-value = 0.2418
$\mu + Ay_t + Bz_t$	vs	$\mu + Ay_t$	p-value = 0.6325
$\mu + Ay_t + Bz_t$	vs	$\mu + Bz_t$	p-value = 0.1864
$\mu + bt + Ay_t + Bz_t$	vs	$\mu + Ay_t + Bz_t$	p-value = 0.8650

From Figure 23 and Table 8 we see that all of the six models fit the minimum flow rate series well, but the only significant improvement over the constant trend model at

Figure 22: GEV MLE fits of the six different trends to the minimum flow rate series from 1957 to 2008 using the sin trend for the minimum precipitation series and linear plus sin trend for the minimum temperature series.

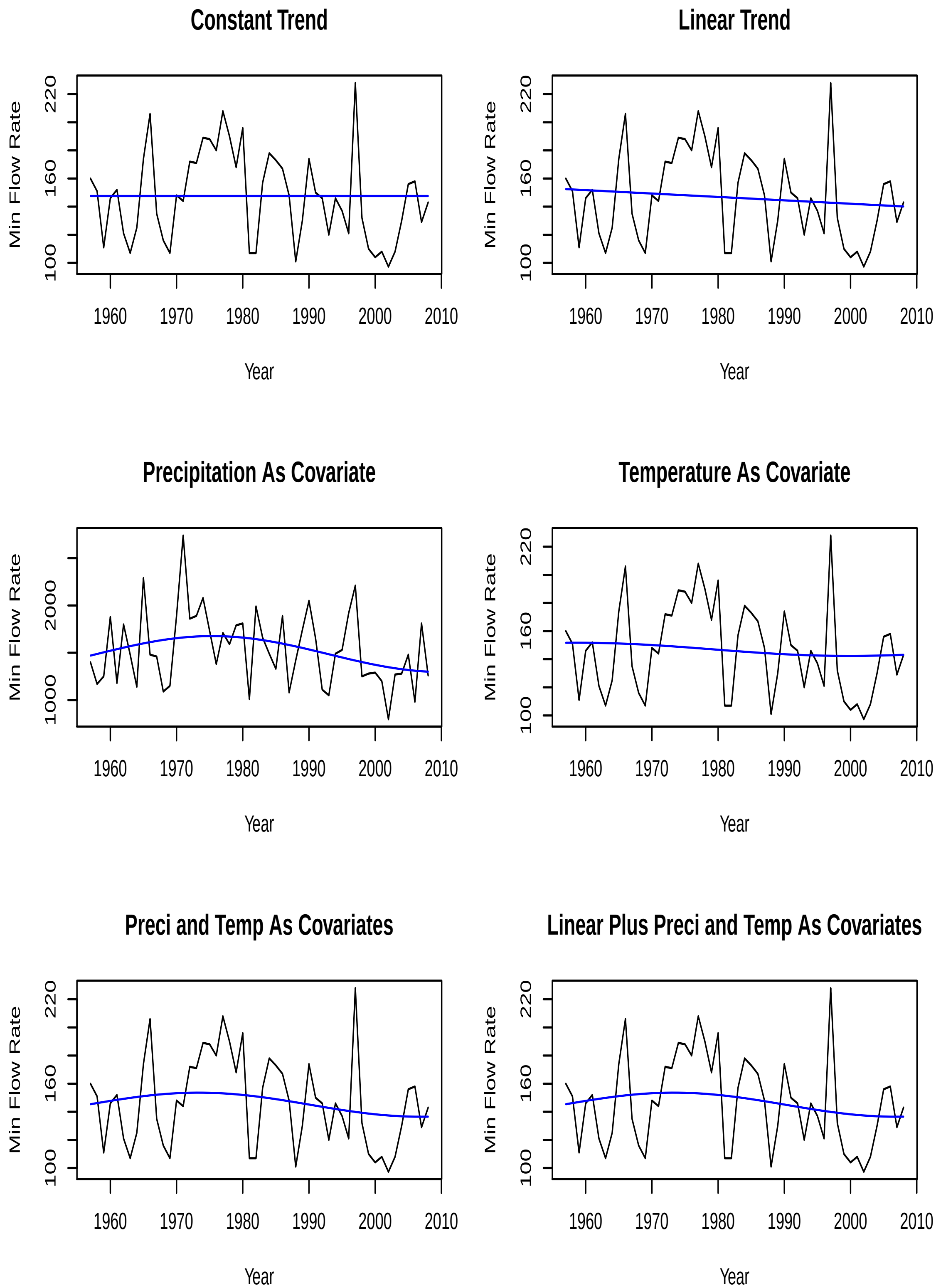
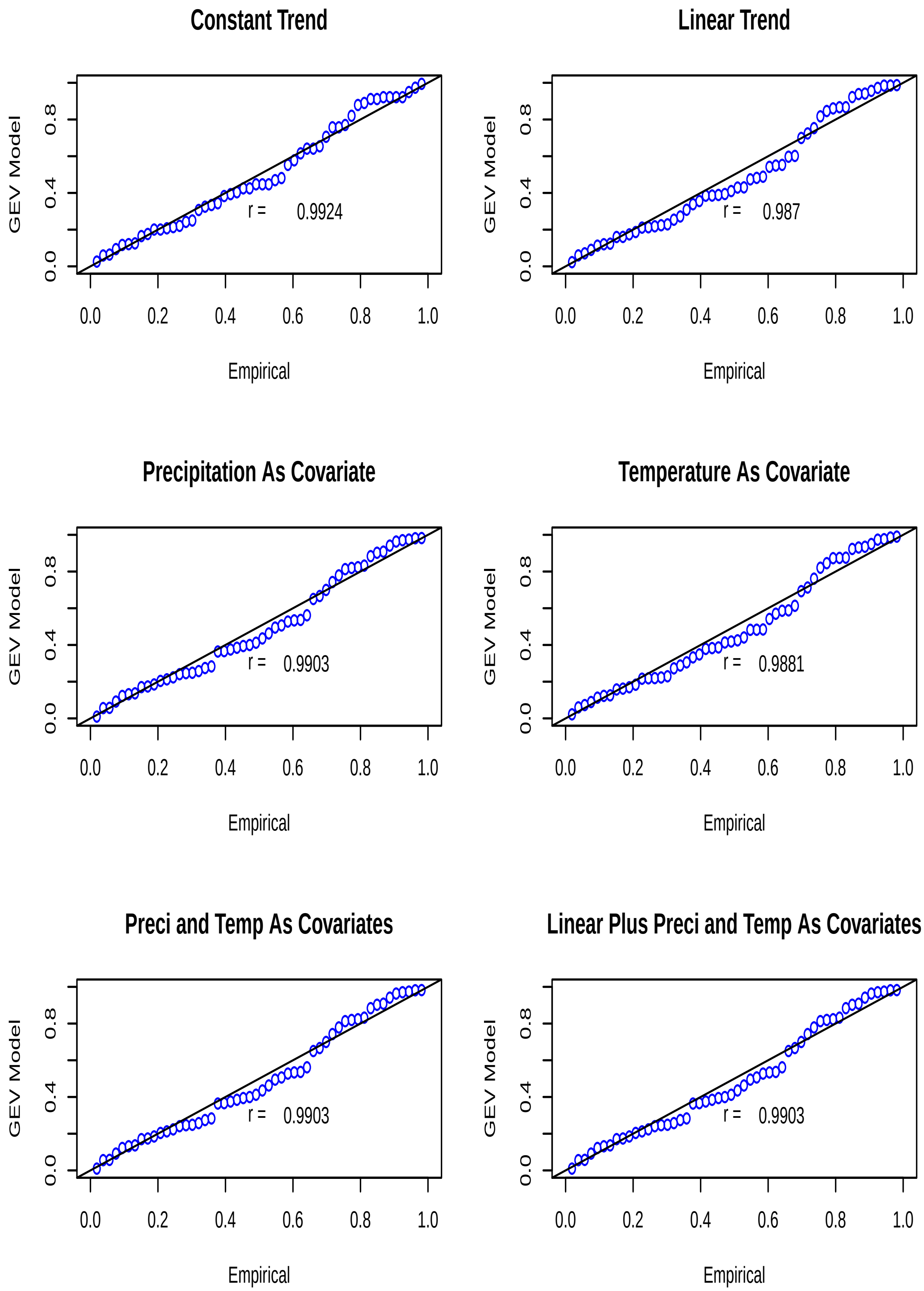


Figure 23: Probability plots of the GEV MLE fits of the six different trends to the minimum flow rate series from 1957 to 2008 using the sin trend for the minimum precipitation series and linear plus sin trend for the minimum temperature series.



significance level $\alpha = 0.05$ is

$$\begin{aligned}
X_t &\sim GEV_{min}(\hat{\mu}(t), \hat{\sigma}, \hat{\xi}), \\
\hat{\mu}(t) &= 101.7926 + 3.9692 y_t, \\
\hat{\sigma} &= 33.3241, \\
\hat{\xi} &= -0.5748, \\
y_t &= \text{the sin trend for the minimum precipitation series} \\
&= 11.1631 - 2.1561 \sin\left(\frac{2\pi}{67.1346} t\right), \ t = 1, \ldots, 48.
\end{aligned} \tag{5.2}$$

We repeat the above analysis with the linear plus sin trend to represent the minimum precipitation series and keep the rest the same. The result is that there is no significant improvement over the constant trend model at significance level $\alpha = 0.05$ (not even at $\alpha = 0.10$ level).

Based on model (5.1) and model (5.2), we can predict future maximum and future minimum flow rates; the results are displayed in Figure 24 and Figure 25. In these two figures, we first predict the means of the maximum and minimum flow rates from 2009 to 2070. Then we subtract the observed maximum and minimum series by their respective means, and denote the residuals for the maximum flow rate series by U_t and for the minimum flow rate series by V_t. We go on to model U_t and V_t using $GEV_{max}(\mu, \sigma, \xi)$ or $GEV_{min}(\mu, \sigma, \xi)$ as possible models (the time or trend effect has been taken out). This leads to the following two models for the residuals:

$$\begin{aligned}
U_t &\sim GEV_{max}(-151.2463, 311.9729, -0.1028), \\
V_t &\sim GEV_{min}(6.3340, 33.3111, -0.5738).
\end{aligned}$$

For U_t, we estimate the 0.025 and 0.975 quantiles of the residual distribution to be $q_{0.025} = -587.0563$ and $q_{0.975} = 803.8426$. For V_t, we do the same thing to get $q_{0.025} = -44.6704$ and $q_{0.975} = 71.0627$. Those estimated quantiles are added to the respective predicted means and the results are plotted in Figure 24 and Figure 25 (the two red curves in each case).

Because the maximum and minimum flow rate series do not have the same periodicities as those of the precipitation and temperature series, we expand our model search by allowing a sin trend in addition to those considered in Table 7 and Table 8. Our expanded modeling gives the following two tables.

From Table 9 and Table 10 we see that no model achieves a significant improvement over the constant trend model at significance level $\alpha = 0.05$. For modeling the maximum flow rate series, there are two borderline significant models (the p-values are 0.0780 and

Figure 24: Prediction of future maximum flow rates based on model (5.1) from 2009 to 2070. The blue curve is the prediction of the mean maximum flow rates, the lower red curve is the prediction of the 0.025 quantiles of the distribution of the maximum flow rates, and the upper red curve is the prediction of the 0.975 quantiles of the distribution of the maximum flow rates.

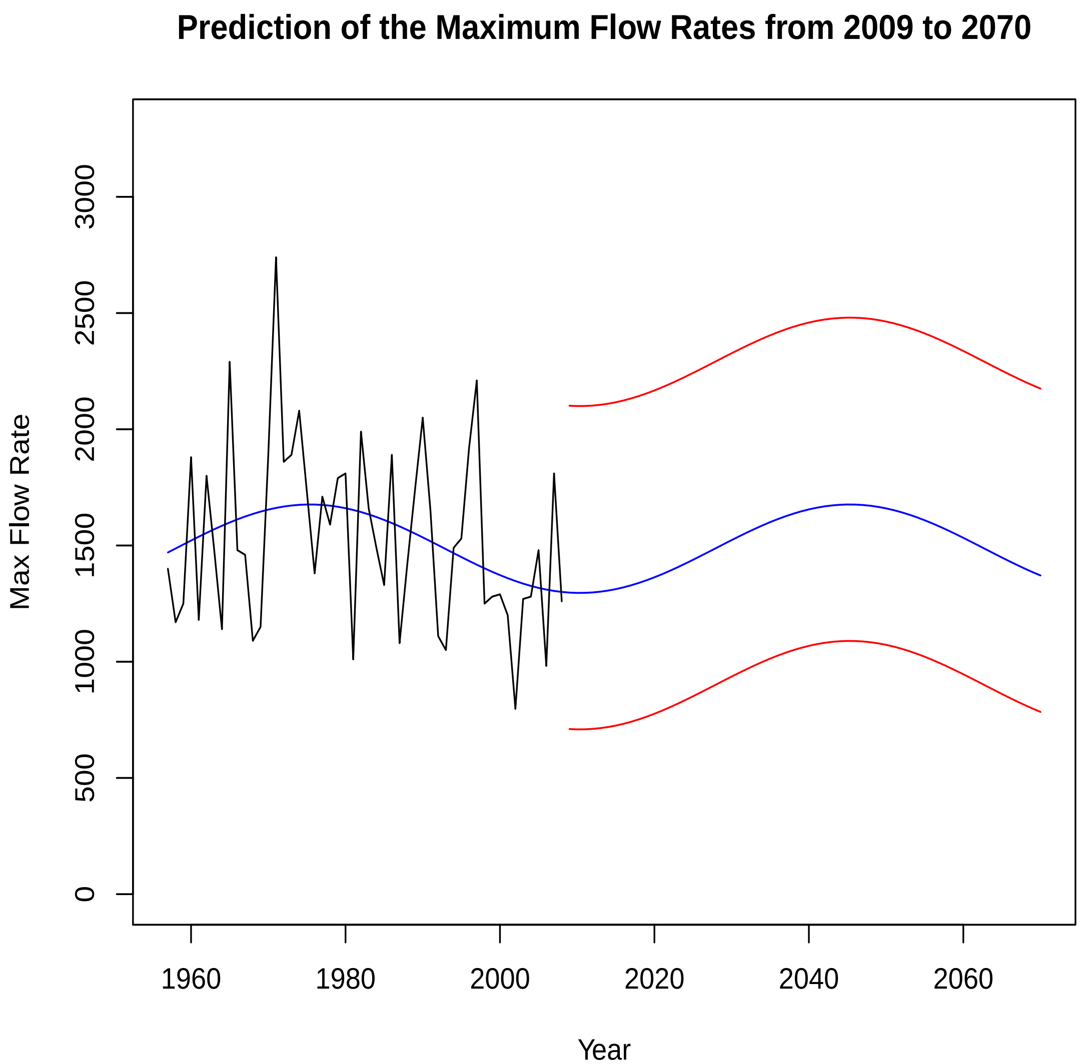


Figure 25: Prediction of future minimum flow rates based on model (5.2) from 2009 to 2070. The blue curve is the prediction of the mean minimum flow rates, the lower red curve is the prediction of the 0.025 quantiles of the distribution of the minimum flow rates, and the upper red curve is the prediction of the 0.975 quantiles of the distribution of the minimum flow rates.

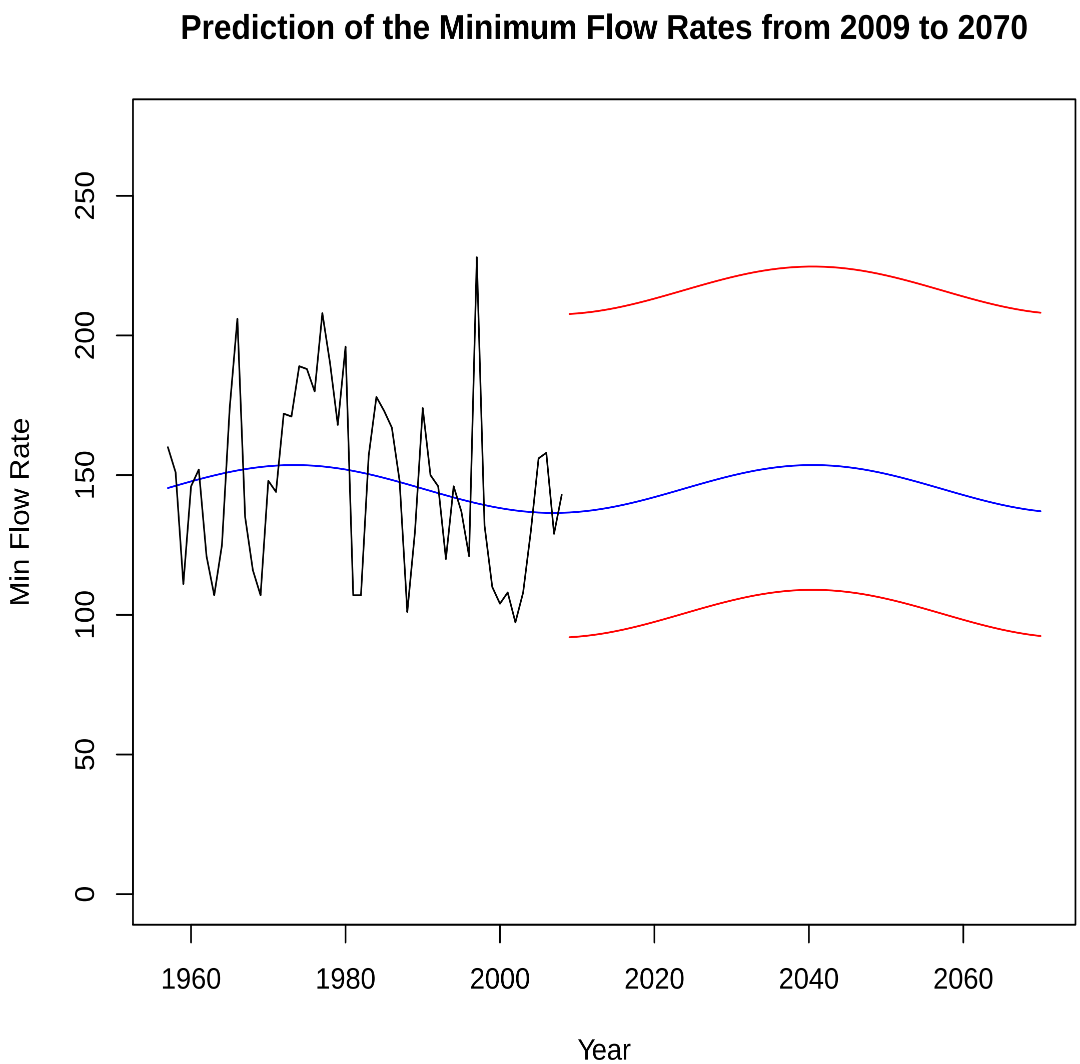


Table 9: Comparison of the expanded six different trends in the GEV model for the maximum flow rate series from 1957 to 2008 using the sin trend for the maximum precipitation and temperature series.

$\mu + bt$	vs	μ	p-value = 0.1503
$\mu + Ay_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	μ	p-value = 0.0780
$\mu + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	μ	p-value = 0.0772
$\mu + Ay_t + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	μ	p-value = 0.1440
$\mu + bt + Ay_t + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	μ	p-value = 0.1982
$\mu + Ay_t + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	$\mu + Ay_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	p-value = 0.8533
$\mu + Ay_t + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	$\mu + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	p-value = 0.9142
$\mu + bt + Ay_t + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	$\mu + Ay_t + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	p-value = 0.4958

0.0772). Since $\alpha = 0.05$ is only used as a guideline, to study a range of possibilities, we list below the models from Table 7 to Table 10 that have p-values less than 0.10.

$$\begin{aligned}
X_t &\sim GEV_{max}(\hat{\mu}(t), \hat{\sigma}, \hat{\xi}), \\
\hat{\mu}(t) &= 8428.3236 - 295.2553 z_t, \\
\hat{\sigma} &= 315.3394, \\
\hat{\xi} &= -0.1065, \\
z_t &= \text{the linear plus sin trend for the maximum temperature series} \\
&= 23.2103 + 0.0030t - 0.5174 \sin\left(\frac{2\pi}{52.5333}t\right), \ t = 1, \ldots, 48.
\end{aligned} \tag{5.3}$$

$$\begin{aligned}
X_t &\sim GEV_{max}(\hat{\mu}(t), \hat{\sigma}, \hat{\xi}), \\
\hat{\mu}(t) &= 1418.9834 + 14.1543 y_t - 66.7391 z_t, \\
\hat{\sigma} &= 313.3758, \\
\hat{\xi} &= -0.1054, \\
y_t &= \text{the sin trend for the maximum precipitation series} \\
&= 92.2616 - 10.7435 \sin\left(\frac{2\pi}{69.8826}t\right), \ t = 1, \ldots, 48, \\
z_t &= \text{the linear plus sin trend for the maximum temperature series} \\
&= 23.2103 + 0.0030t - 0.5174 \sin\left(\frac{2\pi}{52.5333}t\right), \ t = 1, \ldots, 48.
\end{aligned} \tag{5.4}$$

Table 10: Comparison of the expanded six different trends in the GEV model for the minimum flow rate series from 1957 to 2008 using the sin trend for the minimum precipitation series and the linear plus sin trend for the minimum temperature series.

$\mu + bt$	vs	μ	p-value = 0.1073
$\mu + Ay_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	μ	p-value = 0.2771
$\mu + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	μ	p-value = 0.2758
$\mu + Ay_t + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	μ	p-value = 0.3832
$\mu + bt + Ay_t + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	μ	p-value = 0.4781
$\mu + Ay_t + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	$\mu + Ay_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	p-value = 0.5755
$\mu + Ay_t + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	$\mu + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	p-value = 0.5829
$\mu + bt + Ay_t + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	vs	$\mu + Ay_t + Bz_t + A_1 \sin\left(\frac{2\pi}{B_1}t\right)$	p-value = 0.5591

$$\begin{aligned}
X_t &\sim GEV_{max}(\hat{\mu}(t), \hat{\sigma}, \hat{\xi}), \\
\hat{\mu}(t) &= 1434.4469 + 2.4529t + 17.1231y_t - 83.7243z_t, \\
\hat{\sigma} &= 313.4633, \\
\hat{\xi} &= -0.1103, \\
y_t &= \text{the sin trend for the maximum precipitation series} \\
&= 92.2616 - 10.7435 \sin\left(\frac{2\pi}{69.8826}t\right),\ t = 1, \ldots, 48, \\
z_t &= \text{the linear plus sin trend for the maximum temperature series} \\
&= 23.2103 + 0.0030t - 0.5174 \sin\left(\frac{2\pi}{52.5333}t\right),\ t = 1, \ldots, 48.
\end{aligned} \tag{5.5}$$

$$\begin{aligned}
X_t &\sim GEV_{max}(\hat{\mu}(t), \hat{\sigma}, \hat{\xi}), \\
\hat{\mu}(t) &= 1151.3786 + 1.2996y_t + 213.7808 \sin\left(\frac{2\pi}{81.9359}t\right), \\
\hat{\sigma} &= 313.6352, \\
\hat{\xi} &= -0.1036, \\
y_t &= \text{the sin trend for the maximum precipitation series} \\
&= 92.2616 - 10.7435 \sin\left(\frac{2\pi}{69.8826}t\right),\ t = 1, \ldots, 48.
\end{aligned} \tag{5.6}$$

$$
\begin{aligned}
X_t &\sim GEV_{max}(\hat{\mu}(t), \hat{\sigma}, \hat{\xi}), \\
\hat{\mu}(t) &= 1531.4606 - 0.2513 z_t + 225.1076 \sin\left(\frac{2\pi}{82.5418} t\right), \\
\hat{\sigma} &= 311.9086, \\
\hat{\xi} &= -0.1017, \\
z_t &= \text{the linear plus sin trend for the maximum temperature series} \\
&= 23.2103 + 0.0030t - 0.5174 \sin\left(\frac{2\pi}{52.5333} t\right), \ t = 1, \ldots, 48.
\end{aligned} \tag{5.7}
$$

In Figure 26 to Figure 31 we make predictions of the future maximum flow rates from 2009 to 2070 based on models (5.1) and (5.3) to (5.7). In Figure 32 we make predictions of the future minimum flow rates from 2009 to 2070 based on model (5.2). In these figures we add randomly generated maximum or minimum series (in green color).

6 Discussions

In this report we have used the *non-stationary extreme value theory based models* to predict the maximum and minimum flow rates of the Athabasca River at Fort McMurray. Historical precipitation and temperature information was used to build the prediction models and make predictions. All of the computation and graphics are done using an open source and free software called *R*; the Appendix contains the *R* functions we wrote for this report. In the following, we make some specific remarks.

Remarks 1. The maximum and minimum flow rates of the Athabasca River are the so-called (yearly) blocked maximums and minimums, therefore it is natural to study them using extreme value theory based models. In terms of the usual independence and identically distributed requirement on the data to be modeled, we probably have the independence part but almost surely fail to have the identically distributed part (if we had this part, there would be nothing to be modeled because the stationary mean would be the best prediction). The focus of this report is to model the non-stationary means of the flow rates.

Remarks 2. One major weakness of the observed maximum and minimum flow rate series is that they are a little too short, both in the sense of not long enough to contain a longterm climate circle and in the sense of being shorter than the range we want to make predictions. Luckily, we had longer precipitation and temperature series to suggest possible types of trends. We thoroughly explored the linear and cyclic trends.

Figure 26: Prediction of the future maximum flow rates from 2009 to 2070 based on model (5.1). The blue curve is the prediction of the mean maximum flow rates, the lower red curve is the prediction of the 0.025 quantiles of the distribution of the maximum flow rates, and the upper red curve is the prediction of the 0.975 quantiles of the distribution of the maximum flow rates. A randomly generated future maximum flow rate series is added.

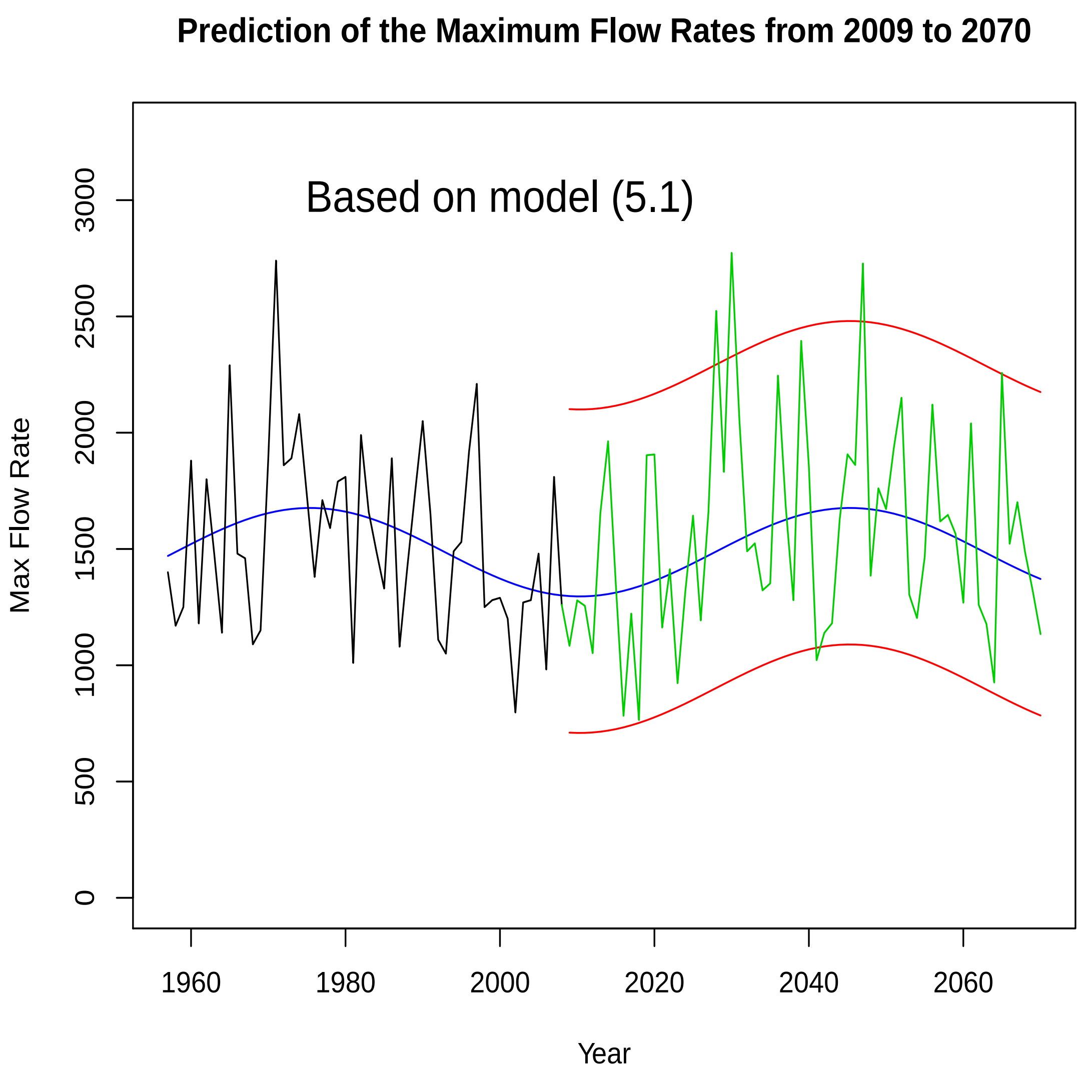


Figure 27: Prediction of the future maximum flow rates from 2009 to 2070 based on model (5.3).

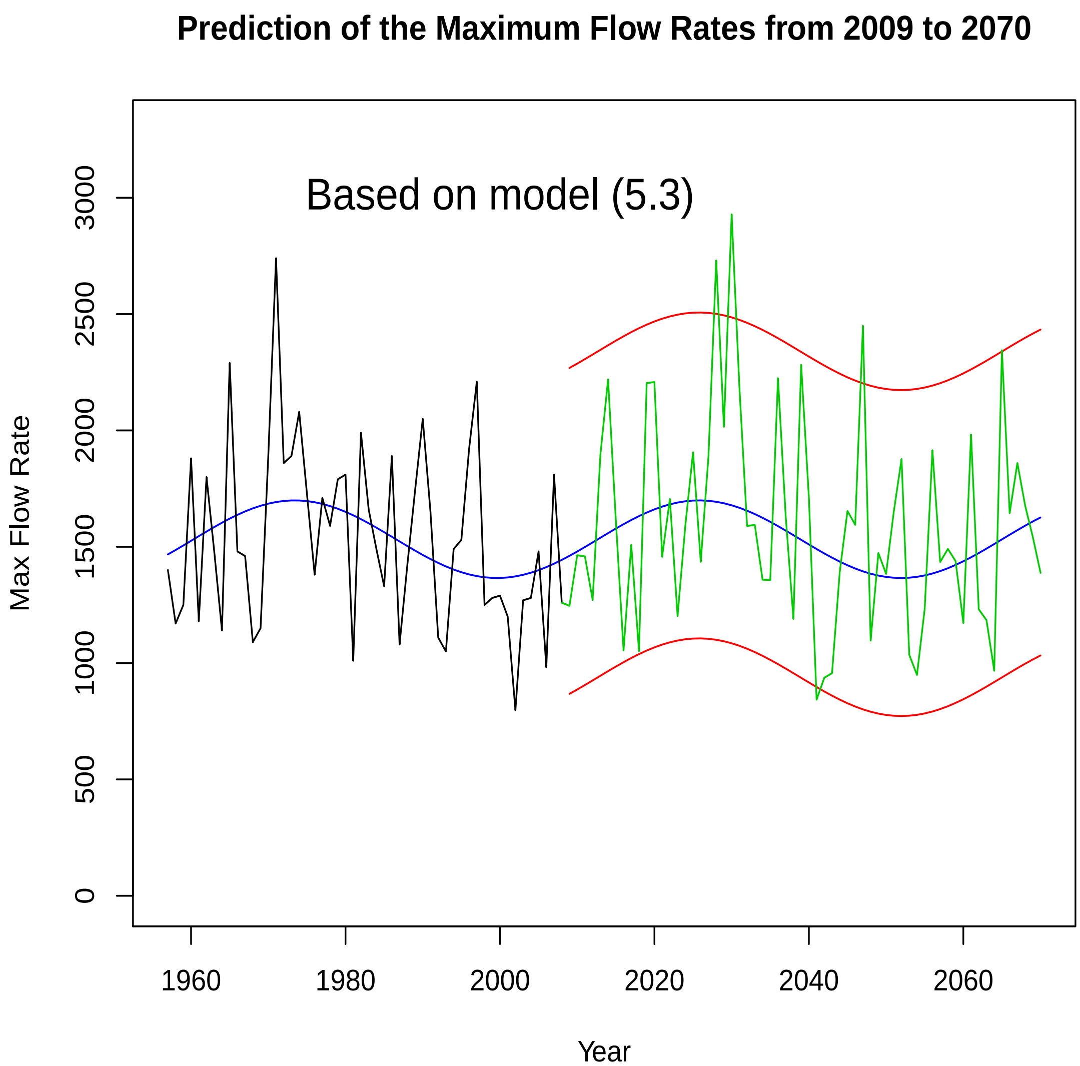


Figure 28: Prediction of the future maximum flow rates from 2009 to 2070 based on model (5.4).

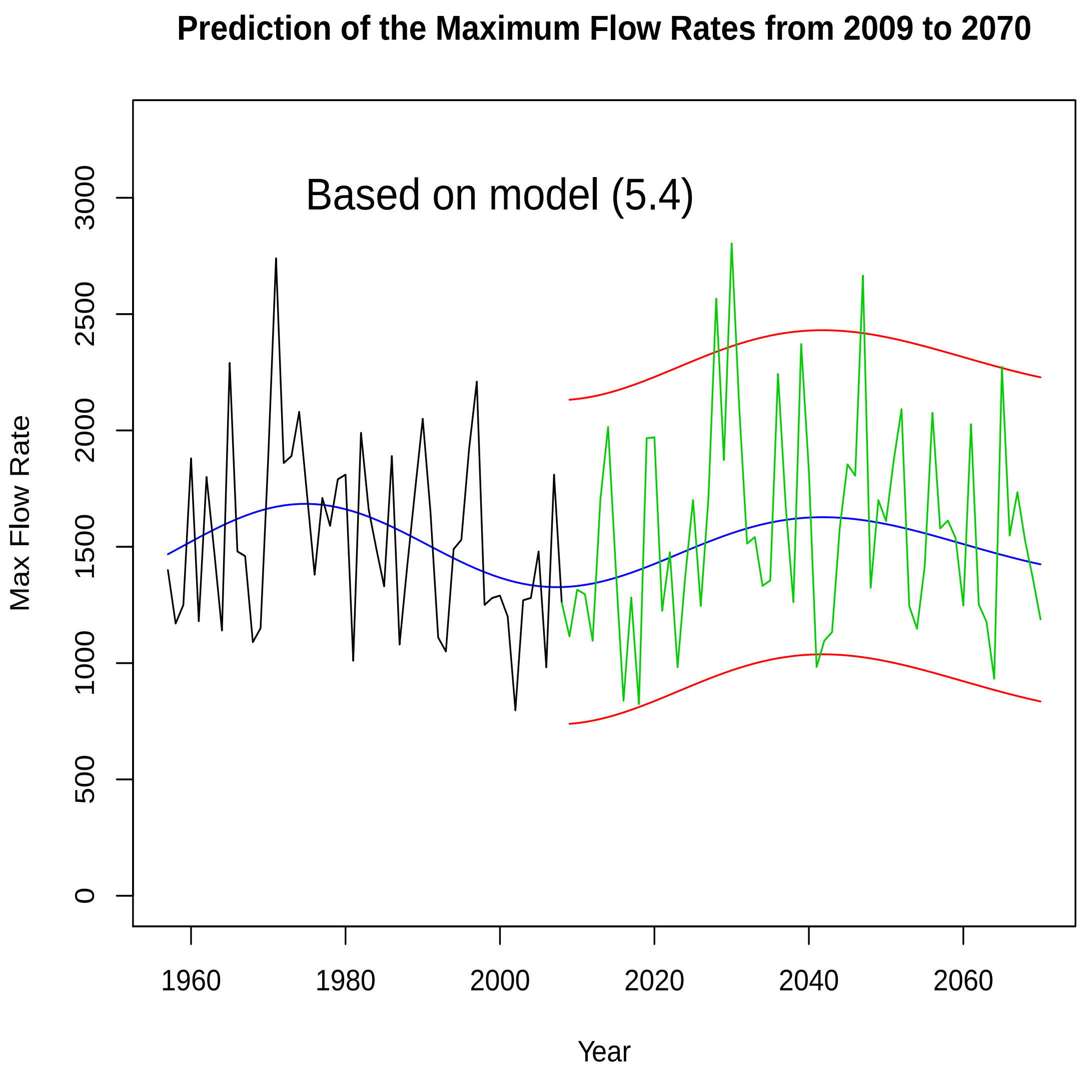


Figure 29: Prediction of the future maximum flow rates from 2009 to 2070 based on model (5.5).

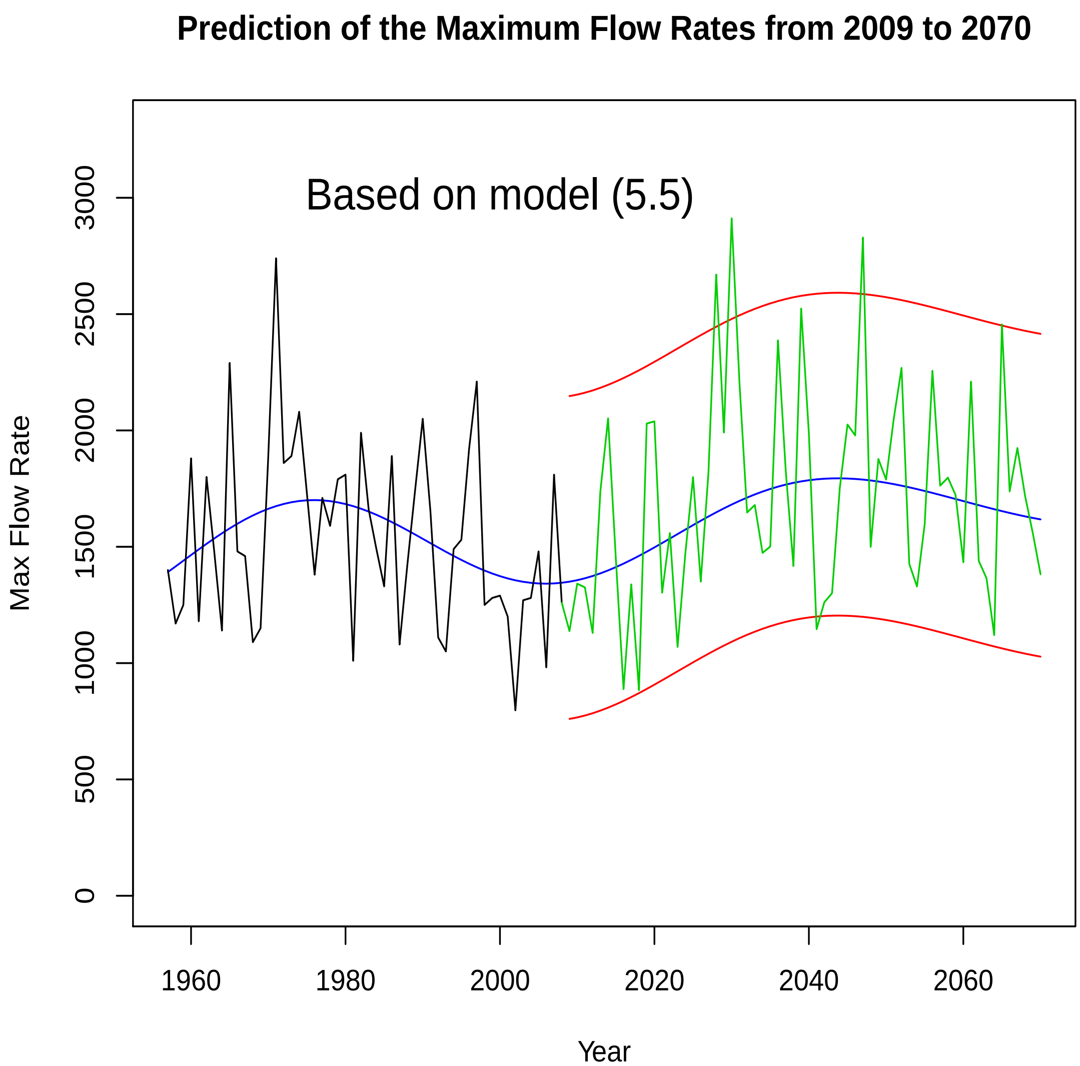


Figure 30: Prediction of the future maximum flow rates from 2009 to 2070 based on model (5.6).

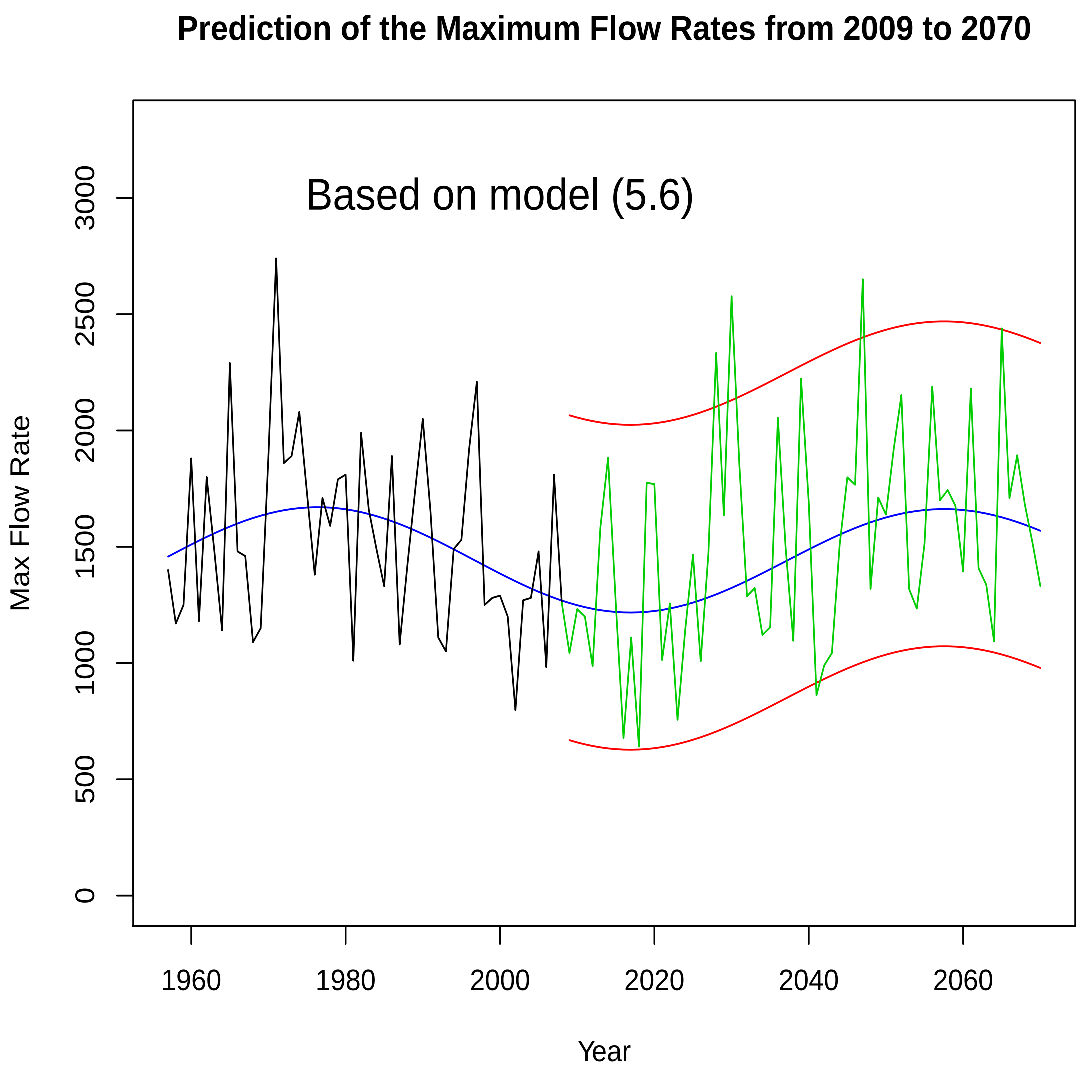


Figure 31: Prediction of the future maximum flow rates from 2009 to 2070based on model (5.7).

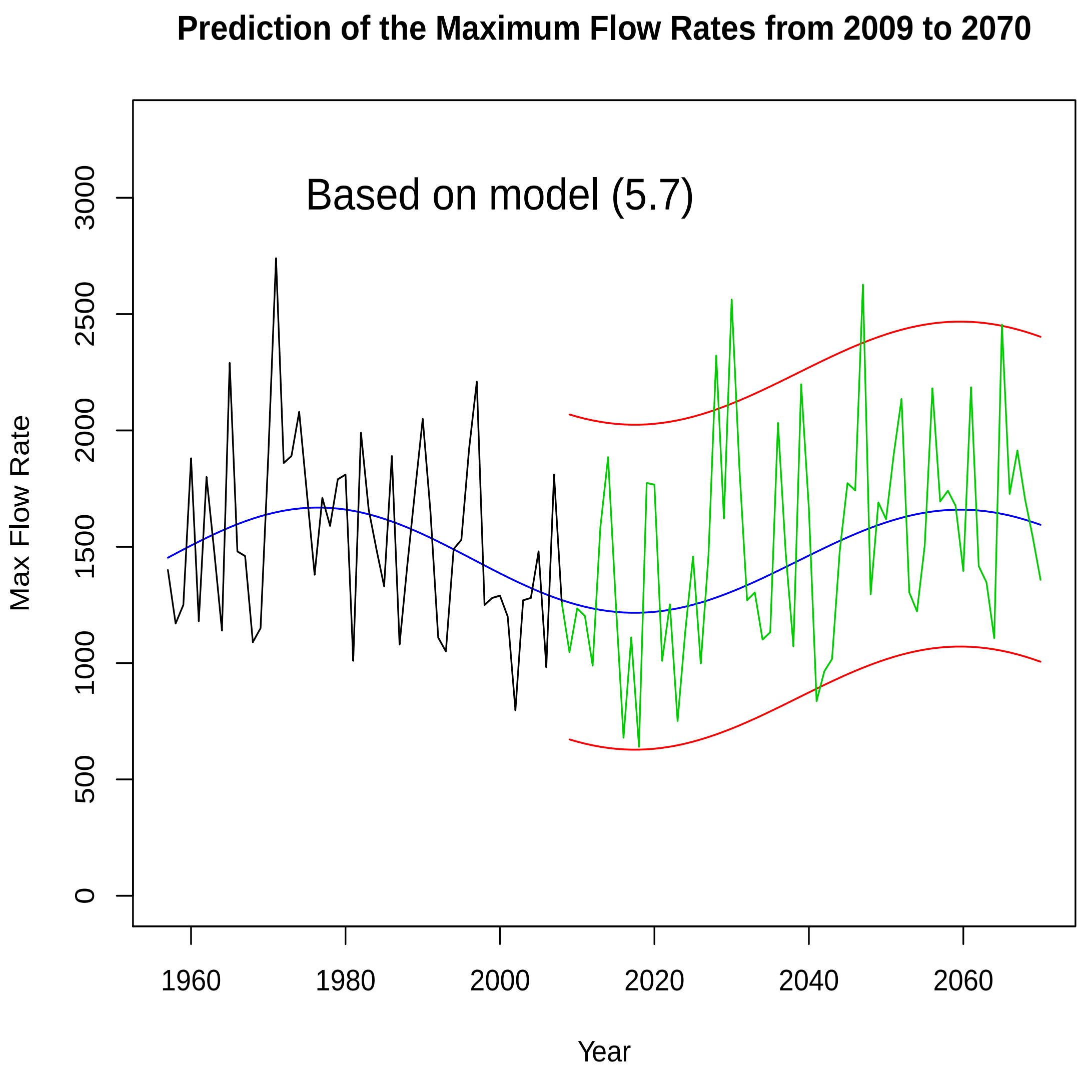
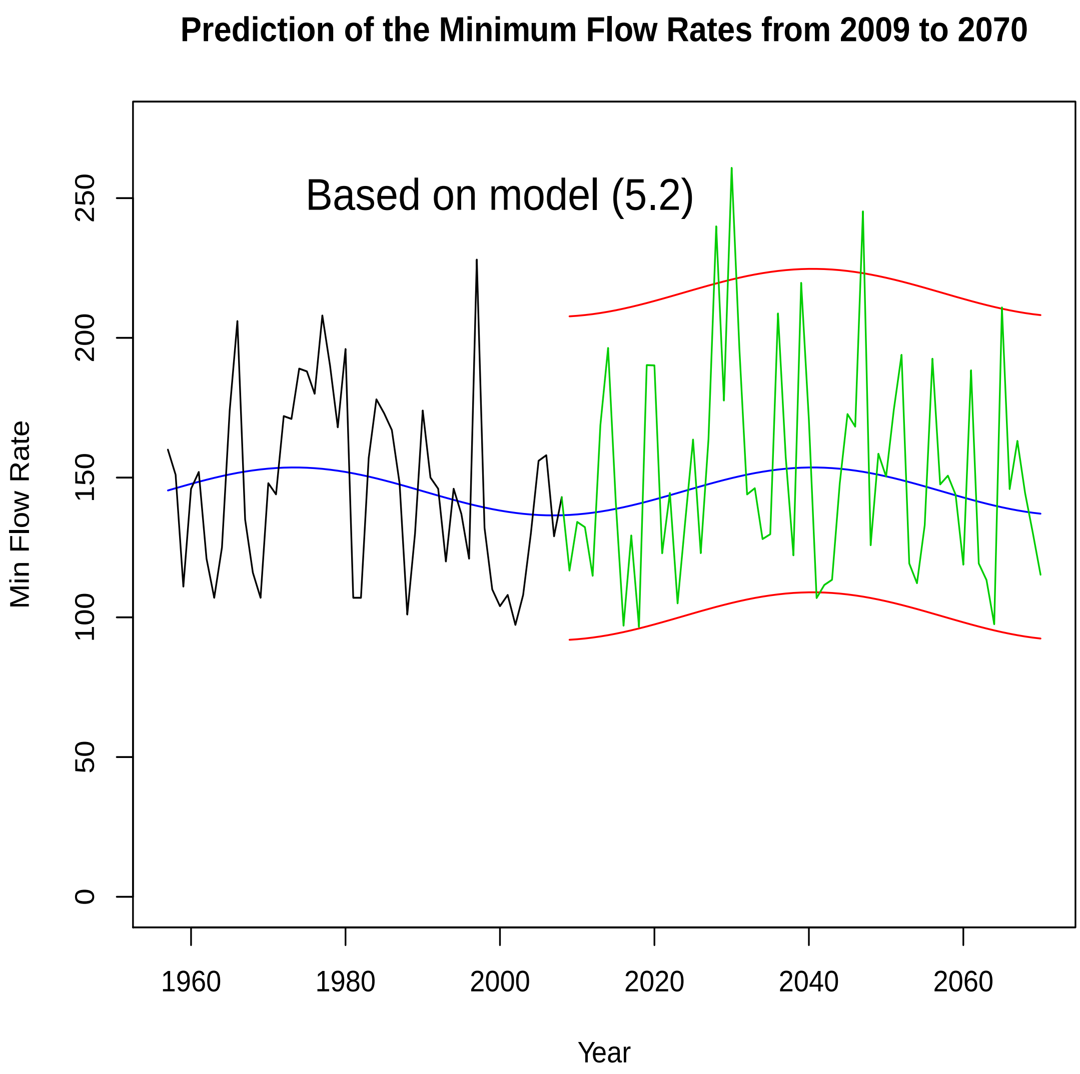


Figure 32: Prediction of the future minimum flow rates from 2009 to 2070 based on model (5.2). The blue curve is the prediction of the mean minimum flow rates, the lower red curve is the prediction of the 0.025 quantiles of the distribution of the minimum flow rates, and the upper red curve is the prediction of the 0.975 quantiles of the distribution of the minimum flow rates. A randomly generated future minimum flow rate series is added.



Remarks 3. A major finding of this report is that the short observed flow rate series were in good agreement with models containing cyclic trends. Of the two types of trends, the linear trend and the sin trend, the observed flow rate series lent little support to the linear trend, but went strongly with the sin trend (in no case was a linear trend model statistically significant, while many of the models with sin trends were significant).

Remarks 4. Another major finding of this report is that the best fitted models (the ones that made the largest improvements over the constant trend model while staying the smallest in size) only contain cyclic trends. This result has serious implications, namely, for the near future and under the natural environmental conditions, both the maximum flow rates and the minimum flow rates are going to be stable. On the other hand, if we reference those statistically less significant models, they seem to tell us that both the maximum flow rates and the minimum flow rates are slightly going up in the near future under the natural environmental conditions.

Remarks 5. Throughout the report we relied on the maximum likelihood approach heavily. In particular, the deviance statistic was used to pick the most significant models. We counted a linear trend with one parameter and counted a sin trend with two parameters, totally ignoring the functional form. In some applications, however, it is the "extreme" of the maximum and minimum flow rate series that matter the most, and the maximum likelihood approach may not lead to the "best" way to measure the fit of a model to a data set.

Remarks 6. Between precipitation and temperature, the former has better power of explaining what was going on in the flow rates. This is a kind of expected because from the scientific point of view, precipitation is the source of river water.

Remarks 7. We obtained way more significant models to describe the maximum flow rate series, and only one (barely) significant model to describe the minimum flow rate series. In other words, we were quite successful using historical precipitation and temperature series to help model the maximum flow rate series, and were not quite successful to model the minimum flow rate series. This does not necessarily imply that we did not model the minimum flow rate series adequately, because being too short to contain prominent features and the cold winter condition are likely related to the root of the issue.

Remarks 8. For the modeling approach used in this report, there is no object way of considering the impacts of various climate change scenarios. However, if there were any important (and clearly observed) climate changes in the past, their impacts were built into our prediction models.

References

[1] Coles, S. (2001). *An Introduction to Statistical Modeling of Extreme Values*. Springer, London.

[2] R Development Core Team (2010). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL http://www.R-project.org.

Appendix

This Appendix contains the R functions written specifically for this report.
Names of the data sets used are:
precmax: maximum precipitation series from 1924 to 2004
ratemax: maximum flow rate series from 1957 to 2008
tempmax: maximum temperature series from 1909 to 2007

precmin: minimum precipitation series from 1924 to 2004
ratemin: minimum flow rate series from 1957 to 2008
tempmin: minimum temperature series from 1909 to 2007

```
findini <- function(x, ind=1, f=60){

# This function finds initial values for the MLE minimization

if(ind == 1) {
fn <- function(theta,x){
a <- theta[1]
b <- theta[2]
u <- c(1:length(x))
sum((x - a - b*u)^2)
                       }
xbar <- mean(x)
ini <- c(xbar, 0)
out <- optim(ini, fn, method="Nelder-Mead", control=list(maxit=1000), x=x)
est <- out$par
              }
if(ind == 2) {
fn <- function(theta,x){
mu<- theta[1]
a <- theta[2]
b <- theta[3]
u <- c(1:length(x))
sum((x - mu - a*sin(2*pi/b*u))^2)
                       }
xbar <- mean(x)
ini <- c(xbar, 0, f)
out <- optim(ini, fn, method="Nelder-Mead", control=list(maxit=1000), x=x)
est <- out$par
              }
if(ind == 3) {
fn <- function(theta,x){
a <- theta[1]
b <- theta[2]
A <- theta[3]
B <- theta[4]
u <- c(1:length(x))
sum((x - a - b*u - A*sin(2*pi/B*(u)))^2)
                       }
xbar <- mean(x)
ini <- c(xbar, 0, 1, f)
out <- optim(ini, fn, method="Nelder-Mead", control=list(maxit=1000), x=x)
```

```
est <- out$par
              }
Est
}




getmeanmax <- function(x, ind = 1, startyear=1909, endyear=2060, tyy=" ",
txx="Year",tmm="Trend Plot for GEV MLE Fit"){

#This function obtains the means of of a fitted GEV model and makes a plot.


if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out1$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], length(y))
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
```

```
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out2$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+mle[4]*y + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}

if(ind == 3) {
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out3$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+ mle[4]*sin(2*pi/mle[5]*y) + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")          }
```

```
if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out4$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+ mle[4]*y + mle[5]*sin(2*pi/mle[6]*y) + mle[2]*(gamma(1-mle[3])
- 1)/mle[3]
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}
invisible(trend)
}
```

```
getmeanmaxnoplot <- function(x, ind = 1, startyear=1909, endyear=2060, tyy=" ",
txx="Year",tmm="Trend Plot for GEV MLE Fit"){

#This function outputs the means of a fitted GEV model for other functions to
use


if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out1$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], length(y))
}

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out2$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
```

```
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+mle[4]*y + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
}

if(ind == 3) {
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out3$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+ mle[4]*sin(2*pi/mle[5]*y) + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
}

if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
```

```
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out4$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+ mle[4]*y + mle[5]*sin(2*pi/mle[6]*y) + mle[2]*(gamma(1-mle[3])
- 1)/mle[3]
}
invisible(trend)
}




getmeanmin <- function(x, ind = 1, startyear=1909, endyear=2060, tyy=" ",
txx="Year",tmm="Trend Plot for GEV MLE Fit"){

#This function obtains the means of fitted GEV models for minimum series

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
```

```
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out1$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], length(y))
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out2$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+mle[4]*y + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}

if(ind == 3) {
```

```
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                         }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out3$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+ mle[4]*sin(2*pi/mle[5]*y) + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}

if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                         }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
```

```
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out4$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+ mle[4]*y + mle[5]*sin(2*pi/mle[6]*y) + mle[2]*(gamma(1-mle[3])
- 1)/mle[3]
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}
invisible(trend)
}




getmeanminnoplot <- function(x, ind = 1, startyear=1909, endyear=2060, tyy=" ",
txx="Year",tmm="Trend Plot for GEV MLE Fit"){

#This function outputs the means of fitted GEV models for minimum series
#No plots are generated

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
```

```
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out1$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], length(y))
}

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                          }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out2$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+mle[4]*y + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
}

if(ind == 3) {
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
```

```
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out3$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+ mle[4]*sin(2*pi/mle[5]*y) + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
}

if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
```

```
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out4$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+ mle[4]*y + mle[5]*sin(2*pi/mle[6]*y) + mle[2]*(gamma(1-mle[3])
- 1)/mle[3]
}
invisible(trend)
}



gevmax <- function(x, ind = 1, txx="Empirical",
tyy="GEV Model",tmm="Probability Plot for GEV MLE Fit"){

# This function estimates 4 different tred models for maximum series.

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
se1 <- sqrt(diag(solve(out1$hessian)))
mle <- out1$par
p <- c(1:n)/(n+1)
zhat <- log(1 + mle[3]*((x-mle[1])/mle[2]))/mle[3]
phat <- sort(exp(-exp(-zhat)))
plot(p, phat, col=4, xlim=c(0,1), ylim=c(0,1), xlab=txx, ylab=tyy, main=tmm)
text(0.5, 0.3, "r = ")
text(0.68, 0.3, round(cor(p,phat),4))
abline(0,1)
print(mle)
             }

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
```

```
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
se2 <- sqrt(diag(solve(out2$hessian)))
mle <- out2$par
p <- c(1:n)/(n+1)
ti <- c(1:n)
zhat <- log(1 + mle[3]*((x-(mle[1]+mle[4]*ti))/mle[2]))/mle[3]
phat <- sort(exp(-exp(-zhat)))
plot(p, phat, col=4, xlim=c(0,1), ylim=c(0,1), xlab=txx, ylab=tyy, main=tmm)
text(0.5, 0.3, "r = ")
text(0.68, 0.3, round(cor(p,phat),4))
abline(0,1)
print(mle)
           }

if(ind == 3) {
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out3$convergence, out3$value))
se3 <- sqrt(diag(solve(out3$hessian)))
mle <- out3$par
p <- c(1:n)/(n+1)
ti <- c(1:n)
zhat <- log(1 + mle[3]*((x-(mle[1] +
mle[4]*sin(2*pi/mle[5]*ti)))/mle[2]))/mle[3]
```

```
phat <- sort(exp(-exp(-zhat)))
plot(p, phat, col=4, xlim=c(0,1), ylim=c(0,1), xlab=txx, ylab=tyy, main=tmm)
text(0.5, 0.3, "r = ")
text(0.68, 0.3, round(cor(p,phat),4))
abline(0,1)
print(mle)
           }

if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out4$convergence, out4$value))
se4 <- sqrt(diag(solve(out4$hessian)))
mle <- out4$par
p <- c(1:n)/(n+1)
ti <- c(1:n)
zhat <- log(1 + mle[3]*((x-(mle[1] + mle[4]*ti +
mle[5]*sin(2*pi/mle[6]*ti)))/mle[2]))/mle[3]
phat <- sort(exp(-exp(-zhat)))
plot(p, phat, col=4, xlim=c(0,1), ylim=c(0,1), xlab=txx, ylab=tyy, main=tmm)
text(0.5, 0.3, "r = ")
text(0.68, 0.3, round(cor(p,phat),4))
abline(0,1)
print(mle)
```

```
            }
}


gevmaxcov <- function(x, c1=3, c2=3){


# This function fits GEV models with maximum precipitattion/temperature as
covariates.

y <- getmeanmaxnoplot(precmax, c1, startyear=1924)[c(49:(49+51))]
z <- getmeanmaxnoplot(tempmax, c2, startyear=1909)[c(49:(49+51))]

xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)

fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))

fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))


fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
u <- 1 + xi*(x-(mu + A*y))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
```

```
                        }
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))


fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
u <- 1 + xi*(x-(mu + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))


fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
u <- 1 + xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))


fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))
```

```
print(out1$par)
print(out2$par)
print(out3$par)
print(out4$par)
print(out5$par)
print(out6$par)


p1 <- 1 - pchisq(2*(-out2$value + out1$value),1)
p2 <- 1 - pchisq(2*(-out3$value + out1$value),1)
p3 <- 1 - pchisq(2*(-out4$value + out1$value),1)
p4 <- 1 - pchisq(2*(-out5$value + out1$value),2)
p5 <- 1 - pchisq(2*(-out6$value + out1$value),3)
p6 <- 1 - pchisq(2*(-out5$value + out3$value),1)
p7 <- 1 - pchisq(2*(-out5$value + out4$value),1)
p8 <- 1 - pchisq(2*(-out6$value + out5$value),1)


print(c(p1,p2,p3,p4,p5,p6,p7,p8),3)

}



gevmaxcov1 <- function(x, c1=3, c2=3){

#This function fits GEV models to the maximum series with both
#Precipitation/temperature covariates and sin trend

y <- getmeanmaxnoplot(precmax, c1, startyear=1924)[c(49:(49+51))]
z <- getmeanmaxnoplot(tempmax, c2, startyear=1909)[c(49:(49+51))]

xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)

fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))

fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
```

```
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))


fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
A1 <- theta[5]
B1 <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))


fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
A1 <- theta[5]
B1 <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))


fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
A1 <- theta[6]
B1 <- theta[7]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
```

```
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))


fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
A1 <- theta[7]
B1 <- theta[8]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5], est[6], est[7])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 0, 60)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))

print(out1$par)
print(out2$par)
print(out3$par)
print(out4$par)
print(out5$par)
print(out6$par)


p1 <- 1 - pchisq(2*(-out2$value + out1$value),1)
p2 <- 1 - pchisq(2*(-out3$value + out1$value),3)
p3 <- 1 - pchisq(2*(-out4$value + out1$value),3)
p4 <- 1 - pchisq(2*(-out5$value + out1$value),4)
p5 <- 1 - pchisq(2*(-out6$value + out1$value),5)
p6 <- 1 - pchisq(2*(-out5$value + out3$value),1)
p7 <- 1 - pchisq(2*(-out5$value + out4$value),1)
p8 <- 1 - pchisq(2*(-out6$value + out5$value),1)


print(c(p1,p2,p3,p4,p5,p6,p7,p8),3)

}



gevmaxcovplot <- function(x, ind=1, c1=3, c2=3, txx="Year", tyy=" ", tmm=" ",
startyear=1){
```

```
#This function plots the results of fitting a GEV model with maximum
precipitation/temperature series as covariates

y <- getmeanmax(precmax, c1, startyear=1924)[c(49:(49+51))]
z <- getmeanmax(tempmax, c2, startyear=1909)[c(49:(49+51))]

if(ind ==1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
mle <- out1$par
ti <- c(1:n)
plot(ti+startyear-1, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear-1, rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], n),
type="l", col=4)
            }

if(ind ==2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
mle <- out2$par
ti <- c(1:n)
plot(ti+startyear-1, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear-1, mle[1] + mle[4]*ti + mle[2]*(gamma(1-mle[3]) - 1)/mle[3],
type="l", col=4)
            }

if(ind == 3) {
```

```
fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
u <- 1 + xi*(x-(mu + A*y))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))
mle <- out3$par
ti <- c(1:n)
plot(ti+startyear-1, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear-1, mle[1] + mle[4]*y + mle[2]*(gamma(1-mle[3]) - 1)/mle[3],
type="l", col=4)
              }

if(ind == 4) {
fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
u <- 1 + xi*(x-(mu + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))
mle <- out4$par
ti <- c(1:n)
plot(ti+startyear-1, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear-1, mle[1] + mle[4]*z + mle[2]*(gamma(1-mle[3]) - 1)/mle[3],
type="l", col=4)
             }

if(ind == 5) {
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
u <- 1 + xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
```

```
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))
mle <- out5$par
ti <- c(1:n)
plot(ti+startyear-1, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear-1, mle[1] + mle[4]*y + mle[5]*z + mle[2]*(gamma(1-mle[3]) -
1)/mle[3], type="l", col=4)
             }

if(ind == 6) {
fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
u <- 1 + xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))
mle <- out6$par
ti <- c(1:n)
plot(ti+startyear-1, x, type="l", xlab=txx, ylab=tyy, main=tmm)
```

```
points(ti+startyear-1, mle[1] + mle[4]*ti + mle[5]*y + mle[6]*z +
mle[2]*(gamma(1-mle[3]) - 1)/mle[3], type="l", col=4)
             }
mle
}



gevmaxest <- function(x, ind = 1){

#This function outputs GEV model estimates for maximum series

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
se1 <- sqrt(diag(solve(out1$hessian)))
mle <- out1$par
             }

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
se2 <- sqrt(diag(solve(out2$hessian)))
mle <- out2$par
           }

if(ind == 3) {
```

```
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out3$convergence, out3$value))
se3 <- sqrt(diag(solve(out3$hessian)))
mle <- out3$par
           }

if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
```

```
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out4$convergence, out4$value))
se4 <- sqrt(diag(solve(out4$hessian)))
mle <- out4$par
           }
mle
}



gevmaxpred <- function(x, ind=1, c1=3, c2=3, txx="Year", tyy=" ", tmm=" ",
endyear=2070){

#This function makes prediction of future flow maximum flow rates

yy <- getmeanmaxnoplot(precmax, c1, startyear=1924, endyear=endyear)
y <- yy[c(49:(49+51))]
zz <- getmeanmaxnoplot(tempmax, c2, startyear=1909, endyear=endyear)
z <- zz[c(49:(49+51))]

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                              }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
mle <- out1$par
ti <- c(1:(n+endyear-2008))
pred <- rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], endyear-1957+1)
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 2) {
fn2 <- function(theta,x){
```

```
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
mle <- out2$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 3) {
fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))
mle <- out3$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]
```

```
newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 4) {
fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))
mle <- out4$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*zz[-c(1:48)] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 5) {
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y + B*z))/sig
```

```
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))
mle <- out5$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[5]*zz[-c(1:48)] + mle[2]*(gamma(1-
mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 6) {
fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0)
```

```
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))
mle <- out6$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[5]*yy[-c(1:48)] + mle[6]*zz[-c(1:48)] +
mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
              }

print(c(q1,q2))
print(mle)
newmle
}



gevmaxpred1 <- function(x, ind=1, c1=3, c2=3, txx="Year", tyy=" ", tmm=" ",
endyear=2070){

#This function predicts future maximum flow rates using
precipitation/temperature
#as covariates and sin trend

yy <- getmeanmaxnoplot(precmax, c1, startyear=1924, endyear=endyear)
y <- yy[c(49:(49+51))]
zz <- getmeanmaxnoplot(tempmax, c2, startyear=1909, endyear=endyear)
z <- zz[c(49:(49+51))]

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                              }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
```

```
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
mle <- out1$par
ti <- c(1:(n+endyear-2008))
pred <- rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], endyear-1957+1)
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
mle <- out2$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }
```

```
if(ind == 3) {
fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
A1 <- theta[5]
B1 <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))
mle <- out3$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[5]*sin(2*pi/mle[6]*ti) +
mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 4) {
fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
A1 <- theta[5]
B1 <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
```

```
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))
mle <- out4$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*zz[-c(1:48)] + mle[5]*sin(2*pi/mle[6]*ti) +
mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 5) {
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
A1 <- theta[6]
B1 <- theta[7]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))
mle <- out5$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[5]*zz[-c(1:48)] +
mle[6]*sin(2*pi/mle[7]*ti) + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
```

```
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 6) {
fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
A1 <- theta[7]
B1 <- theta[8]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
A1 <- theta[6]
B1 <- theta[7]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5], est[6], est[7])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 0, 60)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))
mle <- out6$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[5]*yy[-c(1:48)] + mle[6]*zz[-c(1:48)] +
mle[7]*sin(2*pi/mle[8]*ti) + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
```

```
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
              }

print(c(q1,q2))
print(mle)
newmle
}



gevmaxpred2 <- function(x, ind=1, c1=3, c2=3, txx="Year", tyy=" ", tmm=" ",
endyear=2070){

#This function does the same as gevpred, but adds a randomly generated series

set.seed(2010)

yy <- getmeanmaxnoplot(precmax, c1, startyear=1924, endyear=endyear)
y <- yy[c(49:(49+51))]
zz <- getmeanmaxnoplot(tempmax, c2, startyear=1909, endyear=endyear)
z <- zz[c(49:(49+51))]

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
mle <- out1$par
ti <- c(1:(n+endyear-2008))
pred <- rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], endyear-1957+1)
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,1) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
```

```
            }

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
mle <- out2$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,1) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
            }

if(ind == 3) {
fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
```

```
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))
mle <- out3$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,1) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 4) {
fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))
mle <- out4$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*zz[-c(1:48)] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
```

```
v <- c(x[n], gevsimu(newmle,m,1) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 5) {
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))
mle <- out5$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[5]*zz[-c(1:48)] + mle[2]*(gamma(1-
mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,1) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 6) {
fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
```

```
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))
mle <- out6$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[5]*yy[-c(1:48)] + mle[6]*zz[-c(1:48)] +
mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,1) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
              }

print(c(q1,q2))
print(mle)
newmle
}



gevmaxpred3 <- function(x, ind=1, c1=3, c2=3, txx="Year", tyy=" ", tmm=" ",
endyear=2070){

#This function does the same as gevmaxpred1, but adds a randomly generated
series
```

```
set.seed(2010)

yy <- getmeanmaxnoplot(precmax, c1, startyear=1924, endyear=endyear)
y <- yy[c(49:(49+51))]
zz <- getmeanmaxnoplot(tempmax, c2, startyear=1909, endyear=endyear)
z <- zz[c(49:(49+51))]

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
mle <- out1$par
ti <- c(1:(n+endyear-2008))
pred <- rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], endyear-1957+1)
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,1) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
```

```
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
mle <- out2$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,1) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 3) {
fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
A1 <- theta[5]
B1 <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))
mle <- out3$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[5]*sin(2*pi/mle[6]*ti) +
mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)
```

```
plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,1) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 4) {
fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
A1 <- theta[5]
B1 <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))
mle <- out4$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*zz[-c(1:48)] + mle[5]*sin(2*pi/mle[6]*ti) +
mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,1) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 5) {
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
```

```
B <- theta[5]
A1 <- theta[6]
B1 <- theta[7]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))
mle <- out5$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[5]*zz[-c(1:48)] +
mle[6]*sin(2*pi/mle[7]*ti) + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,1) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 6) {
fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
A1 <- theta[7]
B1 <- theta[8]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
```

```
A1 <- theta[6]
B1 <- theta[7]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5], est[6], est[7])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 0, 60)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))
mle <- out6$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[5]*yy[-c(1:48)] + mle[6]*zz[-c(1:48)] +
mle[7]*sin(2*pi/mle[8]*ti) + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
resid <- x-pred[1:52]

newmle <- gevmaxest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.025))^(-newmle[3])-1)
q2 <- newmle[1]+newmle[2]/newmle[3]*((-log(0.975))^(-newmle[3])-1)

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,1) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
              }

print(c(q1,q2))
print(mle)
newmle

}



gevmin <- function(x, ind = 1, txx="Empirical",
tyy="GEV Model",tmm="Probability Plot for GEV MLE Fit"){

#This function fits GEV models to minimum series


if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
```

```
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
se1 <- sqrt(diag(solve(out1$hessian)))
mle <- out1$par
p <- c(1:n)/(n+1)
zhat <- log(1 - mle[3]*((x-mle[1])/mle[2]))/mle[3]
phat <- sort(exp(-exp(-zhat)))
plot(p, phat, col=4, xlim=c(0,1), ylim=c(0,1), xlab=txx, ylab=tyy, main=tmm)
text(0.5, 0.3, "r = ")
text(0.68, 0.3, round(cor(p,phat),4))
abline(0,1)
print(mle)
             }

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
se2 <- sqrt(diag(solve(out2$hessian)))
mle <- out2$par
p <- c(1:n)/(n+1)
ti <- c(1:n)
zhat <- log(1 - mle[3]*((x-(mle[1]+mle[4]*ti))/mle[2]))/mle[3]
phat <- sort(exp(-exp(-zhat)))
plot(p, phat, col=4, xlim=c(0,1), ylim=c(0,1), xlab=txx, ylab=tyy, main=tmm)
text(0.5, 0.3, "r = ")
text(0.68, 0.3, round(cor(p,phat),4))
abline(0,1)
print(mle)
           }
```

```
if(ind == 3) {
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out3$convergence, out3$value))
se3 <- sqrt(diag(solve(out3$hessian)))
mle <- out3$par
p <- c(1:n)/(n+1)
ti <- c(1:n)
zhat <- log(1 - mle[3]*((x-(mle[1] +
mle[4]*sin(2*pi/mle[5]*ti)))/mle[2]))/mle[3]
phat <- sort(exp(-exp(-zhat)))
plot(p, phat, col=4, xlim=c(0,1), ylim=c(0,1), xlab=txx, ylab=tyy, main=tmm)
text(0.5, 0.3, "r = ")
text(0.68, 0.3, round(cor(p,phat),4))
abline(0,1)
print(mle)
           }

if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
```

```
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out4$convergence, out4$value))
se4 <- sqrt(diag(solve(out4$hessian)))
mle <- out4$par
p <- c(1:n)/(n+1)
ti <- c(1:n)
zhat <- log(1 - mle[3]*((x-(mle[1] + mle[4]*ti +
mle[5]*sin(2*pi/mle[6]*ti)))/mle[2]))/mle[3]
phat <- sort(exp(-exp(-zhat)))
plot(p, phat, col=4, xlim=c(0,1), ylim=c(0,1), xlab=txx, ylab=tyy, main=tmm)
text(0.5, 0.3, "r = ")
text(0.68, 0.3, round(cor(p,phat),4))
abline(0,1)
print(mle)
           }
}



gevmincov <- function(x, c1=3, c2=4){

#This function fits GEV models to minimum series using precipitation/temperature
#as covariates

y <- getmeanminnoplot(precmin, c1, startyear=1924)[c(49:(49+51))]
z <- getmeanminnoplot(tempmin, c2, startyear=1909)[c(49:(49+51))]

xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)

fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
```

```
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))


fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
u <- 1 - xi*(x-(mu + A*y))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))


fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
u <- 1 - xi*(x-(mu + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))


fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
u <- 1 - xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
```

```
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))


fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))

print(out1$par)
print(out2$par)
print(out3$par)
print(out4$par)
print(out5$par)
print(out6$par)


p1 <- 1 - pchisq(2*(-out2$value + out1$value),1)
p2 <- 1 - pchisq(2*(-out3$value + out1$value),1)
p3 <- 1 - pchisq(2*(-out4$value + out1$value),1)
p4 <- 1 - pchisq(2*(-out5$value + out1$value),2)
p5 <- 1 - pchisq(2*(-out6$value + out1$value),3)
p6 <- 1 - pchisq(2*(-out5$value + out3$value),1)
p7 <- 1 - pchisq(2*(-out5$value + out4$value),1)
p8 <- 1 - pchisq(2*(-out6$value + out5$value),1)

print(c(p1,p2,p3,p4,p5,p6,p7,p8),3)
}



gevmincov1 <- function(x, c1=3, c2=4){

#This function fits GEV models to the minimum series with both
#Precipitation/temperature covariates and sin trend


y <- getmeanminnoplot(precmin, c1, startyear=1924)[c(49:(49+51))]
z <- getmeanminnoplot(tempmin, c2, startyear=1909)[c(49:(49+51))]
```

```
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)

fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))

fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))


fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
A1 <- theta[5]
B1 <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))


fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
```

```
xi <- theta[3]
B <- theta[4]
A1 <- theta[5]
B1 <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))


fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
A1 <- theta[6]
B1 <- theta[7]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))


fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
A1 <- theta[7]
B1 <- theta[8]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5], est[6], est[7])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 0, 60)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))

print(out1$par)
print(out2$par)
```

```
print(out3$par)
print(out4$par)
print(out5$par)
print(out6$par)


p1 <- 1 - pchisq(2*(-out2$value + out1$value),1)
p2 <- 1 - pchisq(2*(-out3$value + out1$value),3)
p3 <- 1 - pchisq(2*(-out4$value + out1$value),3)
p4 <- 1 - pchisq(2*(-out5$value + out1$value),4)
p5 <- 1 - pchisq(2*(-out6$value + out1$value),5)
p6 <- 1 - pchisq(2*(-out5$value + out3$value),1)
p7 <- 1 - pchisq(2*(-out5$value + out4$value),1)
p8 <- 1 - pchisq(2*(-out6$value + out5$value),1)


print(c(p1,p2,p3,p4,p5,p6,p7,p8),4)

}



gevmincovplot <- function(x, ind=1, c1=3, c2=3, txx="Year", tyy=" ", tmm=" ",
startyear=1){

#This function plots the fotted result of GEV models for minimum series
#using precipitation/temperature series as covariates

y <- getmeanminnoplot(precmin, c1, startyear=1924)[c(49:(49+51))]
z <- getmeanminnoplot(tempmin, c2, startyear=1909)[c(49:(49+51))]

if(ind ==1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
mle <- out1$par
ti <- c(1:n)
plot(ti+startyear-1, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear-1, rep(mle[1] + mle[2]*(1-gamma(1-mle[3]))/mle[3], n),
type="l", col=4)
            }

if(ind ==2) {
fn2 <- function(theta,x){
mu <- theta[1]
```

```
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
mle <- out2$par
ti <- c(1:n)
plot(ti+startyear-1, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear-1, mle[1] + mle[4]*ti + mle[2]*(1-gamma(1-mle[3]))/mle[3],
type="l", col=4)
            }

if(ind == 3) {
fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
u <- 1 - xi*(x-(mu + A*y))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))
mle <- out3$par
ti <- c(1:n)
plot(ti+startyear-1, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear-1, mle[1] + mle[4]*y + mle[2]*(1-gamma(1-mle[3]))/mle[3],
type="l", col=4)
              }

if(ind == 4) {
fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
u <- 1 - xi*(x-(mu + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
```

```
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))
mle <- out4$par
ti <- c(1:n)
plot(ti+startyear-1, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear-1, mle[1] + mle[4]*z + mle[2]*(1-gamma(1-mle[3]))/mle[3],
type="l", col=4)
             }

if(ind == 5) {
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
u <- 1 - xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))
mle <- out5$par
ti <- c(1:n)
plot(ti+startyear-1, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear-1, mle[1] + mle[4]*y + mle[5]*z + mle[2]*(1-gamma(1-
mle[3]))/mle[3], type="l", col=4)
             }

if(ind == 6) {
fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
```

```
A <- theta[4]
B <- theta[5]
u <- 1 - xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))
mle <- out6$par
ti <- c(1:n)
plot(ti+startyear-1, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear-1, mle[1] + mle[4]*ti + mle[5]*y + mle[6]*z + mle[2]*(1-
gamma(1-mle[3]))/mle[3], type="l", col=4)
             }
mle
}



gevminest <- function(x, ind = 1){

#This function outputs GEV model estimates for maximum series


if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
se1 <- sqrt(diag(solve(out1$hessian)))
mle <- out1$par
             }
```

```
if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                         }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
se2 <- sqrt(diag(solve(out2$hessian)))
mle <- out2$par
            }

if(ind == 3) {
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                         }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out3$convergence, out3$value))
se3 <- sqrt(diag(solve(out3$hessian)))
mle <- out3$par
            }

if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
```

```
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out4$convergence, out4$value))
se4 <- sqrt(diag(solve(out4$hessian)))
mle <- out4$par
           }
mle
}



gevminpred <- function(x, ind=1, c1=3, c2=4, txx="Year", tyy=" ", tmm=" ",
endyear=2070){

#This function makes prediction of future flow minimum flow rates


yy <- getmeanminnoplot(precmin, c1, startyear=1924, endyear=endyear)
y <- yy[c(49:(49+51))]
zz <- getmeanminnoplot(tempmin, c2, startyear=1909, endyear=endyear)
z <- zz[c(49:(49+51))]

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
```

```
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
mle <- out1$par
ti <- c(1:(n+endyear-2008))
pred <- rep(mle[1] + mle[2]*(1 - gamma(1-mle[3]))/mle[3], endyear-1957+1)
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
mle <- out2$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
```

```
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 3) {
fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))
mle <- out3$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 4) {
fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))
```

```
mle <- out4$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*zz[-c(1:48)] + mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 5) {
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))
mle <- out5$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[5]*zz[-c(1:48)] + mle[2]*(1 -
gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 6) {
fn6 <- function(theta,x,y,z){
mu <- theta[1]
```

```
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
A1 <- theta[6]
B1 <- theta[7]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5], est[6], est[7])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 0, 60)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))
mle <- out6$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[5]*yy[-c(1:48)] + mle[6]*zz[-c(1:48)] +
mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
              }

print(c(q1,q2))
print(mle)
newmle
}
```

```
gevminpred1 <- function(x, ind=1, c1=3, c2=4, txx="Year", tyy=" ", tmm=" ",
endyear=2070){

#This function predicts future minimum flow rates using
precipitation/temperature
#as covariates and sin trend


yy <- getmeanminnoplot(precmin, c1, startyear=1924, endyear=endyear)
y <- yy[c(49:(49+51))]
zz <- getmeanminnoplot(tempmin, c2, startyear=1909, endyear=endyear)
z <- zz[c(49:(49+51))]

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
mle <- out1$par
ti <- c(1:(n+endyear-2008))
pred <- rep(mle[1] + mle[2]*(1 - gamma(1-mle[3]))/mle[3], endyear-1957+1)
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
```

```
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
mle <- out2$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 3) {
fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
A1 <- theta[5]
B1 <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))
mle <- out3$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[5]*sin(2*pi/mle[6]*ti) + mle[2]*(1 -
gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))
```

```
plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 4) {
fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
A1 <- theta[5]
B1 <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))
mle <- out4$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*zz[-c(1:48)] + mle[5]*sin(2*pi/mle[6]*ti) + mle[2]*(1 -
gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 5) {
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
A1 <- theta[6]
B1 <- theta[7]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
```

```
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))
mle <- out5$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[5]*zz[-c(1:48)] +
mle[6]*sin(2*pi/mle[7]*ti) + mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
             }

if(ind == 6) {
fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
A1 <- theta[7]
B1 <- theta[8]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
A1 <- theta[6]
B1 <- theta[7]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
```

```
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5], est[6], est[7])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 0, 60)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))
mle <- out6$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[5]*yy[-c(1:48)] + mle[6]*zz[-c(1:48)] +
mle[7]*sin(2*pi/mle[8]*ti) + mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
              }

print(c(q1,q2))
print(mle)
newmle
}



gevminpred2 <- function(x, ind=1, c1=3, c2=4, txx="Year", tyy=" ", tmm=" ",
endyear=2070){

#This function does the same as gevpred, but adds a randomly generated series


set.seed(2010)

yy <- getmeanminnoplot(precmin, c1, startyear=1924, endyear=endyear)
y <- yy[c(49:(49+51))]
zz <- getmeanminnoplot(tempmin, c2, startyear=1909, endyear=endyear)
z <- zz[c(49:(49+51))]

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
```

```
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
mle <- out1$par
ti <- c(1:(n+endyear-2008))
pred <- rep(mle[1] + mle[2]*(1 - gamma(1-mle[3]))/mle[3], endyear-1957+1)
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,2) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
mle <- out2$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))
```

```
plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,2) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 3) {
fn3 <- function(theta,x,y){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))
mle <- out3$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,2) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 4) {
fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
```

```
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
print(c(out4$convergence, out4$value))
mle <- out4$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*zz[-c(1:48)] + mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,2) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 5) {
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))
mle <- out5$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[5]*zz[-c(1:48)] + mle[2]*(1 -
gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
```

```
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,2) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 6) {
fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
A1 <- theta[6]
B1 <- theta[7]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y + B*z))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5], est[6], est[7])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 0, 60)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))
mle <- out6$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[5]*yy[-c(1:48)] + mle[6]*zz[-c(1:48)] +
mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
```

```
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,2) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
              }

print(c(q1,q2))
print(mle)
newmle
}



gevminpred3 <- function(x, ind=1, c1=3, c2=4, txx="Year", tyy=" ", tmm=" ",
endyear=2070){

#This function does the same as gevinxpred1, but adds a randomly generated
series


set.seed(2010)

yy <- getmeanminnoplot(precmin, c1, startyear=1924, endyear=endyear)
y <- yy[c(49:(49+51))]
zz <- getmeanminnoplot(tempmin, c2, startyear=1909, endyear=endyear)
z <- zz[c(49:(49+51))]

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                              }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out1$convergence, out1$value))
mle <- out1$par
ti <- c(1:(n+endyear-2008))
pred <- rep(mle[1] + mle[2]*(1 - gamma(1-mle[3]))/mle[3], endyear-1957+1)
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
```

```
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,2) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
print(c(out2$convergence, out2$value))
mle <- out2$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,2) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 3) {
fn3 <- function(theta,x,y){
mu <- theta[1]
```

```
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
A1 <- theta[5]
B1 <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y)
print(c(out3$convergence, out3$value))
mle <- out3$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[5]*sin(2*pi/mle[6]*ti) + mle[2]*(1 -
gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,2) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 4) {
fn4 <- function(theta,x,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
B <- theta[4]
A1 <- theta[5]
B1 <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,z=z)
```

```
print(c(out4$convergence, out4$value))
mle <- out4$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*zz[-c(1:48)] + mle[5]*sin(2*pi/mle[6]*ti) + mle[2]*(1 -
gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,2) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 5) {
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
A1 <- theta[6]
B1 <- theta[7]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out5$convergence, out5$value))
mle <- out5$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*yy[-c(1:48)] + mle[5]*zz[-c(1:48)] +
mle[6]*sin(2*pi/mle[7]*ti) + mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))

plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
```

```
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,2) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
             }

if(ind == 6) {
fn6 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
A1 <- theta[7]
B1 <- theta[8]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn5 <- function(theta,x,y,z){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
A1 <- theta[6]
B1 <- theta[7]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*y + B*z + A1*sin(2*pi/B1*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini5 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 60)
out5 <- optim(ini5, fn5, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
est <- out5$par
ini6 <- c(est[1], est[2], est[3], 0, est[4], est[5], est[6], est[7])
#ini6 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 0, 0, 60)
out6 <- optim(ini6, fn6, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x,y=y,z=z)
print(c(out6$convergence, out6$value))
mle <- out6$par
ti <- c(1:(n+endyear-2008))
pred <- mle[1] + mle[4]*ti + mle[5]*yy[-c(1:48)] + mle[6]*zz[-c(1:48)] +
mle[7]*sin(2*pi/mle[8]*ti) + mle[2]*(1 - gamma(1-mle[3]))/mle[3]
resid <- x-pred[1:52]

newmle <- gevminest(resid,1)
f <- pred[-c(1:52)]
q1 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.025))^(-newmle[3]))
q2 <- newmle[1]+newmle[2]/newmle[3]*(1 - (-log(1-0.975))^(-newmle[3]))
```

```
plot(seq(1957,endyear,1), pred, col=4, ylim=c(0,1.2*max(x)), type="l", xlab=txx,
ylab=tyy, main=tmm)
points(seq(1957,2008,1), x, type="l")
points(seq(2009,endyear,1), f + q1, type="l", col=2)
points(seq(2009,endyear,1), f + q2, type="l", col=2)
m <- endyear - 2009 + 1
v <- c(x[n], gevsimu(newmle,m,2) + f)
points(seq(2008,endyear,1), v, type="l", col=3)
              }

print(c(q1,q2))
print(mle)
newmle
}



gevtrendmax <- function(x){

# This function assesses the 4 trend models


fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)

fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)

fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
```

```
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)

fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)


p1 <- 1 - pchisq(2*(-out2$value + out1$value),1)
p2 <- 1 - pchisq(2*(-out3$value + out1$value),2)
p3 <- 1 - pchisq(2*(-out4$value + out1$value),3)
p4 <- 1 - pchisq(2*(-out4$value + out2$value),2)
p5 <- 1 - pchisq(2*(-out4$value + out3$value),1)

print(c(p1,p2,p3,p4,p5),3)

}



gevtrendmaxplot <- function(x, ind = 1, tyy="Flow Rate", txx="Year",tmm="Trend
Plot for GEV MLE Fit", startyear=0){

#This function plots the result of the 4 trend models


if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
```

```
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out1$par
n <- length(x)
ti <- c(1:n)
plot(ti+startyear, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear, rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], n),
type="l", col=4)
mle
}

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out2$par
n <- length(x)
ti <- c(1:n)
plot(ti+startyear, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear, mle[1]+mle[4]*ti + mle[2]*(gamma(1-mle[3]) - 1)/mle[3],
type="l", col=4)
mle
}

if(ind == 3) {
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
```

```
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out3$par
n <- length(x)
ti <- c(1:n)
plot(ti+startyear, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+ startyear, mle[1]+ mle[4]*sin(2*pi/mle[5]*ti) + mle[2]*(gamma(1-
mle[3]) - 1)/mle[3], type="l", col=4)
mle
}

if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out4$par
n <- length(x)
```

```
ti <- c(1:n)
plot(ti+startyear, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+ startyear, mle[1]+ mle[4]*ti + mle[5]*sin(2*pi/mle[6]*ti) +
mle[2]*(gamma(1-mle[3]) - 1)/mle[3], type="l", col=4)
mle
}

}



gevtrendmaxplot <- function(x, ind = 1, tyy="Flow Rate", txx="Year",tmm="Trend
Plot for GEV MLE Fit", startyear=0){

#This function plots the result of the 4 trend models


if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                              }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out1$par
n <- length(x)
ti <- c(1:n)
plot(ti+startyear, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear, rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], n),
type="l", col=4)
mle
}

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                         }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
```

```
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out2$par
n <- length(x)
ti <- c(1:n)
plot(ti+startyear, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear, mle[1]+mle[4]*ti + mle[2]*(gamma(1-mle[3]) - 1)/mle[3],
type="l", col=4)
mle
}

if(ind == 3) {
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out3$par
n <- length(x)
ti <- c(1:n)
plot(ti+startyear, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+ startyear, mle[1]+ mle[4]*sin(2*pi/mle[5]*ti) + mle[2]*(gamma(1-
mle[3]) - 1)/mle[3], type="l", col=4)
mle
}

if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
```

```
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out4$par
n <- length(x)
ti <- c(1:n)
plot(ti+startyear, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+ startyear, mle[1]+ mle[4]*ti + mle[5]*sin(2*pi/mle[6]*ti) +
mle[2]*(gamma(1-mle[3]) - 1)/mle[3], type="l", col=4)
mle
}

}



gevtrendmaxplotmatch <- function(x, ind = 1, tyy="Flow Rate",
txx="Year",tmm="Trend Plot for GEV MLE Fit", startyear=1957){

# This function plots the trend before the observed data for modeling maximum
series

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out1$par
n <- length(x)
```

```
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), 100-m, 1)
           }
else {
y <- seq(1, 100, 1)
     }
trend <- rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], length(y))
plot(seq(1909,2008,1), trend, type="l", col=4, ylim=c(min(x),max(x)), xlab=txx,
ylab=tyy, main=tmm)
points(ti, x, type="l")
             }
if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out2$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), 100-m, 1)
           }
else {
y <- seq(1, 100, 1)
     }
trend <- mle[1]+mle[4]*y + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
plot(seq(1909,2008,1), trend, type="l", col=4, ylim=c(min(x),max(x)), xlab=txx,
ylab=tyy, main=tmm)
points(ti, x, type="l")
           }
if(ind == 3) {
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
```

```
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out3$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), 100-m, 1)
           }
else {
y <- seq(1, 100, 1)
     }
trend <- mle[1]+ mle[4]*sin(2*pi/mle[5]*y) + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
plot(seq(1909,2008,1), trend, type="l", col=4, ylim=c(min(x),max(x)), xlab=txx,
ylab=tyy, main=tmm)
points(ti, x, type="l")
           }
if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
```

```
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out4$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), 100-m, 1)
           }
else {
y <- seq(1, 100, 1)
     }
trend <- mle[1]+ mle[4]*y + mle[5]*sin(2*pi/mle[6]*y) + mle[2]*(gamma(1-mle[3])
- 1)/mle[3]
plot(seq(1909,2008,1), trend, type="l", col=4, ylim=c(min(x),max(x)), xlab=txx,
ylab=tyy, main=tmm)
points(ti, x, type="l")
           }
}



gevtrendmaxplotmatch1 <- function(x, ind = 1, tyy="Flow Rate",
txx="Year",tmm="Trend Plot for GEV MLE Fit", startyear=1957, endyear=2060){

#This function plots the trend before and after the maximum observed data
# using precipitation/temperature as covariates.

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 + xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                             }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out1$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- rep(mle[1] + mle[2]*(gamma(1-mle[3]) - 1)/mle[3], length(y))
```

```
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out2$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+mle[4]*y + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}

if(ind == 3) {
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
```

```
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out3$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
      }
trend <- mle[1]+ mle[4]*sin(2*pi/mle[5]*y) + mle[2]*(gamma(1-mle[3]) - 1)/mle[3]
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}

if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 + xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out4$par
```

```
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+ mle[4]*y + mle[5]*sin(2*pi/mle[6]*y) + mle[2]*(gamma(1-mle[3])
- 1)/mle[3]
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}
mle
}



gevtrendmin <- function(x){

#This function fits 4 different trends to GEV models for minimum series

fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)

fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
```

```
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)

fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)

p1 <- 1 - pchisq(2*(-out2$value + out1$value),1)
p2 <- 1 - pchisq(2*(-out3$value + out1$value),2)
p3 <- 1 - pchisq(2*(-out4$value + out1$value),3)
p4 <- 1 - pchisq(2*(-out4$value + out2$value),2)
p5 <- 1 - pchisq(2*(-out4$value + out3$value),1)

print(c(p1,p2,p3,p4,p5),digit=4)

}



gevtrendminplot <- function(x, ind = 1, tyy="Flow Rate", txx="Year",tmm="Trend
Plot for GEV MLE Fit", startyear=0){
```

```
#This function plots 4 different trends fitted to the GEV models for minimum
series

if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                              }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out1$par
n <- length(x)
ti <- c(1:n)
plot(ti+startyear, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear, rep(mle[1] + mle[2]*(1-gamma(1-mle[3]))/mle[3], n),
type="l", col=4)
mle
}

if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                         }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out2$par
n <- length(x)
ti <- c(1:n)
plot(ti+startyear, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+startyear, mle[1]+mle[4]*ti + mle[2]*(1-gamma(1-mle[3]))/mle[3],
type="l", col=4)
mle
}

if(ind == 3) {
fn3 <- function(theta,x){
```

```
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out3$par
n <- length(x)
ti <- c(1:n)
plot(ti+startyear, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+ startyear, mle[1]+ mle[4]*sin(2*pi/mle[5]*ti) + mle[2]*(1-gamma(1-
mle[3]))/mle[3], type="l", col=4)
mle
}

if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
```

```
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out4$par
n <- length(x)
ti <- c(1:n)
plot(ti+startyear, x, type="l", xlab=txx, ylab=tyy, main=tmm)
points(ti+ startyear, mle[1]+ mle[4]*ti + mle[5]*sin(2*pi/mle[6]*ti) +
mle[2]*(1-gamma(1-mle[3]))/mle[3], type="l", col=4)
mle
}

}



gevtrendminplotmatch1 <- function(x, ind = 1, tyy="Flow Rate",
txx="Year",tmm="Trend Plot for GEV MLE Fit", startyear=1957, endyear=2060){

#This function plots the trend before and after the minimum observed data
# using precipitation/temperature as covariates.


if(ind == 1) {
fn1 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
u <- 1 - xi*(x-mu)/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                              }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
ini1 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out1 <- optim(ini1, fn1, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out1$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- rep(mle[1] + mle[2]*(1-gamma(1-mle[3]))/mle[3], length(y))
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}
```

```
if(ind == 2) {
fn2 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,1)
ini2 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2])
#ini2 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1)
out2 <- optim(ini2, fn2, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out2$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+mle[4]*y + mle[2]*(1-gamma(1-mle[3]))/mle[3]
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}

if(ind == 3) {
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out3$par
```

```
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+ mle[4]*sin(2*pi/mle[5]*y) + mle[2]*(1-gamma(1-mle[3]))/mle[3]
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}

if(ind == 4) {
fn4 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
slo <- theta[4]
A <- theta[5]
B <- theta[6]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + slo*ti + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
fn3 <- function(theta,x){
mu <- theta[1]
sig <- theta[2]
xi <- theta[3]
A <- theta[4]
B <- theta[5]
ti <- c(1:length(x))
u <- 1 - xi*(x-(mu + A*sin(2*pi/B*ti)))/sig
if(any(u <= 0) || sig <= 0) return(10^8)
length(x)*log(sig) + (1+1/xi)*sum(log(u)) + sum(u^(-1/xi))
                        }
xbar <- mean(x)
s <- sqrt(var(x))
n <- length(x)
est <- findini(x,2)
ini3 <- c(est[1], sqrt(6)*s/pi, 0.1, est[2], est[3])
#ini3 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 60)
out3 <- optim(ini3, fn3, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
est <- out3$par
ini4 <- c(est[1], sqrt(6)*s/pi, 0.1, 0, est[4], est[5])
#ini4 <- c(xbar - 0.57721*sqrt(6)*s/pi, sqrt(6)*s/pi, 0.1, 0, 0, 60)
out4 <- optim(ini4, fn4, hessian = TRUE,  method="Nelder-Mead",
control=list(maxit=2000), x=x)
mle <- out4$par
n <- length(x)
ti <- c(startyear:(startyear+n-1))
m <- startyear - 1909
if(m != 0) {
```

```
y <- seq(-abs((m-1)), endyear-1909+1-m, 1)
           }
else {
y <- seq(1, endyear-1909+1, 1)
     }
trend <- mle[1]+ mle[4]*y + mle[5]*sin(2*pi/mle[6]*y) + mle[2]*(1-gamma(1-
mle[3]))/mle[3]
plot(seq(1909,endyear,1), trend, type="l", col=4, ylim=c(min(x),max(x)),
xlab=txx, ylab=tyy, main=tmm)
points(ti, x, type="l")
}
mle
}
```