No. ______

CANADA

DEPARTMENT OF MINES AND TECHNICAL SURVEYS

GEOLOGICAL SURVEY OF CANADA
TOPICAL REPORT NO. 48

AQUIFER NOTES

ADAPTED FROM LECTURE NOTES PRESENTED BY G. FERRIS AND S. W. LOHMAN AT THE UNITED STATES GEOLOGICAL SURVEY GROUNDWATER COURSE

BY
R. A. FREEZE

OTTAWA
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PREFACE

These notes have been taken, almost entirely, from two sets of lecture notes presented by John G. Ferris and S.W. Lohman at the United States Geological Survey Groundwater Course in March 1959.

The author has adapted the notes to suit the needs of the Canadian hydrogeologist by making the following changes:

1) Aquifer-testing formulas have been corrected to allow for the use of Imperial gallons in place of U.S. gallons.

2) The metric system has been introduced to supplement the foot-day and gallon-foot-day systems of units.

3) Sample American aquifer tests have been replaced in part by Canadian examples.

TABLE OF CONTENTS.

"When you can measure what you are speaking about and express it in numbers, you know something about it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind. It may be the beginning of knowledge but you have scarcely in your thought advanced to the stage of a science".

--------Lord Kelvin.

"Through and through the world is infected with quantity. To talk sense, is to talk quantities. It is no use saying that the nation is large, -- How large? It is no use saying radium is scarce, -- How scarce? You cannot evade quantity. You may fly to poetry and to music, and quantity and number will face you in your rhythms and octaves. Elegant intellects which despise the theory of quantity are but half developed. They are more to be pitied than blamed".

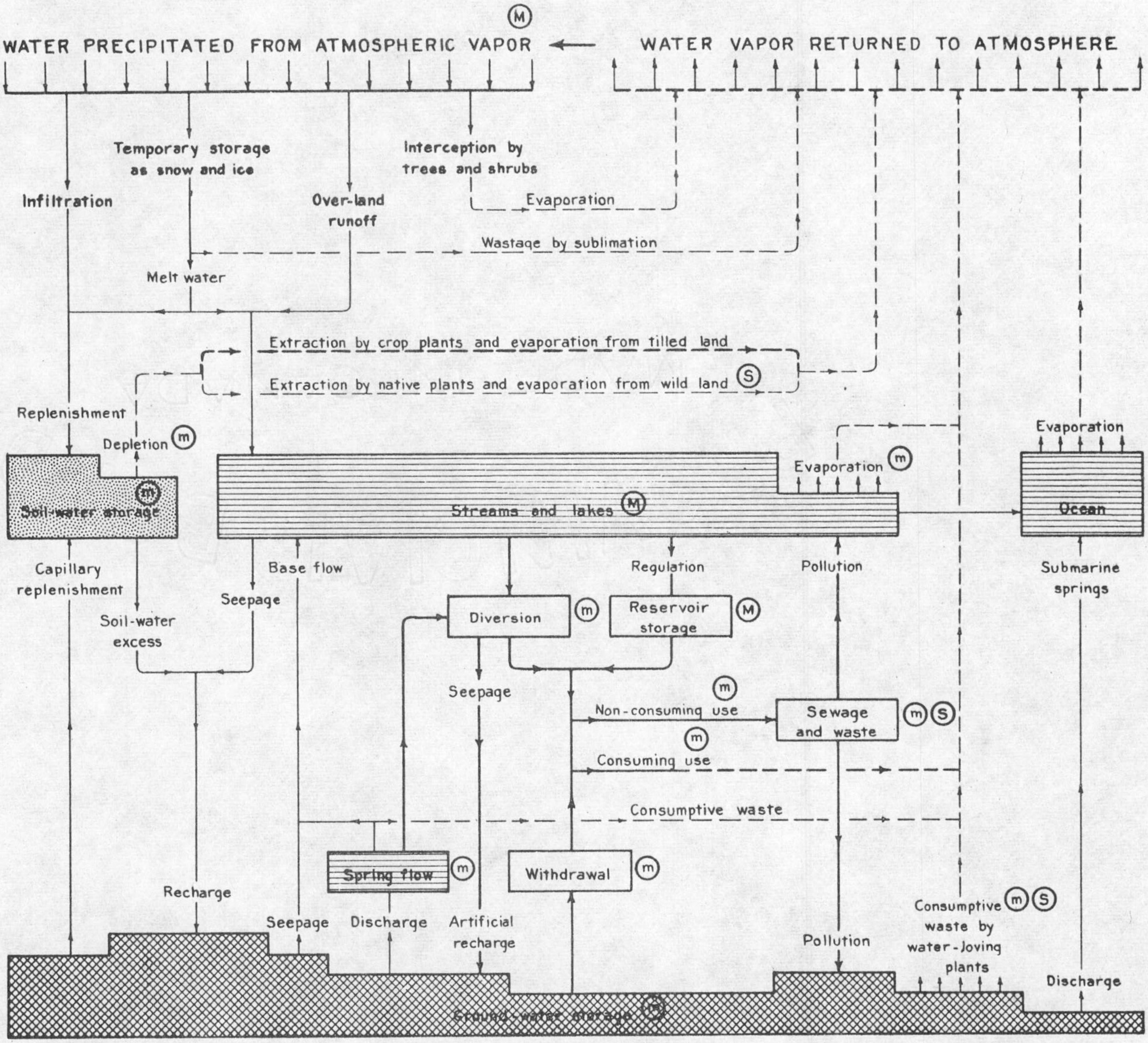
--------A.N. Whitehead.

PART I

CONCEPTS

Contents:

Hydrologic cycle



Solid flow lines indicate movement of water as liquid; broken lines, movement as vapor. Heavy flow lines (lower central part of diagram) indicate man's principal changes in the natural cycle.

M, components of the cycle for which records of measurements are common and fairly extensive, though not everywhere comprehensive; m, components which are not measured readily, and for which more extensive records and improved techniques of measurement are needed; S, components of natural water consumption that can be, or ultimately must be salvaged in substantial part.

Zones of subsurface water, after--

Meinzer[1]	Terzaghi[2]	Versluys[3]
	LAND SURFACE	
Belt of soil water	Zone of discontinuous capillary saturation (Mainly separate rings and envelopes; p = atm)	Pendular stage
Intermediate belt (vadose water)		
Capillary fringe (Fringe water; p < atm)	Zone of semi-continuous capillary saturation (Larger openings unsaturated; p ≤ atm)	Funicular stage
	(p = atm)	
	Zone of continuous capillary saturation (p < atm)	Capillary stage
(p = atm)[4]	WATER TABLE	
Ground water (phreatic water, pleurotic water; p > atm)	Ground water	
Internal water (juvenile water, magmatic water; may be wholly in chemical union)		

Meinzer, brackets at left: Zone of aeration and of suspended (vadose) water (belt of soil water, intermediate belt, capillary fringe); Zone of saturation (ground water); Zone of rock fracture and of interstitial water (zone of aeration and zone of saturation); Zone of rock flowage (internal water).

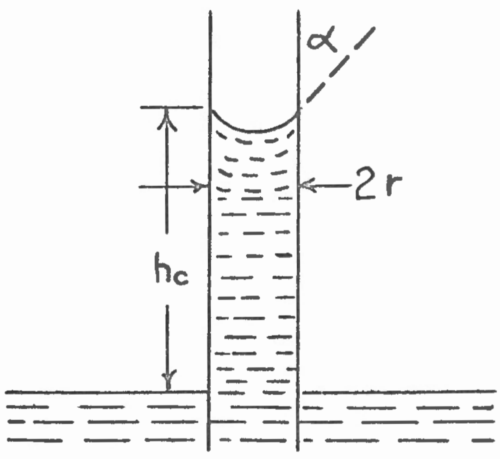
Versluys, bracket at right: Zone of saturation[4] (capillary stage and below).

1/ Meinzer, O. E.: U. S. Geol. Survey Water-Supply Paper 494, p. 23, 1923.
2/ Terzaghi, Karl: Soil moisture and capillary phenomena in soils, chap. 9a in Hydrology, edited by O. E. Meinzer, McGraw-Hill, pp. 331-363, 1942.
3/ Versluys, J., Die Kapillaritat der boden: Internat. Mitt. Bodenkunde, vol. 7, pp. 117-140, 1917.
4/ Hubbert, King, Theory of ground-water motion: Jour. Geol., vol. 48, no. 8, pp. 785-944, Nov.-Dec. 1940.

Surface tension and capillary rise

At an interface between two fluids, molecular forces create a tensile stress in the surface of separation--this stress is known as <u>surface tension</u>. For water against air, this stress is fairly large (0.073 g/cm at 20°C); thus, water can resist hydrostatic tensile stresses of many atmospheres without losing its continuity.

If the lower end of a vertical tube of very small (capillary) diameter is dipped into a liquid, the liquid comes to rest within the tube with its surface either above or below the free liquid surface outside the tube, depending on the composition of the liquid, the material of the tube, and impurities in the liquid or on the tube. If the liquid is clean water and the "tube" is glass or one of the common earth materials, the water will rise in the tube, as in the sketch below.



2r = diameter of tube
h_c = capillary rise
α = "contact angle" between edge of meniscus and wall of tube

Here, pressure is atmospheric at the level of the free water surface both outside and inside the tube; pressure also is atmospheric on the water meniscus within the tube. Thus, weight of the water within the tube is sustained by surface tension in the meniscus, and the raised water column is under tension--that is, the pressure is less than atmospheric.

At equilibrium

$$\pi r^2 \gamma h_c = T\, 2\pi r \cos\alpha \quad (1)$$

(Weight) = (Lift by surface tension)

where γ = density of water at the particular temperature
T = surface tension in g/cm

Then

$$h_c = \frac{2T}{r\gamma}\cos\alpha \quad (2)$$

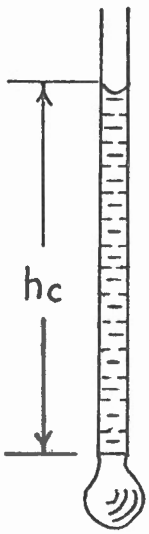
For pure water in clean glass, $\alpha = 0$ and $\cos\alpha = 1$; for room temperature of 20 C, $T = 0.073$ gm/cm and $\gamma = 1$, whence

$$h_c = \frac{0.15}{r} \quad (3)$$

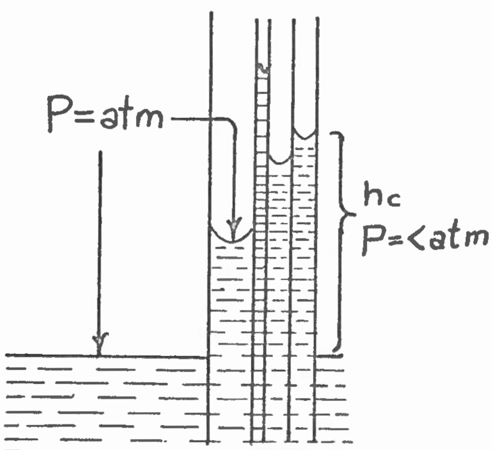
For natural earth materials there are too many variables to express capillary raise in precise terms, but in general it increases as grain size decreases. Terzaghi ("Hydrology," chapt. 9) cites experimental measurements by Atterberg of capillary rise in seven sands at a temperature of 17°C over a term of 72 days. Porosities and void ratios were essentially the same in all seven--41 percent and 0.69, respectively. The measurements, given below, show that capillary rise increased nearly in inverse proportion to the grain size.

Grain size, mm	5-2	2-1	1-0.5	0.5-0.2	0.2-0.1	0.1-0.05	0.05-0.02
h_c, cm	2.5	6.5	13.5	24.6	42.8	105.5	200*

* Still rising after 72 days.



If the capillary tube of the preceding sketch is raised vertically from the liquid, drainage will cease when the meniscus falls to a point about h_c above the lower end of the tube. At the same time a permanent droplet will form at the lower end of the tube as in the sketch at the left. As before, surface tension in the meniscus sustains the weight of the liquid column h_c. Near the lower end of the tube, stress changes from tension in the column to compression in the droplet. Tension in the surface of the droplet acts like an elastic container and transfers the weight of the droplet to the lower end of the tube. Conditions analogous to this example exist in stratified, granular earth materials which are not continuously saturated and in which water is percolating downward from a fine-grained material above to a coarser-grained material below.



Let the sketch at the left represent an idealized section across the wall of a well that taps unconfined ground water, with the water-bearing material replaced by a bundle of vertical capillary tubes of various diameters. Then, the upper surface of the "zone of saturation" in the bundle of capillaries will be minutely irregular and will stand higher than the water surface in the well, by an amount of capillary rise h_c. Here, pressure is atmospheric on the meniscus in each capillary of the bundle, is less than atmospheric within the zone of capillary rise h_c, and is atmospheric at the level of the water surface in the well. Thus, only one precise definition of the water table is possible--that isobaric surface at which hydrostatic pressure is atmospheric. This surface is defined by the level at which unconfined water stands in non-pumped wells. It is not the upper surface of the zone in which all interstices are saturated, except in materials having only very large (supercapillary) openings.

Movement of ground water

<u>Critical velocity</u>.- Index commonly used to determine if flow is laminar or turbulent is the so-called Reynolds number

$$R = \frac{dv\rho}{\mu} = \frac{dv}{\gamma} \qquad (4)$$

where

- d = mean diameter of grains
- v = mean velocity of the moving fluid
- ρ = density of the fluid
- μ = dynamic viscosity of the fluid
- γ = kinematic viscosity of the fluid

If R is less than about 1, the flow is "viscous" or laminar and velocity varies as the first power of the hydraulic gradient. If R is greater than about 10, the flow is turbulent and velocity varies as the one-half power (square root) of the hydraulic gradient. At values of R between 1 and 10, flow may be either laminar or turbulent, depending on the range in size and shape of grains.

<u>Darcy's law</u>.- For steady-state laminar flow in permeable media (when $dh/dt = 0$) Darcy's law may be written

$$Q = kAI = kA\left(\frac{h_1 - h_2}{\ell}\right) = kA\, dh/d\ell \qquad (5)$$

where

- Q = quantity of flow in a given interval of time
- k = a constant[1/]
- A = cross-sectional area through which flow takes place
- I = hydraulic gradient
- h_1 and h_2 = hydrostatic heads at either end of the flow reach
- ℓ = length of the reach
- t = time

Because movement is in the direction of diminishing head, $dh/d\ell$ is considered negative and Darcy's law for unit cross-sectional area may be generalized

$$q = \frac{Q}{A} = -k\, dh/d\ell \quad \text{when} \quad dh/dt = 0 \qquad (6)$$

1/ In the remainder k denotes coefficient of permeability in consistent units; P denotes coefficient of permeability in U.S. Geological Survey units.

Coefficient of permeability.- In equation (5) the constant k is the coefficient of permeability or the transmission constant, a characteristic of the permeable medium.

Then $$k = -\frac{Q}{A\left(\frac{h_1-h_2}{\ell}\right)} \quad \text{or} \quad k = -\frac{q}{dh/d\ell} \qquad (7)$$

Thus, k has the dimensions of $\frac{L^3}{T\,L^2\,L/L} = \frac{L}{T}$ which is a velocity.

Meinzer's coefficient of permeability

$$P = \frac{Q}{IA}\ \text{gpd/ft}^2 \qquad (8)$$

where
- Q = quantity of flow in gallons a day, at 60°F
- I = hydraulic gradient (1) as a ratio $\Delta h/\ell$ or (2) in feet per mile
- A = cross-sectional area in (1) square feet or (2) foot-miles

Thus, P has dimensions of

$$\frac{(\text{gal/day ft}^2(\text{ft/ft})}{(\text{gal/day ft-mile(ft/mile)}} = (\text{gal/day})/\text{ft}^2 = \frac{L}{T}$$

which again is a velocity. In the above equation of dimensions, ft/ft is unit gradient.

Slichter's transmission constant (k, U.S.G.S. Water-Supply Paper 140, p. 11) differs from Meinzer's coefficient of permeability (P) only in that Q is expressed in cubic feet a minute, and thus is in consistent units.

$$k = \frac{\text{ft}^3}{\text{min ft}^2\ \text{ft/ft}} = \text{ft/min} \qquad (9)$$

Coefficient of transmissibility.- The coefficient of transmissibility or transmissivity (Theis) measures the capacity of an entire aquifer to transmit water at the prevailing temperature. In simplest terms it is Meinzer's coefficient of permeability (corrected for temperature) multiplied by the saturated thickness of the aquifer, thus

$$T = P_m = \frac{Q}{IW} \tag{10}$$

where m = thickness of the aquifer, in feet
W = width of aquifer, in feet

Thus, T has dimensions of (gal/day)/ft(ft/ft) = (gal/day)/ft = $\frac{L^2}{T}$

The coefficients P and T now are well entrenched in our literature. However, they conform to neither the foot-pound-second nor the centimeter-gram-second systems. Other workers who deal with movement of fluids through permeable media have established other units of permeability, some of which introduce properties of the fluid (such as viscosity). Some of these units are more logical than ours; therefore, we should not quarrel with their use.

Relation of permeability to grain size.- According to Jacob (Engineering Hydrology, chapt. 5, p. 324, 1950), a coefficient of permeability (k) may be expressed in the form

$$k = \frac{Cd^2\gamma}{\mu} \tag{11}$$

where C = a dimensionless constant depending on physical characteristics of the permeable medium (porosity, range and distribution of grain sizes, shapes of grains, etc.)
d = mean diameter of grains
γ = specific weight (density) of the fluid
μ = viscosity of the fluid

Ignoring properties of fluid (γ and μ), permeability of medium alone may be regarded as

$$k = C'd^2 \tag{12}$$

which is another way of saying that the permeability varies as the square of the grain diameter, and thus has the dimensions L^2. This is in agreement with S. Irmay (On the hydraulic conductivity of unsaturated soils: Am. Geophys. Union Trans., vol. 35, no. 3, equation (12) p. 465, June 1954).

Relation of coefficient of permeability to velocity.- Darcy's law may be written

$$Q = kIA = pAv$$

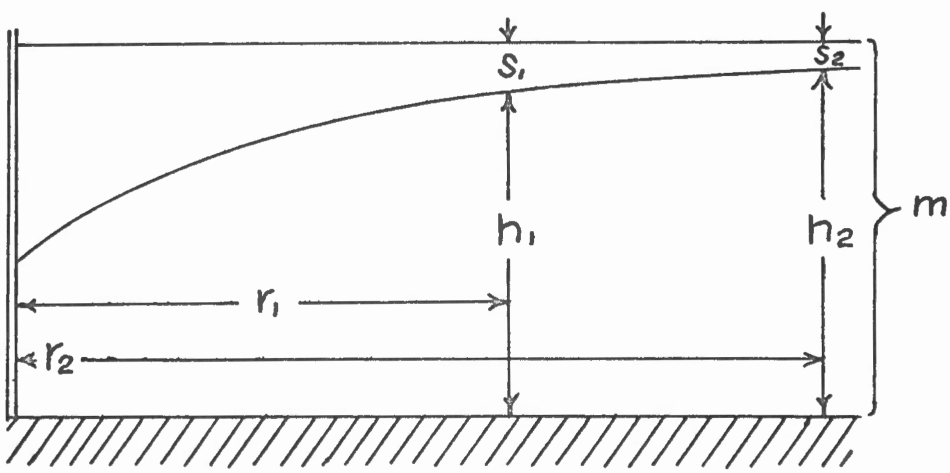
$$v = \frac{kI}{p} \qquad (13)$$

where p = porosity
v = velocity

In Geological Survey units, equation (13) may be written

$$v = \frac{PI}{6.25\ p} \text{ ft/day} \qquad (14)$$

Relation of Darcy's law to Thiem equation.- Let the following sketch represent half the cross section of the cone of depression around a well that has been pumped long enough for the essential establishment of steady-state flow (equilibrium; $dh/dt = 0$)



If the material is homogeneous and if the base of the aquifer and the undisturbed water table are assumed to be parallel and horizontal; then, by Darcy's law, equal quantities of water flow through any two concentric cylinders as at r_1 and r_2, and the quantity of flow is equal to the discharge of the well. Under the assumed conditions of steady-state flow, Darcy's law may be expressed as a differential equation in cylindrical coordinates

$$q = 2\pi krh \ dh/dr$$

collecting constants and separating variables

$$\frac{dr}{r} = \frac{2\pi k}{q} h \ dh \qquad (15)$$

Integrating between the limits r_2 and r_1, and h_2 and h_1

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k}{q}\int_{h_1}^{h_2} h\, dh \qquad (16)$$

$$\log_e \frac{r_2}{r_1} = \frac{2\pi k}{q}\left[\frac{h_2^2-h_1^2}{2}\right]$$

Solving for k, and converting to common logarithms

$$k = \frac{2.303\, q \log_{10} {}^{r_2}/_{r_1}}{\pi\left(h_2^2-h_1^2\right)} \qquad (17)$$

Under artesian conditions (where there is no unwatering) or in thick unconfined aquifers, h_1+h_2 may be assumed essentially equal to 2 m. Then, inasmuch as $h_2^2-h_1^2 = (h_2+h_1)(h_2-h_1)$ and $s_1-s_2 = h_2-h_1$, equation (17) may be rewritten

$$k = \frac{2.303\, q \log_{10} {}^{r_2}/_{r_1}}{2\pi m\,(s_1-s_2)} \qquad (18)$$

In Geological Survey units, noting that T = Pm, equation (18) may be written as the familiar Thiem equation

$$T = \frac{528q \log_{10} {}^{r_2}/_{r_1}}{s_1-s_2} \text{ gpd/ft} \qquad (19)$$

In thin unconfined aquifers in which the drawdown (s) is an appreciable proportion of the thickness (m), however, C. E. Jacob (Notes on determining permeability by pumping tests under water-table conditions: U. S. Geol. Survey processed report, 25 pp., 1944) has shown that the observed drawdowns should be corrected as follows. Note from the sketch that $h_2 = m - s_2$, and $h_1 = m - s_1$. Substituting these values in equation (17) and expanding

$$k = \frac{2.303q \log_{10} {}^{r_2}/_{r_1}}{\pi\left[(m^2 - 2ms_2 + s_2^2) - (m^2 - 2ms_1 + s_1^2)\right]} \qquad (20)$$

For convenience multiply both sides of equation (20) by 2m, which does not alter its value, and simplify

$$k = \frac{2.303q \log_{10} {}^{r_2}/_{r_1}}{2\pi m \left[\left(s_1 - \frac{s_1^2}{2m}\right) - \left(s_2 - \frac{s_2^2}{2m}\right)\right]} \tag{21}$$

In Geological Survey units, equation (21) may be written

$$T = \frac{528q \log_{10} {}^{r_2}/_{r_1}}{s_1' - s_2'} \text{ gpd/ft} \tag{22}$$

where $s' = \text{corrected drawdown} = s - \frac{s^2}{2m}$

Equations (18) and (19) also may be derived directly from Theis' exponential integral (equation 23) for long intervals of discharge (large values of t).

<u>Theis' coefficient of storage</u>.- In 1935 C. V. Theis (The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage: Am. Geophys. Union Trans., 1935, p. 520) introduced for the first time the variable t (time) and the constant S (coefficient of storage) in the following exponential integral (derived from a heat-flow equation)

$$s = \frac{Q}{4\pi T}\int_{\frac{r^2S}{4Tt}}^{\infty}(e^{-u}/u)du \qquad (23)$$

C. E. Jacob later (On the flow of water in an elastic artesian aquifer: Am. Geophys. Union Trans., 1940, pp. 574-586) confirmed Theis' equation (23) from purely hydrologic concepts.

The coefficient of storage (S) was defined by Theis (The significance and nature of the cone of depression in ground-water bodies: Econ. Geol., vol. 33, no. 8, p. 894, December 1938) "as the volume of water, measured in cubic feet, released from storage in each column of the aquifer having a base 1 foot square and a height equal to the thickness of the aquifer, when the water table or other piezometric surface is lowered 1 foot. In water-table bodies, this coefficient of storage <u>for long periods of pumping</u> is approximately the specific yield." (The coefficient of storage S is dimensionless; it assumes the dimensions L^3 only when multiplied by the change in head L and the area L^2). For artesian aquifers the range in S is perhaps 10^{-5} - 10^{-3}; for unconfined aquifers the corresponding range in specific yield is perhaps < 0.1 - 0.3+.

A better understanding of the coefficient of storage S may be had from analysis of the following equation of Jacob (op. cit., p. 577; also chapt. 5 in Engineering Hydraulics, John Wiley & Sons, Inc., p. 334, 1950)

$$S = \theta\gamma m\left(\frac{1}{E_w} + \frac{b}{\theta E_s} + \frac{C}{E_c}\right) \qquad (24)$$

where

θ = porosity of aquifer

γ = specific weight of water per unit area (62.4 lb ft^{-3}/144 $in^2 ft^{-2}$)

m = thickness of aquifer, ft

E_w = bulk modulus of elasticity of water ($3x10^5$ lb in^{-2})

b = effective part of unit area of aquifer that responds elastically. For an uncemented granular aquifer, b is unity; for a solid aquifer, as a limestone having tubular channels, b is apparently equal to the porosity; for a cemented sandstone, b doubtless ranges between these limits.

E_s = bulk modulus of elasticity of the aquifer
C = a dimensionless quantity that depends largely on the thickness, configuration, and distribution of intercalated or adjacent clay beds.
E_c = bulk modulus of compression of the clay.

Within the parentheses of equation (24), the first term denotes the release from storage derived from elastic expansion of the water; the second term indicates that from elastic compression of the aquifer; the third term, where applicable, indicates that from plastic deformation of clay. For practical applications to aquifers reasonably free from clay the third term within the parentheses ordinarily may be ignored--particularly if the time interval involved in determination of S is sufficiently long. Then, for granular aquifers or loosely cemented sandstones for which b = 1 or approximately 1, equation (24) can be simplified to

$$S = \theta \gamma m \left(\beta + \frac{1}{\theta E_s} \right) \qquad (25)$$

where $\beta = \frac{1}{E_w} = 3.3\times10^{-6}\ in^2\ lb^{-1}$

For an example of storage release from expansion of water alone, assume $\theta = 0.3$ and m = 1 ft. Then

$$S = 0.3 \times (62.4 lb\ ft^{-3}/144 in^2 ft^{-2}) \times 1\ ft \times 3.3\times10^{-6} in^2 lb^{-1}$$

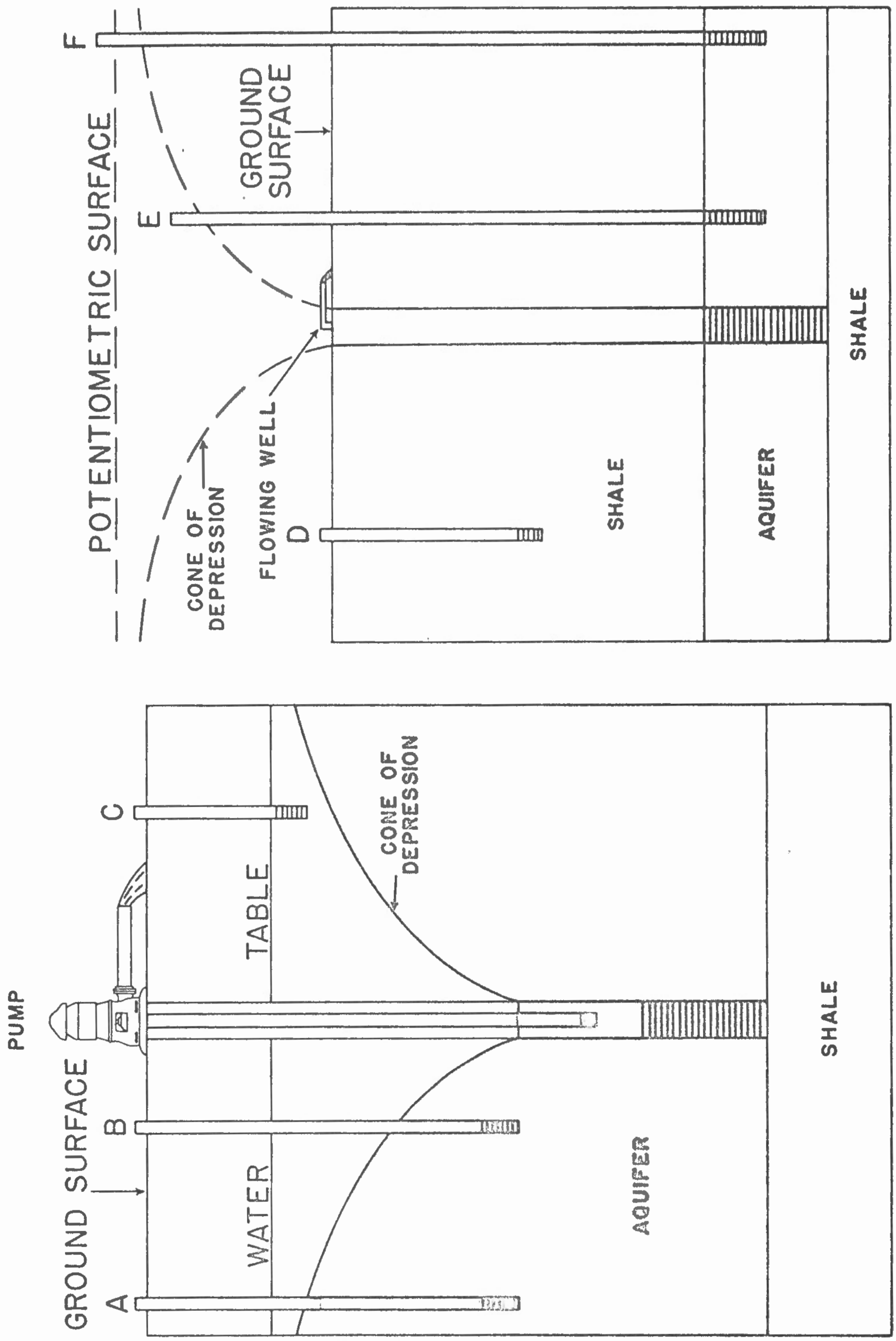
$$= 4.3\times10^{-7}$$

Similarly, for m = 100 ft

$$S = 4.3\times10^{-5}$$

If multiplied by the defining conditions of unit change in head (1 ft) over unit area (1 ft^2) each of the above answers represents the fractional cubic foot of water released under the assumed conditions.

Confined versus unconfined ground water



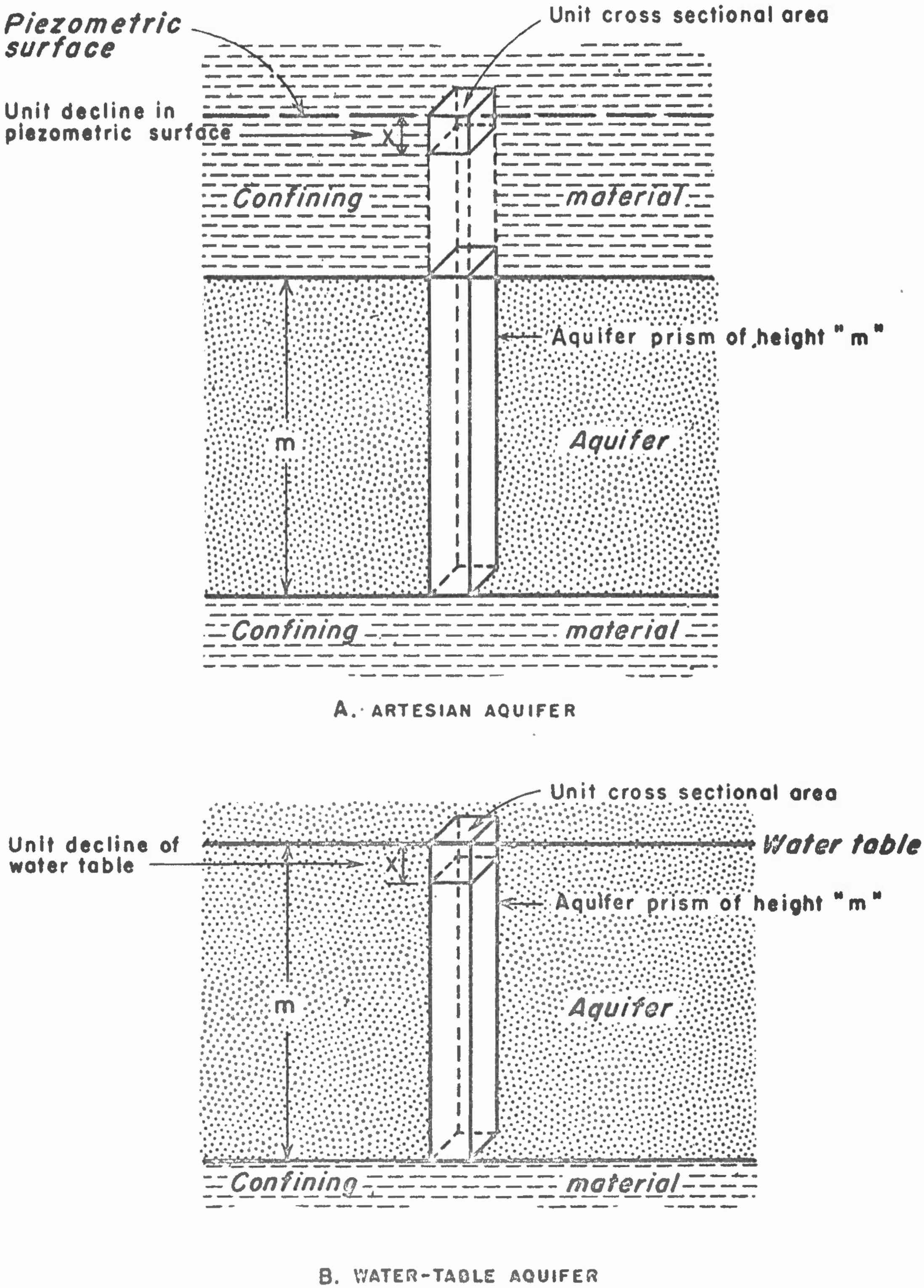


Figure 2 -- Diagrams for explaining the coefficient of storage for artesian and for water-table aquifers

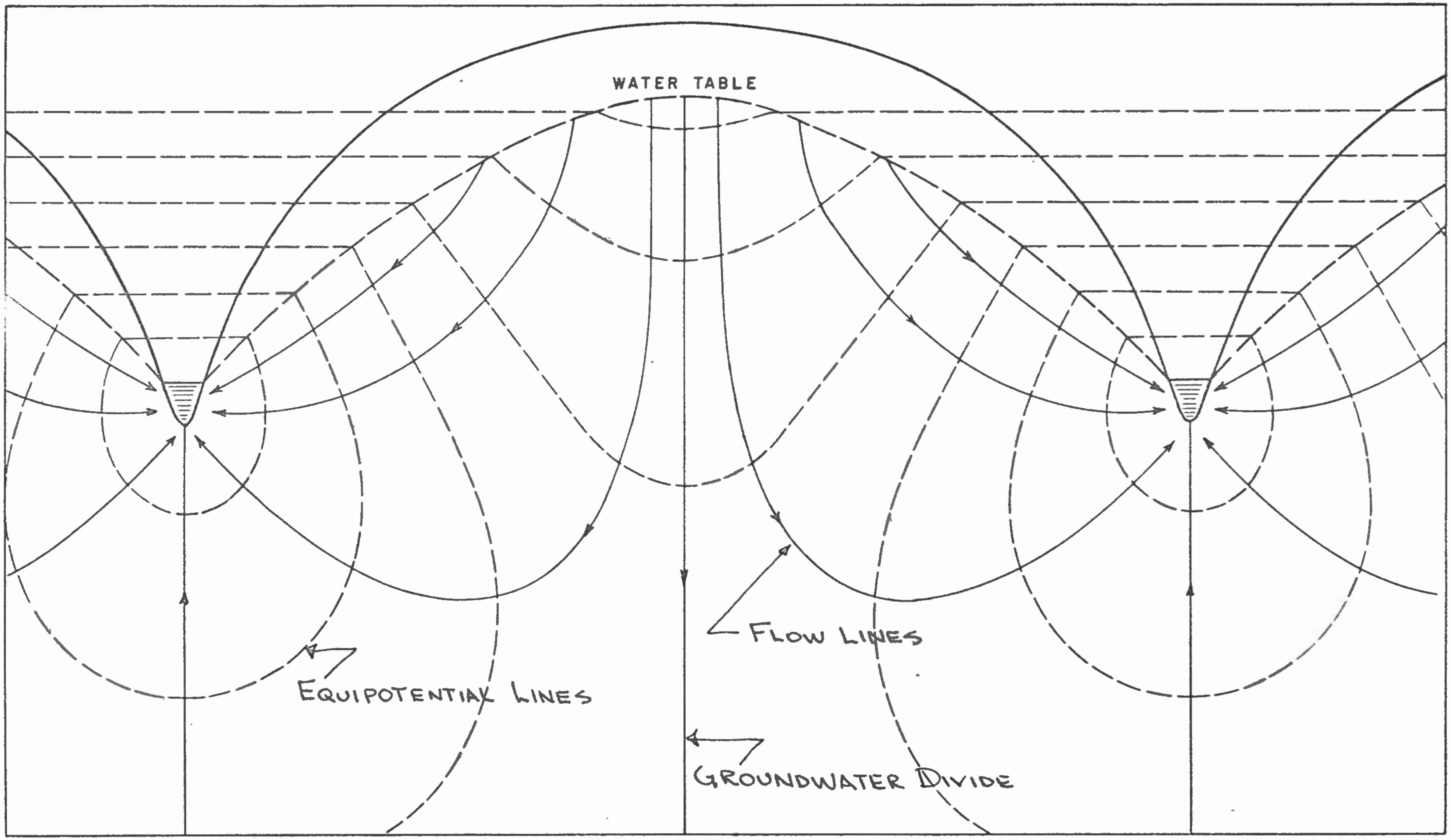


FIG. FLOW NET IN A HOMOGENEOUS WATER TABLE AQUIFER BETWEEN PARALLEL EFFLUENT STREAMS WHEN RECHARGE TO THE AQUIFER IS UNIFORMLY DISTRIBUTED (AFTER HUBBERT).

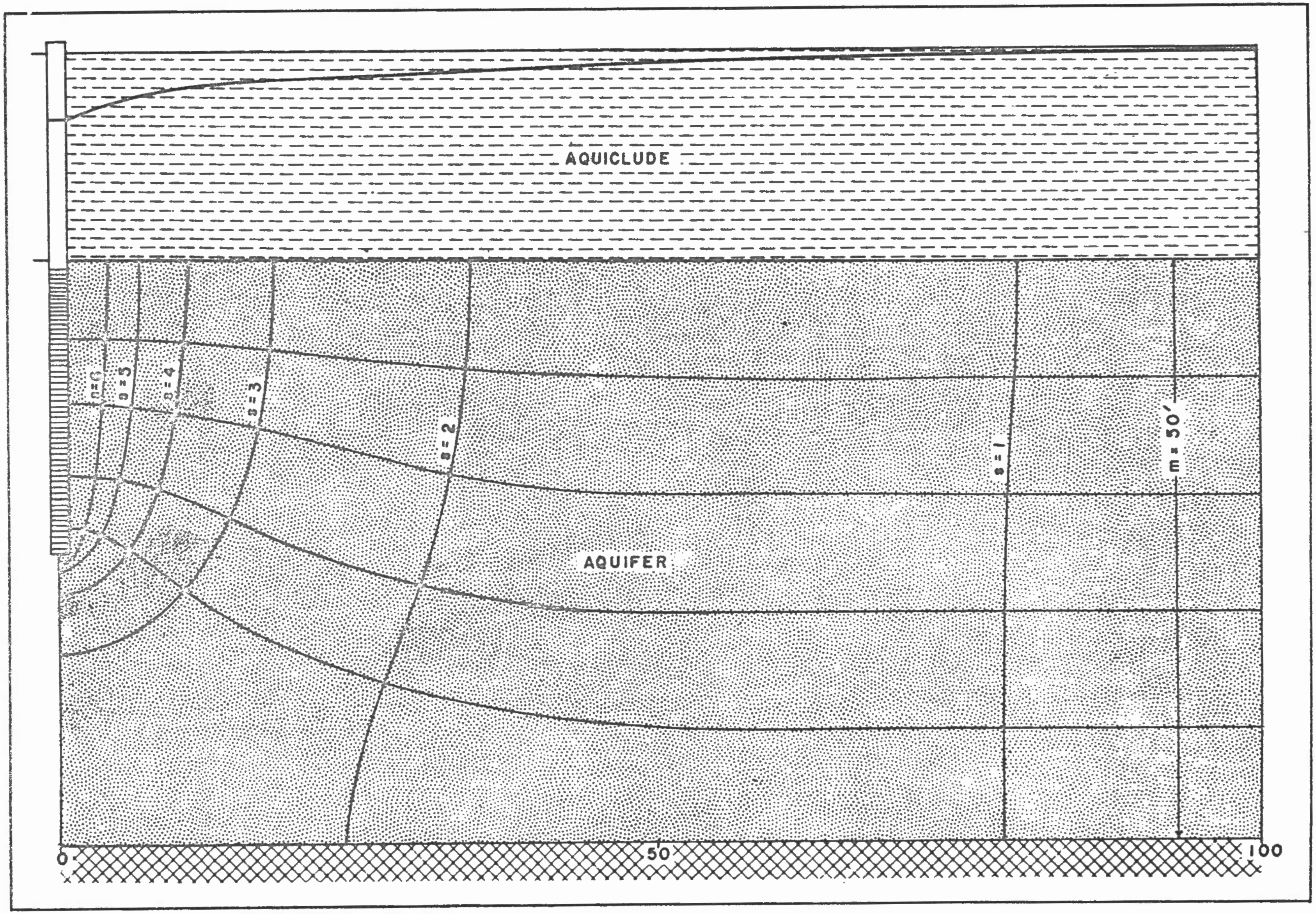


Fig.____ Generalized flow net showing streamlines and equipotential lines in the vicinity of a discharging well with fifty percent penetration in an isotropic sand.

GPO 801664

Subsidence resulting from withdrawal of water or oil

Subsidence of the land surface resulting from withdrawal of ground water or oil is a fairly common phenomenon, but it is more apt to go on undetected in inland areas than along seacoasts--where danger of inundation soon becomes apparent. Among water fields, subsidence of about 6 feet near San Jose, Calif. and 4 feet near Texas City, Texas has occurred. Among oil fields, subsidence of about 2 feet had occurred by 1925 in the Goose Creek field, Texas; of about 6 feet between 1938 and 1947 and 16 feet by 1952 near Wilmington and Long Beach, Calif. (See Gilluly, James and Grant, U. S., Subsidence in the Long Beach Harbor area, Calif.: Geol. Soc. America Bull., vol. 60, pp. 461-530, 1949.) Subsidences of such magnitudes are believed to result mainly from plastic deformation of intercalated or adjacent clay beds.

Lohman proposes a basis for computing the expected <u>elastic</u> subsidence as follows:

From equation (25)

$$\frac{S}{\gamma} = \theta m \beta + \frac{m}{E_s} \tag{26}$$

Then, from Hooke's law that strain is proportional to stress (within the elastic limit), we may write

$$\Delta m = \frac{m}{E_s} \Delta p \text{ or } \frac{m}{E_s} = \frac{\Delta m}{\Delta p} \tag{27}$$

where Δm = subsidence, in feet
Δp = diminutation of pressure, in lb in^{-2}

Then, combining equations (26) and (27)

$$\Delta m = \Delta p \left(\frac{S}{\gamma} - \theta m \beta\right) \tag{28}$$

Thus in a reasonably elastic artesian aquifer where S is known from a pumping or flow test, θ is known from core or sample tests, and m is known from a driller's log or electric log, it is possible to predict the subsidence for a given decline in artesian pressure. For example, from the following values for the Fox Hills sandstone in the Denver artesian basin:

$S = 2\times10^{-4}$
$m = 100$ ft
$\theta = 0.3$
$\Delta p = 100$ lb in^{-2} (231 ft of head; assumed)

$$\Delta m = 10^2 \text{lb in}^{-2}(2\text{x}10^{-4}\text{x}2{,}31 \text{ ft in}^2\text{lb}^{-1} - 0.3\text{x}10^2\text{ft x } 3.3\text{x}10^{-6}\text{in}^2\text{lb}^{-1})$$

$$= 10^2 \text{lb in}^{-2}(4.62\text{x}10^{-4}\text{ft in}^2\text{lb}^{-1} - 10^{-4}\text{ft in}^2\text{lb}^{-1})$$

$$= 3.6\text{x}10^{-2}\text{ft, say } 0.04 \text{ ft}$$

Assume a thick artesian bed in a water or oil field having the following values:

$$S = 10^{-3}$$
$$\theta = 0.3$$
$$m = 1{,}000 \text{ ft}$$
$$\Delta p = 1{,}000 \text{ lb in}^{-2}$$

Then

$$\Delta m = 10^3 \text{lb in}^{-2}(10^{-3}\text{x}2.31\text{ft in}^2\text{lb}^{-1} - 0.3\text{x}10^3\text{ft x } 3.3\text{x}10^{-6}\text{in}^2\text{lb}^{-1})$$

$$= 10^3 \text{lb in}^{-2}(2.31\text{x}10^{-3}\text{ft in}^2\text{lb}^{-1} - 10^{-3}\text{ft in}^2\text{lb}^{-1})$$

$$= 1.3 \text{ ft}$$

These are believed to be reasonable values for expected subsidence resulting from purely elastic compression of the aquifer and elastic expansion of the water. The actual subsidence might be considerably greater owing to plastic deformation of any intercalated or adjacent clay.

If an elastic subsidence has been identified and measured, its value can be inserted into equation (27) to determine the value of E_s. If the latter value proves to be of reasonable magnitude, then elastic compression of the strata becomes a competent explanation of the measured subsidence. E_s also may be determined directly from equation (25) or (26).

Uniform and flashy stream flow from adjacent terranes

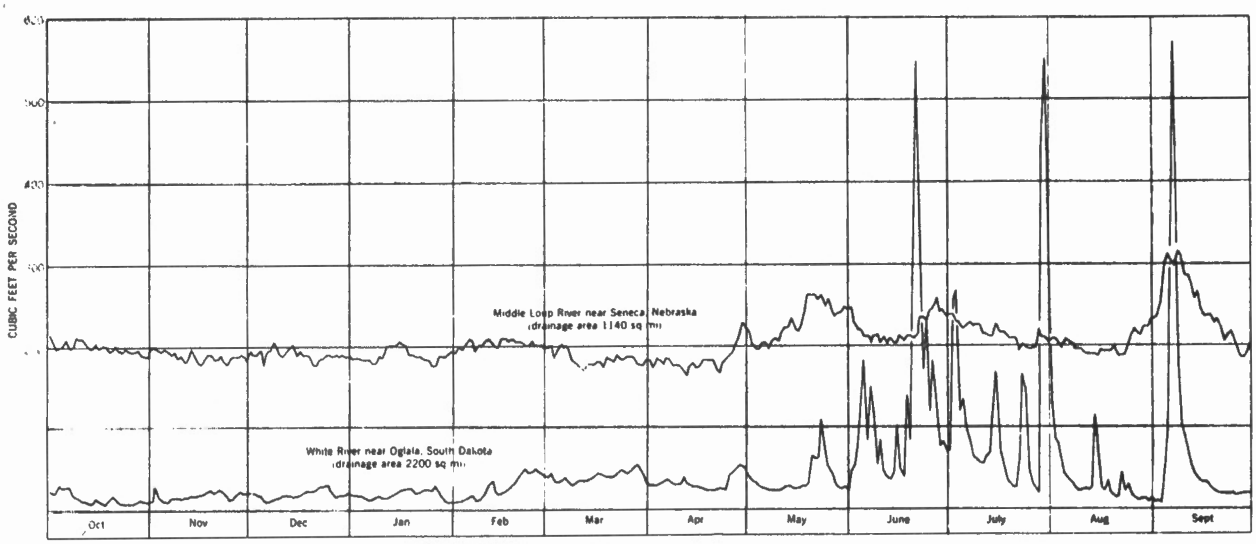


Fig. 2--Accumulated monthly deviations from uniform flow in three streams of central Oregon, in percentage of the mean for the 25 years ending September 30, 1921-1945

Of the preceding two diagrams, the upper (after Lohman, S. W., Sand Hills area, Nebraska, chapt. 5 in The physical and economic foundation of natural resources, pt. IV, subsurface facilities of water management and patterns of supply-type area studies: Interior and Insular Affairs Committee, House of Representatives, United States Congress, fig. 5.7, 1953) contrasts relatively uniform flow of the Middle Loup River, Nebr., with the flashy flow of the White River, S. Dak. The Middle Loup is one of several branches of a stream that drains the Sand Hills, a ground-water reservoir of great volume, whereas the White drains a shale terrane. The two streams are in the same climatic environment, yet during much of the year the flow of the Middle Loup is 10 times the greater. The ratio of greatest flow to least flow (during the year shown) is 1.6 to 1 for the Middle Loup, but is 27.5 to 1 for the White.

The lower diagram (after Piper, A. M.: Am. Geophysical Union Trans., vol. 29, pp. 511-520, 1948) contrasts the John Day, Deschutes, and Metolius Rivers of central Oregon. The John Day drains 7,580 sq mi of rather impermeable terrane. Its flow fluctuates considerably each year and diminished progressively during the recurrent droughts of 1929-1941; its minimum flow has been less than 1 percent of its mean flow. The Deschutes drains 10,500 sq mi. Its flow is strikingly uniform, the minimum of record having been 58 percent of the mean; this uniformity is an effect of large perennial ground-water runoff. The ground-water runoff of the Deschutes comes largely from a part of its basin that is exceptionally permeable. This part is drained largely by a tributary stream, the Metolius River, whose minimum flow of record is 76 percent of the mean. In this tributary, flow seems to lag at least 5 years after fluctuations of precipitation; it actually increased slightly from 1933 into 1938, during a prolonged drought.

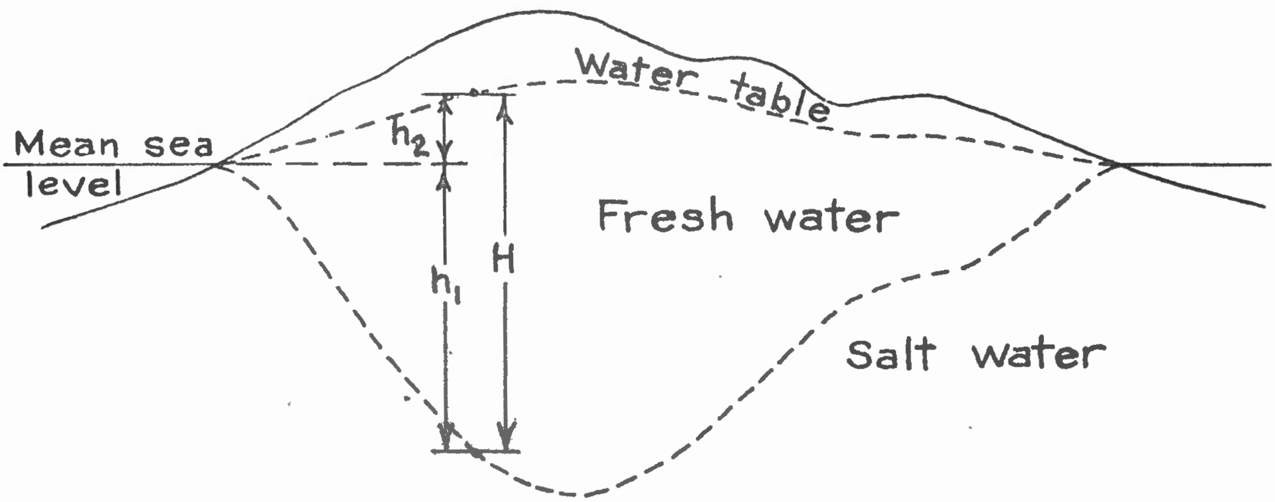
Relation of fresh ground water to salt water along sea coasts

Ghyben-Herzberg principle.- Small oceanic islands and coastal spits, if formed of material that is continuously and moderately permeable, commonly are underlain by a lens-shaped mass of fresh ground water "floating" upon underlying salt water. If hydrostatic equilibrium were attained, the form of the fresh-water mass would accord with Archimedes' principle that a floating body displaces its own weight of liquid. Thus, Badon Ghyben[1] and Herzberg[2] found, apparently independently, that the

1/ Badon Ghyben, W., Nota in verband met de voorgenomen put boring nabij Amsterdam: K. Inst. Ind. Tijdschr, 1888-89, p. 21, The Hague, 1889.

2/ Herzberg, Baurat, Die Wasserversorgung einiger Nordseebader: Jour. Gasbeleuchtung und Wasserversorgung, Jahrg. 44, Munich, 1901.

depth to salt water was roughly a function of the height of the water table above mean sea level, and of the density of the sea water. In the sketch below,



let

H = total thickness of fresh water
h_1 = depth of fresh water below mean sea level
h_2 = height of fresh water above mean sea level
γ_1 = specific gravity of sea water
γ_2 = specific gravity of fresh water (generally 1.000)

Then
$$H = h_1 + h_2 = \frac{\gamma_1}{\gamma_2} h_1 \qquad (29)$$

whence
$$h_2 = \frac{\gamma_1}{\gamma_2} h_1 - h_1 = h_1\left(\frac{\gamma_1}{\gamma_2} - 1\right) \qquad (30)$$

and
$$h_1 = \frac{h_2}{\frac{\gamma_1}{\gamma_2} - 1} \qquad (31)$$

(An average value of γ_1 is about 1.025, whence h_1 = about 40 h_2.)

The foregoing assumes hydrostatic equilibrium, which applies approximately near the center of the lens but does not apply near points of fresh-water discharge into wells or at the coast. Actually, a dynamic equilibrium exists between recharge and discharge, with fresh water moving over the body of salt water3/.

3/ Hubbert, M. K., The theory of ground-water motion: Jour. Geol., vol. 48, pp. 882-884, 924-926, 1940.

Effect of pumping wells.- A well drilled into a "Ghyben-Herzberg" lens may or may not yield fresh water, depending on the depth of the well and of its cone of depression when pumping. The following diagrams show the probable relations between fresh and salt water in a coastal area (after Lohman, S. W., Geology and ground-water resources of the Elizabeth City area, North Carolina: U. S. Geol. Survey Water-Supply Paper 773-A, fig. 3, 1936).

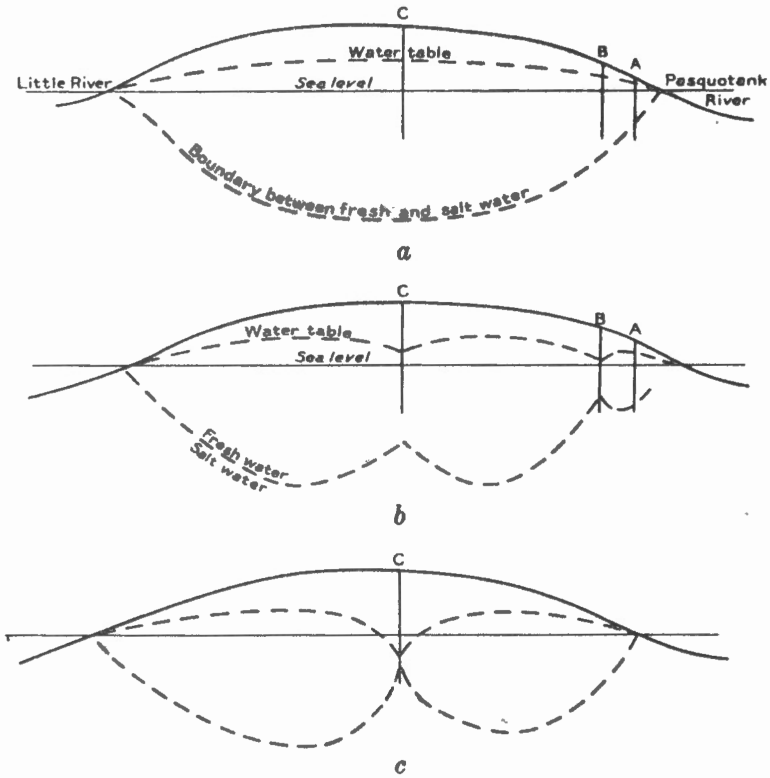


Figure 3. - Diagrams showing probable relations between fresh and salt water in the Elizabeth City area, if it is assumed that the water-bearing materials are homogeneous and permeable both laterally toward the Pasquotank and Little Rivers and downward to considerable depth. (Not drawn to scale.)

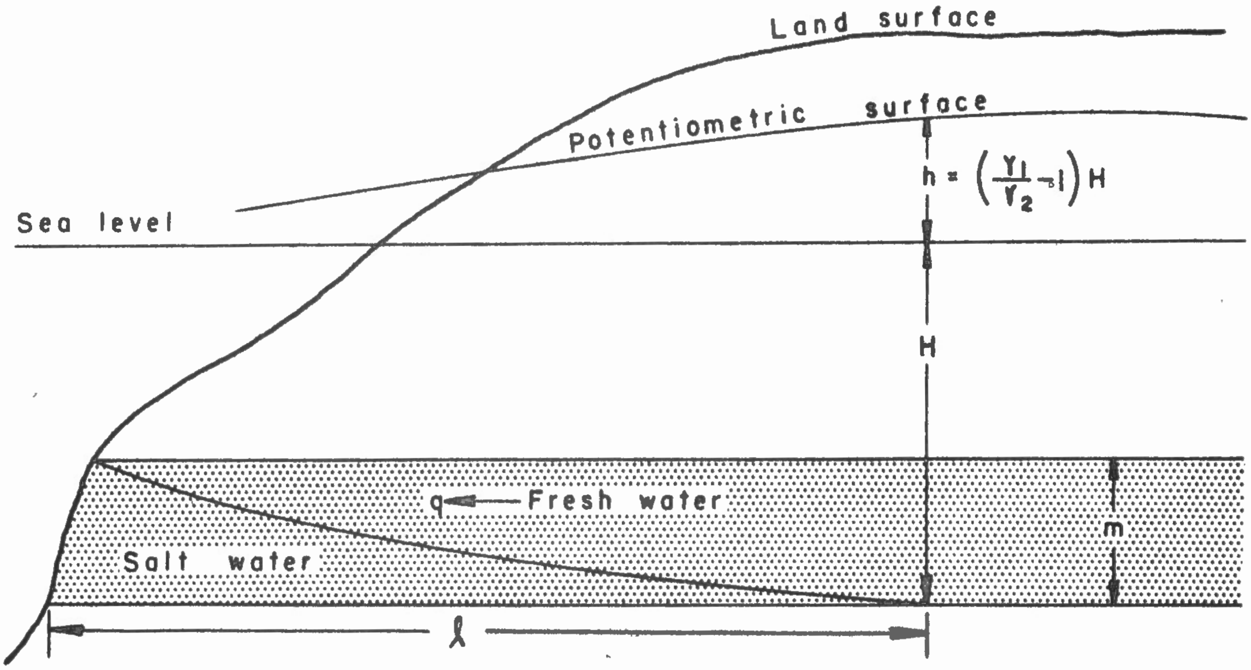
Diagram a shows theoretical relations between fresh and salt water before pumping. A shallow well, A, near either river may encounter salt water even without pumping.

Diagram b shows that when well B is pumped, only a moderate lowering of the head may result in drawing in salt water, whereas in well C an equal lowering of head may not result in drawing in salt water.

Diagram c shows that too great a lowering of the head in well C may also result in drawing in salt water.

Salt-water intrusion into submarine outcrop of artesian aquifer

(After Simpson, T. R., Hotes, F. L., Harder, James A., and Lau, Leung-ku, Final report on sea-water intrusion: Univ. of California Sanitary Engineering Research Laboratory, 49 pp., 10 figs., Sept. 1953.)



$$q = \frac{1}{2} \left(\frac{\gamma_1}{\gamma_2} - 1\right) \frac{m}{\ell} T \qquad (31a)$$

where q = seaward flow of fresh water per unit length of ocean front
γ_1 = specific gravity of sea water
γ_2 = specific gravity of fresh water
m = thickness of aquifer down to lowest depth that must be protected
ℓ = length of salt-water wedge
T = coefficient of transmissibility

In equation (31a), if T is in ft^2/sec, then q is in ft^3/ sec ft (second-feet per foot), m and ℓ are in ft. If T is in gpd/ft, then q also is in gpd/ft.

To prevent sea water from entering an artesian aquifer that has direct access to the sea, as depicted above, the fresh water head must be maintained above sea level a height h, in accordance with the Ghyben-Herzberg principle. The approximate solution (13a) indicates that for an aquifer of thickness m, the maintenance of the fresh water above sea level will result in a seaward leakage of fresh water in the upper part of the aquifer at the rate q for a salt-water wedge that extends inland a distance ℓ. Thus, other factors being equal, ℓ is inversely proportional to q. These relationships are independent of the depth of the aquifer below sea level.

Stratification of salt water and fresh water in sediments

(After Williams, C. C., and Lohman, S. W., Geology and ground-water resources of a part of south-central Kansas, with special reference to the Wichita municipal water supply: Kans. Geol. Survey Bull. 79, p. 181, 1949.)

The following results were obtained from a driven well put down to a depth of 44 feet at a point 250 feet downstream from an "evaporation" pond containing brine with a chloride content of 51,750 parts per million.

Chloride concentrations at given depths in alluvium 250 feet from an "evaporation" pond.

(Depth to water level, 9.5 feet)

Depth (feet)		Chloride concentration (parts per million)
23 - 25		1,900
28 - 30		6,800
33 - 35		39,000
38 - 40		50,300
41 - 43		50,900
43 - 44	(Relatively impermeable)	

The boundary between the fresh and salty waters was found to be much sharper at greater distances from the source of intrusion, as indicated in the following analyses.

Chloride concentrations at given depths in alluvium 750 feet from an "evaporation" pond.

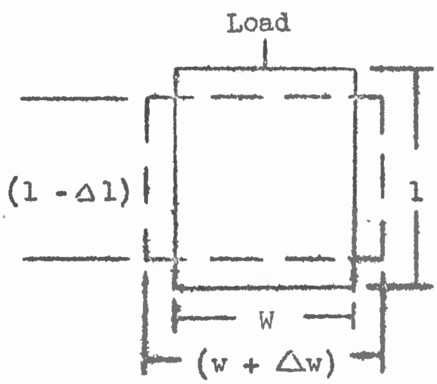
(Depth to water level, 9 feet)

Depth (feet)		Chloride concentration (parts per million)
28 - 30		69
33 - 35		69
38 - 40		71
43 - 45		70
47 - 49		2,240
49 - 50	(Relatively impermeable)	

Elasticity of artesian aquifers

Poisson's ratio:

If a column of rock is subjected to a compressional load along its principal (vertical) axis the column is shortened in the direction of the load, i.e. in the vertical, and at the same time the lateral dimensions of the column increase as shown by the following diagram.



Lateral unit strain = $\frac{\Delta w}{w}$

Longitudinal unit strain = $\frac{\Delta l}{l}$

The ratio of lateral unit strain to longitudinal unit strain is called "Poisson's ratio" and is denoted by the symbol σ (sigma). For most elastic materials its value lies between 0. and 0.5. Gutenberg (1951, p. 378) reports:

> "For most rocks it (Poisson's ratio) is near 1/4. For the interior of the earth Poisson's ratio can be found from the ratio of the velocity V of longitudinal waves to that v of transverse waves. Its value is slightly less than 1/4 for the 'granitic' layer; in the deeper continental layers it is about 1/4, and in the deeper parts of the mantle about 0.3. Inside the core σ seems to be close to 1/2."

REFERENCE: Gutenberg, Beno. Internal constitution of the earth. Dover Publ., Inc. 2nd edition. 1951.

Elasticity of artesian aquifers (cont.)

Seismic velocity related to elasticity

Terminology

Symbol	Description	Unit
ρ	Density of rock	gm./cm.^{3}
σ	Poisson's ratio	--
v	Velocity of transverse seismic wave	cm./sec.
V	Velocity of longitudinal seismic wave	cm./sec.
E_k	Modulus of elasticity (Young's) of aquifer skeleton	dynes/cm.^{2}

Equations:

$$E_k = 2\rho v^2 (1+\sigma) = \frac{\rho V^2 (1+\sigma)(1-2\sigma)}{1-\sigma}$$

When Poisson's ratio σ (sigma) is 0.25, the following equations result:

$$E_k = \frac{5}{2}\rho v^2 = \frac{5}{6}\rho V^2$$

Conversion factors:

1 km. = 10^5 cm. 1 ft. = 30.48 cm.

1 barye = 1 dyne/cm.^{2} = 10^{-6} bar = 1.4504 x 10^{-5} lbs./in.^{2}

1 bar = 10^6 baryes = 10^6 dynes/cm.^{2} = 14.504 lbs./in.^{2}

REFERENCE: Gutenberg, Beno, Internal constitution of the earth: Dover Publications, Inc. 2nd edition. pp. 364-365, 419-420. 1951.

Elasticity of artesian aquifer (cont.)

Seismic wave types

1. Body waves -- travel through the interior of the elastic medium

   A. Longitudinal or P waves -- also known as compressional, condensational, irrotational, primary, or P waves.

      Motion of the particle is along the direction in which the waves advance.

   B. Transverse or S waves -- also known as shear, equivoluminal, secondary, or S waves.

      "A particle in the path of a transverse wave may oscillate in any direction in the plane normal to the direction of advance of the wave. For a transverse wave travelling parallel to the earth's surface, if particles in its path oscillate up and down, the wave is given a special symbol SV; if the particles oscillate in the horizontal plane, it is called SH."

2. Surface waves -- travel on the surface of the elastic medium

   A. Rayleigh or R waves -- a combined longitudinal and transverse wave with its plane of oscillation at right angles to the surface and parallel to the direction of propogation.

   B. Love or Q waves -- a special type of transverse wave with its plane of oscillation in the formation boundary.

   C. Hydrodynamic or H waves -- so called by Leet, who first recognized same on record of earth motion following the first atomic bomb test in New Mexico.

   D. Coupled or C waves -- so called by Leet, who first described same in 1939.

REFERENCES: Leet, L. Don, Earth Waves: Harvard Monographs in Applied Science, No. 2, 1950. pp. 44-57.

Heiland, C. A., Geophysical Exploration: Prentice Hall, Inc., New York, New York. 1946. pp. 450-452.

Porosity of artesian aquifer

Terminology:

Symbol	Description	Unit
b	Proportion of contact area between aquifer and aquiclude over which hydrostatic pressure is effective	--
m	Thickness of aquifer	feet
B	Barometric efficiency	--
E_k	Modulus of elasticity (Young's) of aquifer skeleton	lbs./ft.^{2}
E_w	Modulus of elasticity (bulk) of water	lbs./ft.^{2}
S	Coefficient of storage	--
γ_o	Specific weight of water	lbs./ft.^{3}
θ	Porosity of aquifer	--

Equations: The following equations are based on the assumption that there is no leakage between the aquifer and the confining beds, no water of compaction from intercalated clays or shales, and no gas dispersed throughout the aquifer.

$$m = \frac{S\,E_k}{\gamma_o}(1 - B) \qquad \theta = \frac{E_w}{E_k}\left[\frac{E_k\,S}{\gamma_o\,m} - b\right]$$

$$\theta = \frac{E_w}{E_k}\left[\frac{1}{(1 - B)} - b\right] \qquad \theta = \frac{S\,E_w}{\gamma_o\,m}\left[1 - b\,(1 - B)\right]$$

Permeability

Terminology:

Symbol	Description	Ft.-sec.	Gal.-ft.-day	m.-sec.
k	Permeability of aquifer	ft/sec.		cm/sec.
P	Permeability of aquifer		gpd/ft^2	
Q	Quantity of flow	ft^3/sec.	g.p.d.	cm^3/sec.
A	Cross-sectional area of flow	ft^2	ft^2	cm^2.
I	Hydraulic gradient $= dh/dl = \frac{h_1 - h_2}{l}$	dimension-less	dimension-less	dimension-less

Equations: Ft.-day or m.-sec. system

$$k = - \frac{Q}{A \left(\frac{h_1 - h_2}{l}\right)}$$

$$= \frac{-Q}{dh/dl}$$

Gal.-ft.-day system:

$$P = \frac{Q}{IA} *$$

* at 60° F.

Note: Almost all conceivable combinations of units have been proposed for the relationship:

$$P = \frac{\text{volume of flow}}{(\text{time})\ (\text{area})\ (\text{hydraulic gradient})}$$

Permeability (cont)

In general, Darcy's equation is:

$$Q = \frac{k\,A\,\Delta p}{\mu\,L} \qquad\qquad k = \frac{Q\,\mu\,L}{A\,\Delta p}$$

where:
- Q = flow of fluid
- k = permeability of porous media
- A = cross-sectional area of flow
- $\Delta p/L$ = pressure gradient in the direction of flow
- μ = viscosity of fluid

If:
- $Q = 1\ cm^3/sec.$
- $A = 1\ cm^2$
- $\mu = 1$ centipoise
- $\Delta p/L = 1$ atmosphere/cm

then $k = 1$ darcy

Note: at 68°F (20°C) water has $\mu = 1.005$ centipoises.

Dimensional analysis of k:

- $Q = L^3/t$
- $L = L$
- $\mu = M/Lt$
- $A = L^2$
- Δp = force/area = mass x acc/area = $M/(Lt)^2$

$$k = \frac{Q\,\mu\,L}{A\,\Delta p} = \frac{(L^3/t)\,(M/Lt)\,(L)}{(L^2)\,(M/Lt^2)} = L^2$$

If all factors are taken into account (i.e. - viscosity and pressure gradient), k will have units of $(length)^2$. This concept has value in the fields of petroleum engineering and soil mechanics.

TABLE 1.—CONVERSION TABLE FOR PERMEABILITY UNITS

Units	darcy $\frac{1\ cc}{sec.cm^2\ (atm/cm)}$	$\frac{1\ cc}{sec.cm^2\ (dyne/cm^2)/cm}$	$\frac{1\ ft^3}{sec.ft^2\ (atm/ft)}$	$\frac{1\ ft^3}{sec.ft^2\ (psi/ft)}$	$\frac{1\ cc\ H_2O\ (20°C)}{sec.cm^2\ (1\ cm\ H_2O)/cm}$	$\frac{1\ gal\ H_2O\ (20°C)}{min.\ ft^2\ (1\ ft\ H_2O)/ft}$	$\frac{1\ bbl\ H_2O\ (20°C)}{day.\ ft^2\ (psi/ft)}$
darcy $\frac{1\ cc}{sec.\ cm^2\ (atm/cm)}$ =	1	9.8692×10^{-7}	1.0764×10^{-3}	7.3243×10^{-5}	9.6301×10^{-4}	1.4181×10^{3}	1.1215
$\frac{1\ cc}{sec.\ cm^2\ (dyne/cm^2)/cm}$ =	1.0132×10^{6}	1	1.0908×10^{3}	74.211	9.7576×10^{2}	1.4369×10^{4}	1.1364×10^{6}
$\frac{1\ ft^3}{sec.\ ft^2\ (atm/ft)}$ =	9.2903×10^{2}	9.1688×10^{-4}	1	6.8046×10^{-2}	0.89467	13.174	1.0419×10^{3}
$\frac{1\ ft^3}{sec.\ ft^2\ (psi/ft)}$ =	1.3653×10^{4}	1.3474×10^{-2}	14.696	1	13.148	1.9360×10^{2}	1.5312×10^{4}
$\frac{1\ cc\ H_2O\ (20°C)}{sec.\ cm^2\ (1\ cm\ H_2O/cm)}$ =	1.0384×10^{3}	1.0248×10^{-3}	1.1178	7.6063×10^{-2}	1	14.726	1.1646×10^{3}
$\frac{1\ gal\ H_2O\ (20°C)}{min.\ ft^2\ (1\ ft\ H_2O/ft)}$ =	70.519	6.9596×10^{-5}	7.5906×10^{-2}	5.1651×10^{-3}	6.7910×10^{-2}	1	79.087
$\frac{1\ bbl\ H_2O\ (20°C)}{day.\ ft^2\ (psi/ft)}$ =	0.89165	8.7999×10^{-7}	9.5977×10^{-4}	6.5308×10^{-5}	8.5865×10^{-4}	1.2644×10^{-2}	1

First four units refer to a fluid of 1 cp viscosity; viscosity of water at 20°C is taken as 1.005 cp.

PART II

FORMULAS AND EXAMPLES

Contents:

Thiem formula

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
s_1, s_2	Drawdown or recovery of water level at distances r_1, r_2 respectively	feet	feet	metres
r_1, r_2	Distances from pumped well	feet	feet	metres
re	Distance intercept at s = 0	feet	feet	metres
t	Time since pumping started or stopped	days	days	seconds
Q	Rate of discharge by pumped well	ft^3/day	g.p.m.	m^3/sec.
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft	m^2/sec.
S	Coefficient of storage	dimension-less	dimension-less	dimension-less

Equations:

Ft.-day or m.-sec. system:

$$T = \frac{Q \log_e r_2/r_1}{2\pi(s_1 - s_2)}$$

or

$$T = \frac{2.30\ Q \log_{10} r_2/r_1}{2\pi(s_1 - s_2)}$$

$$S = \frac{2.25\ T\ t}{r_e^2}$$

Gal.-ft.-day system:

$$T = \frac{527.7\ Q \log_{10} r_2/r_1}{(s_1 - s_2)}$$

$$S = \frac{0.36\ T\ t}{r_e^2}$$

Theis non-equilibrium formula

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
Δs	Drawdown or recovery of water level	feet	feet	metres
r	Distance from pumped well	feet	feet	metres
t	Time since pumping started or stopped	days	days	seconds
Q	Rate of discharge by pumped well	ft^3/day	g.p.m.	m^3/sec.
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft	m^2/sec.
S	Coefficient of storage	dimension-less	dimension-less	dimension-less

Equations:

Ft.-day or m.-sec. system:

$$\Delta s = \frac{Q\,W(u)}{4\pi T}$$

$$W(u) = \int_{\frac{r^2 S}{4Tt}}^{\infty} \frac{e^{-u}}{u}\,du$$

$$u = \frac{r^2 S}{4Tt}$$

Gal.-ft.-day system:

$$\Delta s = \frac{114.6\,Q\,W(u)}{T}$$

$$W(u) = \int_{\frac{1.56 r^2 S}{Tt}}^{\infty} \frac{e^{-u}}{u}\,du$$

$$u = \frac{1.56\,r^2 S}{T t}$$

where $W(u) = -0.577216 - \log_e u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} \ldots\ldots$

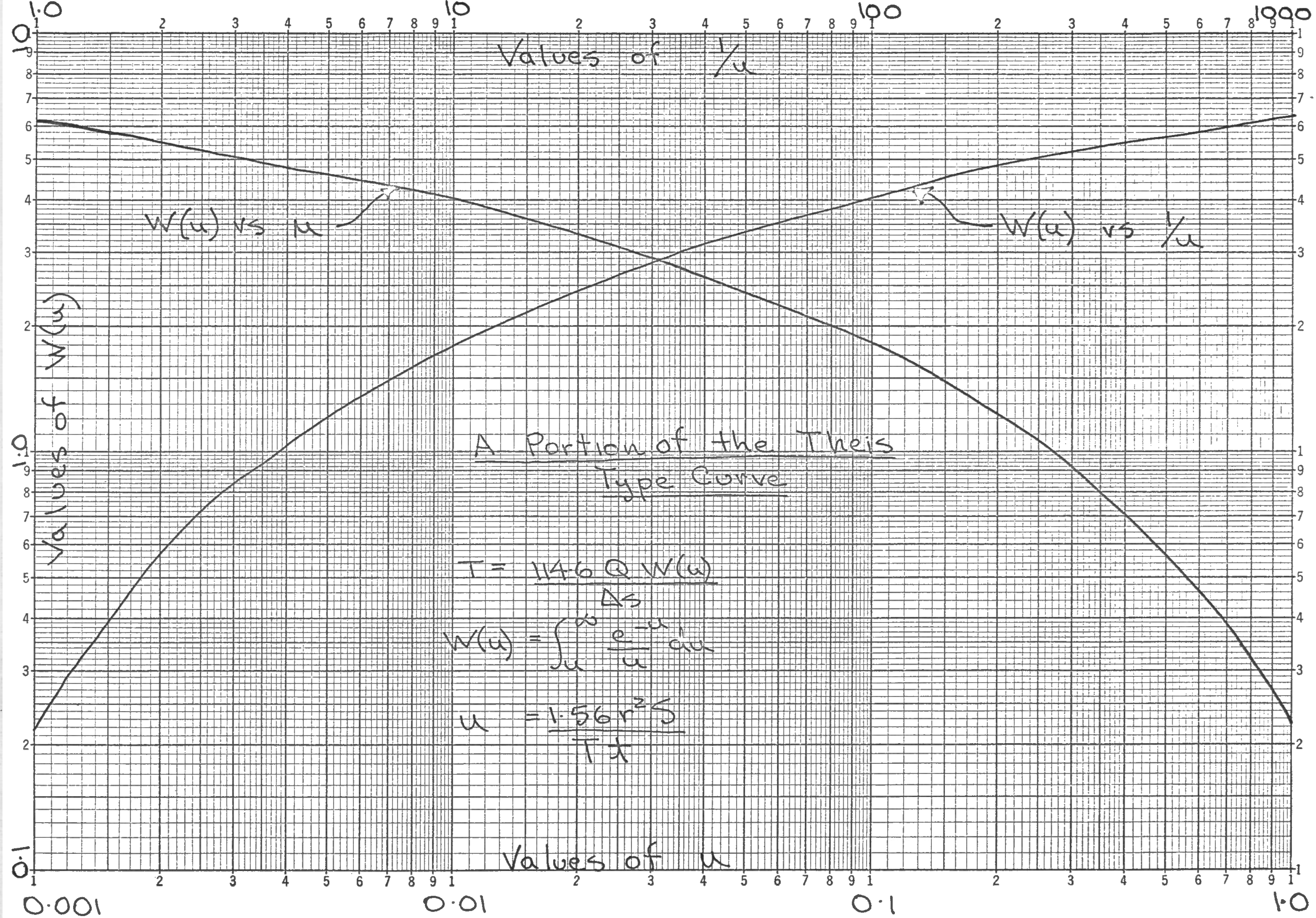

Values of 1/u
W(u) vs u
W(u) vs 1/u
Values of W(u)
A Portion of the Theis Type Curve
T = 114.6 Q W(u) / Δs
W(u) = ∫u^∞ (e^-u / u) du
u = 1.56 r²S / Tt
Values of u
1.0
10
100
1000
0.001
0.01
0.1
1.0
0.1

TABLE 1

VALUES OF W(u) FOR NON-EQUILIBRIUM FORMULA

N \ u	$N\times10^{-15}$	$N\times10^{-14}$	$N\times10^{-13}$	$N\times10^{-12}$	$N\times10^{-11}$	$N\times10^{-10}$	$N\times10^{-9}$	$N\times10^{-8}$	$N\times10^{-7}$	$N\times10^{-6}$	$N\times10^{-5}$	$N\times10^{-4}$	$N\times10^{-3}$	$N\times10^{-2}$	$N\times10^{-1}$	N
1.0	33.9616	31.6590	29.3564	27.0538	24.7512	22.4486	20.1460	17.8435	15.5409	13.2383	10.9357	8.6332	6.3315	4.0379	1.8229	0.2194
1.1	33.8662	31.5637	29.2611	26.9585	24.6559	22.3533	20.0507	17.7482	15.4456	13.1430	10.8404	8.5379	6.2363	3.9436	1.7371	.1860
1.2	33.7792	31.4767	29.1741	26.8715	24.5689	22.2663	19.9637	17.6611	15.3586	13.0560	10.7534	8.4509	6.1494	3.8576	1.6595	.1584
1.3	33.6992	31.3966	29.0940	26.7914	24.4889	22.1863	19.8837	17.5811	15.2785	12.9759	10.6734	8.3709	6.0695	3.7785	1.5889	.1355
1.4	33.6251	31.3225	29.0199	26.7173	24.4147	22.1122	19.8096	17.5070	15.2044	12.9018	10.5993	8.2968	5.9955	3.7054	1.5241	.1162
1.5	33.5561	31.2535	28.9509	26.6483	24.3458	22.0432	19.7406	17.4380	15.1354	12.8328	10.5303	8.2278	5.9266	3.6374	1.4645	.1000
1.6	33.4916	31.1890	28.8864	26.5838	24.2812	21.9786	19.6760	17.3735	15.0709	12.7683	10.4657	8.1634	5.8621	3.5739	1.4092	.08631
1.7	33.4309	31.1283	28.8258	26.5232	24.2206	21.9180	19.6154	17.3128	15.0103	12.7077	10.4051	8.1027	5.8016	3.5143	1.3578	.07465
1.8	33.3738	31.0712	28.7686	26.4660	24.1634	21.8608	19.5583	17.2557	14.9531	12.6505	10.3479	8.0455	5.7446	3.4581	1.3098	.06471
1.9	33.3197	31.0171	28.7145	26.4119	24.1094	21.8068	19.5042	17.2016	14.8990	12.5964	10.2939	7.9915	5.6906	3.4050	1.2649	.05620
2.0	33.2684	30.9658	28.6632	26.3607	24.0581	21.7555	19.4529	17.1503	14.8477	12.5451	10.2426	7.9402	5.6394	3.3547	1.2227	.04890
2.1	33.2196	30.9170	28.6145	26.3119	24.0093	21.7067	19.4041	17.1015	14.7989	12.4964	10.1938	7.8914	5.5907	3.3069	1.1829	.04261
2.2	33.1731	30.8705	28.5679	26.2653	23.9628	21.6602	19.3576	17.0550	14.7524	12.4498	10.1473	7.8449	5.5443	3.2614	1.1454	.03719
2.3	33.1286	30.8261	28.5235	26.2209	23.9183	21.6157	19.3131	17.0106	14.7080	12.4054	10.1028	7.8004	5.4999	3.2179	1.1099	.03250
2.4	33.0861	30.7835	28.4809	26.1783	23.8758	21.5732	19.2706	16.9680	14.6654	12.3628	10.0603	7.7579	5.4575	3.1763	1.0762	.02844
2.5	33.0453	30.7427	28.4401	26.1375	23.8349	21.5323	19.2298	16.9272	14.6246	12.3220	10.0194	7.7172	5.4167	3.1365	1.0443	.02491
2.6	33.0060	30.7035	28.4009	26.0983	23.7957	21.4931	19.1905	16.8880	14.5854	12.2828	9.9802	7.6779	5.3776	3.0983	1.0139	.02185
2.7	32.9683	30.6657	28.3631	26.0606	23.7580	21.4554	19.1528	16.8502	14.5476	12.2450	9.9425	7.6401	5.3400	3.0615	.9849	.01918
2.8	32.9319	30.6294	28.3268	26.0242	23.7216	21.4190	19.1164	16.8138	14.5113	12.2087	9.9061	7.6038	5.3037	3.0261	.9573	.01686
2.9	32.8968	30.5943	28.2917	25.9891	23.6865	21.3839	19.0813	16.7788	14.4762	12.1736	9.8710	7.5687	5.2687	2.9920	.9309	.01482
3.0	32.8629	30.5604	28.2578	25.9552	23.6526	21.3500	19.0474	16.7449	14.4423	12.1397	9.8371	7.5348	5.2349	2.9591	.9057	.01305
3.1	32.8302	30.5276	28.2250	25.9224	23.6198	21.3172	19.0146	16.7121	14.4095	12.1069	9.8043	7.5020	5.2022	2.9273	.8815	.01149
3.2	32.7984	30.4958	28.1932	25.8907	23.5881	21.2855	18.9829	16.6803	14.3777	12.0751	9.7726	7.4703	5.1706	2.8965	.8583	.01013
3.3	32.7676	30.4651	28.1625	25.8599	23.5573	21.2547	18.9521	16.6495	14.3470	12.0444	9.7418	7.4395	5.1399	2.8668	.8361	.008939
3.4	32.7378	30.4352	28.1326	25.8300	23.5274	21.2249	18.9223	16.6197	14.3171	12.0145	9.7120	7.4097	5.1102	2.8379	.8147	.007891
3.5	32.7088	30.4062	28.1036	25.8010	23.4985	21.1959	18.8933	16.5907	14.2881	11.9855	9.6830	7.3807	5.0813	2.8099	.7942	.006970
3.6	32.6806	30.3780	28.0755	25.7729	23.4703	21.1677	18.8651	16.5625	14.2599	11.9574	9.6548	7.3526	5.0532	2.7827	.7745	.006160
3.7	32.6532	30.3506	28.0481	25.7455	23.4429	21.1403	18.8377	16.5351	14.2325	11.9300	9.6274	7.3252	5.0259	2.7563	.7554	.005448
3.8	32.6266	30.3240	28.0214	25.7188	23.4162	21.1136	18.8110	16.5085	14.2059	11.9033	9.6007	7.2985	4.9993	2.7306	.7371	.004820
3.9	32.6006	30.2980	27.9954	25.6928	23.3902	21.0877	18.7851	16.4825	14.1799	11.8773	9.5748	7.2725	4.9735	2.7056	.7194	.004267
4.0	32.5753	30.2727	27.9701	25.6675	23.3649	21.0623	18.7598	16.4572	14.1546	11.8520	9.5495	7.2472	4.9482	2.6813	.7024	.003779
4.1	32.5506	30.2480	27.9454	25.6428	23.3402	21.0376	18.7351	16.4325	14.1299	11.8273	9.5248	7.2225	4.9236	2.6576	.6859	.003349
4.2	32.5265	30.2239	27.9213	25.6187	23.3161	21.0136	18.7110	16.4084	14.1058	11.8032	9.5007	7.1985	4.8997	2.6344	.6700	.002969
4.3	32.5029	30.2004	27.8978	25.5952	23.2926	20.9900	18.6874	16.3848	14.0823	11.7797	9.4771	7.1749	4.8762	2.6119	.6546	.002633
4.4	32.4800	30.1774	27.8748	25.5722	23.2696	20.9670	18.6644	16.3619	14.0593	11.7567	9.4541	7.1520	4.8533	2.5899	.6397	.002336
4.5	32.4575	30.1549	27.8523	25.5497	23.2471	20.9446	18.6420	16.3394	14.0368	11.7342	9.4317	7.1295	4.8310	2.5684	.6253	.002073
4.6	32.4355	30.1329	27.8303	25.5277	23.2252	20.9226	18.6200	16.3174	14.0148	11.7122	9.4097	7.1075	4.8091	2.5474	.6114	.001841
4.7	32.4140	30.1114	27.8088	25.5062	23.2037	20.9011	18.5985	16.2959	13.9933	11.6907	9.3882	7.0860	4.7877	2.5268	.5979	.001635
4.8	32.3929	30.0904	27.7878	25.4852	23.1826	20.8800	18.5774	16.2748	13.9723	11.6697	9.3671	7.0650	4.7667	2.5068	.5848	.001453
4.9	32.3723	30.0697	27.7672	25.4646	23.1620	20.8594	18.5568	16.2542	13.9516	11.6491	9.3465	7.0444	4.7462	2.4871	.5721	.001291
5.0	32.3521	30.0495	27.7470	25.4444	23.1418	20.8392	18.5366	16.2340	13.9314	11.6289	9.3263	7.0242	4.7261	2.4679	.5598	.001148
5.1	32.3323	30.0297	27.7271	25.4246	23.1220	20.8194	18.5168	16.2142	13.9116	11.6091	9.3065	7.0044	4.7064	2.4491	.5478	.001021
5.2	32.3129	30.0103	27.7077	25.4051	23.1026	20.8000	18.4974	16.1948	13.8922	11.5896	9.2871	6.9850	4.6871	2.4306	.5362	.0009086
5.3	32.2939	29.9913	27.6887	25.3861	23.0835	20.7809	18.4783	16.1758	13.8732	11.5706	9.2681	6.9659	4.6681	2.4126	.5250	.0008086
5.4	32.2752	29.9726	27.6700	25.3674	23.0648	20.7622	18.4596	16.1571	13.8545	11.5519	9.2494	6.9473	4.6495	2.3948	.5140	.0007198.
5.5	32.2568	29.9542	27.6516	25.3491	23.0465	20.7439	18.4413	16.1387	13.8361	11.5336	9.2310	6.9289	4.6313	2.3775	.5034	.0006409
5.6	32.2388	29.9362	27.6336	25.3310	23.0285	20.7259	18.4233	16.1207	13.8181	11.5155	9.2130	6.9109	4.6134	2.3604	.4930	.0005708
5.7	32.2211	29.9185	27.6159	25.3133	23.0108	20.7082	18.4056	16.1030	13.8004	11.4978	9.1953	6.8932	4.5958	2.3437	.4830	.0005085
5.8	32.2037	29.9011	27.5985	25.2959	22.9934	20.6908	18.3882	16.0856	13.7830	11.4804	9.1779	6.8758	4.5785	2.3273	.4732	.0004532
5.9	32.1866	29.8840	27.5814	25.2789	22.9763	20.6737	18.3711	16.0685	13.7659	11.4633	9.1608	6.8588	4.5615	2.3111	.4637	.0004039
6.0	32.1698	29.8672	27.5646	25.2620	22.9595	20.6569	18.3543	16.0517	13.7491	11.4465	9.1440	6.8420	4.5448	2.2953	.4544	.0003601
6.1	32.1533	29.8507	27.5481	25.2455	22.9429	20.6403	18.3378	16.0352	13.7326	11.4300	9.1275	6.8254	4.5283	2.2797	.4454	.0003211
6.2	32.1370	29.8344	27.5318	25.2293	22.9267	20.6241	18.3215	16.0189	13.7163	11.4138	9.1112	6.8092	4.5122	2.2645	.4366	.0002864
6.3	32.1210	29.8184	27.5158	25.2133	22.9107	20.6081	18.3055	16.0029	13.7003	11.3978	9.0952	6.7932	4.4963	2.2494	.4280	.0002555
6.4	32.1053	29.8027	27.5001	25.1975	22.8949	20.5923	18.2898	15.9872	13.6846	11.3820	9.0795	6.7775	4.4806	2.2346	.4197	.0002279
6.5	32.0898	29.7872	27.4846	25.1820	22.8794	20.5768	18.2742	15.9717	13.6691	11.3665	9.0640	6.7620	4.4652	2.2201	.4115	.0002034
6.6	32.0745	29.7719	27.4693	25.1667	22.8641	20.5616	18.2590	15.9564	13.6538	11.3512	9.0487	6.7467	4.4501	2.2058	.4036	.0001816
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6.9	32.0300	29.7275	27.4249	25.1223	22.8197	20.5171	18.2145	15.9119	13.6094	11.3068	9.0043	6.7023	4.4059	2.1643	.3810	.0001293
7.0	32.0156	29.7131	27.4105	25.1079	22.8053	20.5027	18.2001	15.8976	13.5950	11.2924	8.9899	6.6879	4.3916	2.1508	.3738	.0001155
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7.2	31.9875	29.6849	27.3823	25.0797	22.7771	20.4746	18.1720	15.8694	13.5668	11.2642	8.9617	6.6598	4.3636	2.1246	.3599	.00009219
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7.7	31.9203	29.6178	27.3152	25.0126	22.7100	20.4074	18.1048	15.8022	13.4997	11.1971	8.8946	6.5927	4.2970	2.0623	.3280	.00005263
7.8	31.9074	29.6048	27.3023	24.9997	22.6971	20.3945	18.0919	15.7893	13.4868	11.1842	8.8817	6.5798	4.2842	2.0503	.3221	.00004707
7.9	31.8947	29.5921	27.2895	24.9869	22.6844	20.3818	18.0792	15.7766	13.4740	11.1714	8.8689	6.5671	4.2716	2.0386	.3163	.00004210
8.0	31.8821	29.5795	27.2769	24.9744	22.6718	20.3692	18.0666	15.7640	13.4614	11.1589	8.8563	6.5545	4.2591	2.0269	.3106	.00003767
8.1	31.8697	29.5671	27.2645	24.9619	22.6594	20.3568	18.0542	15.7516	13.4490	11.1464	8.8439	6.5421	4.2468	2.0155	.3050	.00003370
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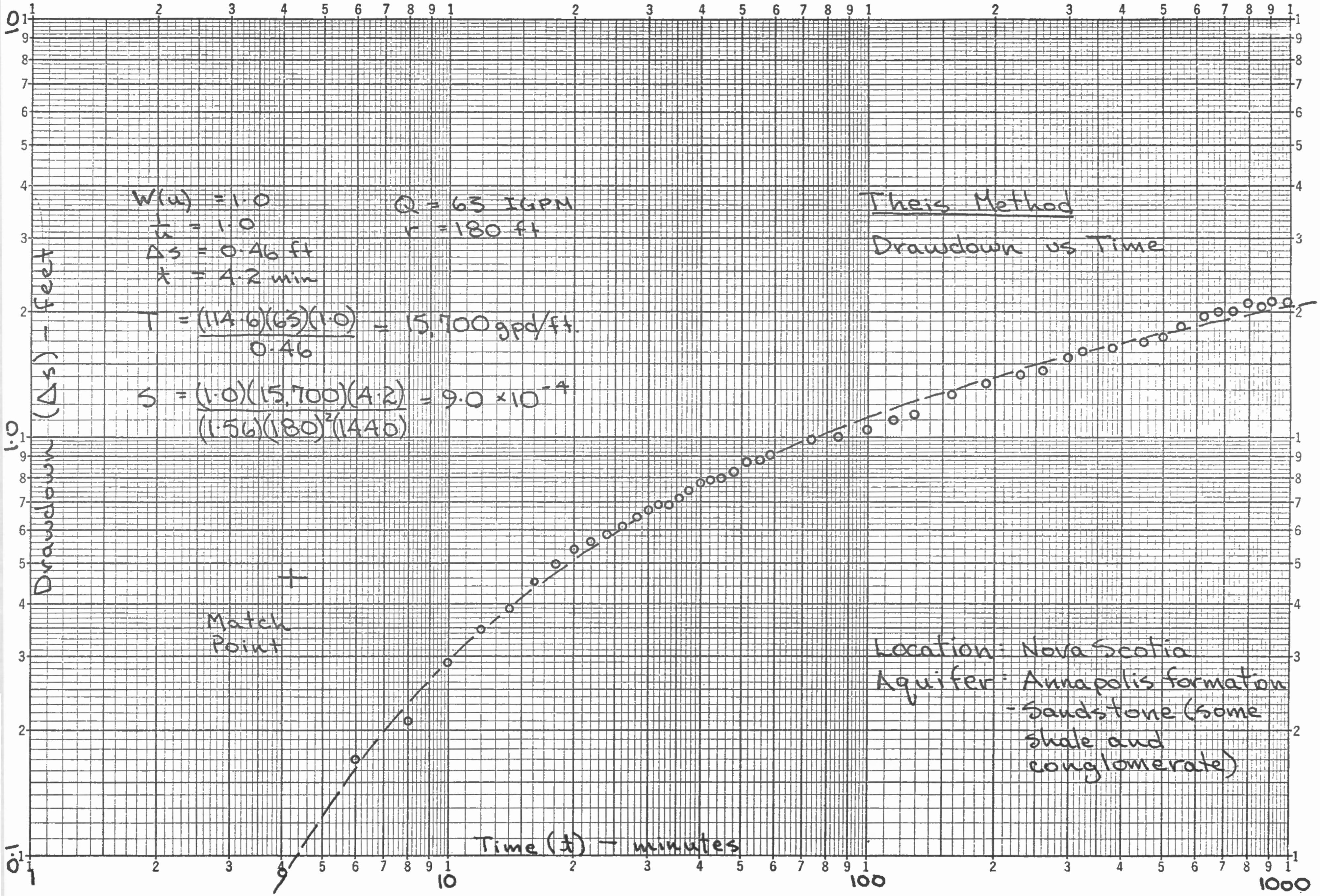
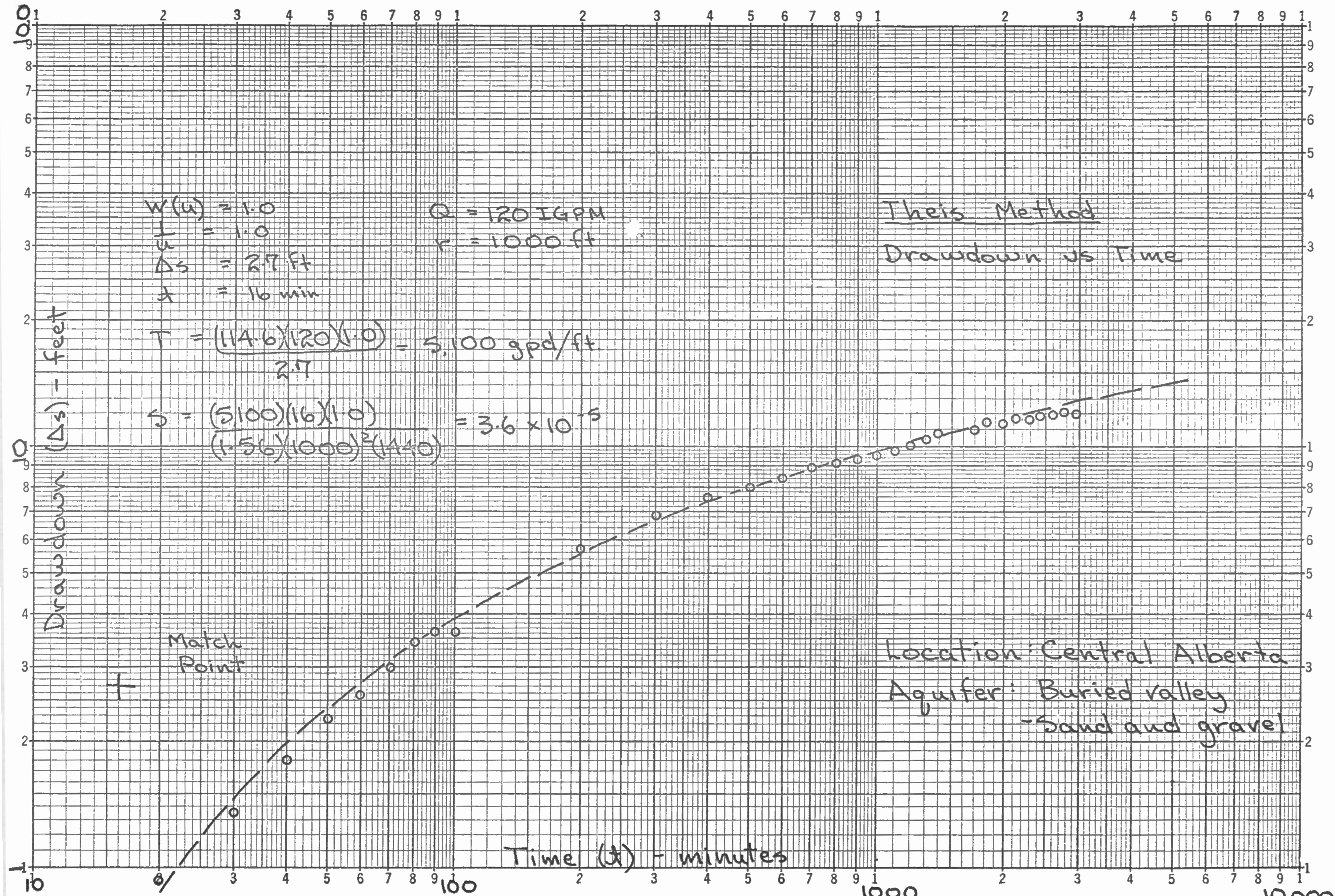
8.3	31.8453	29.5427	27.2401	24.9375	22.6350	20.3324	18.0298	15.7272	13.4246	11.1220	8.8195	6.5177	4.2226	1.9930	.2943	.00002699
8.4	31.8333	29.5307	27.2282	24.9256	22.6230	20.3204	18.0178	15.7152	13.4126	11.1101	8.8076	6.5057	4.2107	1.9820	.2891	.00002415
8.5	31.8215	29.5189	27.2163	24.9137	22.6112	20.3086	18.0060	15.7034	13.4008	11.0982	8.7957	6.4939	4.1990	1.9711	.2840	.00002162
8.6	31.8098	29.5072	27.2046	24.9020	22.5995	20.2969	17.9943	15.6917	13.3891	11.0865	8.7840	6.4822	4.1874	1.9604	.2790	.00001936
8.7	31.7982	29.4957	27.1931	24.8905	22.5879	20.2853	17.9827	15.6801	13.3776	11.0750	8.7725	6.4707	4.1759	1.9498	.2742	.00001733
8.8	31.7868	29.4842	27.1816	24.8790	22.5765	20.2739	17.9713	15.6687	13.3661	11.0635	8.7610	6.4592	4.1646	1.9393	.2694	.00001552
8.9	31.7755	29.4729	27.1703	24.8678	22.5652	20.2626	17.9600	15.6574	13.3548	11.0523	8.7497	6.4480	4.1534	1.9290	.2647	.00001390
9.0	31.7643	29.4618	27.1592	24.8566	22.5540	20.2514	17.9488	15.6462	13.3437	11.0411	8.7386	6.4368	4.1423	1.9187	.2602	.00001245
9.1	31.7533	29.4507	27.1481	24.8455	22.5429	20.2404	17.9378	15.6352	13.3326	11.0300	8.7275	6.4258	4.1313	1.9087	.2557	.00001115
9.2	31.7424	29.4398	27.1372	24.8346	22.5320	20.2294	17.9268	15.6243	13.3217	11.0191	8.7166	6.4148	4.1205	1.8987	.2513	.000009988
9.3	31.7315	29.4290	27.1264	24.8238	22.5212	20.2186	17.9160	15.6135	13.3109	11.0083	8.7058	6.4040	4.1098	1.8888	.2470	.000008948
9.4	31.7208	29.4183	27.1157	24.8131	22.5105	20.2079	17.9053	15.6028	13.3002	10.9976	8.6951	6.3934	4.0992	1.8791	.2429	.000008018
9.5	31.7103	29.4077	27.1051	24.8025	22.4999	20.1973	17.8948	15.5922	13.2896	10.9870	8.6845	6.3828	4.0887	1.8695	.2387	.000007185
9.6	31.6998	29.3972	27.0946	24.7920	22.4895	20.1869	17.8843	15.5817	13.2791	10.9765	8.6740	6.3723	4.0784	1.8599	.2347	.000006439
9.7	31.6894	29.3868	27.0843	24.7817	22.4791	20.1765	17.8739	15.5713	13.2688	10.9662	8.6637	6.3620	4.0681	1.8505	.2308	.000005771
9.8	31.6792	29.3766	27.0740	24.7714	22.4688	20.1663	17.8637	15.5611	13.2585	10.9559	8.6534	6.3517	4.0579	1.8412	.2269	.000005173
9.9	31.6690	29.3664	27.0639	24.7613	22.4587	20.1561	17.8535	15.5509	13.2483	10.9458	8.6433	6.3416	4.0479	1.8320	.2231	.000004637

(From U. S. Geological Survey Water-Supply Paper 887)


Theis Method
Drawdown vs Time
W(u) = 1.0
1/u = 1.0
Δs = 0.46 ft
t = 4.2 min
Q = 63 IGPM
r = 180 ft
T = (114.6)(63)(1.0) / 0.46 = 15,700 gpd/ft.
S = (1.0)(15,700)(4.2) / (1.56)(180)²(1440) = 9.0 × 10⁻⁴
Match Point
Location: Nova Scotia
Aquifer: Annapolis formation
- Sandstone (some shale and conglomerate)
Drawdown (Δs) — feet
Time (t) — minutes
0.1
1.0
10
1
10
100
1000



Theis Method
Drawdown vs Time
W(u) = 1.0
1/u = 1.0
Δs = 2.7 ft
t = 16 min
Q = 120 IGPM
r = 1000 ft
T = (114.6)(120)(1.0) / 2.7 = 5,100 gpd/ft
S = (5100)(16)(1.0) / (1.56)(1000)²(1440) = 3.6 × 10⁻⁵
Match Point
Location: Central Alberta
Aquifer: Buried valley
-Sand and gravel
Drawdown (Δs) - feet
Time (t) - minutes
10
100
1000
10,000

Asymptotic expression for Theis non-equilibrium formula - Jacob Method.

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
Δs	Drawdown or recovery of water level	feet	feet	metres
r	Distance from pumped well	feet	feet	metres
t	Time since pumping started or stopped	days	days	seconds
Q	Rate of discharge by pumped well	ft^3/day	g.p.m.	m^3/sec.
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft	m^2/sec.
S	Coefficient of storage	dimensionless	dimensionless	dimensionless

Equations:

Ft.-day or m.-sec. system:

$$\Delta s = \frac{Q}{4\pi T} \log_e \frac{2.25\, T\, t}{r^2 S}$$

or

$$\Delta s = \frac{2.30\, Q}{4\pi T} \log_{10} \frac{2.25\, T\, t}{r^2\, S}$$

$$T = \frac{2.30\, Q}{4\pi\, \Delta s}$$

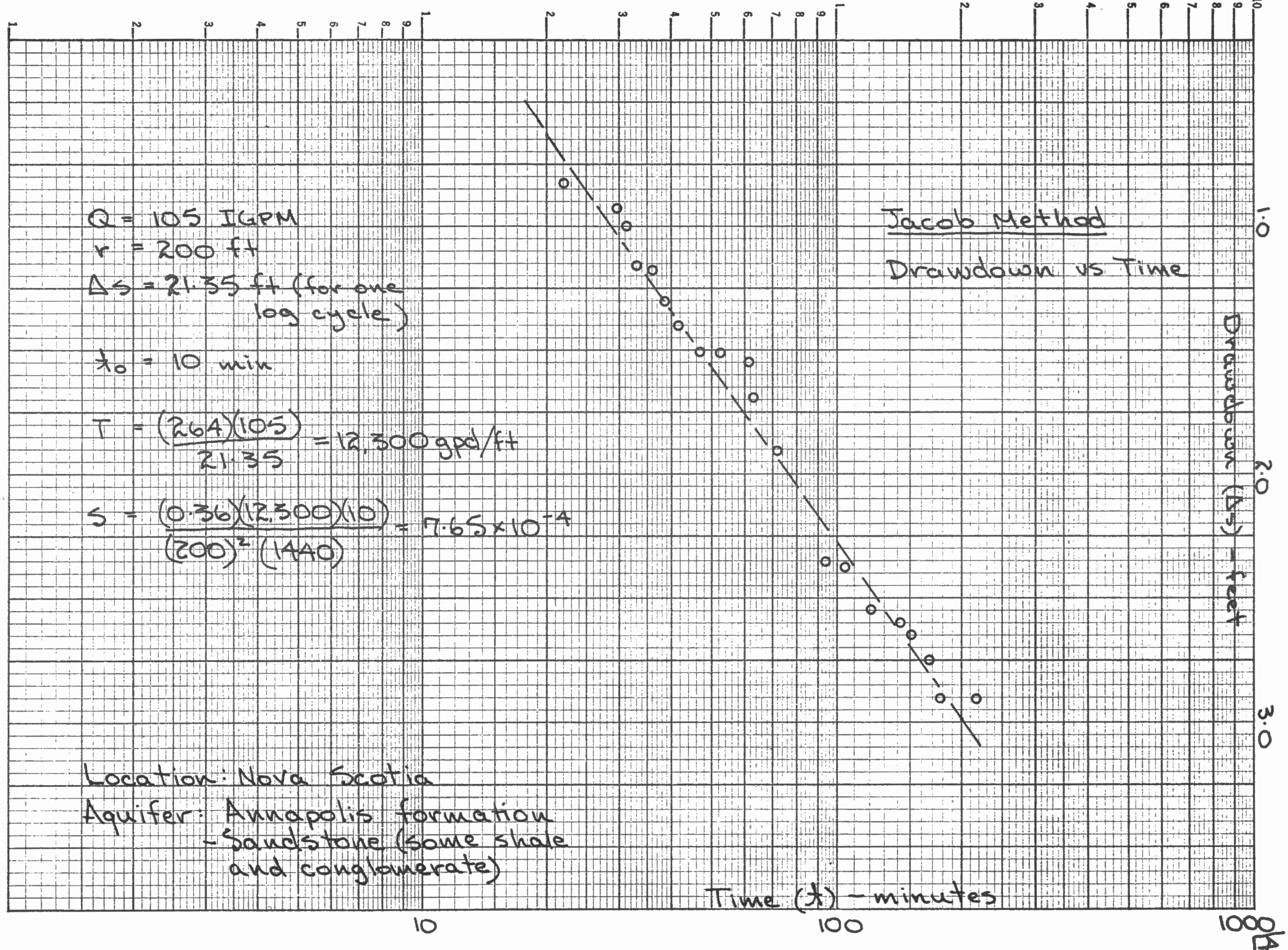
$$S = \frac{2.25\, T\, t_o}{r^2}$$

Gal.-ft.-day system:

$$\Delta s = \frac{264\, Q}{T} \log_{10} \frac{0.36\, T\, t}{r^2\, S}$$

$$T = \frac{264\, Q}{\Delta s}$$

$$S = \frac{0.36\, T\, t_o}{r^2}$$


Jacob Method
Drawdown vs Time
Q = 105 IGPM
r = 200 ft
Δs = 21.35 ft (for one log cycle)
t₀ = 10 min
T = (264)(105) / 21.35 = 12,300 gpd/ft
S = (0.36)(12,300)(10) / (200)² (1440) = 7.65×10⁻⁴
Location: Nova Scotia
Aquifer: Annapolis formation - Sandstone (some shale and conglomerate)
Time (t) - minutes
10
100
1000
Drawdown (Δs) - feet
1.0
2.0
3.0

Jacob - Theis formula

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
s_1, s_2	Drawdown or recovery of water level at time t_1, t_2 respectively	feet	feet	metres
r	Distance from pumped well	feet	feet	metres
t_o	Time intercept at s = 0	days	days	seconds
t_1, t_2	Times since pumping started and stopped	days	days	seconds
Q	Rate of discharge by pumped well	ft^3/day	g.p.m.	m^3/sec.
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft	m^2/sec.
S	Coefficient of storage	dimension-less	dimension-less	dimension-less

Equations:

Ft.-day or m.-sec. system:

$$T = \frac{Q \log_e t_2/t_1}{4\pi(s_2 - s_1)}$$

or

$$T = \frac{2.30\ Q \log_{10} t_2/t_1}{4\pi(s_2 - s_1)}$$

$$S = \frac{2.25\ T\ t_o}{r^2}$$

Gal.-ft.-day system:

$$T = \frac{264\ Q \log_{10} t_2/t_1}{s_2 - s_1}$$

$$S = \frac{0.36\ T\ t_o}{r^2}$$

Theis recovery formula

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
s^1	Residual drawdown of water level	feet	feet	metres
t	Time since pumping started	days	days	seconds
t^1	Time since pumping stopped	days	days	seconds
Q	Rate of discharge by pumped well	ft^3/day	g.p.m.	m^3/sec.
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft	m^2/sec.

Equations:

Ft.-day or m.-sec. system:

$$T = \frac{Q \log_e t/t^1}{4 \pi s^1}$$

or

$$T = \frac{2.30\ Q \log_{10} t/t^1}{4 \pi\ s^1}$$

Gal.-ft. day system:

$$T = \frac{264\ Q \log_{10} t/t^1}{s^1}$$

Leaky Artesian Aquifer
Non-steady state

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
Δs	Drawdown or recovery of water level	feet	feet	metres
r	Distance from pumped well	feet	feet	metres
t	Time since pumping started or stopped	days	days	seconds
Q	Rate of discharge by pumped well	ft^3/day	g.p.m.	$m^3/sec.$
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft	$m^2/sec.$
S	Coefficient of storage	dimension-less	dimension-less	dimension-less
P^1	Coefficient of vertical permeability of leaky bed	$ft^3/day/ft^2$	$g/day/ft^2$	$m^3/day/m^2$
m^1	Thickness of leaky bed	feet	feet	metres

Equations:

Ft.-day or m.-sec. system:

$$\Delta s = \frac{Q\,W\,(u,\ ^r/B)}{4\pi T}$$

$$u = \frac{r^2 S}{4\,T\,t}$$

$$^r/B = \frac{r}{\sqrt{T/\,(P'/m')}}$$

Gal.-ft.-day system:

$$\Delta s = \frac{114.6\,Q\,W\,(u,\ ^r/B)}{T}$$

$$u = \frac{1.56\,r^2 S}{T\,t}$$

$$^r/B = \frac{r}{\sqrt{T/\,(P'/m')}}$$

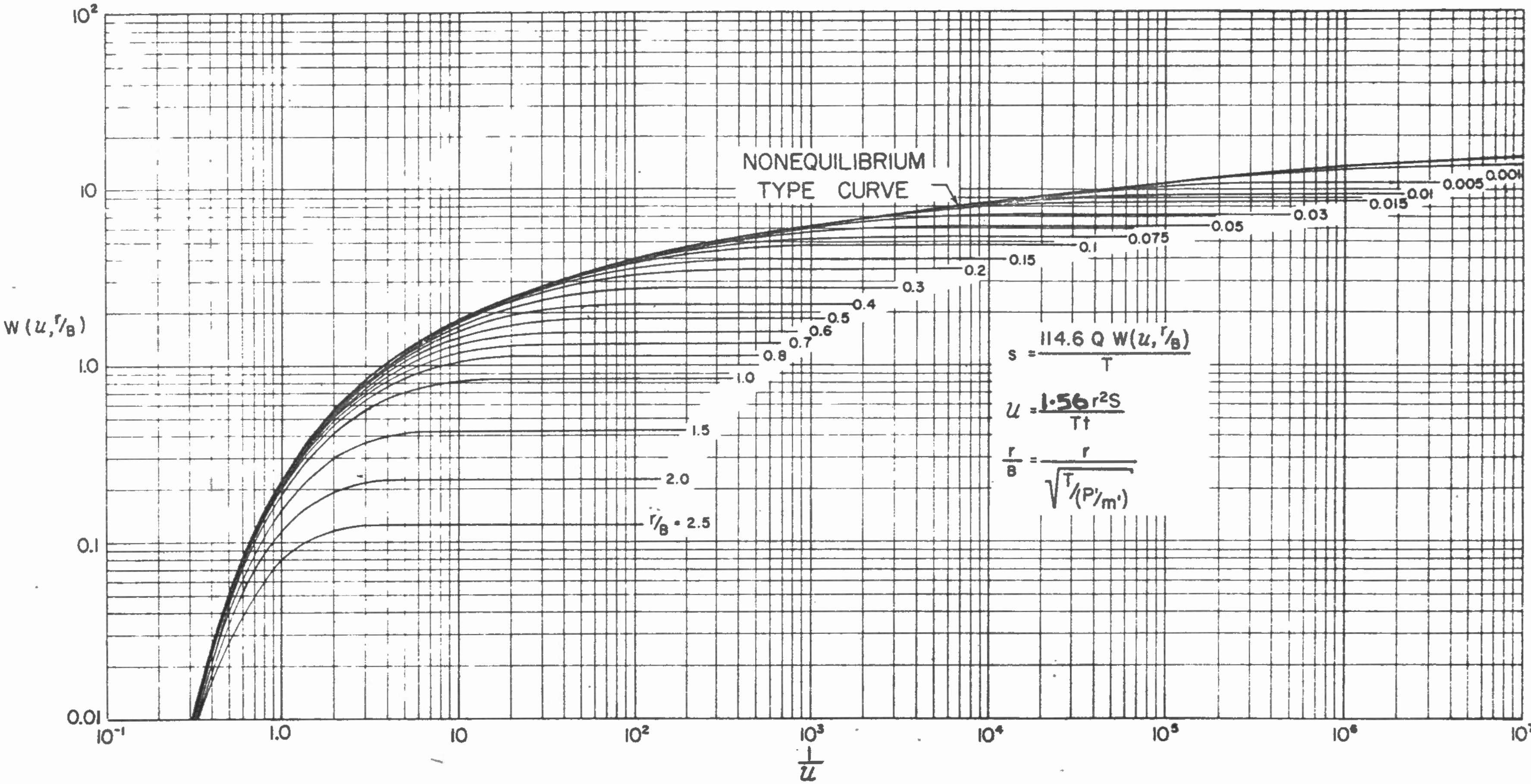
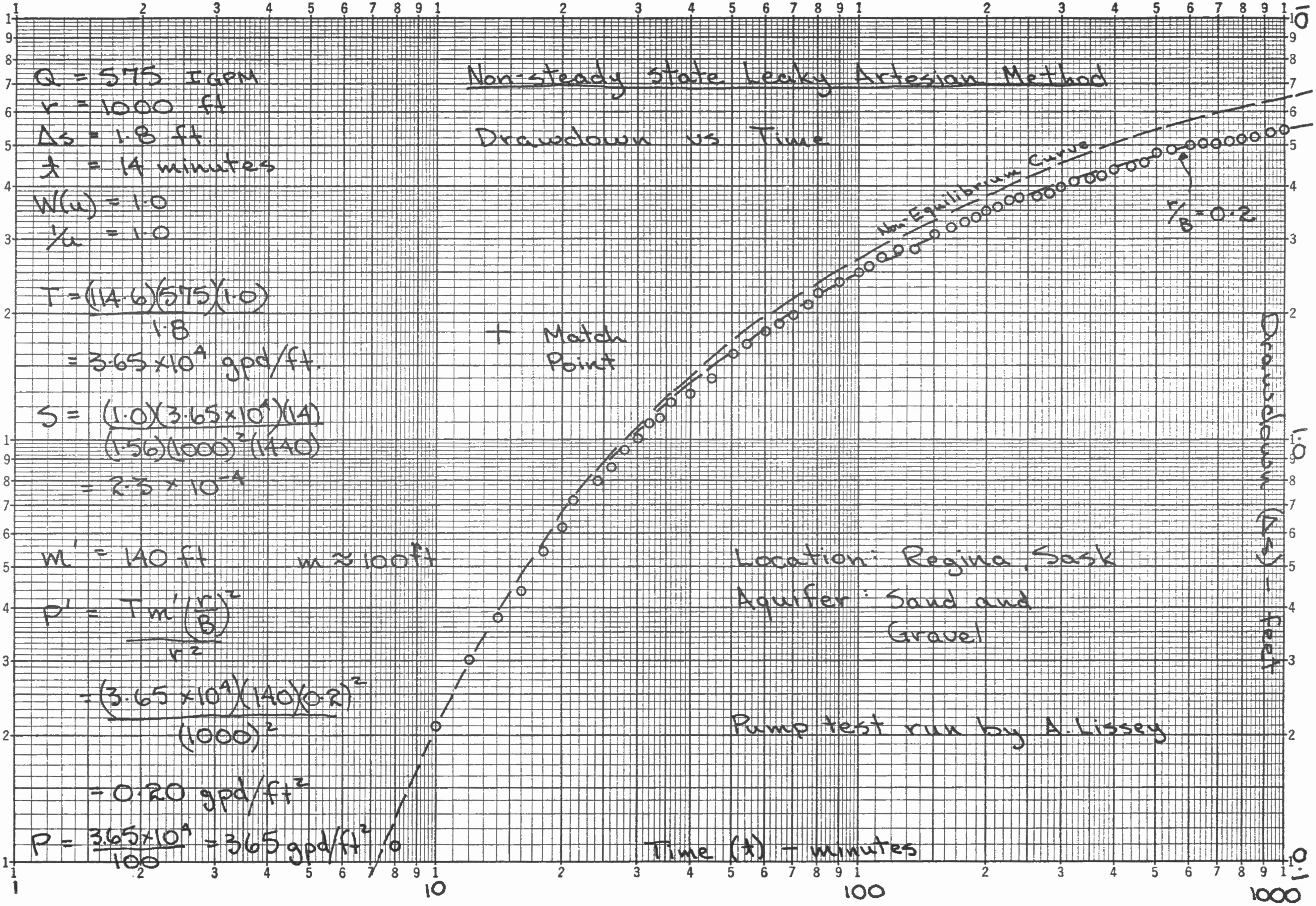


FIGURE 1 NONSTEADY-STATE LEAKY ARTESIAN TYPE CURVES

Non-steady state Leaky Artesian Method

Drawdown vs Time



$Q = 575$ IGPM

$r = 1000$ ft

$\Delta s = 1.8$ ft

$t = 14$ minutes

$W(u) = 1.0$

$1/u = 1.0$

$$T = \frac{(14.6)(575)(1.0)}{1.8} = 3.65 \times 10^4 \text{ gpd/ft.}$$

$$S = \frac{(1.0)(3.65 \times 10^4)(14)}{(1.56)(1000)^2(1440)} = 2.3 \times 10^{-4}$$

$m' = 140$ ft $\qquad m \approx 100$ ft

$$P' = \frac{T m' \left(\frac{r}{B}\right)^2}{r^2} = \frac{(3.65 \times 10^4)(140)(0.2)^2}{(1000)^2} = 0.20 \text{ gpd/ft}^2$$

$$P = \frac{3.65 \times 10^4}{100} = 365 \text{ gpd/ft}^2$$

Location: Regina, Sask

Aquifer: Sand and Gravel

Pump test run by A. Lissey

Leaky Artesian Aquifer
Steady State

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
Δs	Drawdown or recovery of water level	feet	feet	metres
r	Distance from pumped well	feet	feet	metres
t	Time since pumping started	days	days	seconds
Q	Rate of discharge by pumped well	ft^3/day	g.p.m.	$m^3/sec.$
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft	$m^2/sec.$
P'	Coefficient of vertical permeability of leaky bed	$ft^3/day/ft^2$	$g/day/ft^2$	$m^3/sec/m^2$
m'	Thickness of leaky bed	feet	feet	metres

Equations:

Ft.-day or m.-sec. system:

$$T = \frac{Q\,K_o\,(x)}{2\pi\Delta s}$$

$$X = {}^{r}/B = \frac{r}{\sqrt{T/\,(P'/m')}}$$

Gal.-ft.-day system:

$$T = \frac{229\,Q\,K_o\,(x)}{\Delta s}$$

$$X = {}^{r}/B = \frac{r}{\sqrt{T/\,(P'/m')}}$$

where

$$K_o\,(x) = -\left(y + \log_e \frac{x}{2}\right) I_o\,(x) + \left[\frac{1}{1!}\right]^2 \left[\frac{x}{2}\right]^2 + \left[\frac{1}{2!}\right]^2 \left[\frac{x}{2}\right]^4 \left(1 + \frac{1}{2}\right) + \left[\frac{1}{3!}\right]^2 \left[\frac{x}{2}\right]^6 \left(1 + \frac{1}{2} + \frac{1}{3}\right) + \ldots\ldots$$

$$I_o\,(x) = 1 + \left[\frac{x}{2}\right]^2 (1!)^2 + \left[\frac{x}{2}\right]^4 (2!)^2 + \left[\frac{x}{2}\right]^6 (3!)^2 + \ldots\ldots$$

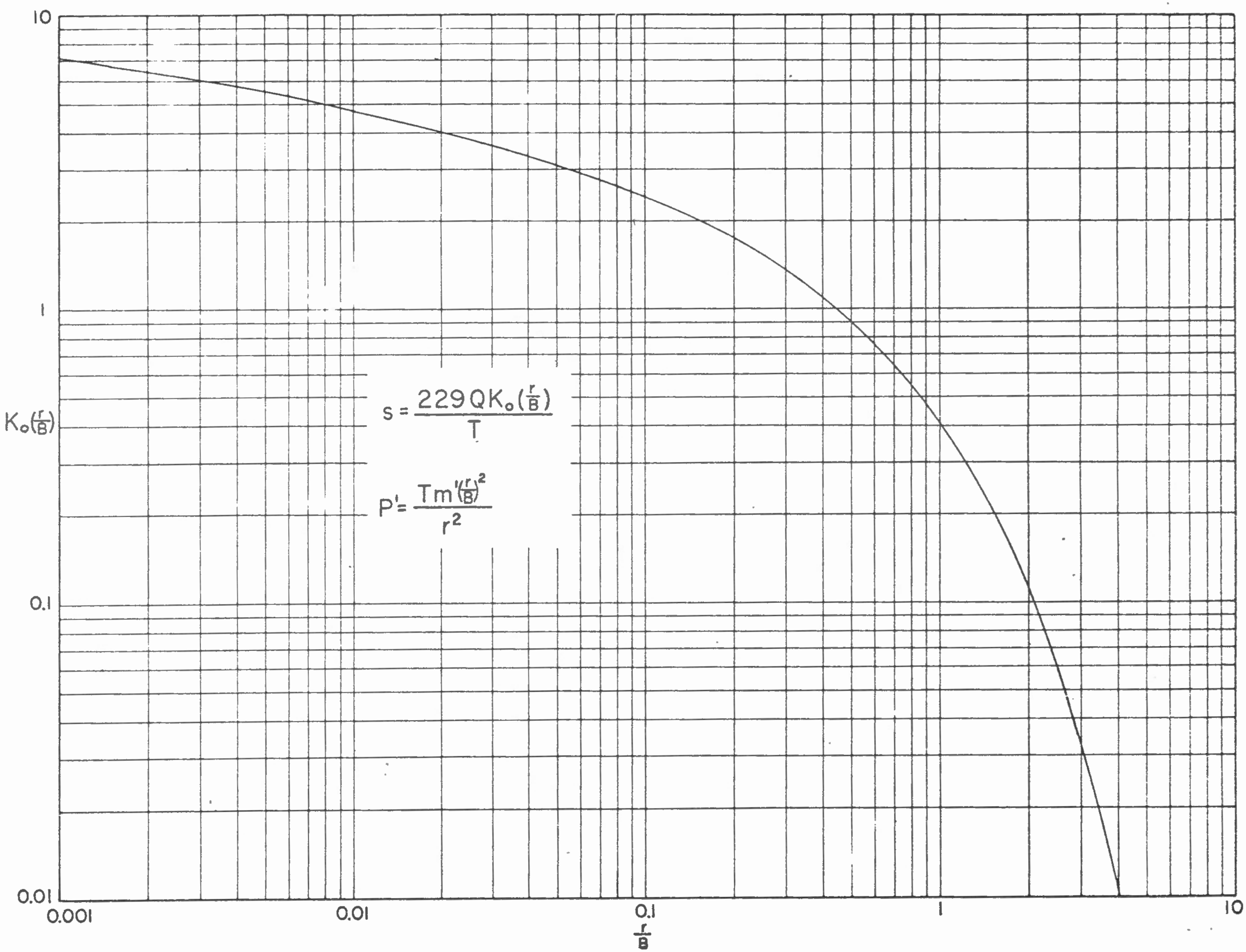


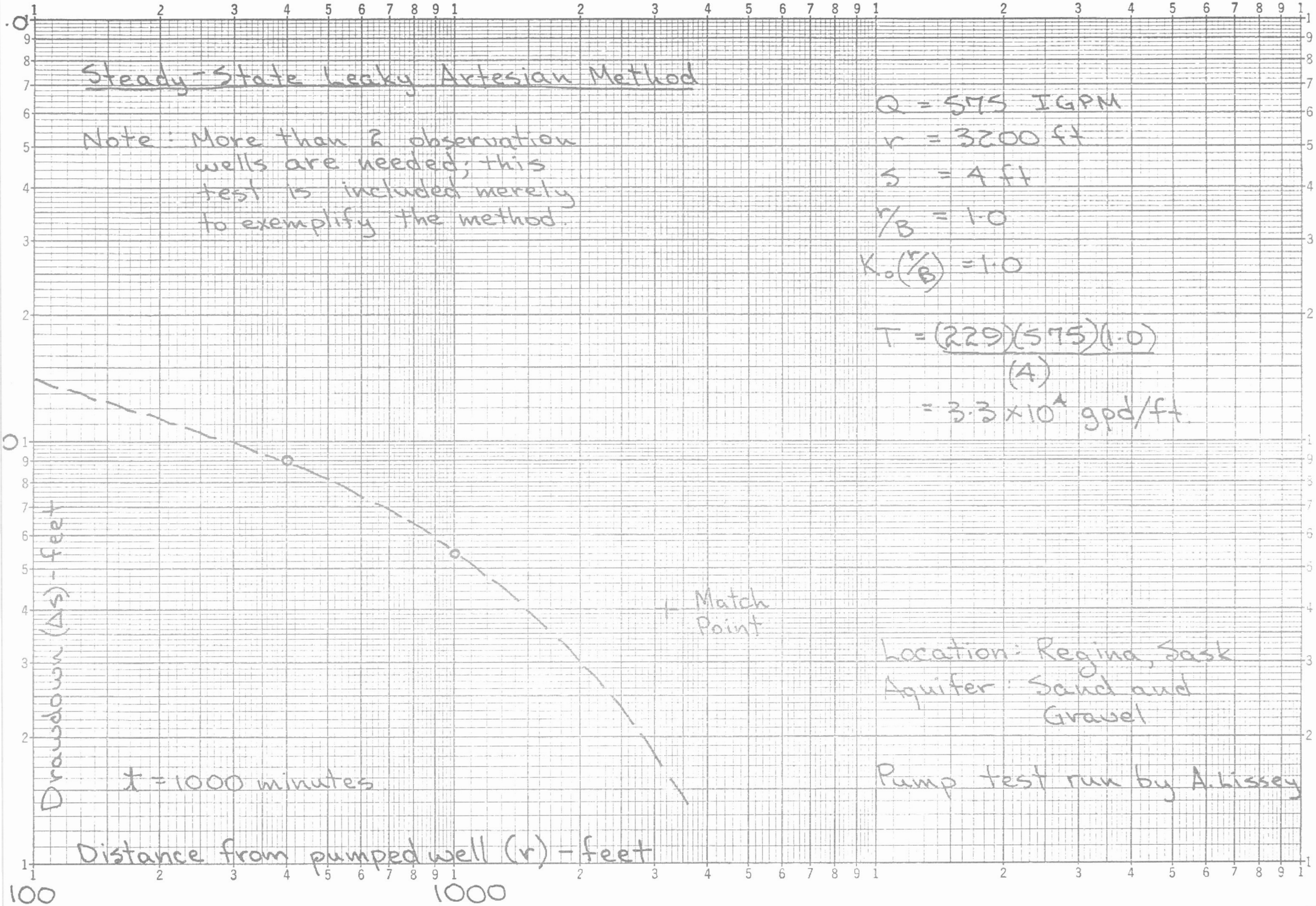
FIGURE 2 STEADY-STATE LEAKY ARTESIAN TYPE CURVE

Values of the modified Bessel function of the second kind of 0 order, $K_0(x)$ /1

x /2	$K_0(x)$ x10⁰	$K_0(x)$ $x10^{-1}$	$K_0(x)$ $x10^{-2}$	x /2	$K_0(x)$ x10⁰	$K_0(x)$ $x10^{-1}$	$K_0(x)$ $x10^{-2}$	x /2	$K_0(x)$ x10⁰	$K_0(x)$ $x10^{-1}$	$K_0(x)$ $x10^{-2}$
1.0	0.4210	2.4271	4.7212	4.0	.01116	1.1145	3.3365	7.0	.0004248	.6605	2.7798
1.1	0.3656	2.3333		4.1	.009980	1.0930		7.1	.0003817	.6501	
1.2	0.3185	2.2479		4.2	.008927	1.0721		7.2	.0003431	.6399	
1.3	0.2782	2.1695		4.3	.007988	1.0518		7.3	.0003084	.6300	
1.4	0.2437	2.0972		4.4	.007149	1.0321		7.4	.0002772	.6202	
1.5	0.2138	2.0300		4.5	.006400	1.0129		7.5	.0002492	.6106	
1.6	.1880	1.9674		4.6	.005730	0.9943		7.6	.0002240	.6012	
1.7	.1655	1.9088		4.7	.005132	.9761		7.7	.0002014	.5920	
1.8	.1459	1.8537		4.8	.004597	.9584		7.8	.0001811	.5829	
1.9	.1288	1.8018		4.9	.004119	.9412		7.9	.0001629	.5740	
2.0	.1139	1.7527	4.0285	5.0	.003691	.9244	3.1142	8.0	.0001465	.5653	2.6475
2.1	.1008	1.7062		5.1	.003308	.9081		8.1	.0001317	.5568	
2.2	.08927	1.6620		5.2	.002966	.8921		8.2	.0001185	.5484	
2.3	.07914	1.6199		5.3	.002659	.8766		8.3	.0001066	.5402	
2.4	.07022	1.5798		5.4	.002385	.8614		8.4	.00009588	.5321	
2.5	.06235	1.5415		5.5	.002139	.8466		8.5	.00008626	.5242	
2.6	.05540	1.5048		5.6	.001918	.8321		8.6	.00007761	.5165	
2.7	.04926	1.4697		5.7	.001721	.8180		8.7	.00006983	.5088	
2.8	.04382	1.4360		5.8	.001544	.8042		8.8	.00006283	.5013	
2.9	.03901	1.4036		5.9	.001386	.7907		8.9	.00005654	.4940	
3.0	.03474	1.3725	3.6235	6.0	.001244	.7775	2.9329	9.0	.00005088	.4867	2.5310
3.1	.03095	1.3425		6.1	.001117	.7646		9.1	.00004579	.4796	
3.2	.02759	1.3136		6.2	.001003	.7520		9.2	.00004121	.4727	
3.3	.02461	1.2857		6.3	.0009001	.7397		9.3	.00003710	.4658	
3.4	.02196	1.2587		6.4	.0008083	.7277		9.4	.00003339	.4591	
3.5	.01960	1.2327		6.5	.0007259	.7159		9.5	.00003006	.4524	
3.6	.01750	1.2075		6.6	.0006520	.7043		9.6	.00002706	.4459	
3.7	.01563	1.1832		6.7	.0005857	.6930		9.7	.00002436	.4396	
3.8	.01397	1.1596		6.8	.0005262	.6820		9.8	.00002193	.4333	
3.9	.01248	1.1367		6.9	.0004728	.6711		9.9	.00001975	.4271	

/1 Values of $K_0(x)$ from 0.01 to 5.00 taken from Table VII pp. 266-270, Bessel functions pt. 1, Mathematical Tables Vol. VI British Assoc. for Advancement of Sci. Cambridge University Press, 1937. Values of $K_0(x)$ from 5.1 to 9.9 taken from Table X and Table VI, pp. 314-315, A treatise on Bessel Functions and their Application to Physics by Andrew Gray, G. B. Mathews and T. M. Macrobert. Mc Millan and Co. Ltd., London 1931.

/2 When x = 1.0 the value of $K_0(x)$ is 0.4210; for x = 0.1, $K_0(x)$ is 2.4271; for x = 0.01, $K_0(x)$ is 4.7212.


Steady-State Leaky Artesian Method
Note: More than 2 observation wells are needed; this test is included merely to exemplify the method.
Q = 575 IGPM
r = 3200 ft
s = 4 ft
r/B = 1.0
K₀(r/B) = 1.0
T = (229)(575)(1.0) / (4)
= 3.3 x 10⁴ gpd/ft
Match Point
Location: Regina, Sask
Aquifer: Sand and Gravel
Pump test run by A. Lissey
t = 1000 minutes
Drawdown (Δs) - feet
Distance from pumped well (r) - feet
.01
0.1
100
1000

Recharge at uniform rate and steady radial flow.

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
Δs	Drawdown of water level	feet	feet	metres
r	Distance from pumped well	feet	feet	metres
Q	Rate of discharge by pumped well	ft^3/day	g.p.m.	$m^3/sec.$
W	Effective average rate of recharge	ft/day	in/yr.	m/sec.
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft.	$m^2/sec.$

Equations:

Ft.-day or m.-sec. system:

$$T = \frac{Q\,R(z)}{4\pi\,\Delta s}$$

$$Z = \frac{\pi r^2 W}{Q}$$

Gal.-ft.-day system:

$$T = \frac{114.6\,Q\,R(z)}{\Delta s}$$

$$Z = \frac{r^2 W}{320{,}000\,Q}$$

where $R(z) = (Z - \log_e Z - 1)$

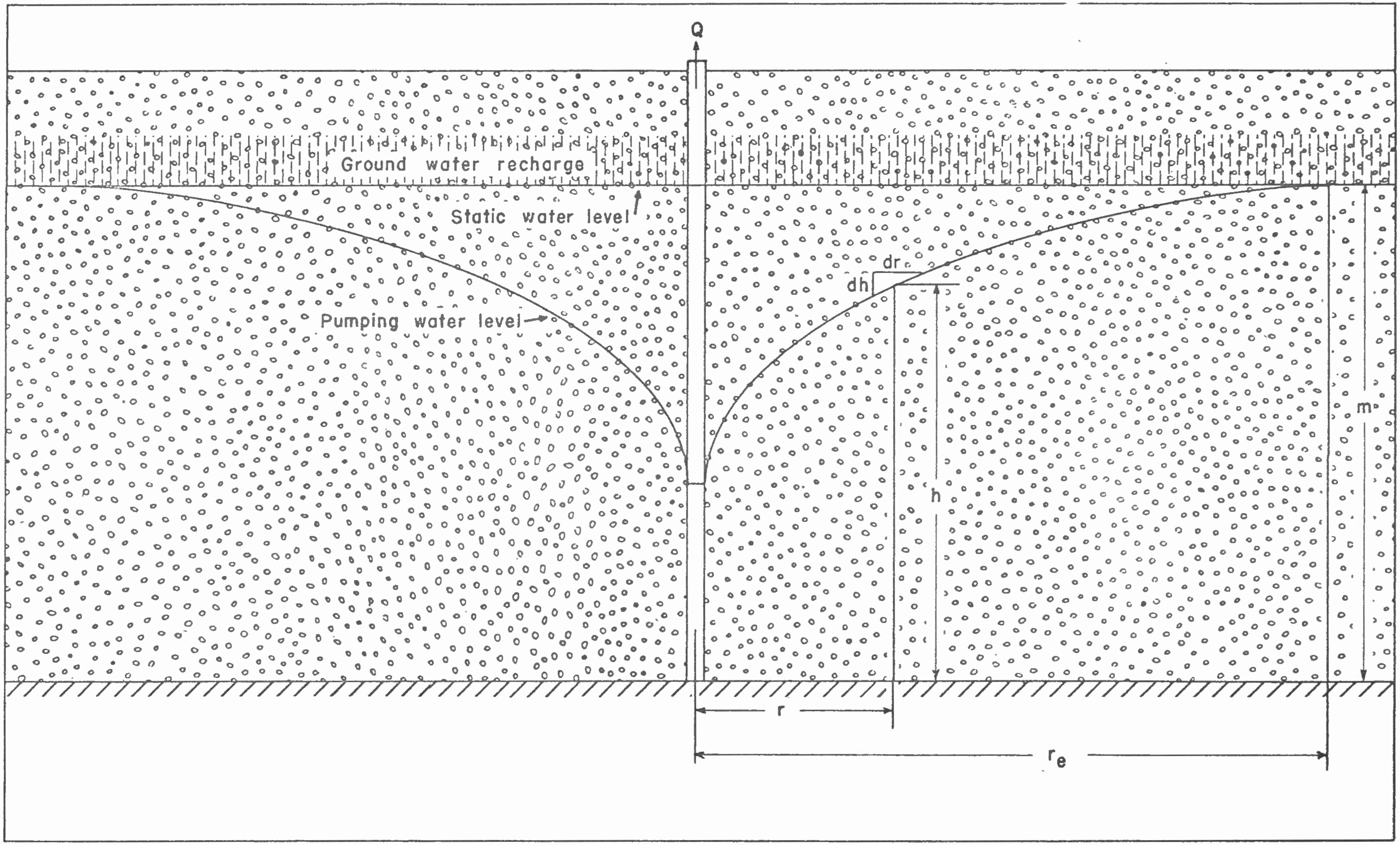
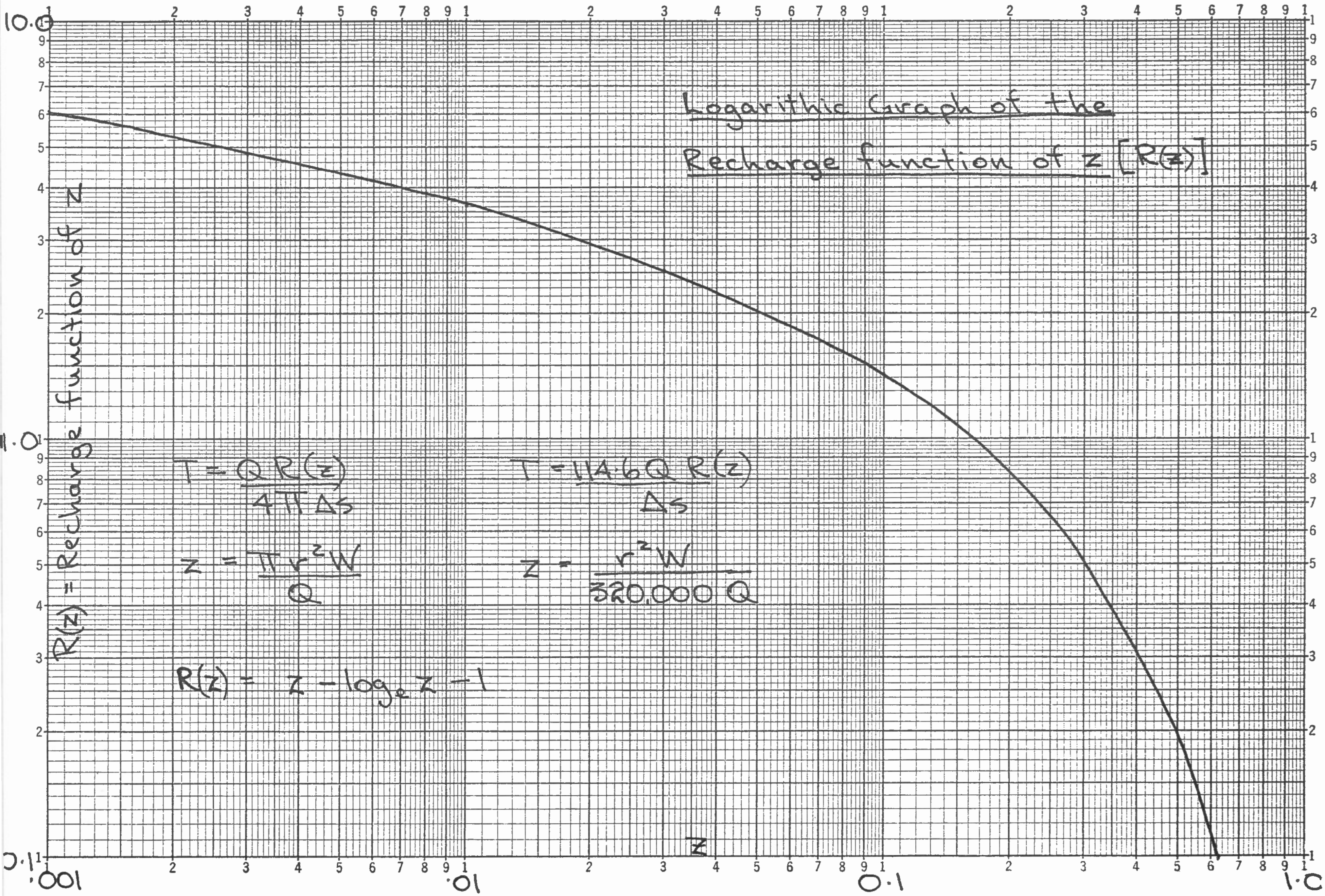


Figure Section showing steady radial flow toward a well of constant discharge in a water-table aquifer that is uniformly recharged at a constant rate.

Table . - Values of the recharge function R(z)

$$R(z) = (z - \log_e z - 1)$$

z	$\log_{10} z$	$\log_e z$	R(z)
1.0	0	0	0
0.8	9.9031 - 10 = -0.0969	-0.2232	0.0232
0.6	9.7782 - 10 = - .2218	- .5108	.1108
0.4	9.6021 - 10 = - .3979	- .9164	.3164
0.3	9.4771 - 10 = - .5229	-1.2042	.5042
0.2	9.3010 - 10 = - .6990	-1.6098	.8098
0.1	9.0000 - 10 = -1.0000	-2.3030	1.4030
0.08	8.9031 - 10 = -1.0969	-2.5262	1.6062
0.06	8.7782 - 10 = -1.2218	-2.8138	1.8738
0.04	8.6021 - 10 = -1.3979	-3.2194	2.2594
0.03	8.4771 - 10 = -1.5229	-3.5072	2.5372
0.02	8.3010 - 10 = -1.6990	-3.9128	2.9328
0.01	8.0000 - 10 = -2.0000	-4.6060	3.6160
0.008	7.9031 - 10 = -2.0969	-4.8292	3.8372
0.006	7.7782 - 10 = -2.2218	-5.1168	4.1228
0.004	7.6021 - 10 = -2.3979	-5.5224	4.5264
0.003	7.4771 - 10 = -2.5229	-5.8102	4.8132
0.002	7.3010 - 10 = -2.6990	-6.2158	5.2178
0.001	7.0000 - 10 = -3.0000	-6.9090	5.9100


Logarithic Graph of the
Recharge function of z [R(z)]
R(z) = Recharge function of z
z
10.0
1.0
0.1
·001
·01
0·1
1·0
T = QR(z) / 4πΔs
z = πr²W / Q
R(z) = z − logₑz − 1
T = 114·6Q R(z) / Δs
z = r²W / 320,000 Q

Drain formulas
Constant Discharge

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
Δs	Drawdown or recovery of water level	feet	feet	metres
x	Distance from drain	feet	feet	metres
t	Time since draining started or stopped	days	days	seconds
Q	Net gain or loss per unit length of drain	ft^3/day/ft	g.p.d./ft.	m^3/sec/ft
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft.	m^2/sec.
S	Coefficient of storage	dimension-less	dimension-less	dimension-less

Equations:

Ft.-day or m.-sec. system:

$$T = \frac{Q \times D_q(u)}{2\Delta s}$$

$$u^2 = \frac{x^2 S}{4Tt}$$

Gal.-ft.-day system:

$$T = \frac{Q \times D_q(u)}{2\Delta s}$$

$$u^2 = \frac{1.56\, x^2 S}{T t}$$

$$\text{where } D_q(u) = \left[\frac{e^{-u^2}}{u\sqrt{\pi}} - 1 + \frac{2}{\sqrt{\pi}}\int_0^u e^{-u^2}\,du\right]$$

Table .--Values of D(u), u, and u² for the constant-discharge drain function

$$D(u) = \left[\frac{e^{-u^2}}{u\sqrt{\pi}} - 1 + \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2}\, du\right]$$

u	u^2	D(u)
0.0500	0.0025	10.32
0.0600	0.0036	8.468
0.0700	0.0049	7.109
0.0800	0.0064	6.130
0.0900	0.0081	5.331
0.1000	0.010	4.714
0.1140	0.013	4.008
0.1265	0.016	3.532
0.1414	0.020	3.079
0.1581	0.025	2.657
0.1732	0.030	2.354
0.1871	0.035	2.109
0.2000	0.040	1.943
0.2236	0.050	1.658
0.2449	0.060	1.441
0.2646	0.070	1.282
0.3000	0.090	1.049
0.3317	0.110	0.8810
0.3605	0.130	0.7598
0.4000	0.160	0.6284
0.4359	0.190	0.5324
0.4796	0.230	0.4384
0.5291	0.280	0.3517
0.5745	0.330	0.2895
0.6164	0.380	0.2434
0.6633	0.440	0.2008
0.7071	0.500	0.1837
0.7616	0.580	0.1345
0.8124	0.660	0.1094
0.8718	0.760	0.0864
0.9486	0.900	0.0623
1.0000	1.000	0.0507

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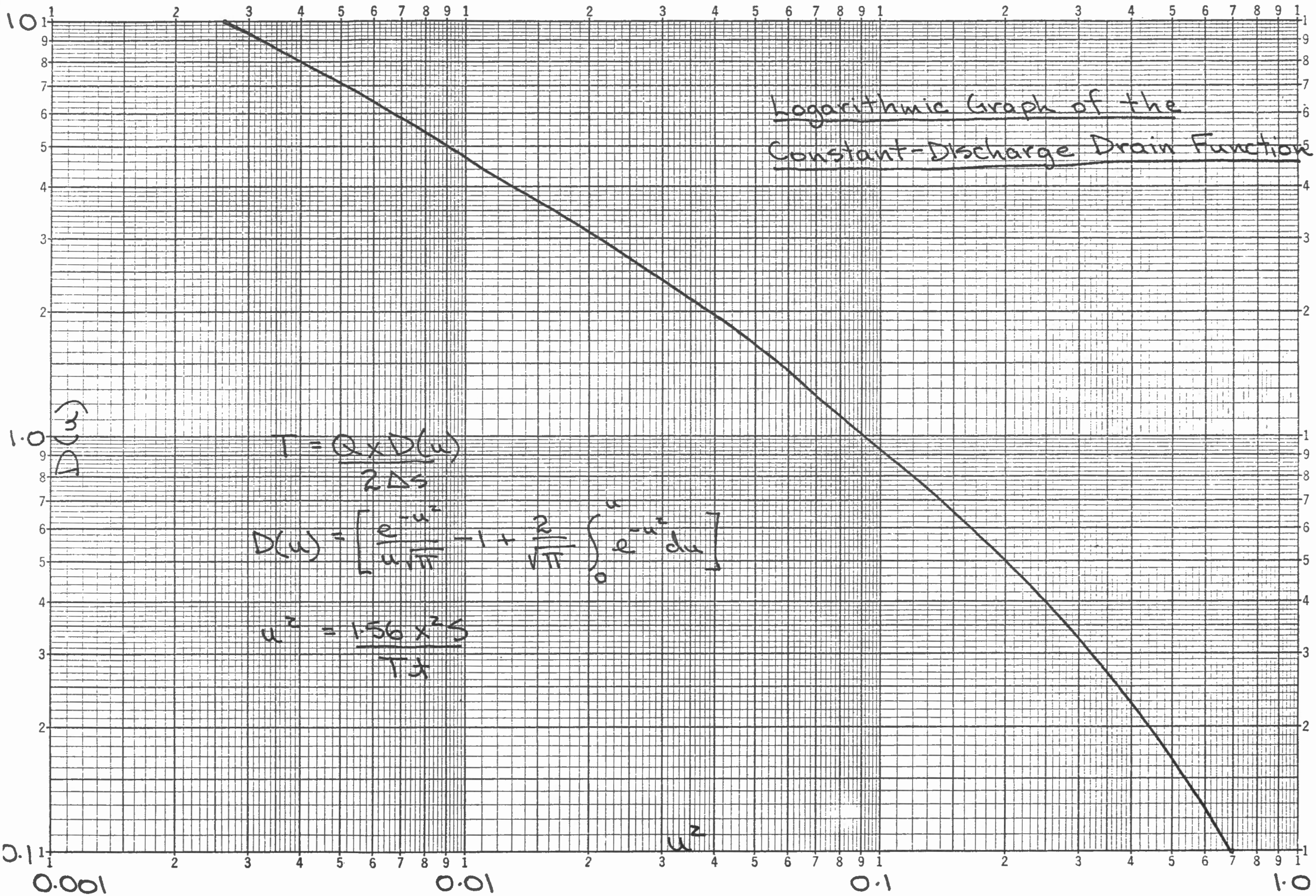
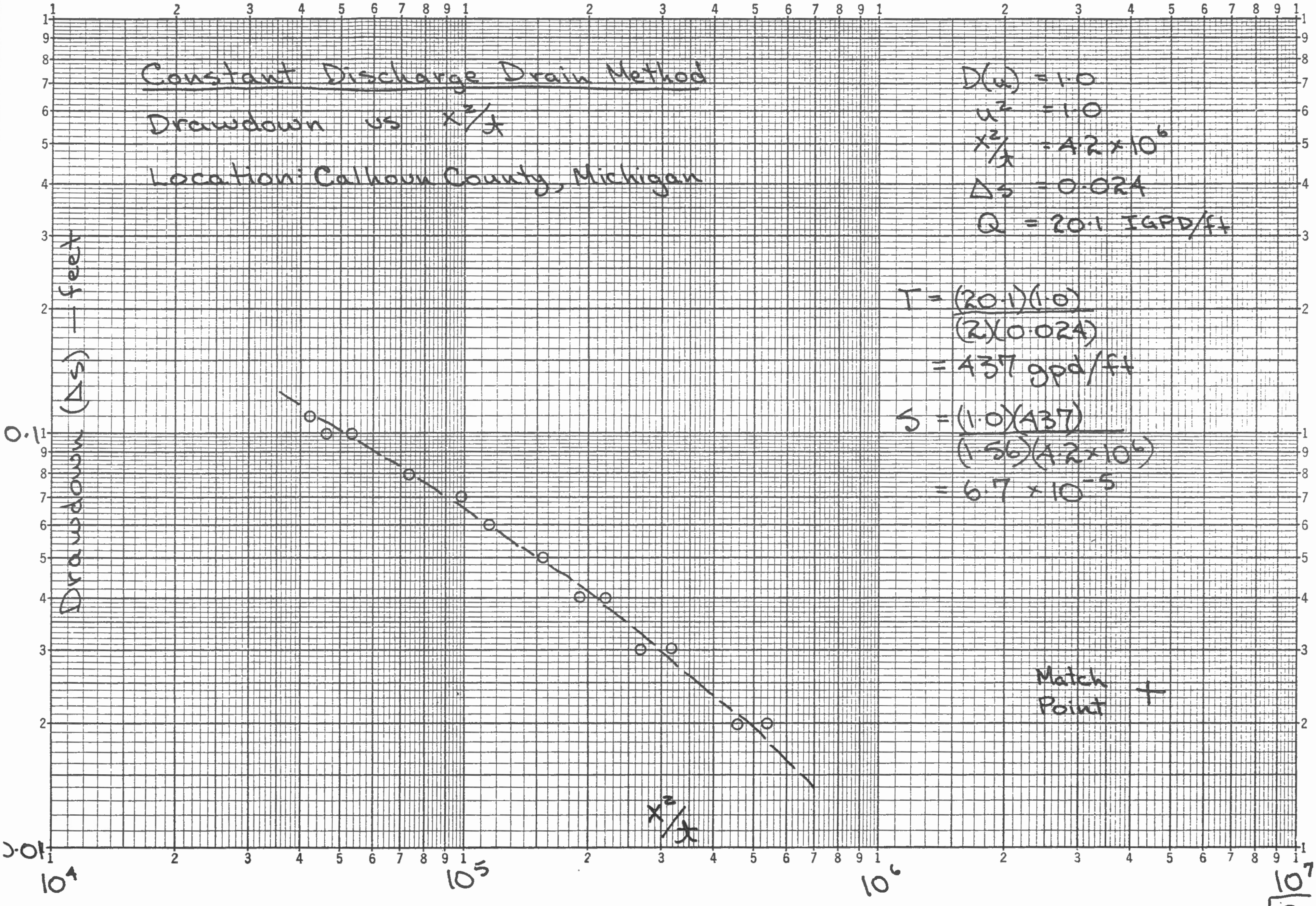

Logarithmic Graph of the
Constant-Discharge Drain Function
$D(u)$
u^2
0.001
0.01
0.1
1.0
10
1.0
0.1
$T = \frac{Q \times D(u)}{2 \Delta s}$
$D(u) = \left[\frac{e^{-u^2}}{u\sqrt{\pi}} - 1 + \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du \right]$
$u^2 = \frac{1.56\, x^2 S}{T t}$



$D(u) = 1.0$

$u^2 = 1.0$

$x^2/t = 4.2 \times 10^6$

$\Delta s = 0.024$

$Q = 20.1$ IGPD/ft

$$T = \frac{(20.1)(1.0)}{(2)(0.024)} = 437 \text{ gpd/ft}$$

$$S = \frac{(1.0)(437)}{(1.56)(4.2 \times 10^6)} = 6.7 \times 10^{-5}$$

Drain formulas
Constant Head.

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
Δs	Drawdown or recovery of water level	feet	feet	metres
So	Change in drain or stream stage	feet	feet	metres
x	Distance from drain or stream	feet	feet	metres
t	Time since stage was changed at drain or stream	days	days	seconds
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft	m^2/sec.
S	Coefficient of storage	dimension-less	dimension-less	dimension-less

Equations:

Ft.-day or m.-sec. system:

$$\Delta s = S_o \, D_d \, (u)$$

$$u^2 = \frac{x^2 S}{4 T t}$$

Gal.-ft.-day system:

$$\Delta s = S_o \, D_d \, (u)$$

$$u^2 = \frac{1.56 \, x^2 S}{T t}$$

where $D_d \, (u) = \left[1 - \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du \right]$

Cyclic formulas
Amplitude - ratio method

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
S_r	Range or double-amplitude of water level fluctuation in observation well	feet	feet	metres
S_o	Half-range or amplitude of fluctuation in tidal body	feet	feet	metres
X	Distance from tidal water	feet	feet	metres
t_o	Period of tidal motion	days	days	seconds
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft	m^2/sec.
S	Coefficient of storage	dimension-less	dimension-less	dimension-less

Equations:

Ft.-day or m.-sec. system:

$$T = \frac{0.594\ \overline{\Delta x}^{2}\ S}{t_o}$$

Gal.-ft.-day system:

$$T = \frac{3.72\ \overline{\Delta x}^{2}\ S}{t_o}$$

where $$x = \frac{\log_{10}(S_r/2S_o)}{x_1}\,1 - \frac{\log_{10}(S_r/2S_o)}{x_2}\,2$$

Cyclic formulas
Time-lag method

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
x	Distance from tidal water	feet	feet	metres
t_o	Period of tidal motion	days	days	seconds
t_1	Time lag in water level response of observation well	days	days	seconds
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft	m^2/sec.
S	Coefficient of storage	dimension-less	dimension-less	dimension-

Equations:

Ft.-day or m.-sec. system:

$$T = \frac{t_o}{4\pi} \left[\frac{x}{t_1}\right]^2 S$$

Gal.-ft.-day system:

$$T = 0.50\, t_o \left\{\frac{x}{t_1}\right\}^2 S$$

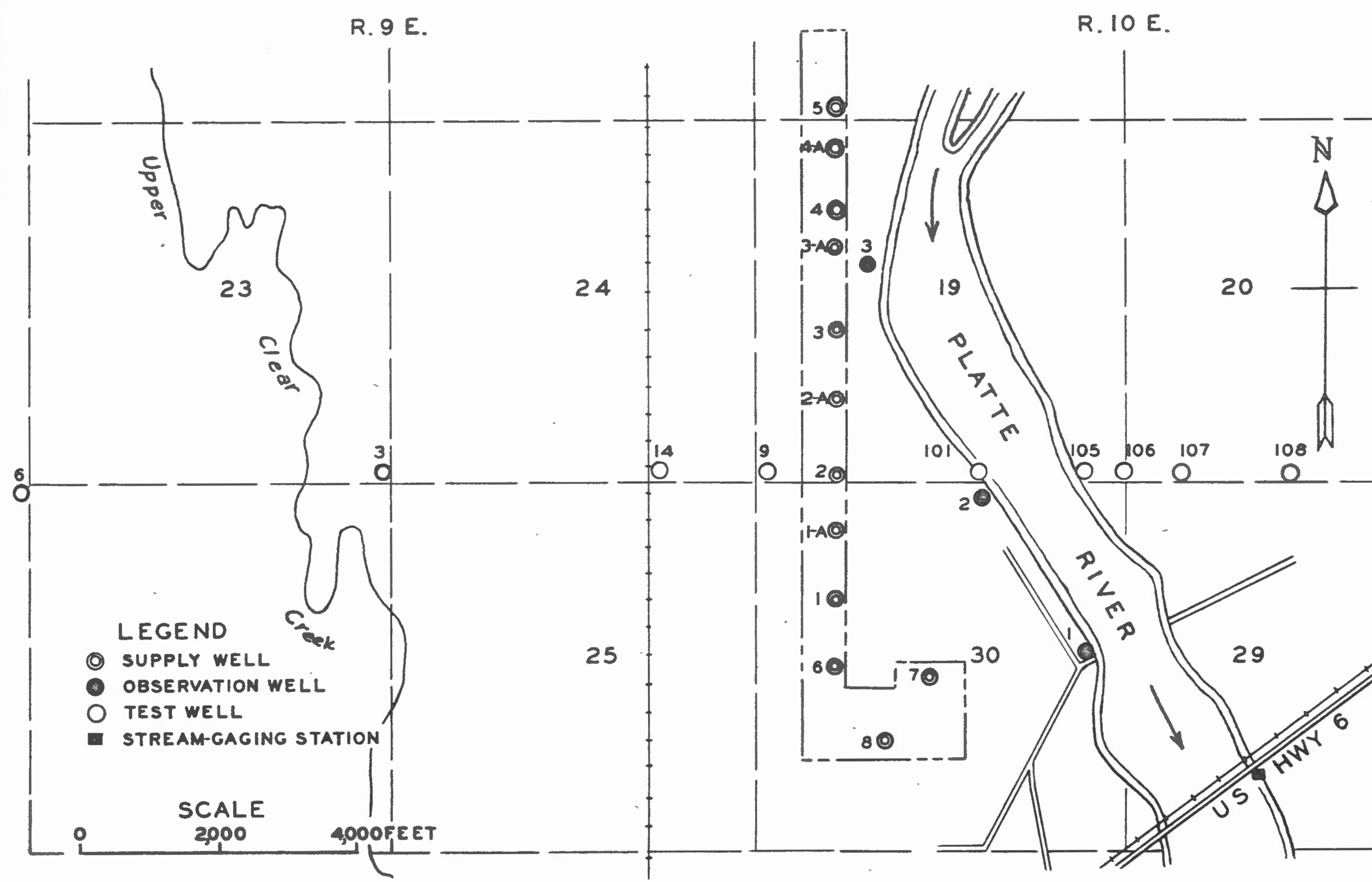


FIGURE 1.-MAP SHOWING A PORTION OF THE CITY OF LINCOLN WELL STATION AT ASHLAND, NEBR.

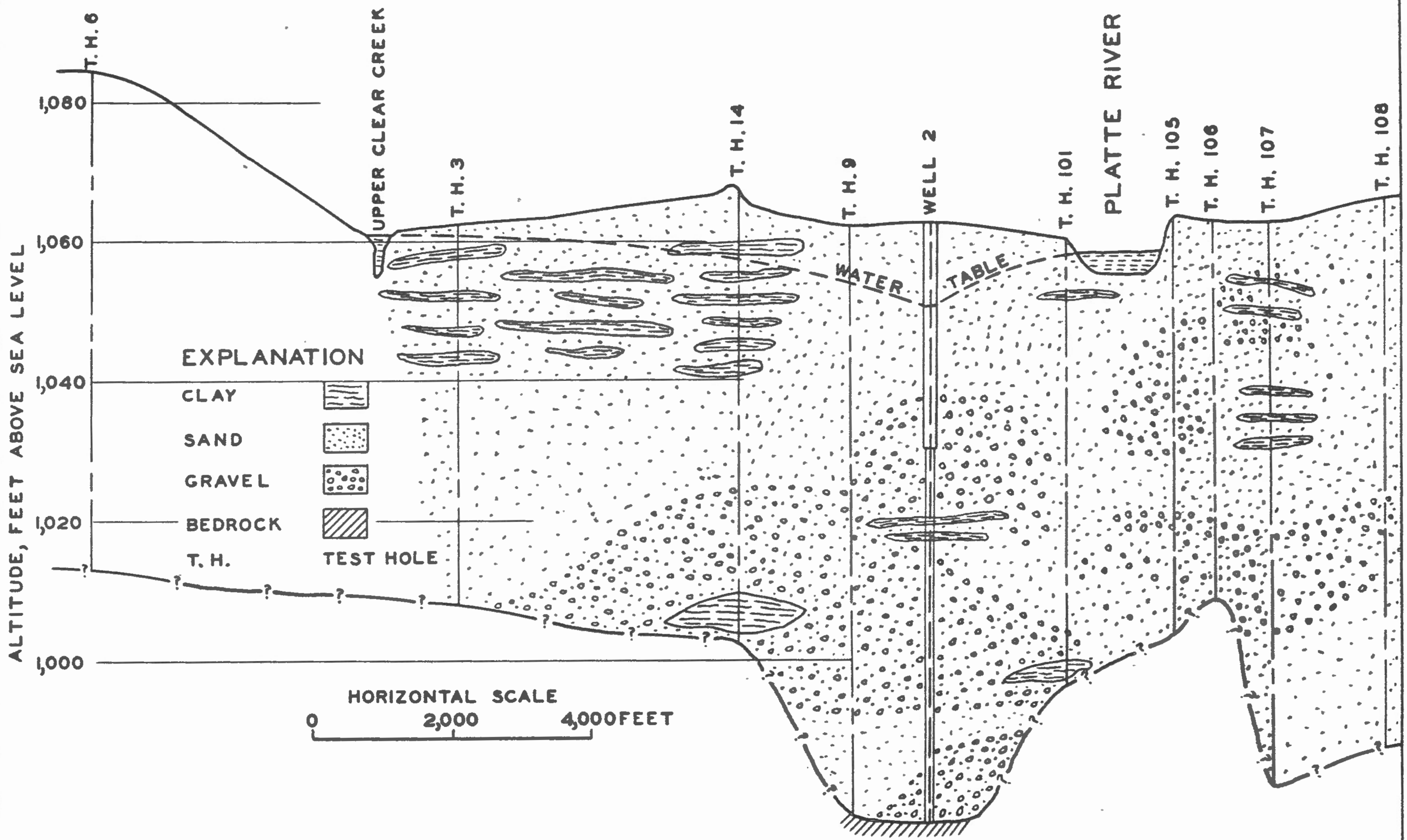


FIGURE 2.- GENERALIZED GEOLOGIC SECTION FROM WEST TO EAST THROUGH SUPPLY WELL 2 OF THE CITY OF LINCOLN AT THE ASHLAND, NEBR. WELL STATION.

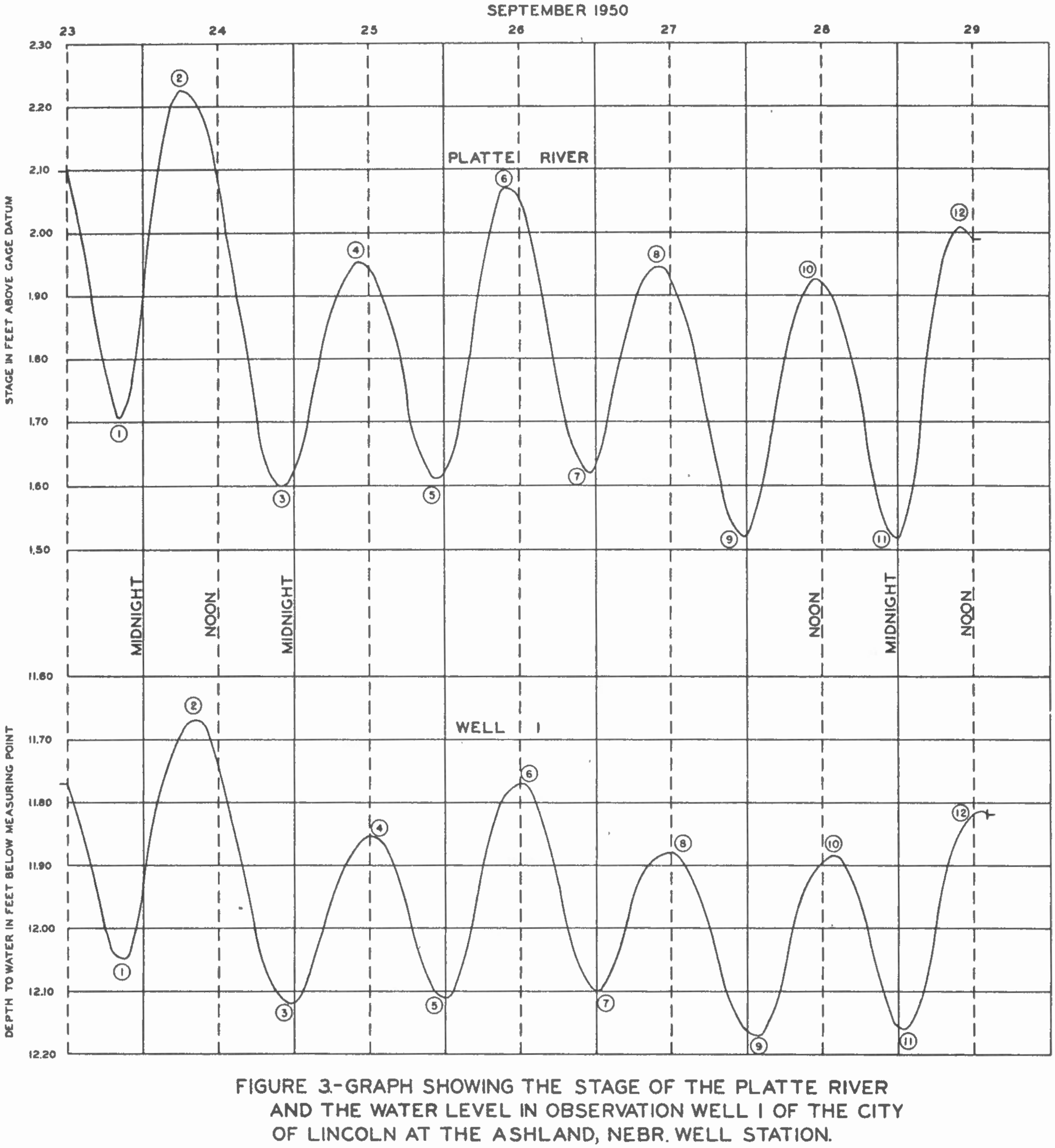


FIGURE 3.-GRAPH SHOWING THE STAGE OF THE PLATTE RIVER AND THE WATER LEVEL IN OBSERVATION WELL 1 OF THE CITY OF LINCOLN AT THE ASHLAND, NEBR. WELL STATION.

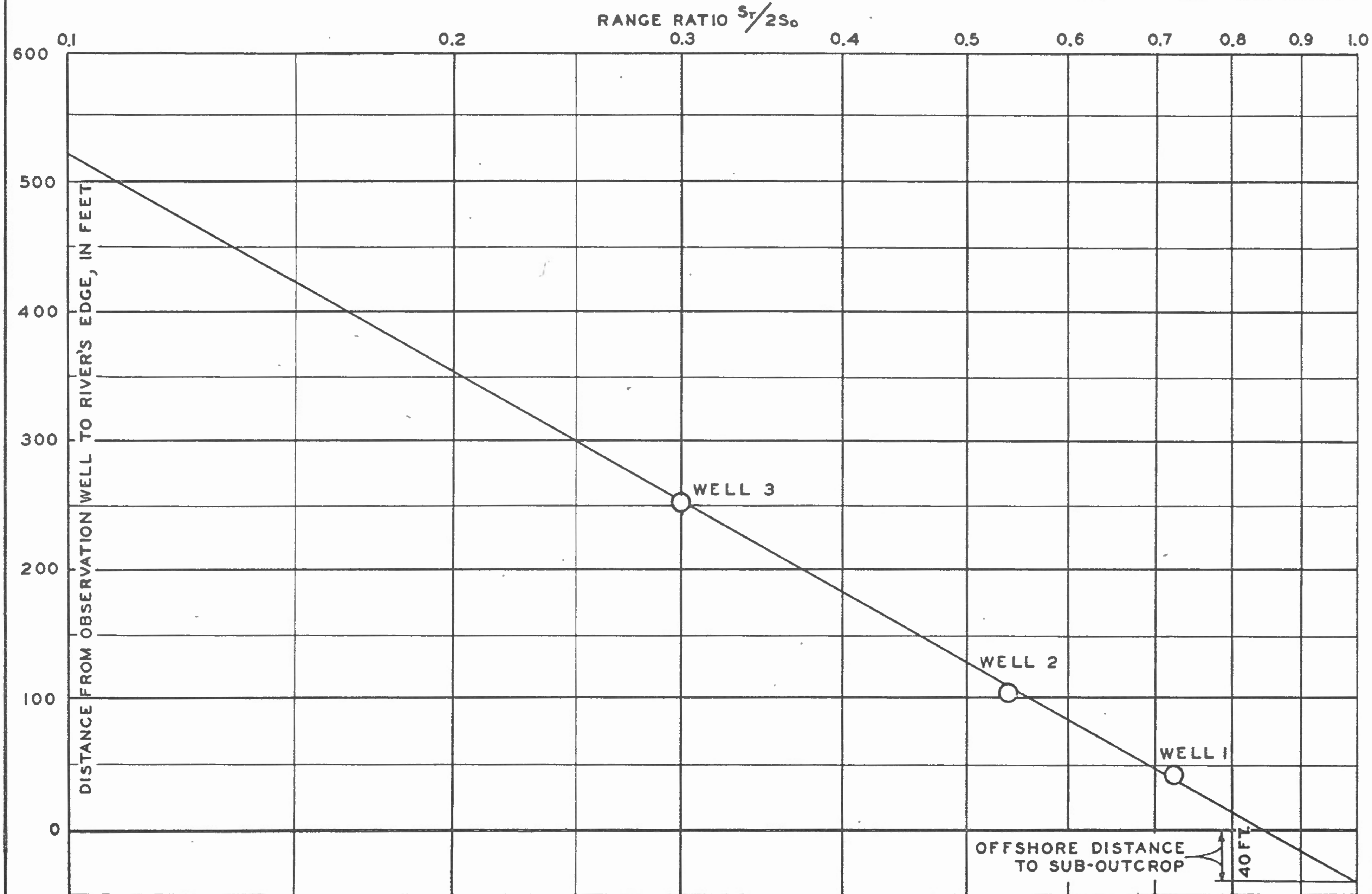


FIGURE 4.-SEMILOGARITHMIC PLOT OF THE RATIO OF GROUND-WATER STAGE TO STREAM STAGE VS. THE DISTANCE OF THE OBSERVATION WELL FROM THE CYCLIC STREAM.

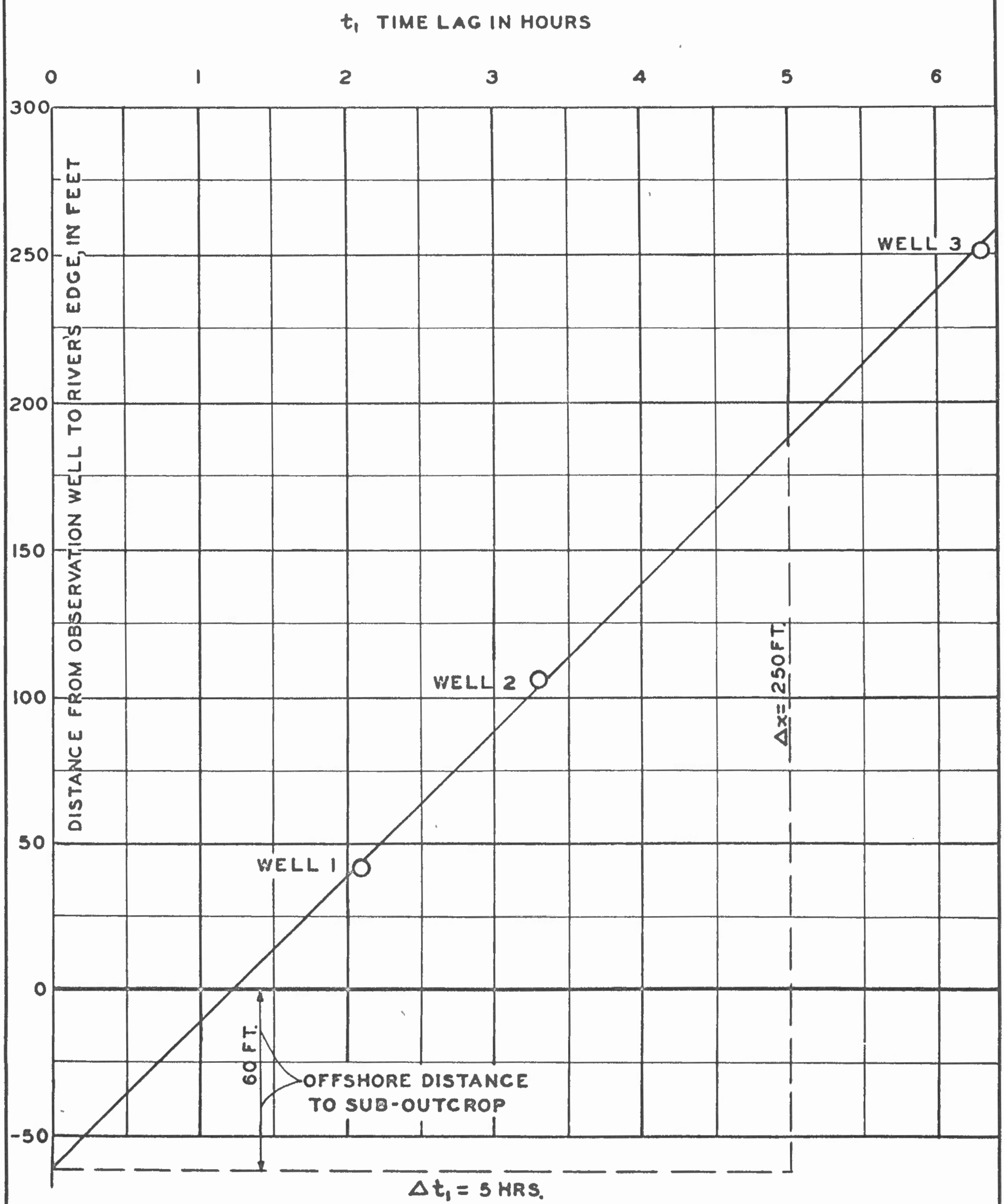


FIGURE 5.- PLOT OF TIME LAG VS. DISTANCE OF OBSERVATION WELL FROM SUB-OUTCROP.

Piezometric Parabola

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
h_o	Altitude of water level above stream datum	feet	feet	metres
x	Distance from well to stream	feet	feet	metres
a	Distance from stream to ground-water divide	feet	feet	metres
qb	Ground-water discharge	$ft^3/day/ft^2$	$c.f.s./mi^2$	$m^3/sec/m^2$
W	Effective average ground-water recharge	ft/day	in/yr	m/sec.
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft	$m^2/sec.$

Equations:

Ft.-day or m.-sec. system:

$$\frac{T}{qb} = \left(\frac{x}{h_o}\right) a - \left(\frac{x^2}{2\,h_o}\right)$$

$$\frac{T}{W} = \left(\frac{x}{h_o}\right) a - \left(\frac{x^2}{2\,h_o}\right)$$

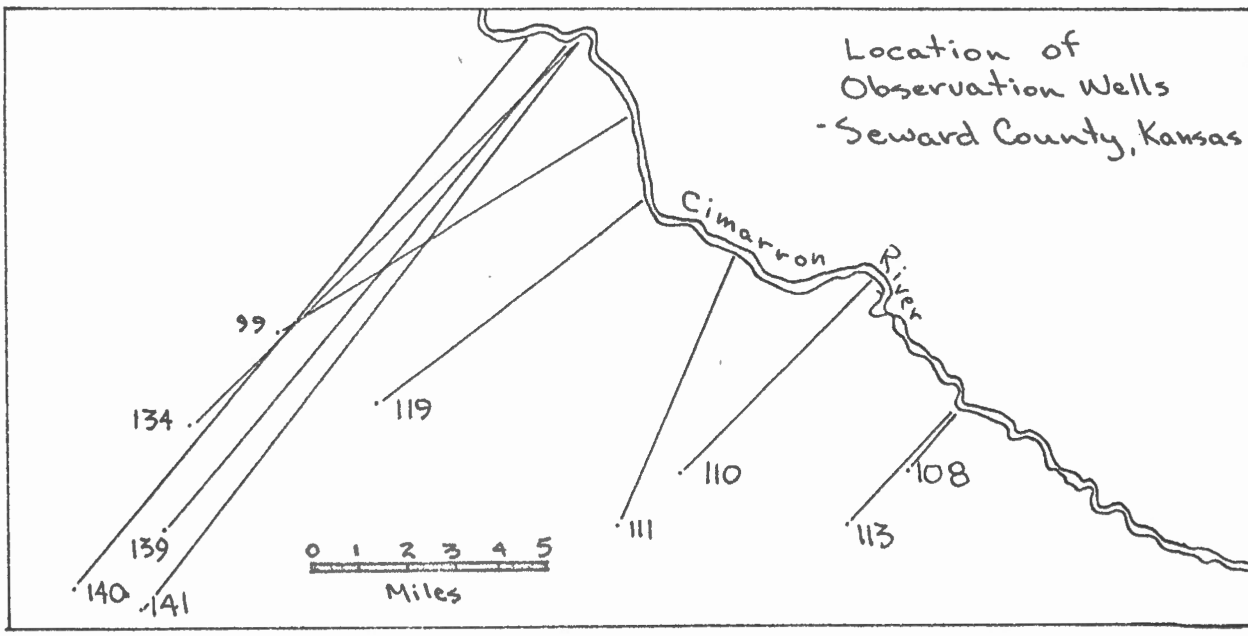
Gal.-ft.-day system:

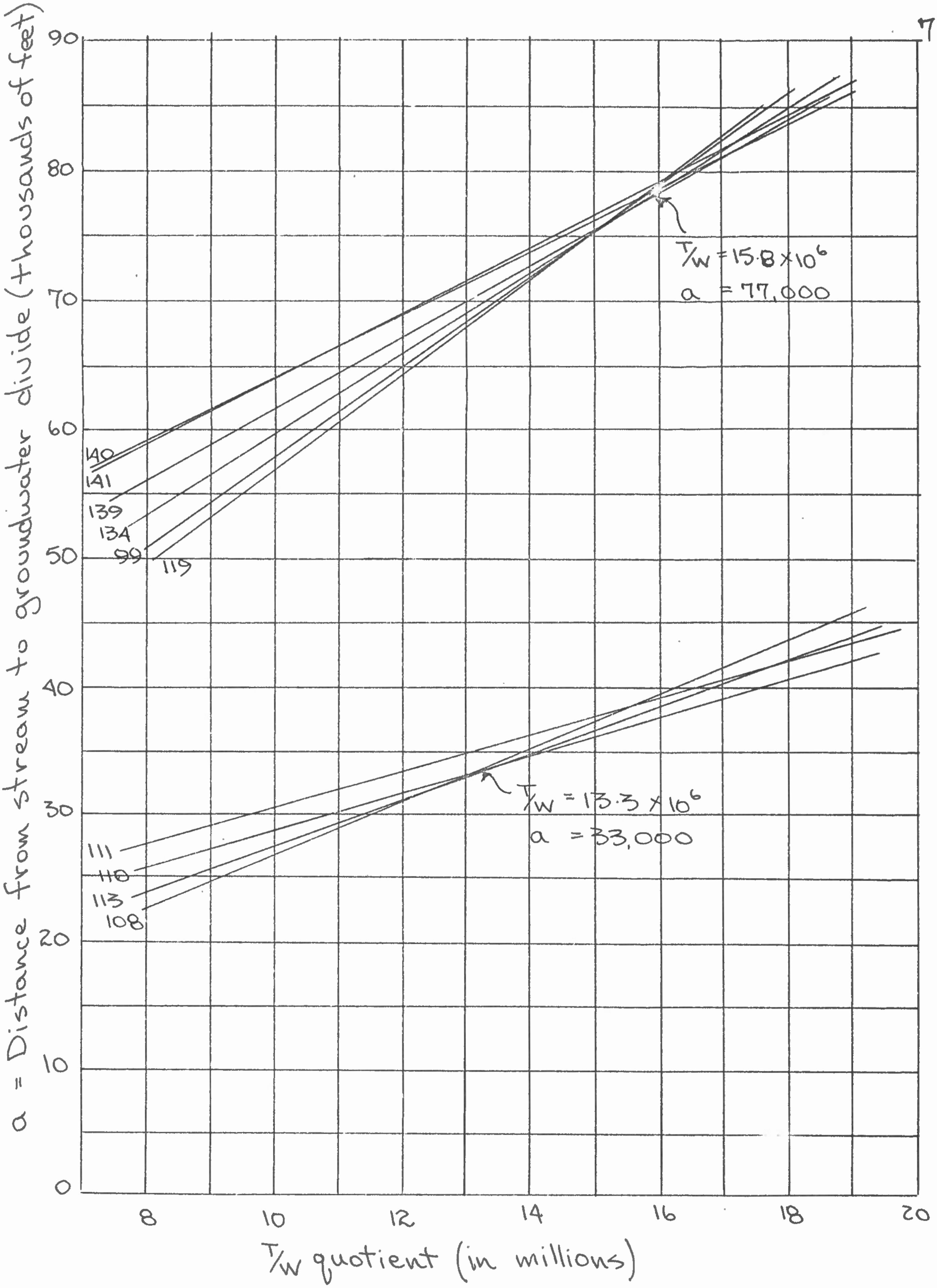
$$\frac{T}{0.0193\,qb} = \left(\frac{x}{h_o}\right) a - \left(\frac{x^2}{2\,h_o}\right)$$

$$\frac{700\,T}{W} = \left(\frac{x}{h_o}\right) a - \left(\frac{x^2}{2\,h_o}\right)$$

Table .--Altitude of water level and distance from river for application of water-table profile equations to observation wells in Seward County, Kansas.

Well number	Depth of well (feet)	Altitude of water level above m.s.l. (feet)	h_o Altitude of water level above river (feet)	Distance of well from river (feet)	x^2	x/h_o	$x^2/2h_o$
119	148	2744	146	40000	160×10^7	274	~~1~~55×10^5
99	143	2777	154	43000	185×10^7	279	$60 \times 10^{}$~~6~~^{5}
134	114	2822	176	55400	307×10^7	315	87×10^5
139	123	2831	185	66800	447×10^7	361	121×10^5
141	93	2835	190	76200	581×10^7	400	153×10^5
140	106	2845	190	79000	624×10^7	416	164×10^5
111	193	2619	45	33150	110×10^7	737	121×10^5
108	225	2526	24	11600	13.4×10^7	483	28×10^5
113	216	2536	34	19550	38×10^7	575	56×10^5
110	222	2580	40	28000	78×10^7	700	98×10^5





Graphical solution of observation well equations for application of profile method Seward County, Kansas.

Velocify of Groundwater motion.
(A) Toward a discharging well in an infinite aquifer of uniform thickness.

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
m	Saturated thickness of aquifer	feet	feet	metres
r	Distance from pumped well	feet	feet	metres
r_w	radius of pumped well	feet	feet	metres
t_w	time of traverse to pumped well	days	days	seconds
v	Velocity of groundwater at distance r	ft/day	ft/day	m/sec.
Q	Rate of discharge by pumped well	ft^3/day	g.p.m.	m^3/sec.
θ	Porosity of aquifer	%	%	%

Equations:

Ft.-day or m.-sec. system:

$$v = \frac{Q}{2\pi m \theta}\left[\frac{1}{r}\right]$$

$$t_w = \frac{\pi m \theta}{Q}\left[r^2 - r_w^2\right]$$

Gal.-ft.-day system:

$$v = \frac{36.7\ Q}{m\theta}\left[\frac{1}{r}\right]$$

$$t_w = \frac{m\theta}{73.5\ Q}\left[r^2 - r_w^2\right]$$

when r_w is small relative to r:

$$t_w = \frac{\pi m \theta r^2}{Q}$$

$$t_w = \frac{m \theta r^2}{73.5\ Q}$$

Velocity of groundwater motion
(B) Along the shortest path from a perennial stream toward a discharging well in an infinite aquifer of uniform thickness.

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
m	Saturated thickness of aquifer	feet	feet	metres
r	Distance from pumped well	feet	feet	metres
r_w	Radius of pumped well	feet	feet	metres
t_w	Time of traverse to pumped well via shortest path from stream	days	days	seconds
v	Velocity of groundwater at r	ft/day	ft/day	m/sec.
A	Distance from stream to pumped well	feet	feet	metres
Q	Rate of discharge by pumped well	ft^3/day	g.p.m.	m^3/sec.
θ	Porosity of aquifer	%	%	%

Equations:

Ft.-day or m.-sec. system:

$$v = \frac{Q}{\pi m \theta}\left[\frac{A}{2Ar - r^2}\right]$$

$$t_w = \frac{\pi m \theta A^2}{Q}\left[\frac{2}{3} - \frac{r_w^2}{A^2} + \frac{r_w^3}{3A^3}\right]$$

when r_w is small relative to A:

$$t_w = \frac{2\pi m \theta A^2}{3Q}$$

Gal.-ft.-day system:

$$v = \frac{73.5\, Q}{m \theta}\left[\frac{A}{2Ar - r^2}\right]$$

$$t_w = \frac{m \theta A^2}{73.5\, Q}\left[\frac{2}{3} - \frac{r_w^2}{A^2} + \frac{r_w^3}{3A^3}\right]$$

$$t_w = \frac{m \theta A^2}{110.2\, Q}$$

Flow toward a well from a nearby stream Rorabaugh formulas.

Terminology:

Symbol	Description	Ft.-day	Gal.-ft.-day	m.-sec.
r	Distance from observation well to pumped well	feet	feet	metres
Δs	Drawdown in observation well	feet	feet	metres
$\emptyset$	Angle included between line from pumped well to image well and line from pumped well to observation well	---	----	------
A	Distance from pumped well to line source	feet	feet	metres
Q	Discharge of pumped well	ft^3/day	g.p.m.	$m^3/sec.$
T	Coefficient of transmissibility	ft^2/day	g.p.d./ft	$m^2/sec.$

Equations:

Ft.-day or m.-sec. systems:

$$T = \frac{230\, Q \log_{10} \dfrac{\sqrt{4A^2 + r^2 - 4Ar(\cos \emptyset)}}{r}}{2\pi \Delta s}$$

Gal.-ft.-day system:

$$T = \frac{527.7\, Q \log_{10} \dfrac{\sqrt{4A^2 + r^2 - 4Ar(\cos \emptyset)}}{r}}{\Delta s}$$

If the observation well is on the riverward line $\phi = 0^{\circ}$ and:

Ft.-day or
M.-sec. system:

$$T = \frac{2.30\ Q \log_{10}\left[\frac{2A-r}{r}\right]}{2\pi \Delta s}$$

Gal.-ft.-day system:

$$T = \frac{527.7\ Q \log_{10}\left[\frac{2A-r}{r}\right]}{\Delta s}$$

If on the landward line $\phi = 180^{\circ}$ and:

Ft.-day or
m.-sec. system:

$$T = \frac{2.30\ Q \log_{10}\left[\frac{2A+r}{r}\right]}{2\pi \Delta s}$$

Gal.-ft.-day system:

$$T = \frac{527.7\ Q \log_{10}\left[\frac{2A+r}{r}\right]}{\Delta s}$$

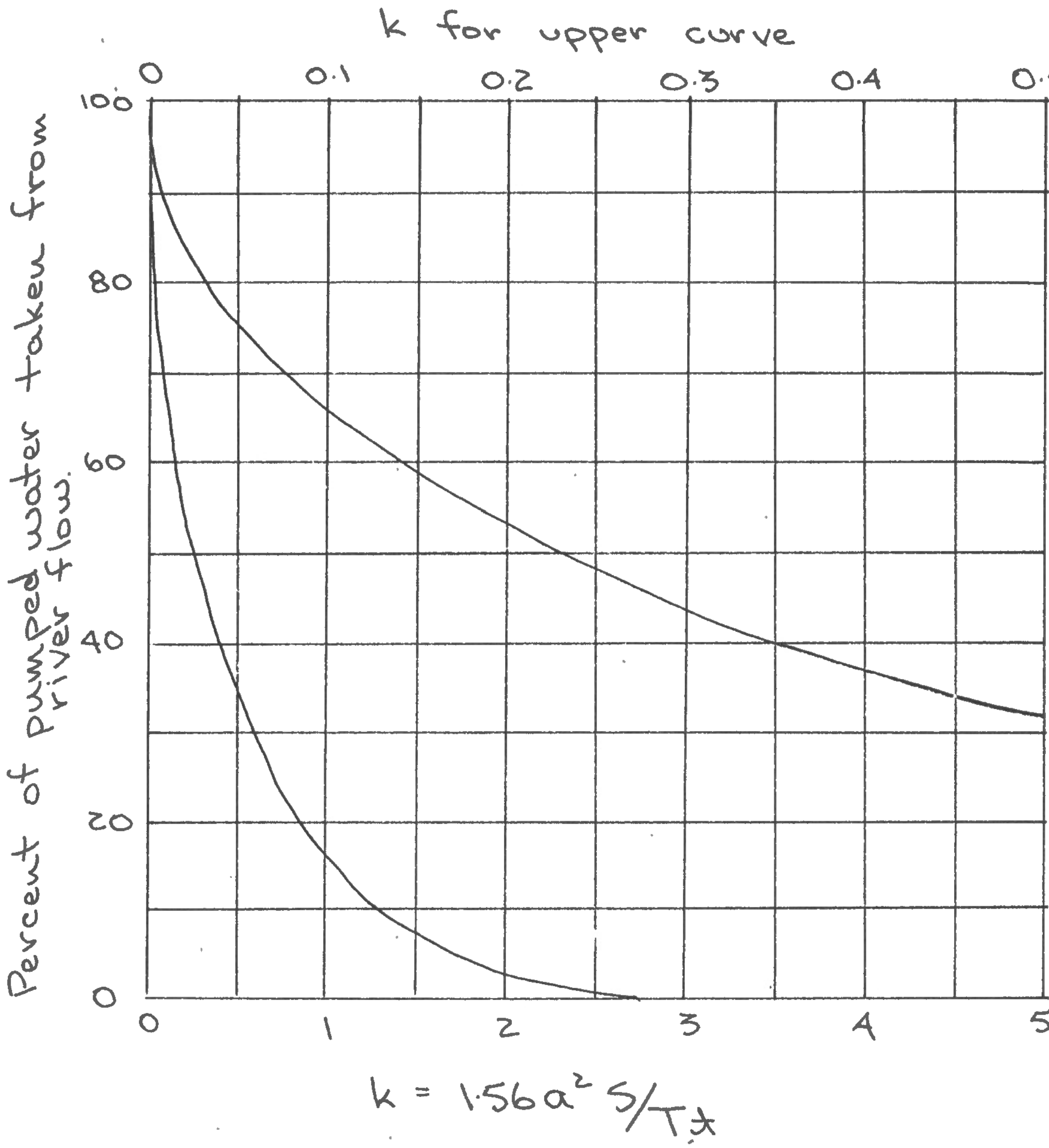
If on a line parallel to the line source $\phi = 90^{\circ}$ or 270° and:

Ft.-day or
m.-sec. system:

$$T = \frac{2.30\ Q \log_{10}\left[\frac{\sqrt{4A^2 + r^2}}{r}\right]}{2\pi \Delta s}$$

Gal.-ft.-day system:

$$T = \frac{527.7\ Q \log_{10}\left[\frac{\sqrt{4A^2 + r^2}}{r}\right]}{\Delta s}$$

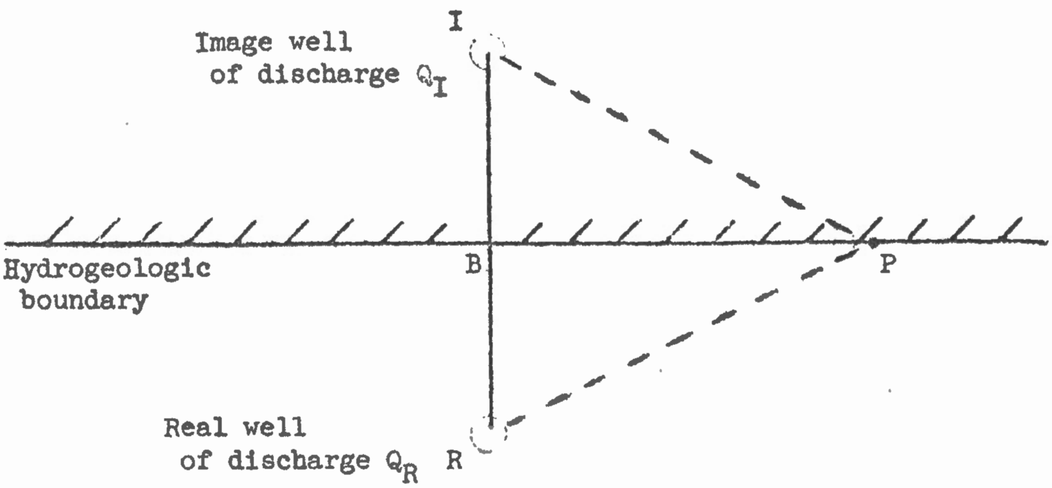


Graph showing the relationship between Theis' k factor and the percent of pumped water taken from river flow when a well is pumped in a semi-infinite aquifer that is bounded by a perennial stream. (Theis, 1941)

Image Well Theorems
No. 1

Theorem:

An image well, placed so that the strike of a hydrogeologic boundary marks the perpendicular bisector of a line drawn from image well to real well, will produce at any point on that boundary a change in potential equal to that produced by the real well.



Given: IB = BR; IBR ⊥ BP; $Q_R = Q_I$

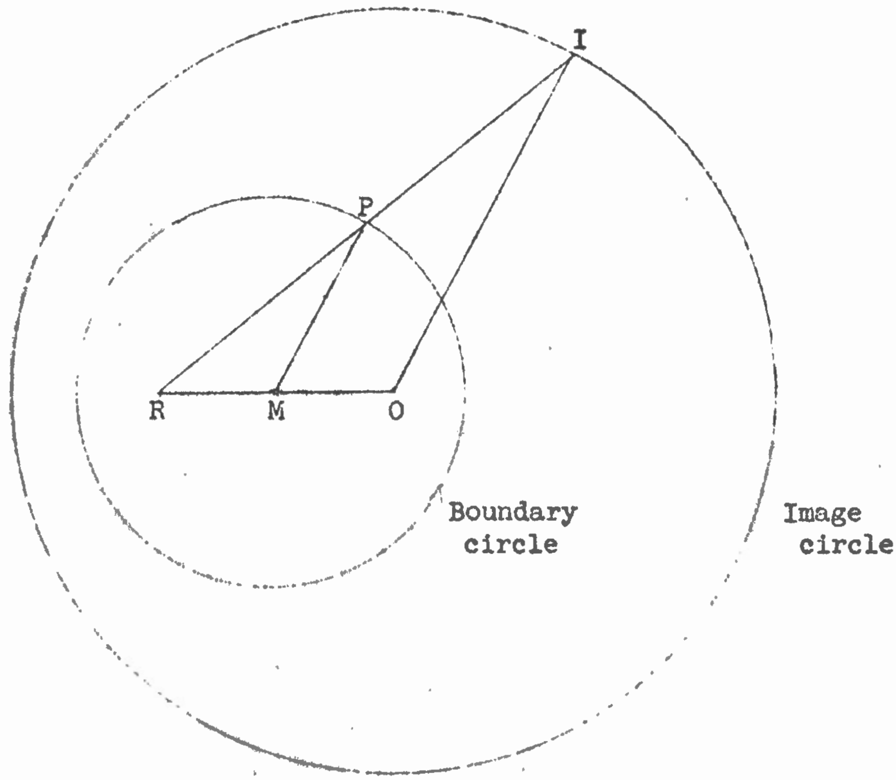
To prove: $s_R = s_I$ at any point P, on boundary

Statement	Reason
1. IB = BR	1. Given
2. BP = BP	2. Identity
3. ∠IBP = ∠RBP	3. All right ∠s are equal
4. △IBP ≅ △RBP	4. Two sides and incl. ∠ are equal
5. PI = PR	5. Corresp. parts of ≅ △s are equal
6. $Q_I = Q_R$	6. Given
7. $s_I = s_R$	7. Drawdowns are equal at points equi distant from wells of equal discharge

Image Well Theorems
No. 2

Theorem:

If the center of a circle is located at the midpoint (M) of a line drawn from a real well (R) to an observation well (O), and if the circumference of that circle intersects the hydrogeologic boundary at the midpoint of the line drawn from the real well (R) to its primary image well (I), then the radius of that circle is equal to one-half of the distance from the observation well (O) to the primary image well (I).



Given: RP = PI, RM = MO

To prove: $MP = \frac{OI}{2}$

Statement	Reason
1. RP = PI	1. Given
2. RM = MO	2. Given
3. $\angle PRM = \angle IRO$	3. Identity
4. $\triangle PRM \sim \triangle IRO$	4. Two sides proportional and included angle equal
5. $PM = \frac{OI}{2}$	5. Corresponding sides of similar triangles are proportional in the same ratio as other corresponding sides

Image-well distances

Terminology:

Symbol	Description
r_i	Distance from observation well to image well
r_r	Distance from observation well to real well
t_i	Time required for image well to develop a given draw-down at the observation well
t_r	Time required for real well to develop the same draw-down at the observation well

Equation:

$$r_i = r_r \sqrt{\frac{r_r^2/t_r}{r_r^2/t_i}}$$

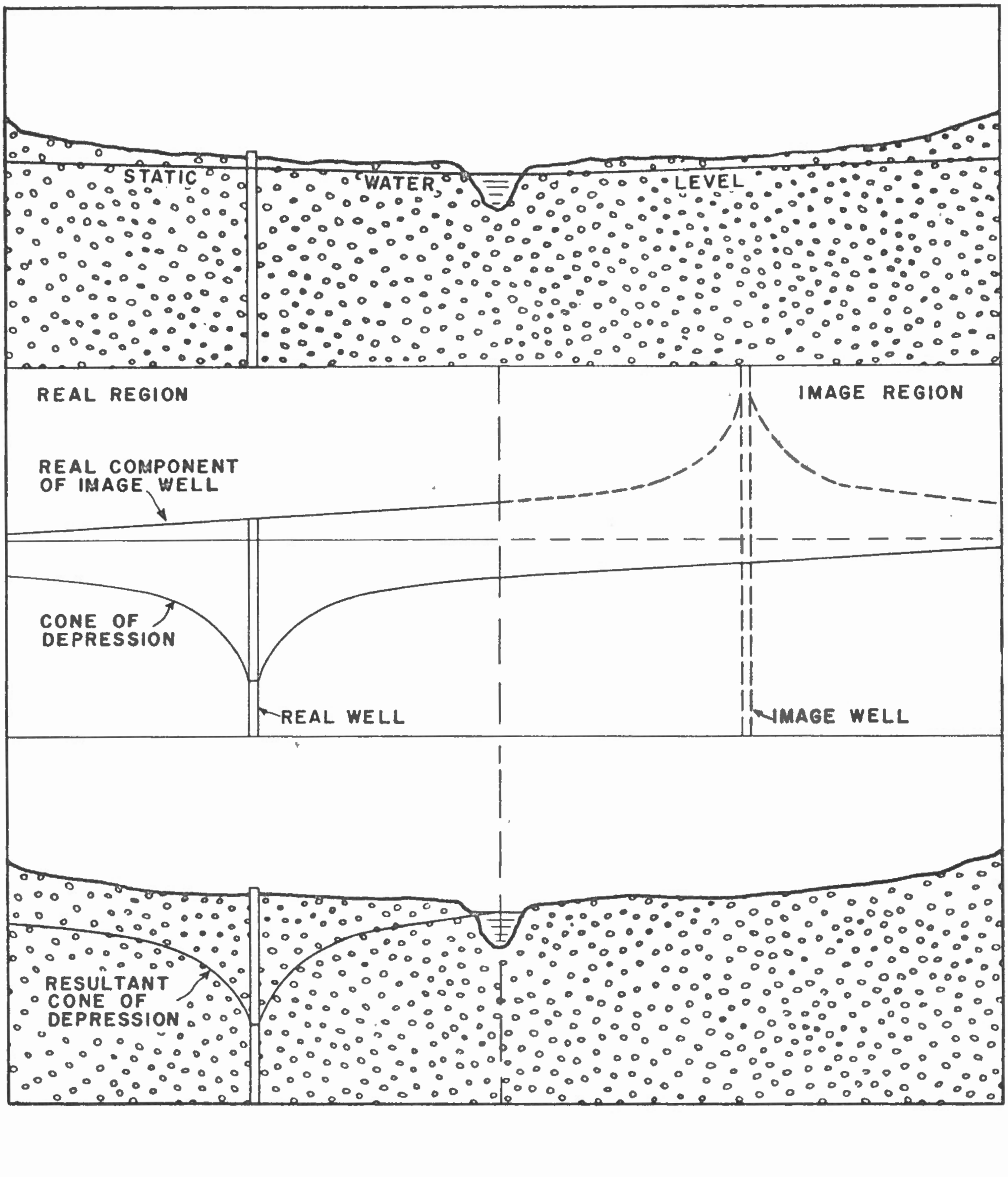


FIGURE IDEALIZED SECTION THROUGH AN AQUIFER INTERSECTED BY A PERENNIAL STREAM AND DIAGRAM SHOWING COMPONENTS OF EQUIVALENT HYDRAULIC SYSTEM FOR SOLUTION OF THE PROBLEM OF A WELL DISCHARGING FROM THE AQUIFER. SECTION AND PROFILE DIAGRAM ARE TAKEN PERPENDICULAR TO THE STREAM.

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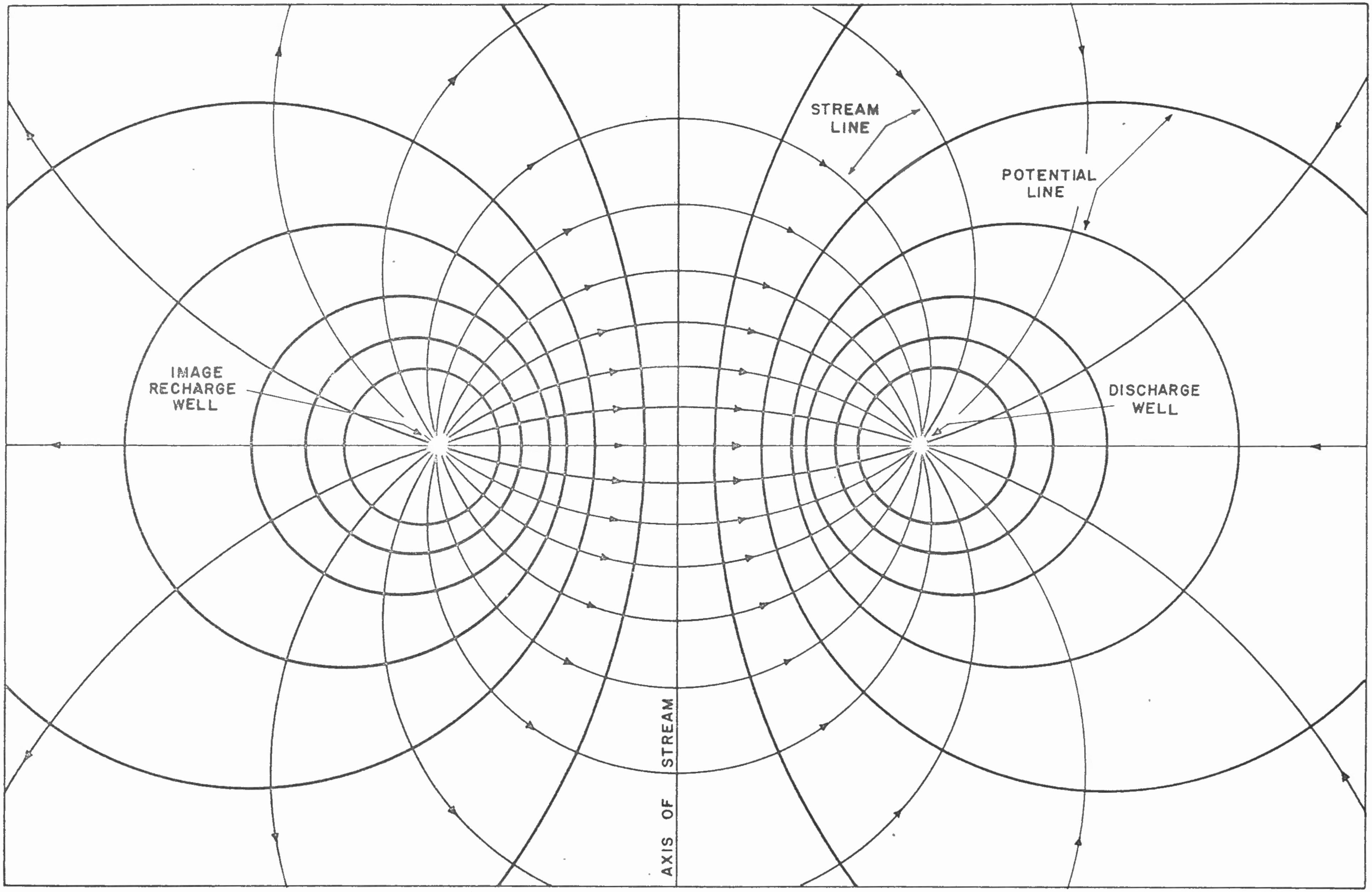


FIGURE GENERALIZED FLOW NET SHOWING STREAM LINES AND POTENTIAL LINES IN THE VICINITY OF A WELL DEPENDENT UPON INDUCED INFILTRATION FROM A NEARBY STREAM. (AFTER SLICHTER, P. 340, 1899)

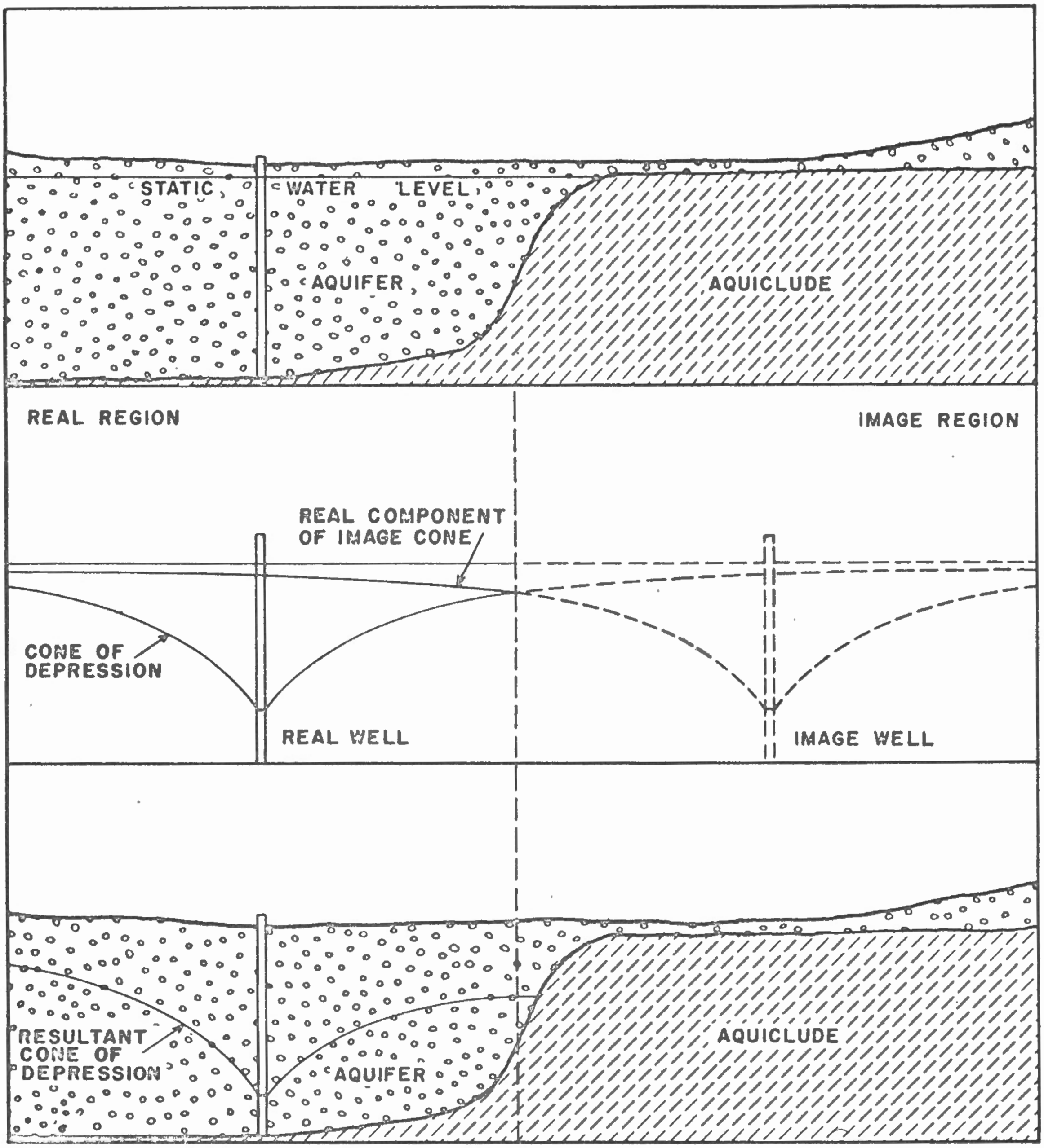


FIGURE IDEALIZED SECTION THROUGH AN AQUIFER BOUNDED BY AN IMPERVIOUS FORMATION AND DIAGRAM SHOWING COMPONENTS OF EQUIVALENT HYDRAULIC SYSTEM FOR THE SOLUTION OF THE PROBLEM OF A WELL DISCHARGING FROM THE AQUIFER. SECTION AND PROFILE DIAGRAM ARE TAKEN PERPENDICULAR TO THE IMPERVIOUS BOUNDARY.

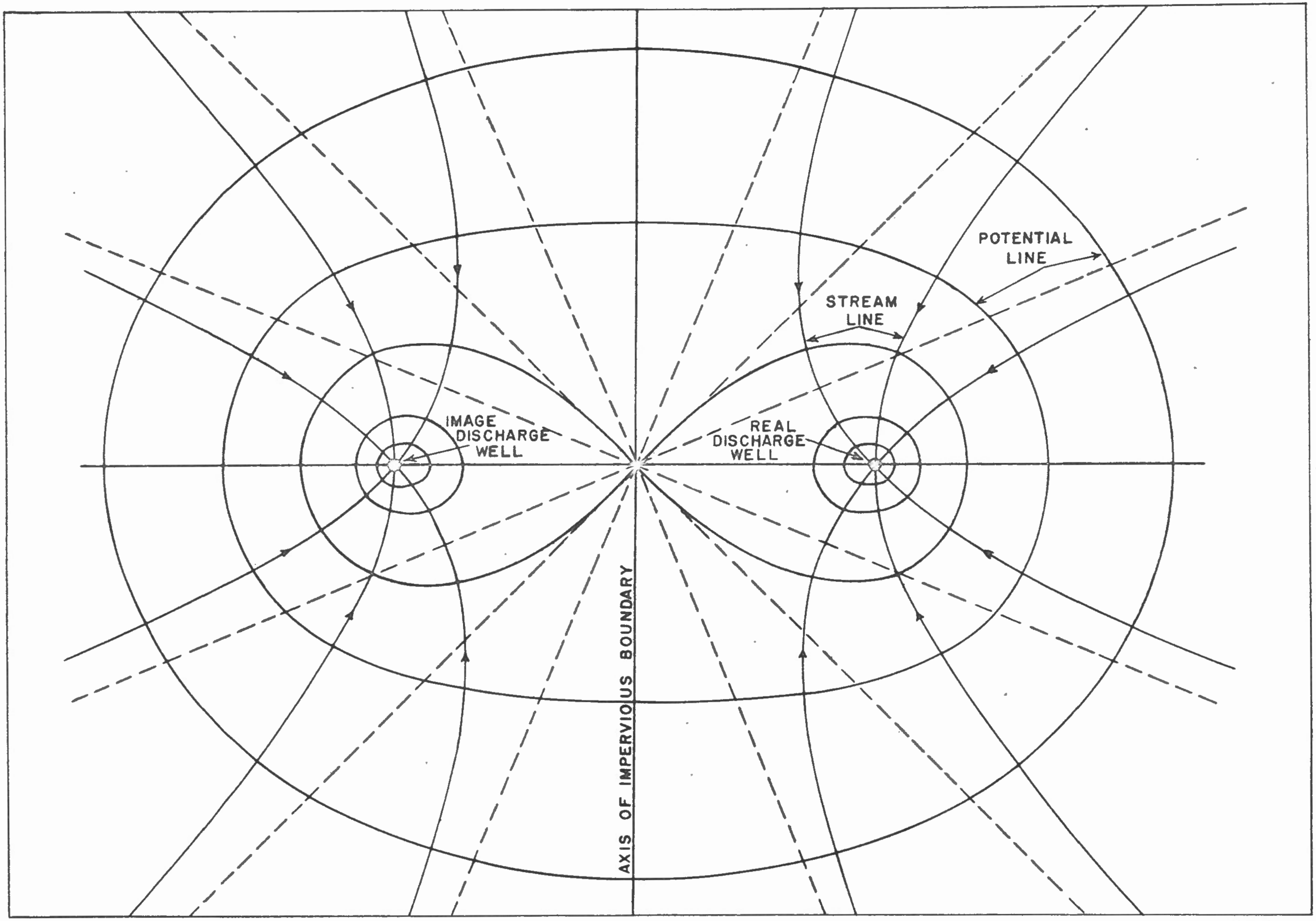


FIGURE GENERALIZED FLOW NET SHOWING STREAM LINES AND POTENTIAL LINES IN THE VICINITY OF A DISCHARGING WELL LOCATED NEAR AN IMPERVIOUS BOUNDARY. (AFTER SLICHTER, P. 342, 1899)

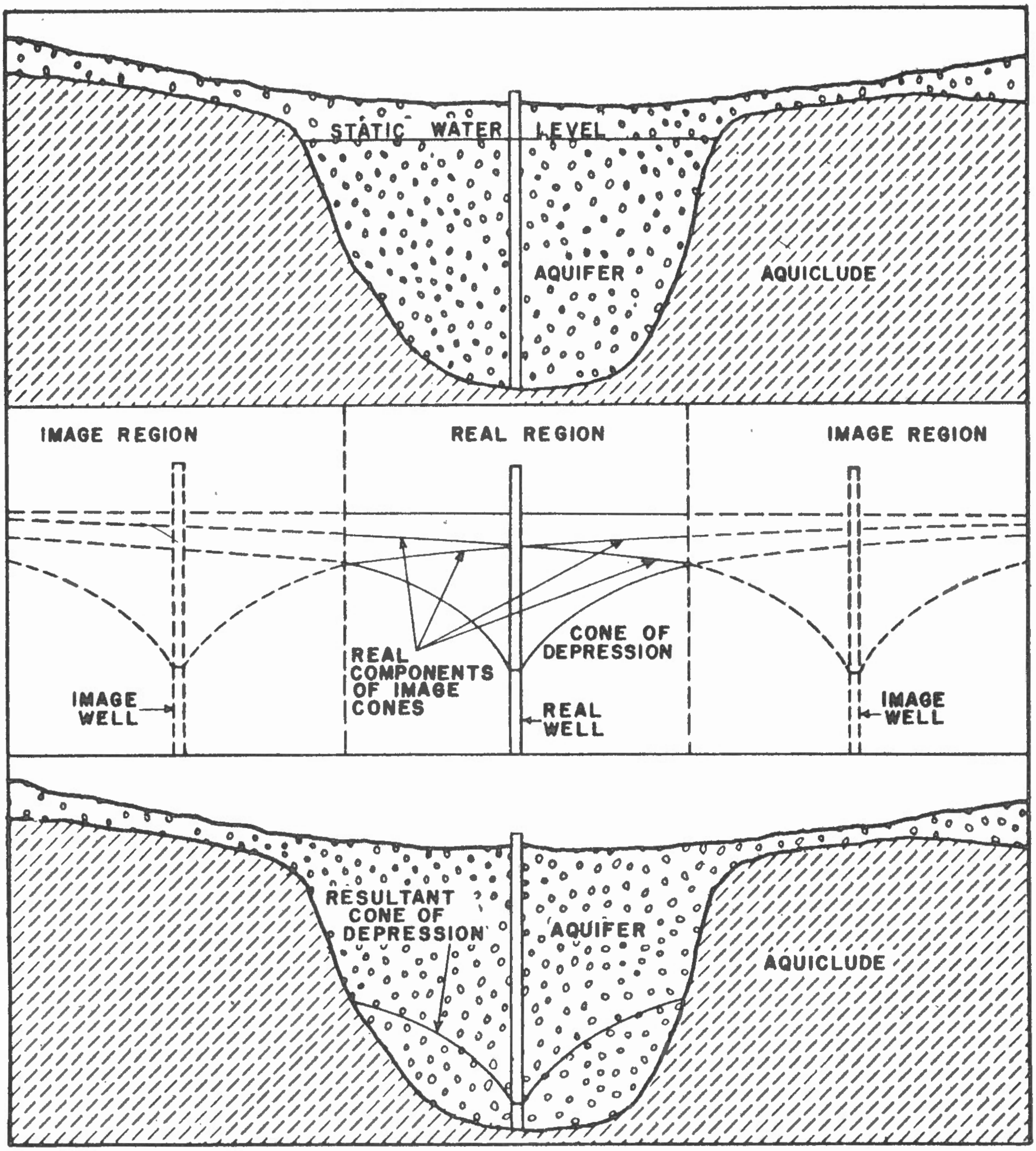


FIGURE IDEALIZED SECTION THROUGH AN UNDER FLOW CHANNEL BOUNDED BY IMPERVIOUS FORMATIONS AND DIAGRAM SHOWING PRIMARY COMPONENTS OF EQUIVALENT HYDRAULIC SYSTEM FOR THE SOLUTION OF THE PROBLEM OF A WELL DISCHARGING FROM THE AQUIFER. DRAWDOWN COMPONENTS OF HIGHER ORDER IMAGE WELLS ARE NOT ILLUSTRATED. SECTION AND PROFILE DIAGRAM ARE TAKEN PERPENDICULAR TO THE IMPERVIOUS BOUNDARIES.

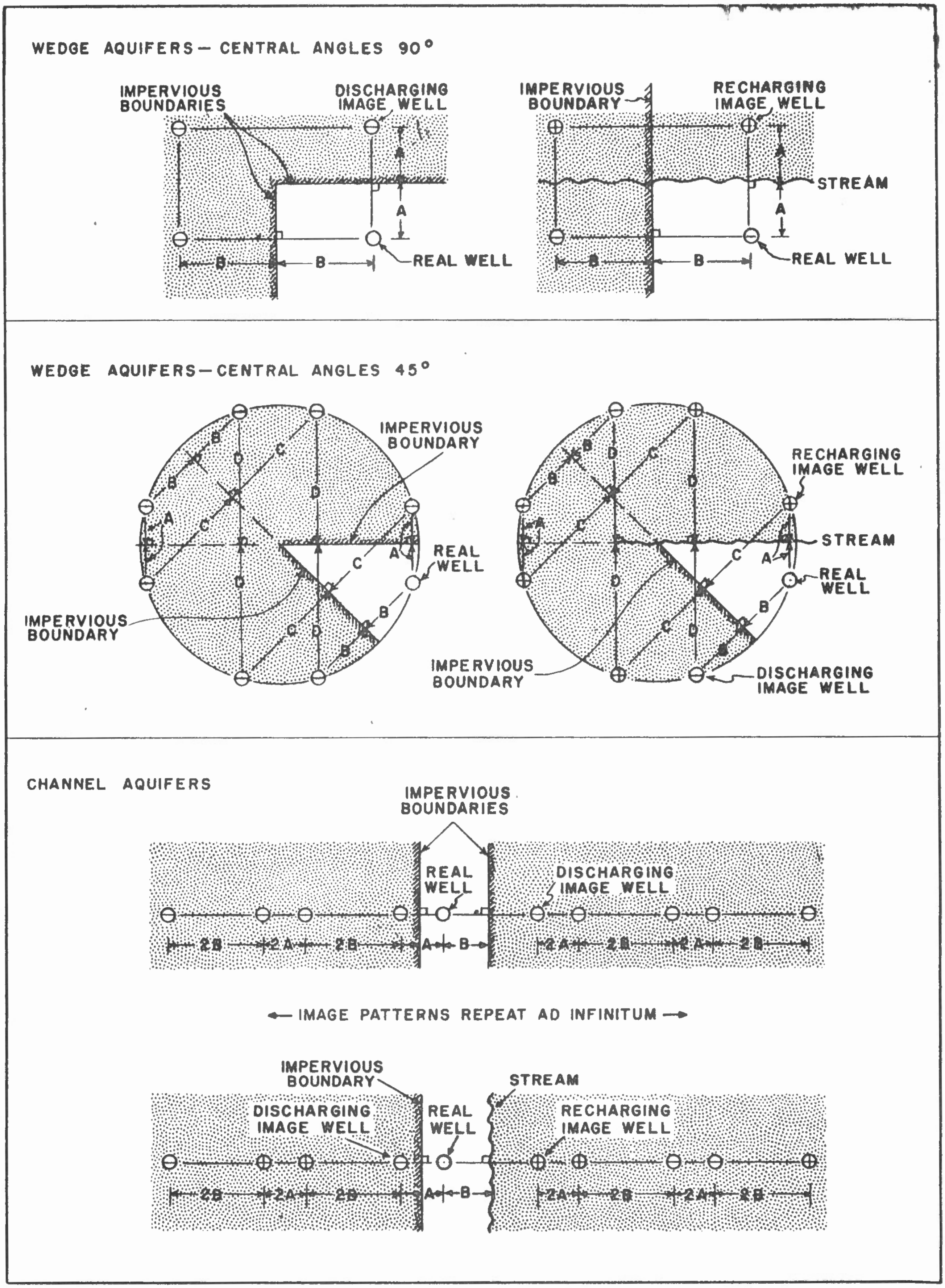
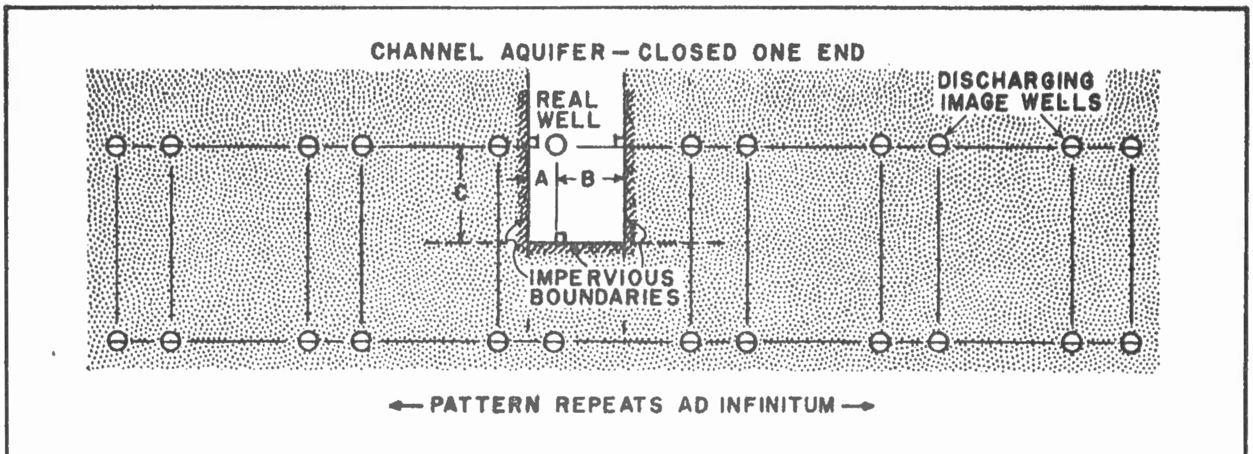


FIGURE DIAGRAMS OF IMAGE WELL SYSTEMS FOR SELECTED WEDGE AND CHANNEL AQUIFERS.



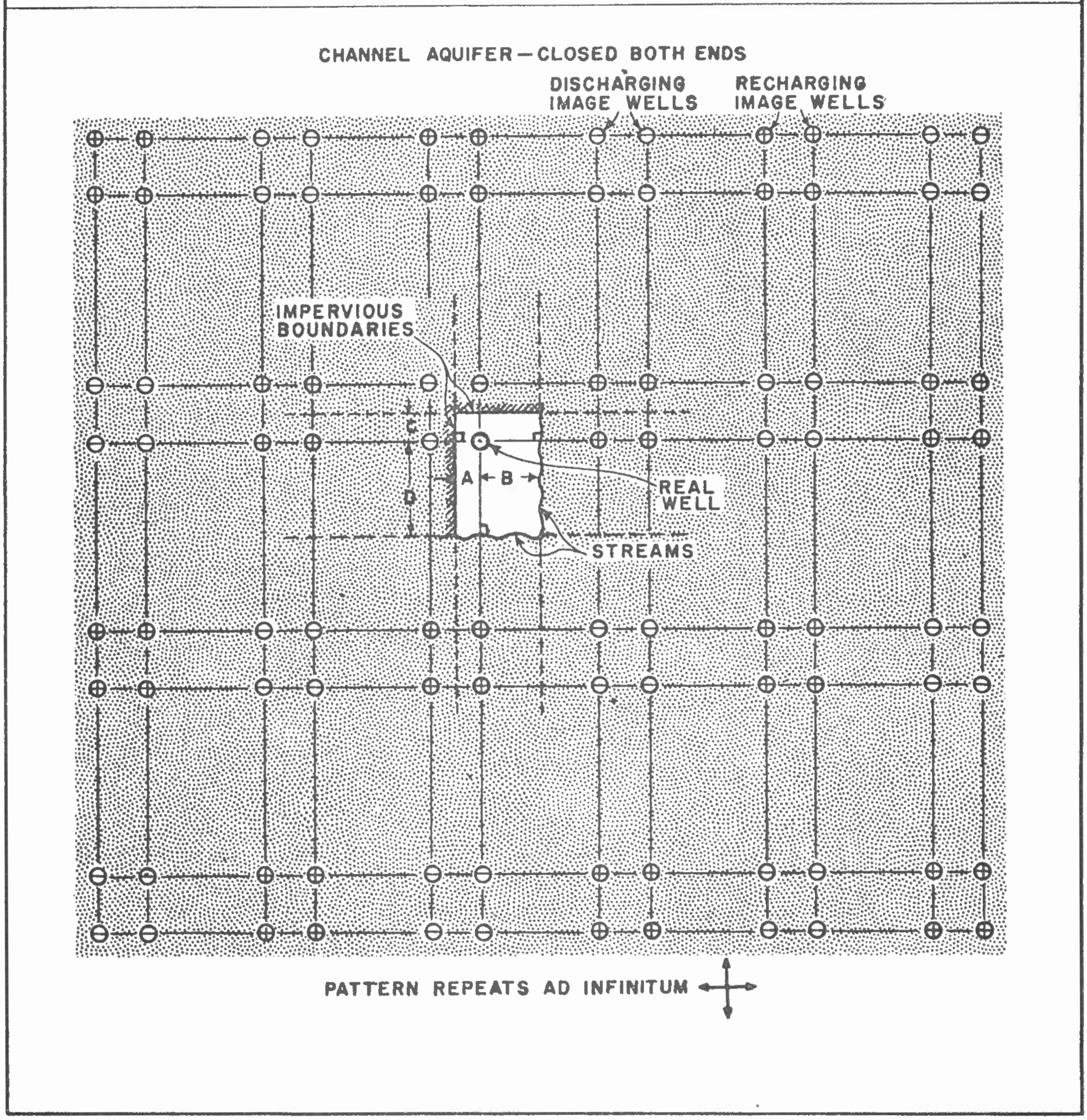


FIGURE DIAGRAMS OF IMAGE WELL SYSTEMS FOR TWO SELECTED CHANNEL AQUIFERS.

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PART III

RESULTS OF A GROUNDWATER SURVEY

Contents:

Page

"The source of water derived from wells"

Under the above title C. V. Theis (Civil Eng., vol. 10, pp. 277-280, 1940) has stated concisely the hydrologic principles upon which depend much of our present quantitative approach to ground-water problems. The statements that follow are summarized from these principles.

The essential factors that determine the response of an aquifer to development by wells are:

1. Distance to, and character of, the recharge
2. Distance to the locality of natural discharge
3. Character of the cone of depression in the aquifer, which depends upon coefficients of transmissibility (T) and of storage (S)

Prior to any development by wells, an aquifer is in a state of approximate dynamic equilibrium in that over the years recharge and discharge essentially balance. Pumping from wells upsets this condition of equilibrium, but unless the draft is excessive, a new dynamic equilibrium may be established by:

1. Loss of ground-water storage
2. Increase in recharge (natural or artificial)
3. Decrease in natural discharge
4. Combination of these

From equation 23 it follows that the drawdown (s) is directly proportional to the rate of discharge (Q) and inversely proportional to the coefficient of transmissibility (T); thus (using k as a constant of proportionality):

$$s = kQ \tag{32}$$

$$s = \frac{k}{T} \tag{33}$$

Furthermore, after any given time of pumping (t), excepting very short intervals, the area of influence (πr^2) is independent of the rate of discharge and inversely proportional to the coefficient of storage (S), thus:

$$\pi r^2 = \left(\frac{k}{S}\right)_t \tag{34}$$

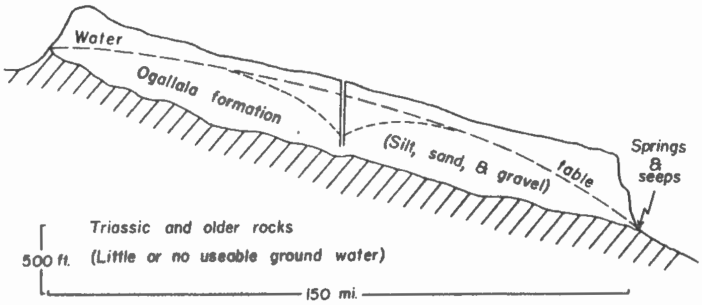
In an artesian aquifer the coefficient of storage is no more than a small fraction of that in an unconfined aquifer; hence, the cone of depression in an artesian aquifer may spread from a hundred to several thousand times faster than in an unconfined aquifer. Thus, excepting aquifers that are very extensive, a new equilibrium in an artesian aquifer may be established soon after development begins. Accordingly, such an artesian aquifer generally can be treated essentially as a unit in any measures for conservation of ground water.

Conversely, in an extensive unconfined aquifer where the development occurs at great distances from the areas of recharge and of natural discharge, for a considerable period of time most of the water is derived from storage and equilibrium is reestablished very slowly. A large unconfined aquifer of this type generally cannot be treated as a unit for conservation measures; rather, it must be treated generally as a number of distinct sub-units.

Thorough knowledge of these hydrologic principles plus the gathering and proper interpretation of the pertinent field data should permit the solution of virtually any quantitative ground-water problems, although in some areas the solution may be very difficult.

Some Chosen Examples

Southern High Plains of Texas-New Mexico



<u>Aquifer</u>.- Tertiary stream deposits (Ogallala formation); thickness, featheredge to 600 feet, average 300 feet; moderately permeable; rests on relatively impermeable rocks.

<u>Recharge</u>.-Solely from scanty precipitation; cut off from upstream sources of water by western escarpment. Theis estimated rate at 1/20-1/2 inch per year, order of 75,000 acre feet a year.

<u>Storage capacity</u>.- Very large--order of 400,000,000 acre feet.

<u>Natural discharge</u>.- Almost entirely by springs and seeps along eastern escarpment--equal to recharge.

<u>Withdrawal by pumping</u>.- Has increased from less than 100,000 acre feet in 1934 to nearly 5,000,000 acre feet in 1954 (largely for irrigation). Withdrawal in 1955 reduced to 4,100,000 acre feet because of favorable precipitation. In diagram, withdrawal represented by one large well.

Salvaged rejected recharge.- Essentially none; water table lies 50 feet or more beneath most of area, thus more than ample space provided for all possible natural recharge.

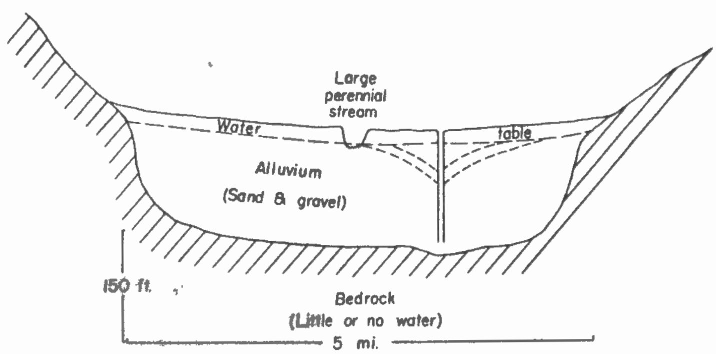
Salvaged natural discharge.- Essentially none; gradient toward eastern escarpment virtually unaffected. Even if all natural discharge could be salvaged, it would amount to less than 1/65 of the withdrawal.

Result.- Virtually all water being pumped from storage (mined); water levels declining.

Possibility of reestablishing equilibrium.-
1. Artificial recharge from ephemeral ponds through recharge wells; being tried, but if successful, would be sufficient only to retard decline in water level.
2. Imported water--by lifting several hundred feet from Canadian River; considered too costly.
3. Conservancy district, to which most counties belong, seeks to retard depletion by limiting withdrawals and proper spacing of future wells.

For additional details, see: Gaum, Carl H., High Plains, or Llano Estacado, Texas-New Mexico, chapter 6 in reference given on Geol. 12.

Large valley of perennial stream in humid region



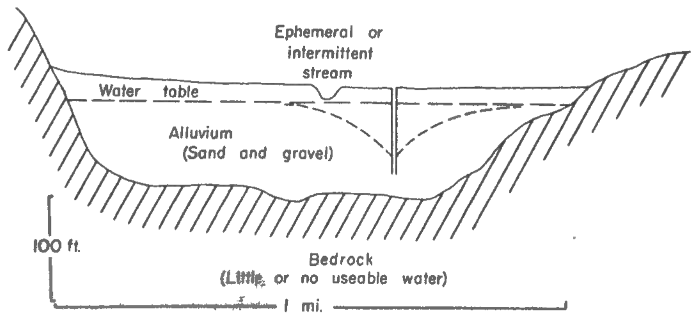
Setting.- Thick, permeable alluvium filling valley cut into shale; permeable channel beneath large perennial stream; shallow water table; many phreatophytes; moderately heavy precipitation.

Sources of water to large ground-water development.-
1. Withdrawal from storage--creates cone of depression.
2. Salvaged rejected recharge--lowering of water table provides more room for recharge from precipitation, hence, reduces or prevents surface runoff.
3. Salvaged natural discharge-
   a. By lowering water table beneath phreatophytes.
   b. By decreasing gradient toward stream and, hence, decreasing discharge of ground water into stream.

4. Recharge directly from stream--Sources 1-3 may suffice but, if not, cone of depression will continue to spread until it intersects stream, then gradient will be reversed and stream water will percolate downward through permeable channel and move toward wells (depicted as one well in diagram), permitting very large withdrawals.

Limitations- Amount of withdrawals essentially limited to amount of streamflow, provided that:
1. Lowering of water table does not impair or destroy useful phreatophytes.
2. Reduction in streamflow does not interfere with established water rights (as in western states).

Valley of ephemeral stream in semi-arid region

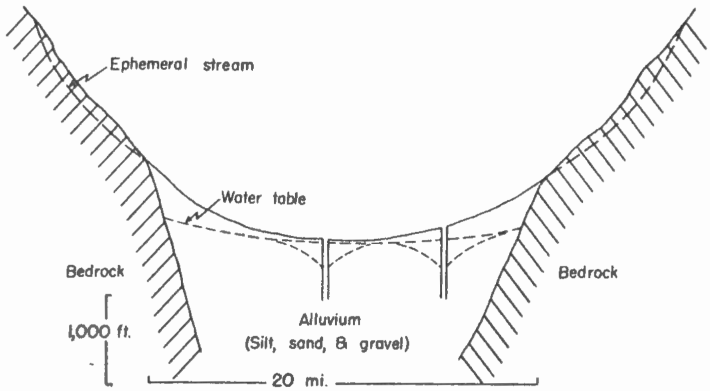


Setting.- Moderately thick, permeable alluvium filling valley cut into shale; permeable channel bottom; water table beneath reach of most phreatophytes--latter limited to cottonwood trees along banks of stream; precipitation, 15 inches a year; stream dry most of year but floods after heavy rains or cloudbursts in high headwater region.

Sources of water to moderate development of ground water for irrigation.-
1. Withdrawal from storage--creates cone of depression.
2. Salvaged rejected recharge from precipitation--none, as there is more than enough room for meager recharge from low precipitation.
3. Salvaged natural discharge--very small, as phreatophytes limited to cottonwood trees along bank of stream.
4. Recharge directly from stream--may be very large provided that water table is kept sufficiently lowered by pumping to provide room for all or most of floodwater, which percolates rapidly downward through permeable channel. Functions effectively as evaporation-free flood-control reservoir.

Limitations-
1. Relation of flood frequency to water needs.
2. Water table must be lowered enough to provide adequate storage space, yet kept high enough for economical well operation.
3. Procedure has been very successful thus far in ephemeral stream valleys of eastern Colorado, such as Big Sandy Creek Valley.

Closed desert basin



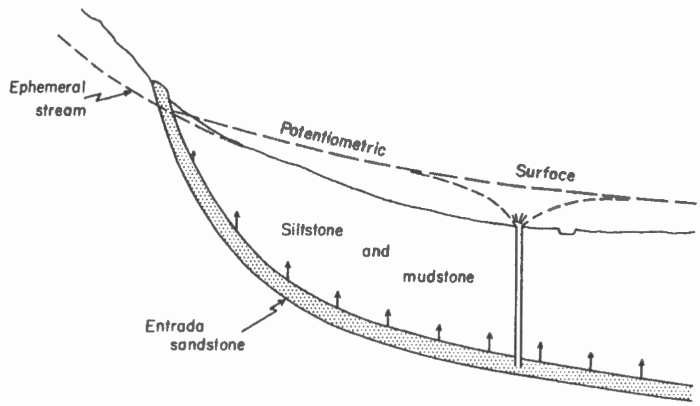
Setting.- Large desert basin receiving 3-5 inches of precipitation annually, surrounded by high mountains that receive 20-30 inches of precipitation; aquifer--thick bolson deposits built up by coalescing alluvial fans, coarse and permeable near mountains, fine-grained at and near playa; water table--at or near surface at playa, deep near mountains; streams--all ephemeral; phreatophytes--only near playa in middle of basin (salt grass, etc.).

Sources of water to moderate ground-water development.-

1. Withdrawal from storage--creates cone of depression.
2. Salvaged rejected recharge--essentially none, for virtually all the precipitation that falls in valley evaporates. Recharge comes entirely from small ephemeral streams that head in surrounding high mountains.
3. Salvaged natural discharge.
   a. By development near playa (well A)--may be large, by lowering water table below reach of transpiration by phreatophytes and direct evaporation from playa.
   b. By development along border of basin (well B)--some salvage by reducing gradient toward playa.
4. Limitations-
   a. Water near playa, where maximum development possible, generally is too highly mineralized for most uses.
   b. Development of water of good quality in border areas may be limited in quantity and by pumping lift.
5. Possible remedial measures-
   a. Destroy phreatophytes near playa (difficult and costly).
   b. Build retention dams in canyons of bordering mountains so that flash floods that normally reach and temporarily flood playa may be stored and released slowly for recharge in the permeable heads of alluvial fans. Has been successful in California and elsewhere.

Grand Junction, Colo. artesian basin

There are three artesian aquifers in the Grand Junction artesian basin, but only one, the Entrada sandstone, is depicted and discussed below.



Setting.- Aquifer--Entrada sandstone, Jurassic, fine-grained, cemented with $CaCO_3$, 150 feet thick, T = 150 gpd/ft, S = 5×10^{-5}; precipitation--$7\frac{1}{2}$ inches a year; recharge--only at crossing of outcrops by alluvium of small ephemeral streams; natural discharge--only by small leakage upward through aquitard (Summerville and Morrison formations); initial artesian head--variable depending upon topography, but as much as 160 feet above land surface.

Sources of water to wells.-

1. Withdrawal from artesian storage (S)--limited, but accounts for much of water recovered.
2. Salvaged rejected recharge--very small, steepening of gradient from recharge area may salvage some water, but amount from this source limited by small transmitting capacity (T) of aquifer.
3. Salvaged natural discharge--very small within area of development, limited by very low permeability of aquitard.

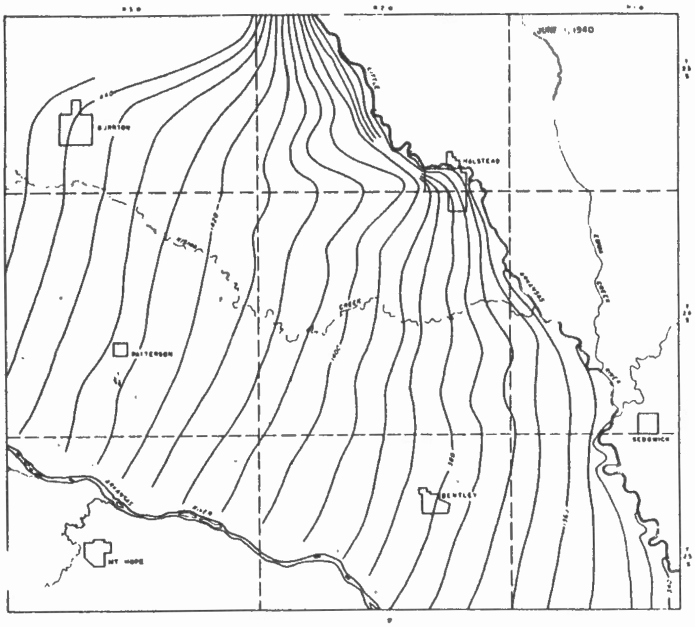
Result.- Continuing decline in artesian head, which has been accelerated by:

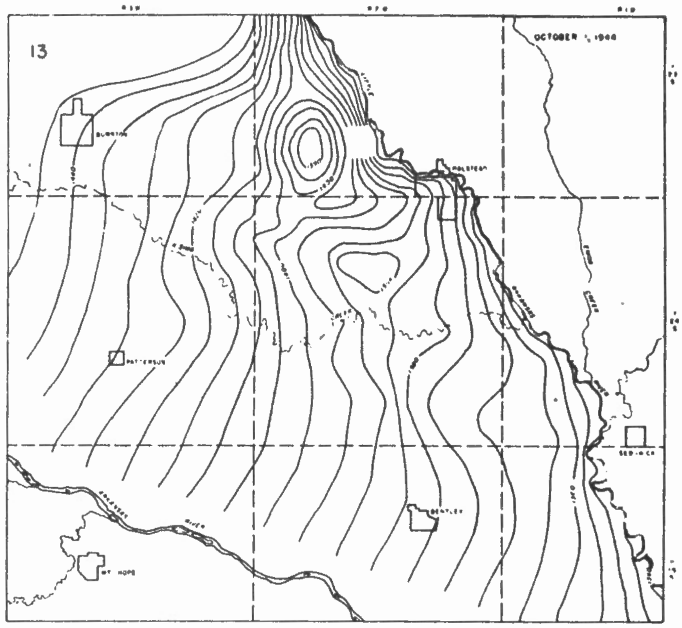
1. Drilling of many additional wells.
2. Mutual interference between closely spaced wells.
3. Use of pumps by some well owners whose wells have stopped flowing or have reduced appreciably in flow.

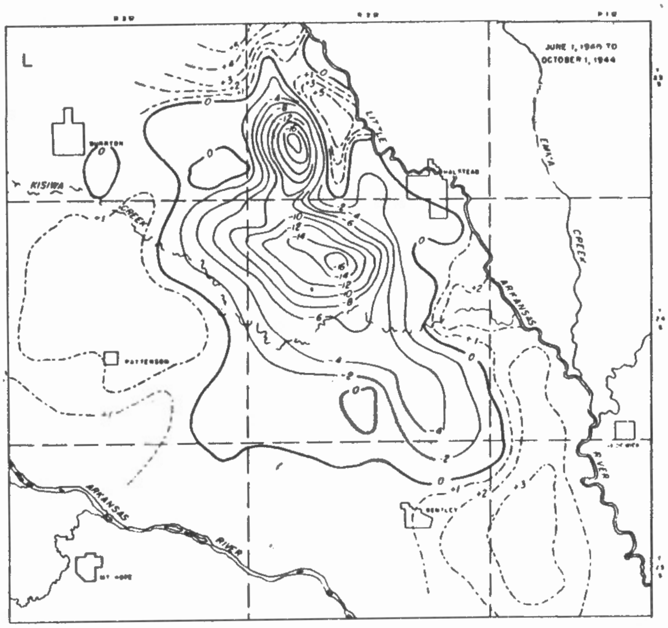
Computing changes in ground-water storage from

1.-Maps showing lines of equal change in water level

(After Williams, C. C., and Lohman, S. W.)



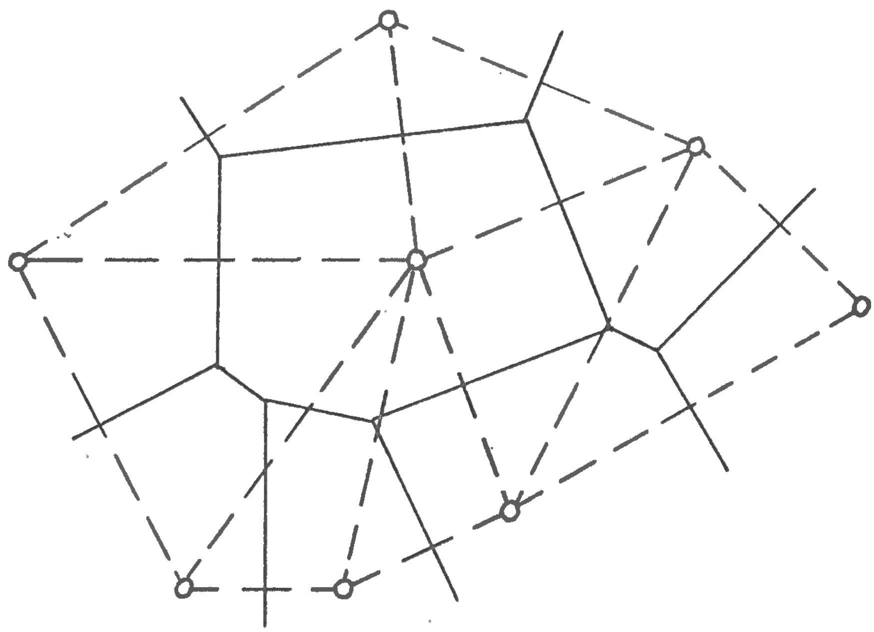




Superposition of water-table contour maps made before pumping began (June 1, 1940) and after 4 years of pumping (Oct. 1, 1944) allows construction of map at left showing lines of equal change in water level. From such maps and average value of specific yield, changes in ground-water storage are readily computed.

2.-Thiessen polygons

For computing mean height of water table, change in ground-water storage, and the like, it is a common practice to weight the observed data from each observation well according to a polygonal area of influence for that well. "Thiessen polygons" are simple to construct, although procedures differ in some details. A common method is as follows:

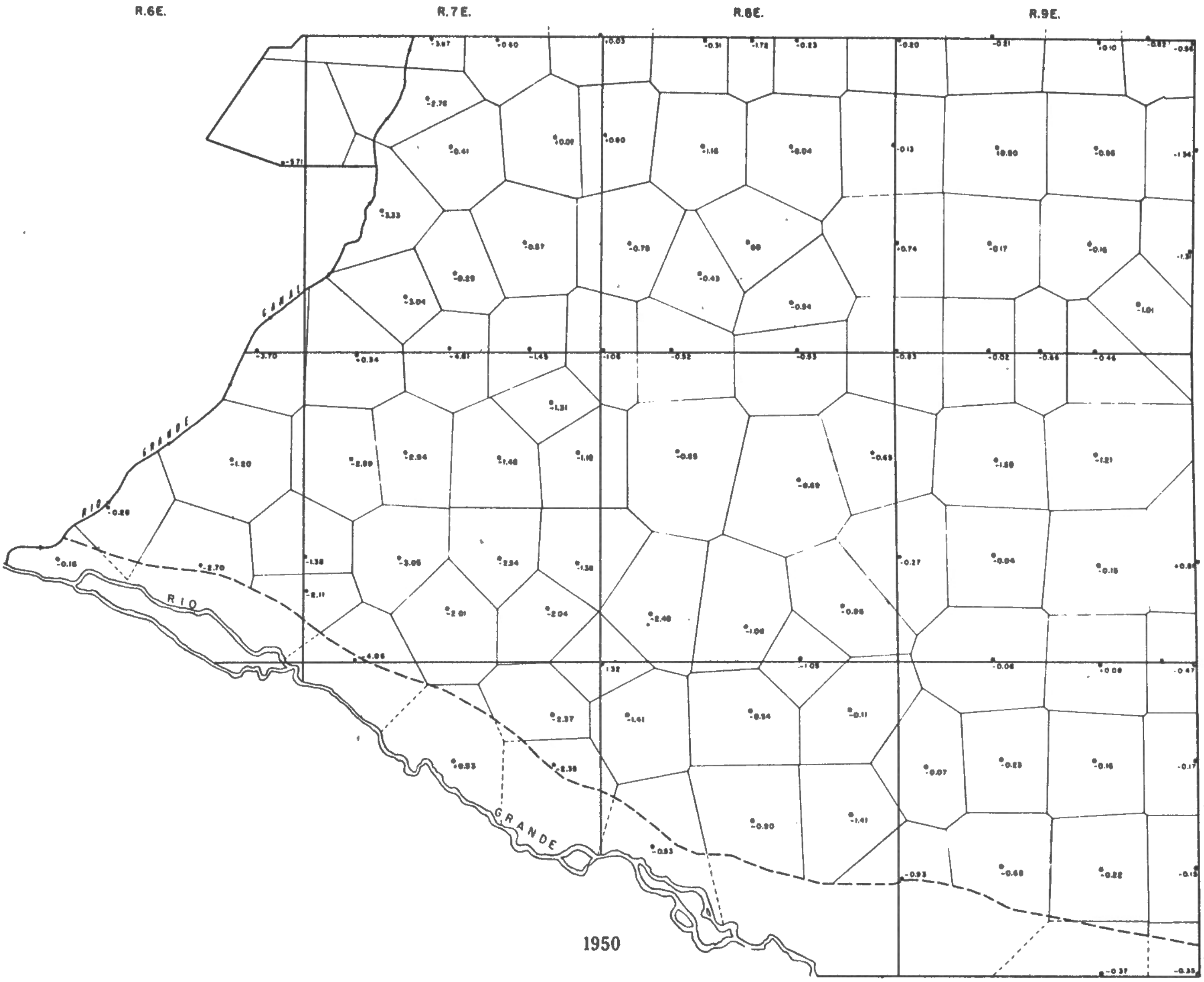


In the typical array above, join observation wells by rays (dashed lines) dividing the area into a network of triangles whose sides are as short as possible, and whose angles ordinarily are not obtuse. At the midpoint of each ray, erect a perpendicular and extend these perpendiculars to intersect one another (solid lines). The intersecting perpendiculars define the polygon of influence around each well. Areas of the polygons are determined readily by planimeter.

Storage change in a given area may be computed at intervals such as a quarter, half year, or year. Here the same polygonal areas should be used in each successive computation, but a complication arises wherever an observation well is not represented in the observed data. In this event, it is convenient to interpolate the missing data from the observations at adjacent wells rather than to construct new polygons, as the end result generally is essentially the same.

Other complications arise where the water table or potentiometric surface is uneven. Here, it is undesirable that a polygon span a sharp ridge or deep trough of that surface. Considerable judgment can be exercised in building up the observation-well net to avoid this contingency.

An example is given below of a map of part of the San Luis Valley, Colorado, showing changes in water level within such polygonal areas. (After Powell, W. J., Ground-water resources of the San Luis Valley, Colorado: U. S. Geol. Survey Water-Supply Paper in press, pl. 10.)



Depth-to-water maps

(After Frye, J. C., and Fishel, V. C., Ground water in southwestern Kansas: Kans. Geol. Survey, fig. 5, 1949.)

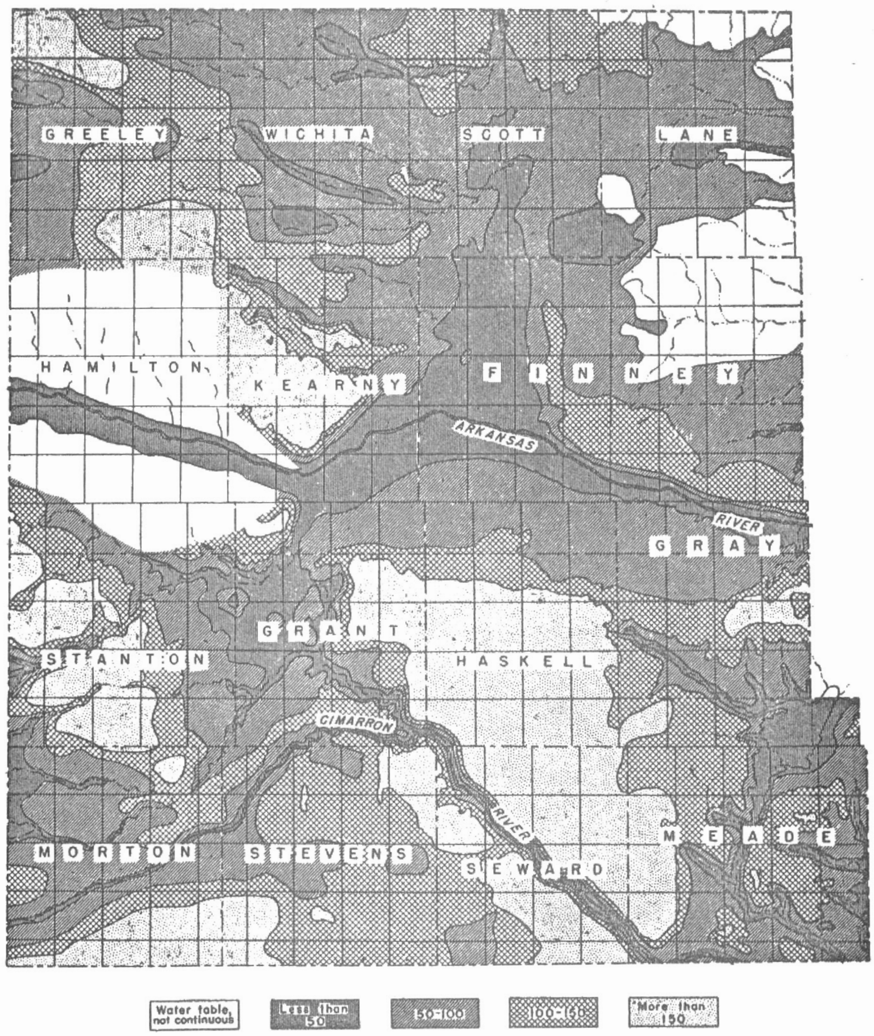


Figure 5. Map showing depths to water level below land surface in southwestern Kansas. (Compiled by W. W. Wilson)

Maps of bedrock topography

(After Latta, B. F., Geology and ground-water resources of Finney and Gray Counties, Kansas: Kans. Geol. Survey Bull. 55, fig. 7, 1944.)

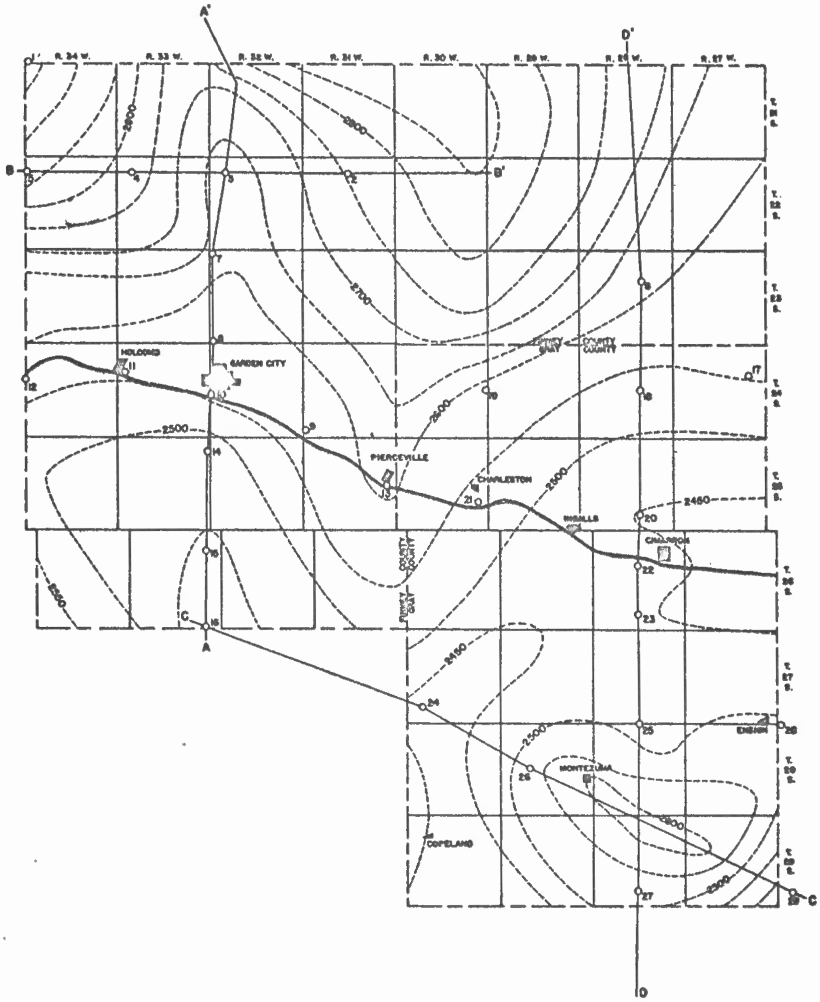


Fig. 7. Map of Finney and Gray counties showing by means of contours (dashed lines) the shape and slope of the pre-Tertiary surface, location of test holes (numbered circles), and location of cross sections shown in figures 8 and 9.

In some areas, particularly glaciated regions, such maps may show buried stream valleys that may contain productive aquifers. If aquifer lies above bedrock and is all water-bearing, superposition of water-table contour maps on contour maps of bedrock topography permits construction of saturated-thickness maps.

Maps showing thickness of saturated material

(After Frye, J. C., and Fishel, V. C.)

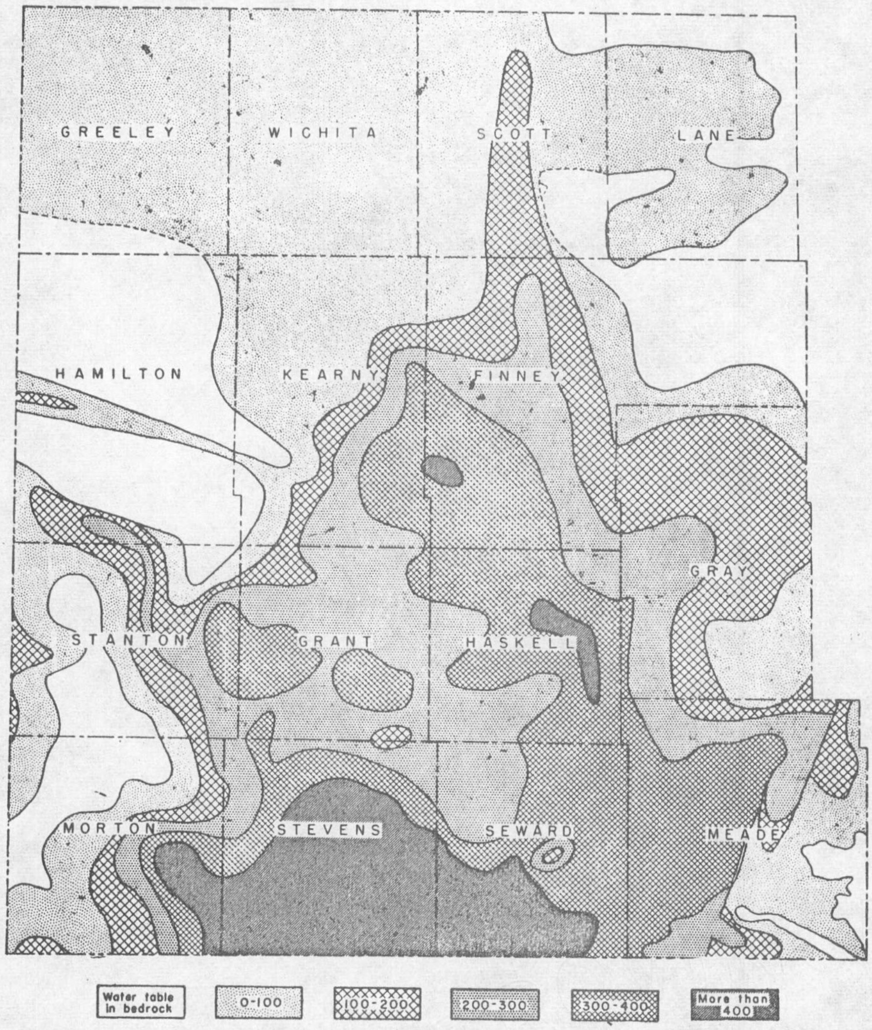


Figure 4. Map showing thickness of the Pleistocene and Ogallala deposits that are saturated with water in southwestern Kansas. Saturated sandstones of the Permian and Cretaceous rocks are not included. The quantity of water in storage is, in general, proportional to the saturated thickness of water-bearing material. (Compiled by Glenn Prescott)

Such maps can be constructed from information given in driller's logs or electric logs, or in the manner described. If specific yield is known, such maps also can be used to determine quantity of ground water in storage.

Maps showing availability of ground-water

(After Lohman, S. W., and others, Ground-water supplies in Kansas available for national defense industries: Kans. Geol. Survey Bull. 41, pt. 2, fig. 2, 1942. See also Lohman, S. W., and Burtis, V. M., General availability of ground water and depth to water level in the Arkansas, White, and Red River Basins: U. S. Geol. Survey Hydrologic Investigations Atlas HA-3, 1953.)

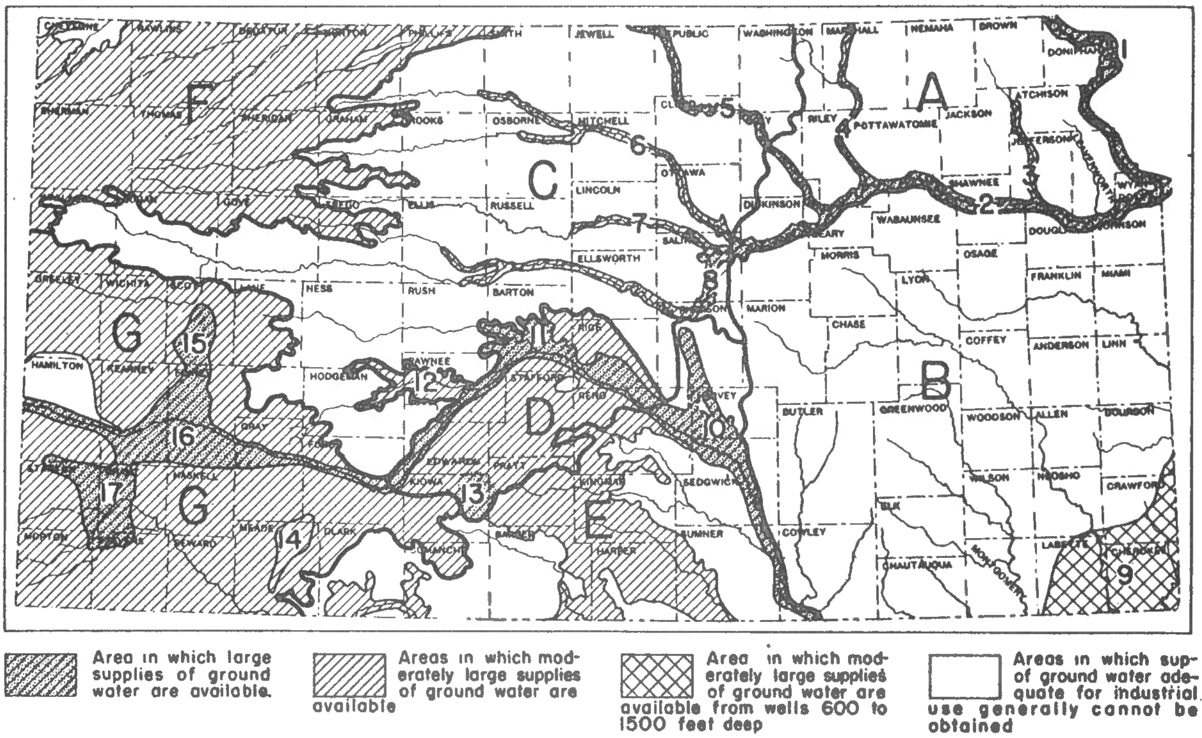


FIGURE 2. Map of Kansas showing by patterns the areas in which may be obtained supplies of ground water adequate for national defense industries. Areas designated by letters or numbers are discussed separately in the text.

Isometric block diagram

(After Harshbarger, J. W., Repenning, C. A., and Callahan, J. T., The Navajo Country, Arizona-Utah-New Mexico, chapt. 7 in the physical and economic foundation of natural resources, pt. IV, Subsurface facilities of water management and patterns of supply-type area studies: Interior and Insular Affairs Committee, House of Representatives, United States Congress, p. 119, 1953).

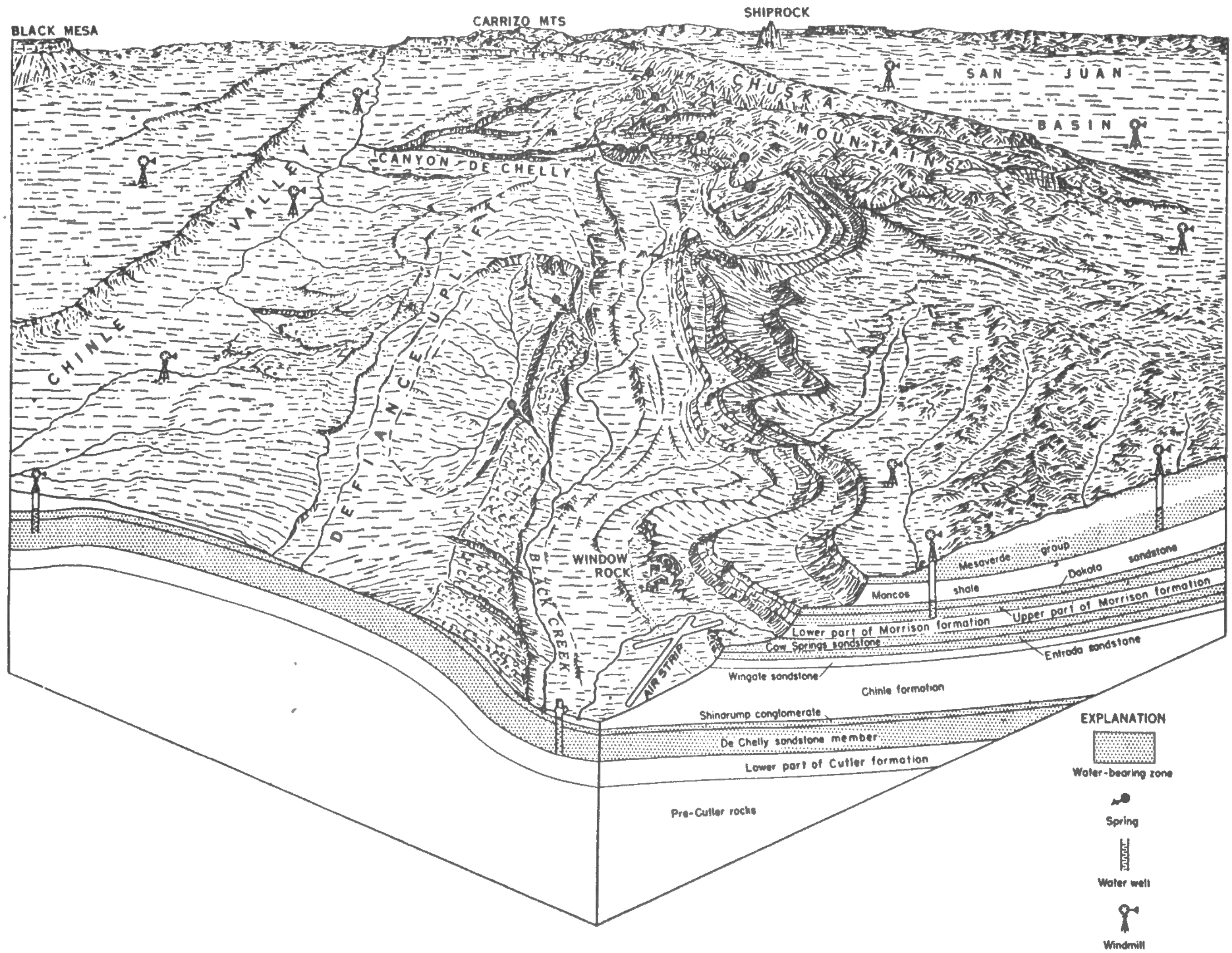


FIGURE 7.10.—*Water-bearing formations of the Defiance Uplift, Chuska Mountains, and vicinity.* This perspective sketch shows the relation between the layered rocks and the occurrence of ground water.

PART IV

GROUNDWATER EQUIPMENT

Contents:

GROUND WATER EQUIPMENT

A. I. Johnson
February, 1955*
(revised)

The proper equipment, in good condition, may mean the difference between obtaining the correct answer to a ground water problem, or arriving at false conclusions. Personnel should always be on the lookout for the application of new materials, equipment and techniques to our work. The following is a partial list of equipment which the engineer and geologist uses, or may find useful, in conducting a ground water investigation.

A. Office equipment

1. Stereoscopes

   a. Lens type

   b. Mirror type

   c. Prism type

2. Photogrammetric projectors

3. Photogrammetric plotters

4. Stereocomparagraph

5. Stereoelevation meter

6. Stereoslope meter

7. Super-Duper Dipper

8. Pantograph

9. Isometrograph

10. Spacing divider

11. Planimeter

12. Map measure

13. Proportional dividers

14. Electric analog plotter

*For distribution at GWSC. Official use only.

A. Office equipment (cont'd)

15. Calculator or slide rule
16. Microscope
17. Sand identification tube

B. Surveying equipment

1. Brunton compass
2. Hand level
3. Abney level
4. Altimeter
5. Engineer's level - dumpy, Wye
6. Transit
7. Alidade
8. Theodolite

C. Meteorological equipment

1. Thermometer - maximum and minimum
2. Rain and snow-gage - recording, non-recording
3. Evaporation pan
4. Wind direction indicator
5. Anemometer
6. Pyrheliometer
7. Hygrometer
8. Psychrometer
9. Hygrothermograph
10. Thermograph
11. Barograph

D. Equipment for the measurement of water level or pressure

1. Steel tape
2. Electric tape
3. Water-level recorder - float type, pressure type
4. Pressure gage
5. Manometer - mercury, ink-well, flexible
6. Soil plug
7. Water-level indicator
8. Plumbers tools and pipe fittings
9. Gage shelters
10. Observation well signs

E. Equipment for the measurement of running water

1. Volumetric equipment
2. Weir - free or submerged, contracted or suppressed; rectangular, 90° V-notch, trapezoidal or "cipolletti"
3. Orifice - free or submerged, contracted or suppressed
4. Venturi meter
5. Flow-nozzle
6. Parshall flume - full, modified
7. Pitot tube
8. Current meter - Price, Pygmy, Hoff
9. Sparling meter
10. California Pipe method
11. Cox Flow meter
12. Collins' Flow gage
13. Staff gage

F. Well exploration equipment

1. Depth-to-water
2. Electrical conductivity survey
3. Velocity survey
4. Temperature survey
5. Well calipers
6. Casing finder
7. Water samplers
8. Portable pH meter

G. Well construction equipment

1. Pipe and casing - metal, plastic
2. Perforators
3. Well screens and drive points
4. Pumps - windmill, plunger type, turbine, centrifugal, submersible, jet, propeller

H. Subsurface exploration equipment

1. Hand augers
2. Power augers
3. Drive-core samplers
4. Coring - diamond, drilled shot
5. Drilling - cable-tool, hydraulic rotary, pneumatic rotary
6. Jetting
7. Electric loggers
8. Seismic survey
9. Resistivity survey
10. Gamma-ray logging

WELL SCREEN SLOT AND GAUZE DESIGNATION

Cook or Johnson Screens

Slot No.	Opening Equivalent in inches	Gauze No.
6	0.006	90
7	0.007	80
8	0.008	70
10	0.010	60
12	0.012	50
14	0.014	
16	0.016	
18	0.018	40
20	0.020	
25	0.025	30
30	0.030	
35	0.035	20
40	0.040	
50	0.050	
60	0.060	
70	0.070	
80	0.080	
100	0.100	1/10 in.
125	0.125	1/8 in.
187	0.187	3/16 in.

Layne Shutter Screens

Slot No.	Opening Equivalent in inches
1	0.205
2	0.180
3	0.155
4	0.130
5	0.105
6	0.080
7	0.055
8	0.030