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SUBSURFACE MODELLING FOR GEOTHERMAL PRODUCTION AND ENERGY STORAGE

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ABSTRACT

The general concept of heat storage by pumping fluid at an elevated temperature into a deep aquifer and then later withdrawing the hot fluid has been examined by means of mathematical models.

The major findings are the following:

1. Thermal energy can be stored in suitable aquifers with efficiencies of 80 to 90 percent.

2. The best aquifer-aquitard systems for storage purposes are those which lead to most spherical temperature disturbances. Thus thick aquifers with high porosity and leaky aquitards are preferable.

3. Problems with the produced fluid temperature are encountered if a simple in-out system is used. The fluid temperature history can be usefully improved by conditioning achieved by added input for the initial input phase.

4. Conditioning does not have deleterious effects on storage efficiency in the long run.

The findings are interpreted as indicating that such storage is quite feasible provided thermal energy storage is deemed desirable within the current energy economic milieu.

RÉSUMÉ

Des modèles mathématiques sont utilisés pour étudier le concept général du stockage de chaleur dans un aquifère profond en introduisant, par pompage, un fluide à température élevée et, par après, en retirant le fluide chaud. Les principaux résultats sont les suivants:

1. Le rendement d'énergie thermique emmagasinée dans un aquifère convenable peut être de 80 à 90 pourcent.

2. Les meilleurs systèmes d'aquifères et d'aquicludes pour le stockage sont ceux qui produisent les perturbations thermiques les plus sphériques. Les aquifères épais de grande porosité et les aquicludes qui manquent d'étanchéité sont donc préférables.

3. Des problèmes avec la température du fluide de production surviennent si un système simple d'injection et de production est utilisé. L'évolution de la température du fluide peut être améliorée utilement en augmentant l'alimentation pendant la phase initiale d'injection.

4. Ce conditionnement n'a pas d'effets nuisibles sur l'efficacité à long terme du stockage.

L'interprétation des résultats indique que ce stockage est faisable, pourvu que l'emmagasinage d'énergie thermique est désirable à l'intérieur du présent milieu économique de l'énergie.

SUBSURFACE MODELLING FOR GEOTHERMAL PRODUCTION AND ENERGY STORAGE

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K_H	hydraulic conductivity
h	hydraulic head
n	an outward directed normal
dB	a differential element of boundary
t	time
S_S	specific storativity
dV	a differential element of volume
K_T	thermal conductivity
T	temperature
T	transmissivity
P	density of fluid
C	specific heat of fluid
S	storativity
n	porosity
P_A	density of aquifer
C_A	specific heat of aquifer
P_R	density of aquifer rock
C_R	specific heat of aquifer rock
r	radius
b	aquifer thickness

1) Introduction

The following report is a final report on a study of the general behavior and factors affecting the storage of heat in aquifers. The general concept involved is that heat could possibly be stored for some length of time by simply pumping fluid at an elevated temperature into a suitable aquifer and then later withdrawing the hot fluid.

The practicality of constructing such storage systems turns on several factors. These factors include: (i) availability of a sufficient supply of hot fluid; (ii) occurence of an aquifer of appropriate properties; and (iii) a use for the heat energy that would be stored. The rather indefinite specification of "an aquifer of suitable properties" was taken to cover such considerations as: (i) the aquifer having sufficient transmissivity and size to handle the expected flows and volumes of fluid desired; (ii) the aquifer being located at a practical depth so that well costs would not be prohibitively high; and (iii) that the efficiency of storage be sufficiently high as to enable economic operation of the system. The present work focuses on the latter consideration (storage efficiency), and does not explore the question of where suitable aquifers might be found.

The questions of heat sources and uses, while not part of the present work, are worth considering in order to establish the context of the work. The most obvious sources of low grade waste heat are thermal, (eg. coal fired) power stations. Given that these generating stations are typically 30 to 40% efficient in terms of thermal to electrical energy conversion it is apparent that a great deal of thermal energy is being discarded via smoke stacks and cooling water. While the exact

distribution of losses will vary with station design and operation, it appears that the amount of energy available in the form of hot water will be comparable to the rated capacity of the generating station. Other potential sources of waste thermal energy include pumping stations and some industrial processes. Suitable industrial processes would most likely be ones involving the use of high grade thermal energy such as brick or cement factories. Industries of this type are rather scarce in the prairies but may occur in other parts of the country in conjunction with suitable geological formations.

The principle use for low grade thermal energy is thought to be space heating. In this regard aquifer stored thermal energy could be viewed as interchangeable with geothermal energy. The economics of distribution suggest that it would be strongly preferable to apply the heat energy to large buildings, (ie. apartment complexes, office buildings, large institutions) rather than to attempt to district heating of individual dwellings. In cases where large amounts of heat are available heating greenhouse complexes might be considered. Very few industrial processes require low grade thermal energy although it may be feasible in some instances to top up the stored fluid to suitable temperature.

Storage of energy does not, as such, create any additional energy but rather saves energy for use at a future time. In the space heating example presumably no heating energy would be required during summer months. The storage system could then save this otherwise unusable energy for use during winter months or approximately double the energy available for heating provided storage were efficient. An additional benefit might be a reduction of thermal pollution arising from dumping

the hot waste water during summer months. There could also be an opportunity factor if a heat source were available during only part of the year. Collection of solar energy would fall into this category. It is doubtful that such sources would play a significant part in large scale systems.

Assuming there is motivation to store the thermal energy it is apparent that the feasibility of doing so will turn upon the efficiency of storage. That is, for every unit of energy put into the system what fraction can be recovered? A second question that is somewhat less obvious is at what temperature can the energy be recovered? The following modelling work has been undertaken to seek answers to these questions and to obtain insight into the sensitivities of such storage systems.

2) Proposed and accomplished work

The original proposal divided the anticipated work into six tasks. These tasks were as follows:

Task 1. Develop basic model

Establish a computer model of a simple aquifer bounded by impermeable aquicludes. Describe fluid flow by Theis equation. Model vertical conductive heat flow across aquifer boundaries into or out of boundary strata.

Task 2. Model vertical fluid flow

Incorporate vertical changes of lithology that include aquifers and aquitards. Model vertical fluid and heat flow between aquitards and aquifers.

Task 3. Model vertical boundary effects

Incorporate realistic geothermal gradient as part of initial boundary conditions. Incorporate a variety of strata above and below the producing or storage reservoir interval. Incorporate surface effect.

Task 4. Develop higher order solutions.

Develop improved Integrated Finite Difference Method (IFDM) techniques. Improve accuracy in time discretization. Seek alternate system solvers.

Task 5. Incorporate "minor" effects.

Include dependence of system parameters such as fluid density and possibly specific heat on temperatures and pressures.

Task 6. Multiple aquifers and other topics of interest

Model fluid flow from multiple aquifer system with a range of hydrostatic heads. Incorporate other topics of interest which might include: (i) buoyancy effects if density of injected liquid differs from density of produced fluid; (ii) heat flow from a subjacent or superjacent aquifer containing fluid with high rates of regional flow but virtually no fluid connection with the storage aquifer. This task grouping was included on a as time and practically permits basis.

The above proposals were of course made a priori. With the usual wisdom of hindsight, and in the interests of attaining the best results for effort expended, it was found expedient to make a number of changes in detail. Tasks 1), 2), and 3) were completed basically as intended although some negative findings permitted simplifications to the model. Task 4) was found to be very much larger than anticipated. This should

not be interpreted in a negative sense since the improvements obtainable have also turned out to be much greater than expected. An inconvenient consequence of the magnitude of this task is that it has not been possible to link together the diverse improvements into a coherent overall scheme within the permitted time frame. It is expected that research will continue in this area so as bring the improvements to full fruition.

Fairly extensive work was done on Task 5) in terms of assembling the required data and functional relationships. The findings of the previous tasks indicated that the impact of the "minor" effects would be so slight as to be within the probable numerical error of the model constructed. Implementation of these modifications was consequently suspended.

The multiple aquifer aspect of the suggested research of Task 6) was formulated in terms of the parameters applying to the University of Regina geothermal well. As discussed below under section 7 this task proved beyond the capabilities of the computational machinery available during the time frame of the present work, when formulated in terms of conventional techniques.

3) The radial model

3.1) Formulation of radial model

The simplest representation of an aquifer storage system is one in which only radial flows are considered. Realism would be improved by introducing vertical heat flows occur by means of thermal conduction. Suppression of vertical heat flow, however, gives a clearer insight into the loss characteristics inherent in the aquifer. For this reason the

vertical thermal conductivity was assumed to be zero in this first model.

If only radial fluid flow from a single well is being considered the fluid flow can be described by the Theis solution. This modelling technique has previously been applied to geothermal modelling and is known to be computationally efficient. Since this solution was known it was felt that it would be desirable to implement a numerical solution so that comparison of accuracy could be made. This proved a fortunate choice.

The aquifer storage system requires two partial differential equations (PDE's) for its description. These equations, in integral form, are the flow equation

$$\oint_B K_H \nabla h \cdot \hat{n} dB = \frac{\partial}{\partial t} \int_V S_S h dv \tag{1}$$

which describes the fluid flow of the system, and the energy, (or convective-diffusion) equation

$$\oint_B K_T \nabla T \cdot \hat{n} dB + \oint_B \rho c K_H T \nabla h \cdot \hat{n} dB = \frac{\partial}{\partial t} \int_V \rho_A c_A T dv \tag{2}$$

which describes the flows of energy within the system. The equations in their more familiar differential form are

$$\nabla^2 h = \frac{S}{T} \frac{\partial h}{\partial t} \tag{3}$$

and

$$K_T \nabla^2 T + \rho c K_H \nabla (T \cdot \nabla h) = \frac{\partial}{\partial t} \rho_A c_A T \tag{4}$$

respectively, where

$$p_A c_A = npc + (1-n)p_R c_R \tag{5}$$

It is convenient to adopt a radial coordinate grid system to describe radial flow with respect to a single well. If the flow equation is spatially discretized by use of the integrated finite difference method (IFDM) the semi-discretized result is

$$\left[\frac{2T'}{SW_i \Delta r_{i-1}}\left(r_i - \frac{\Delta r_{i-1}}{2}\right)\right] h_{i-1}$$

$$- \left[\frac{2T'}{SW_i \Delta r_{i-1}}\left(r_i - \frac{\Delta r_{i-1}}{2}\right) + \frac{2T'}{SW_i \Delta r_i}\left(r_i + \frac{\Delta r_i}{2}\right)\right] h_i$$

$$+ \left[\frac{2T'}{SW_i \Delta r_i}\left(r_i + \frac{\Delta r_i}{2}\right)\right] h_{i+1} = \frac{\partial h_i}{\partial t} \tag{6}$$

where

$$\Delta r_i = r_{i+1} - r_i$$

$$\Delta r_{i-1} = r_i - r_{i-1}$$

and

$$W_i = \left(r_i + \frac{\Delta r_i}{2}\right)^2 - \left(r_i - \frac{\Delta r_{i-1}}{2}\right)^2$$

The general two point time discretization is obtained by equating a weighted sum of the spatial derivatives for present and future times to the time derivative of the function. That is, for the flow equation the temporal semi-discretization becomes

$$\frac{\partial h_i}{\partial t} = \frac{T'}{S} \nabla^2 h_i^{n+1} + (1-\lambda)\frac{T'}{S} \nabla^2 h_i^{n} \tag{7}$$

The Crank-Nicolson scheme is obtained if $\lambda = 1/2$ and the fully implicit scheme if $\lambda = 1$. The fully implemented discretized flow equation is then

$$
\begin{aligned}
&- \left\{ \frac{2\lambda T' \Delta t}{SW_i \Delta r_{i-1}} \left(r_i - \frac{\Delta r_{i-1}}{2} \right) \right\} h_{i-1}^{n+1} \\
&+ \left\{ 1 + \frac{2\lambda T' \Delta t}{SW_i} \left[\frac{1}{\Delta r_{i-1}} \left(r_i - \frac{\Delta r_{i-1}}{2} \right) + \frac{1}{\Delta r_i} \left(r_i + \frac{\Delta r_i}{2} \right) \right] \right\} h_i^{n+1} \\
&- \left\{ \frac{2\lambda T' \Delta t}{SW_i \Delta r_i} \left(r_i + \frac{\Delta r_i}{2} \right) \right\} h_{i+1}^{n+1} = \\
&\left\{ \frac{2(1-\lambda) T' \Delta t}{SW_i \Delta r_{i-1}} \left(r_i - \frac{\Delta r_{i-1}}{2} \right) \right\} h_{i-1}^{n} \\
&+ \left\{ 1 - \frac{2(1-\lambda) T' \Delta t}{SW_i} \left[\frac{1}{\Delta r_{i-1}} \left(r_i - \frac{\Delta r_{i-1}}{2} \right) + \frac{1}{\Delta r_i} \left(r_i + \frac{\Delta r_i}{2} \right) \right] \right\} h_i^{n} \\
&+ \left\{ \frac{2(1-\lambda) T' \Delta t}{SW_i \Delta r_i} \left(r_i + \frac{\Delta r_i}{2} \right) \right\} h_{i+1}^{n} \qquad (8).
\end{aligned}
$$

A corresponding but somewhat more complicated result obtains for the discretization of the energy equation. In the case of the energy equation considerable simplification results from choosing $\lambda = 1$. This is a desirable course of action since the fully implicit scheme exhibits much less overshoot at the front than does the Crank-Nicolson scheme.

The radial flow equation proves to be quite difficult to solve accurately by numerical means. Use of the Crank-Nicolson scheme from the start results in violent oscillations in the solutions that damp out very slowly. Since new shocks are created every time the well pumping is reversed the solution is effectively worthless. The fully implicit scheme results in a solution with very minor oscillations, however, since the method is ony first order accurate the accuracy of solution

for hydraulic head values is not very good. A combination of the time discretization techniques was found to give the best results. The fully implicit scheme is used for the first time step after initiation or reversal of pumping. The weighting factor, λ, is then reduced over a number of time steps to a value of 1/2 so that the second order Crank-Nicolson scheme is used for most of the computations. This approach yields hydraulic head values that are about 1.5% in error for the first kilometer of the solution with the exception of the head predicted for the well face which is typically in error by 25 to 30%. Fortunately the hydraulic gradients predicted by the above scheme are accurate to within 0.1% over the spatial range of interest. Since the hydraulic gradients are the critical values required in the energy equation the above scheme was felt to be adequate for present purposes, although this would not necessarily be the case in more complex problems. The error characteristics certainly provide stimulus for seeking better solution schemes.

3.2) Results of the radial model

The purely radial flow model is clearly not a very realistic storage model and for that reason rather little effort was put into producing results with it. Representative temperature history curves for outtaken fluid are depicted in figure one. The temperature history curves are given for several storage cycles and show two features of particular interest. These are: that a marked decline in the temperature of the outtaken fluid occurs in the latter part of the withdrawal phase; and that the temperature declines become less severe with repetition of the storage cycles. Improvement in outtake temperature occurs rapidly at first but quickly slows down.

Storage efficiency versus cycles are given in table one. Storage efficiency has been defined as the percentage ratio of sensible energy outtaken to sensible energy input for a given storage cycle. Sensible energy was taken to be the thermal energy contained in the working fluid above 33°C. In the present case 60°C fluid was injected into a 33°C aquifer. Two points are to be made of the storage efficiency results: firstly that the storage efficiencies are quite high; and secondly, as might be expected from the temperature history curves, the efficiency improves with successive storage cycles.

Table One

Aquifer Storage Efficiency (radial model)

Cycle	% storage efficiency
1	85.6
3	91.1
5	92.9
10	94.6

The presence of an auto conditioning effect led to consideration of how this improvement might be enhanced or speeded. Investigation of this matter was, however, not pursued for the radial model.

4) The cylindrical model

4.1) Development of the cylindrical model

The same governing equations, (equations 1) and 2)) that describe the physics of the radial model also apply to a cylindrical model. The coordinate system differs from the radial model in that the cylindrical shell elements are now divided in the vertical dimension and flows in the vertical sense are allowed for in the discretization of the equations.

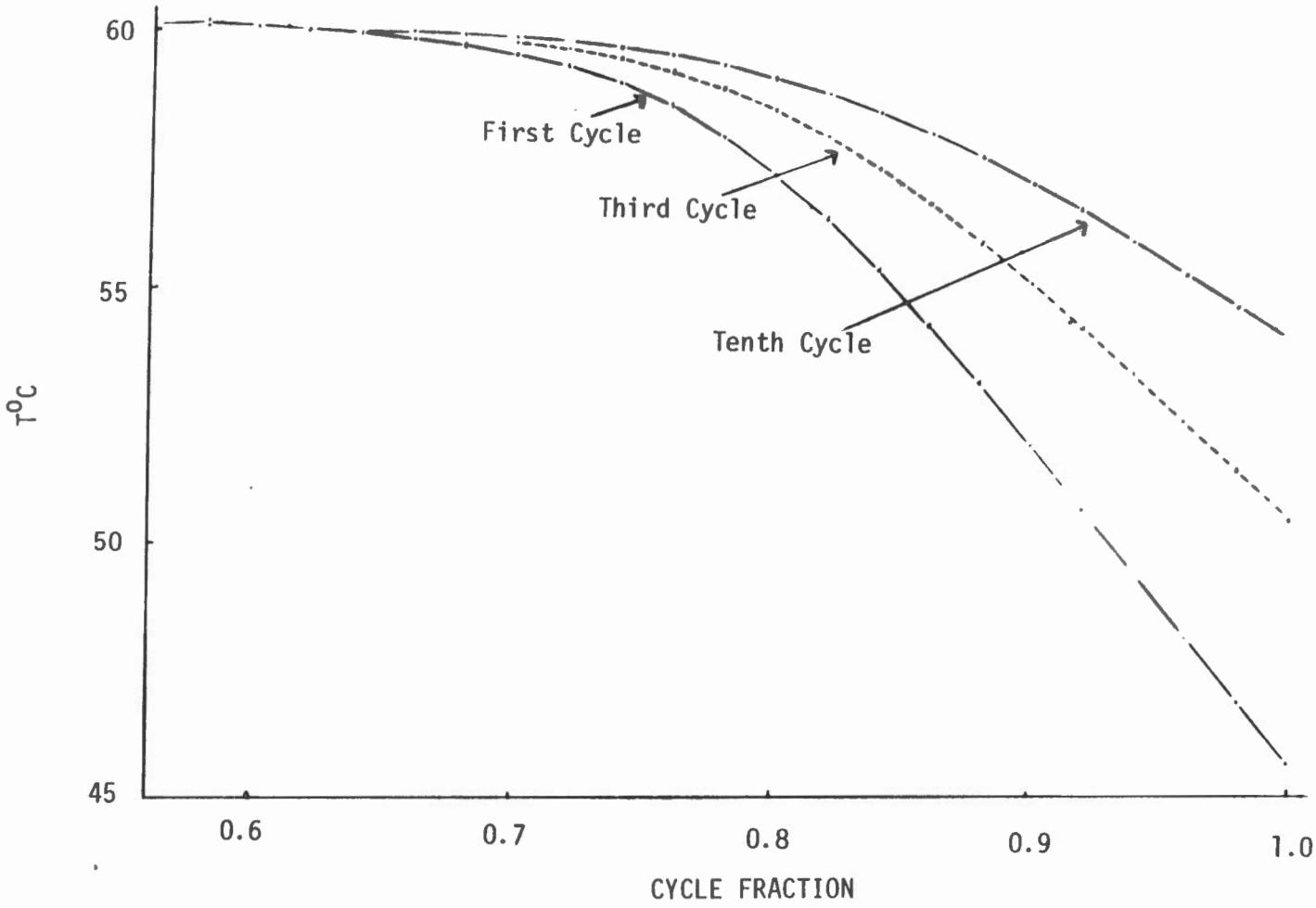


Figure One

Temperature History (Radial Model)

Original specification of the problem called for inclusion of varied geological structure, geothermal gradient, and possible surface effects. These were investigated to a limited degree as will be commented on in the results section below. In the interests of improving understanding of the consequences of changing system parameters a somewhat simplified system was used for most runs. In the simplified model the geothermal gradient was omitted and a single variety of vertical boundary strata assumed. These assumptions permit a reduction of the size of the numerical problem by use of a plane of symmetry through the centre of the aquifer. Most model runs investigating model sensitivity to parameter changes were made using the reduced version of the cylindrical model.

4.2) Cylindrical storage model results

4.2.1) The standard model

A "typical" storage aquifer was assumed as a standard case for the study. The standard storage aquifer was assumed to be of constant thickness, horizontal, and of effectively infinite horizontal extent. The parameters were generally assumed to be macroscopically constant except for hydraulic conductivity. The hydraulic conductivities were assumed to be anisotropic with horizontal hydraulic conductivity being substantially larger than the vertical hydraulic conductivity. Similar assumptions were made with respect to the aquitards. Regional flow was presumed negligable. The specific parameter values used in the standard run are given in Appendix I. It was also assumed that the volume of input fluid was equal to the volume of outtaken fluid for any given cycle. While this does not require the input and outtake phases of a

given cycle to be of equal length this assumption was used for most runs. Brief experimentation with unequal input and outtake phases using the radial model indicated that very small decreases in storage efficiency are to be expected for unequal phases.

Storage efficiencies and terminal temperatures are shown in figure two. Storage efficiency has been defined as the percentage ratio of sensible heat energy outtaken to sensible heat energy input for a given cycle. Sensible heat being taken to be the heat content of the fluid above 33^{0}C. For the standard run input fluid temperature was set to 60^{0} C and the background temperature of the aquifer and environs set to 30^{0} C. Terminal temperature is simple the temperature of the outtaken fluid at the end of the outtake phase.

The essential features of the storage efficiency curve are that it starts at a reasonably high storage efficiency, (75%) and rises rapidly for the first few years. The rate of improvement slows with time so that improvements in annual cycle efficiencies increase quite slowly after about ten years. The storage efficiency is approximately 88% on the tenth cycle and reaches 90% after approximately 23 years. These values are felt to be quite encouraging insofar as storage efficiency determines the practicality of aquifer thermal storage.

The terminal temperature curve shows behavior similar to the storage efficiency curve in that improvement is rapid for the first few cycles and then tends to level off. While the improvement characteristics are useful the actual values of terminal temperatures are somewhat disquieting. The terminal temperature of the first cycle is approximately 41^{0}C. This means 8^{0}C for the sensible heat range as

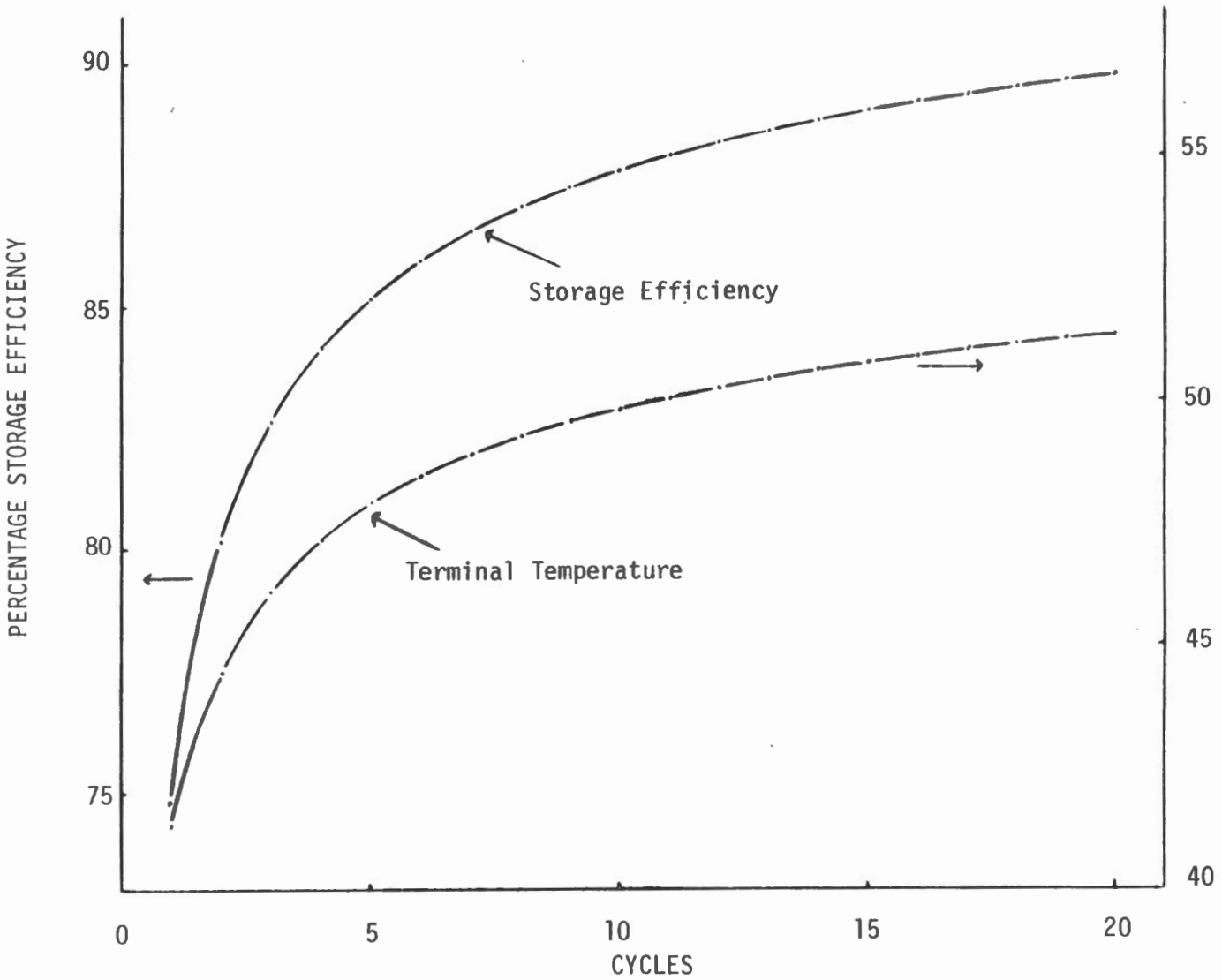


Figure Two

Standard Run

compared to 27°C in the input fluid. Thus the fluid produced at the end of the outtake phase contains only 30% as much sensible heat per unit volume as does the fluid produced at the start of the outtake phase. In addition to the reduction of sensible energy content the temperature decline is likely to result in a reduction of heat exchanger efficiency or a substantially reduced coefficient of performance if heat pumps are used.

Admittedly the terminal temperature occurs only at the end of the heating season when lesser amounts of heating energy should be required. The values cited above therefore indicate a rather bleaker picture then is the reality. Nevertheless the temperature of the produced fluid could constitute a serious problem if substantial temperature declines occur well within the heating season.

Temperature history curves for the first, third, and tenth cycles of the standard model are shown in figure three. The curves can be described as having three characteristic periods. These are: a brief period during which the temperature of the produced fluid is essentially at input temperature; a rather lengthier period, (40 to 50% of outtake phase) in which the temperature of the produced fluid, while somewhat below input temperature, declines very little; and an end period in which temperature decline is fairly rapid. These characteristics may be explained as follows.

The first period results from hot fluid flowing back through a fully warmed aquifer. The second period corresponds to cooled fluid flowing back over a largely warm aquifer and harvesting heat from the aquitards that had been stored there during the input phase. The third

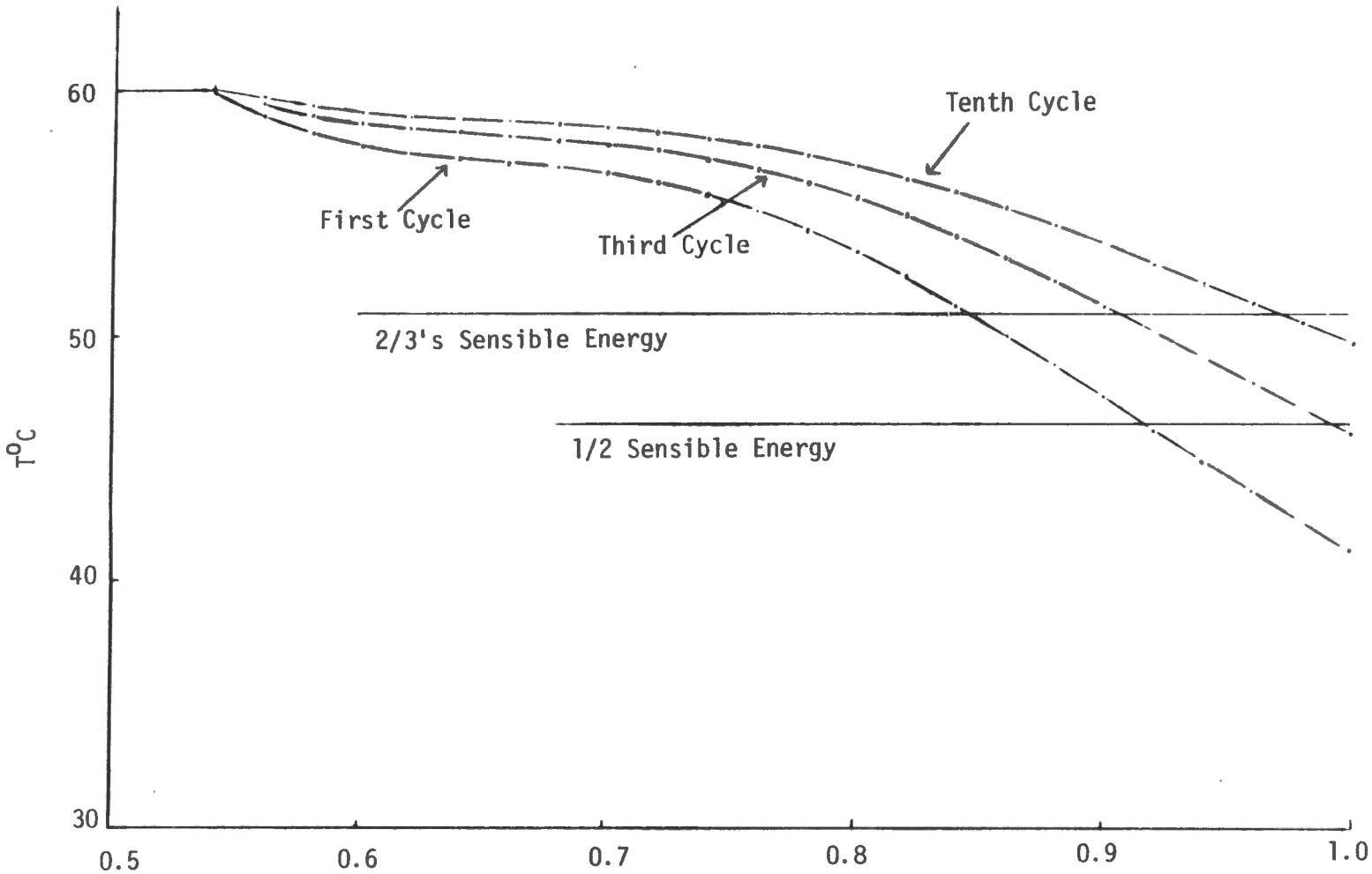


Figure Three

Temperature Histories (Standard Run)

period is one in which the harvest of heat from the aquitards is largely complete and cooled water is flowing back through an increasingly cooled aquifer.

As with the efficiency curves the temperature history curves can be said to improve with additional cycles. The first cycle is such that the sensible heat context per unit volume drops to 2/3's input value after 0.69 of the outtake phase, and 1/2 the input value by 0.83 of the outtake phase. The temperature history curve for the third cycle shows considerable improvement with the sensible heat content per unit volume dropping below 2/3's input value after 0.81 of the outtake phase, and dropping below 1/2 only at the very end of the phase. By the tenth cycle the sensible heat content per unit volume of produced fluid drops below 2/3's input value for only a brief time at the end of the outtake phase.

The importance of the decline in sensible heat content of the produced fluid unquestionably depends on the climatic characteristics of the project site. For a location such as Regina it is suspected that the declines occur uncomfortably early in the heating season and are of bothersomely large magnitude for at least the first two or three cycles. In addition the variation in system behavior may pose substantial engineering problems in system design. These inherent difficulties with the start up aspects of the standard model stimulated interest in the question of how the temperature characteristics of the produced fluid might be bettered.

4.2.2) The conditioning effects

The observation that the quality of the storage system improves with successive cycles, (ie. the system is auto-conditioning in a posi-

tive sense) holds forth the possibility of speeding or enhancing the conditioning process. It may also be observed that if only part of the input volume of fluid were withdrawn that the temperature at the end of this shortened outtake phase would be higher than if a normal outtake phase were used. Since shortening the duration of outtake phase poses major difficulties the approximately equivalent alternative of lengthening the initial input phase by starting it early was considered. In this approach to improving the temperature characteristics of the storage system it is assumed that the additional input would be sacrificed to a preheating process. Ideally one would be creating a pad of warm water outside the normal storage volume that can be allowed to gradually dissipate. Two a priori concerns that arise concerning this conditioning process are that long term efficiency is not adversely affected and that the conditioning does not in some sense wear out.

Temperature history curves for the first storage cycle corresponding to 0.1 and 0.2 year additional initial inputs are shown in figure four. The evidence of improvements in the temperature of produced fluid during the latter half of the outtake phase is readily apparent. In particular the strong temperature declines are postponed until later in the outtake phase and terminal temperature drops are less severe. The decline in sensible heat content per unit volume to below 2/3's original is shifted from 0.69 of the outtake phase in the no conditioning case to 0.82 for 0.1 year case. The 0.2 year conditioning case drops below 2/3's initial sensible heat content per unit volume only briefly near the end of the outtake phase. The improvement in terminal temperature for 0.2 year conditioning is such that the sensible heat content at the end of the first cycle is twice as great as in the unconditioned case.

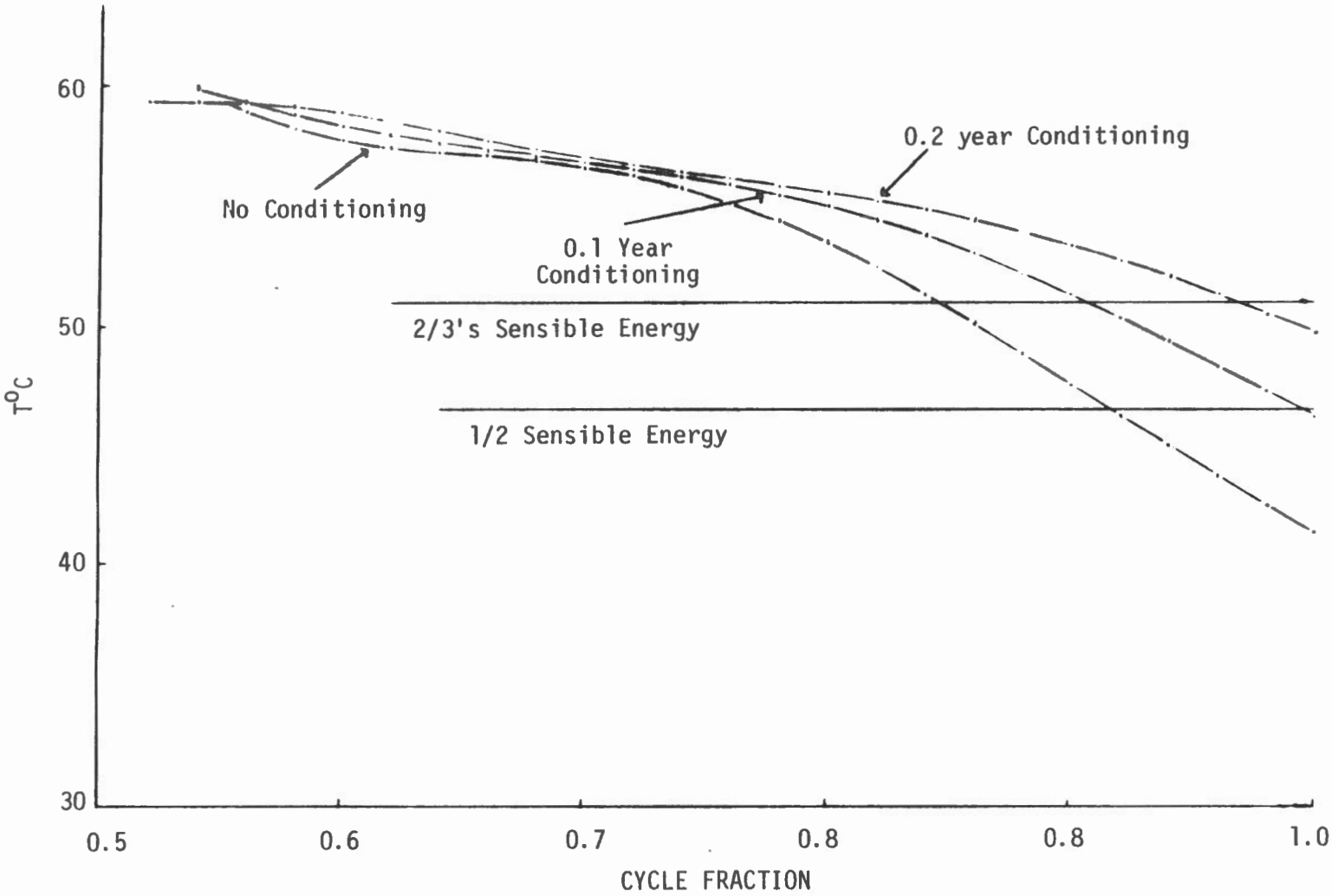


Figure Four

Conditioning Effect (First Cycles)

Temperature history curves for the 0.2 year conditioning case are shown in figure five. The improvements in temperature show the same general pattern of improvement with repeated cycles as does the unconditioned case. An important difference is that there is a small drop in terminal temperature from the first cycle to the second cycle. Improvement in the more important midrange is, however, continuous. From a system design point of view the conditioned system has the merit of varying much less than the unconditioned system.

Storage efficiencies for the unconditioned and different conditioned systems are given in figure six. The additional initial input used for conditioning has been assigned to the first cycle. This causes a substantial drop in storage efficiency for the first cycle of the conditioned systems. This drop in initial cycle efficiency is rapidly compensated for by a decided improvement in the storage efficiency of the second and subsequent cycles. The storage efficiencies of the conditioned and unconditioned systems draw closer together with repeated cycles so that long term efficiency is very little effected by conditioning.

Terminal temperatures of successive cycles for conditioned and unconditioned cycles are shown in figure seven. In general conditioning results in substantial improvements to the terminal temperature of the initial cycle although the process is apparently subject to diminishing returns with increasing amounts of conditioning. If what might be regarded as large amounts of conditioning, (0.2 years) are used there is a drop in the terminal temperature from the first to second cycles. Thereafter continuous improvement in terminal temperature is predicted.

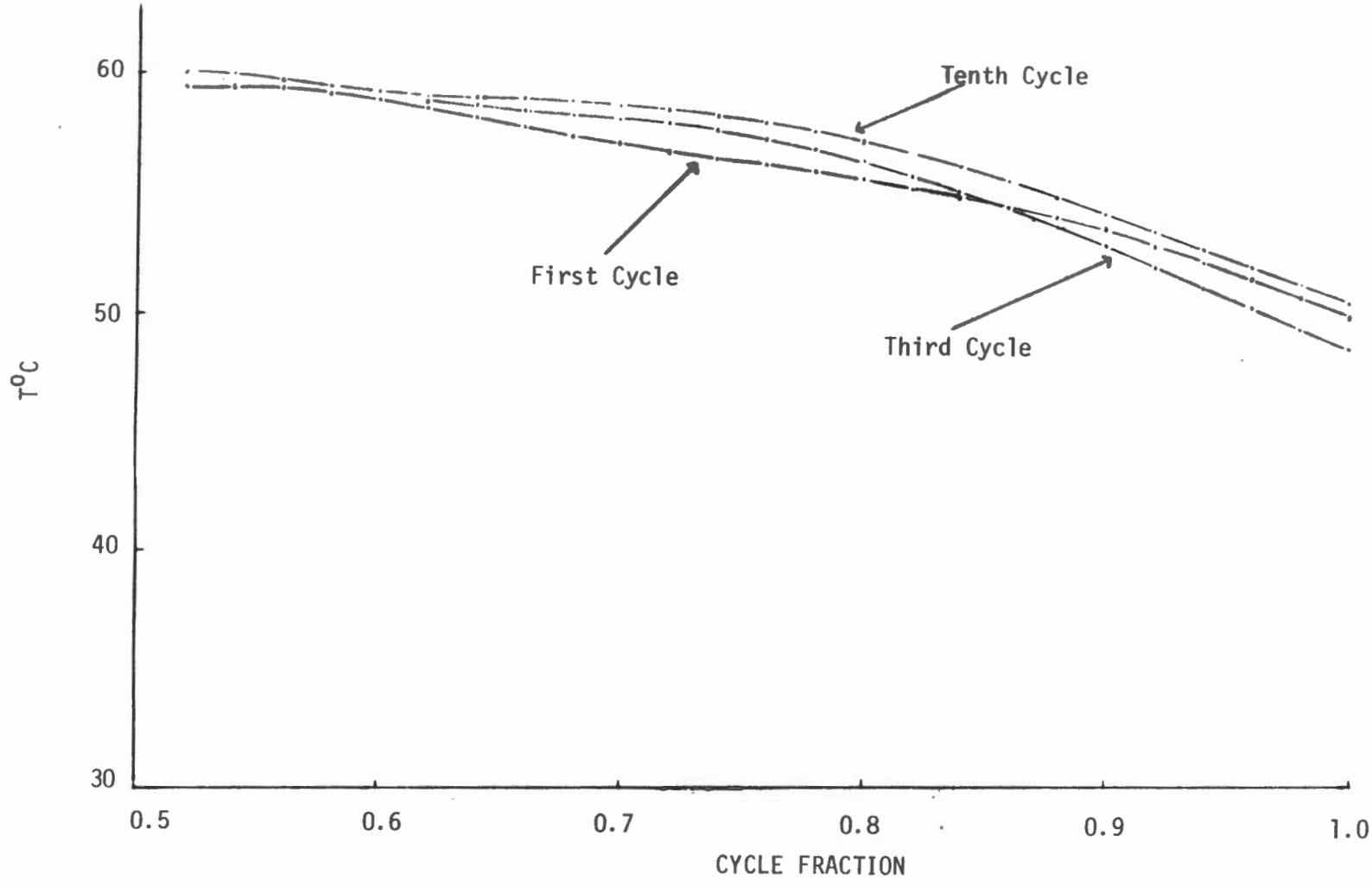


Figure Five

Temperature History (0.2 Year Conditioning)

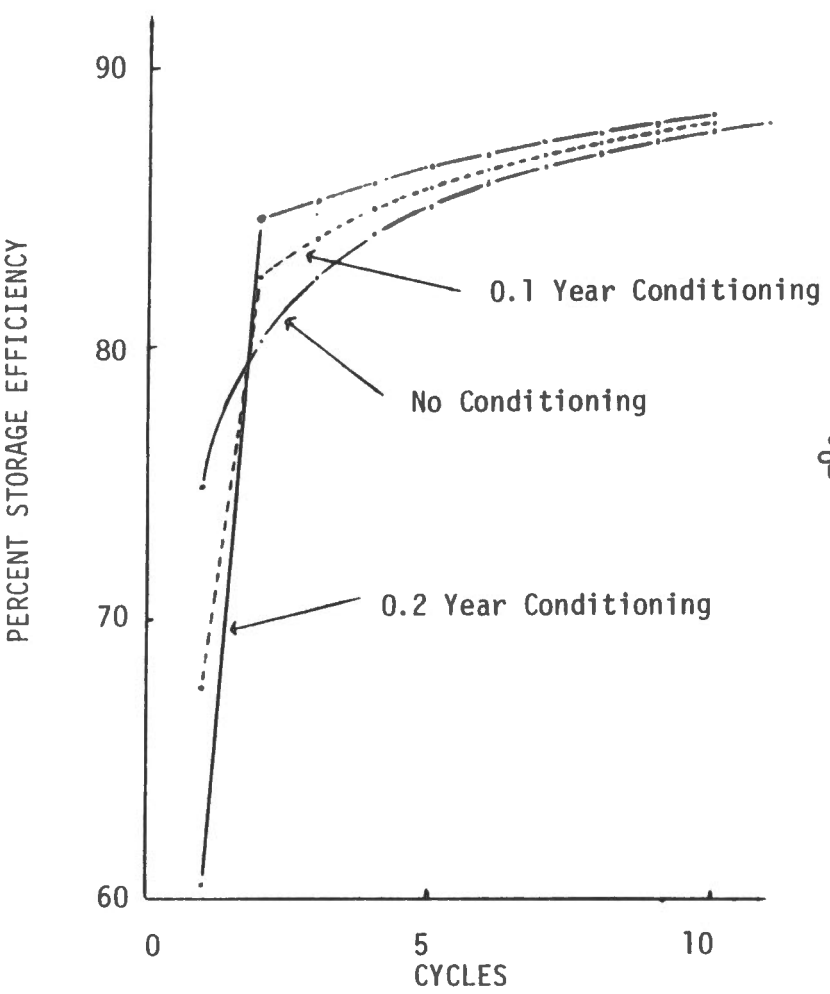


Figure Six

Conditioning and Storage Efficiency

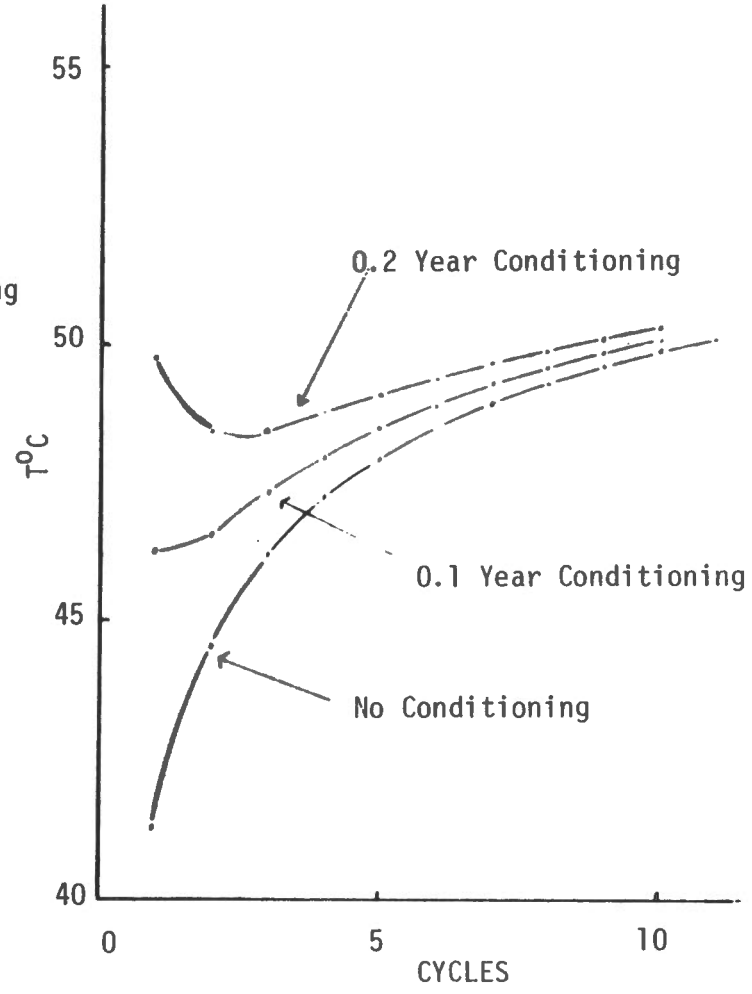


Figure Seven

Conditioning and Terminal Temperatures

Terminal temperatures of conditioned systems are always higher than those of unconditioned systems.

4.2.3) Model sensitivity

It is reasonable to expect that some aquifers might be preferable to others for use as storage systems. In order to explore the question of what are desirable aquifer properties number of model runs using different properties than the standard run were made. The process was carried out by varying one parameter at a time. This way of proceding also allows one to investigate the question of model sensitivities, that is to determine which, if any, parameter changes cause large changes in the model results. Some thirteen cases were considered. These were:

1) the standard model
2) standard with 0.1 year conditioning
3) standard with 0.? year conditioning
4) increase pumping rates to 1.5 times standard
5) decrease pumping rates to 0.5 times standard
6) increase aquifer thickness to 1.5 times standard
7) decrease aquifer thickness to 0.5 times standard
8) increase vertical hydraulic conductivity of aquitard to 10 times standard
9) decrease vertical hydraulic conductivity of aquitard to 0.1 times standard
10) increase porosity of aquifer to 0.30
11) decrease porosity of aquifer to 0.20
12) decrease aquifer original temperature to 20°C
13) increase temperature of input fluid to 80°C

The results of these runs are given in the following tables. Table two gives storage efficiencies for individual cycles for the various

Table Two

Annual Storage Efficiencies (%)

cycle/ case	1	2	3	4	5	6	7	8	9	10
1	74.8	80.1	82.6	84.1	85.1	85.9	86.5	87.0	87.4	87.8
2	67.6	82.6	83.9	85.0	85.7	86.4	86.9	87.3	87.7	88.0
3	60.5	84.7	85.3	85.9	86.5	86.9	87.3	87.7	88.0	88.3
4	76.0	81.2	83.6	85.1	86.1	86.8	87.4	87.9	88.3	88.7
5	71.8	77.5	80.0	81.6	82.7	83.5	84.1	84.6	85.1	85.4
6	77.4	82.2	84.4	85.7	86.6	87.3	87.8	88.3	88.6	88.9
7	66.1	72.9	76.1	78.0	79.4	80.5	81.3	82.0	82.5	83.0
8	75.2	80.5	82.9	84.3	85.4	86.1	86.7	87.2	87.6	87.9
9	74.6	80.0	82.5	84.0	85.0	85.8	86.4	86.9	87.3	87.6
10	75.5	80.7	83.1	84.6	85.6	86.4	86.9	87.4	87.8	88.1
11	74.1	79.5	82.1	83.6	84.7	85.5	86.1	86.6	87.0	87.3
12	66.4	73.5	76.8	78.8	80.2	81.3	82.0	82.7	83.2	83.7
13	75.9	81.0	83.4	84.8	85.8	86.5	87.1	87.6	88.0	88.3

Table Three

Cumulative Efficiencies to Date

cycle/ case	1	2	3	4	5	6	7	8	9	10
1	74.8	77.5	79.2	80.4	81.4	82.1	82.7	83.3	83.7	84.1
2	67.6	75.1	78.0	79.8	81.0	81.9	82.6	83.2	83.7	84.1
3	60.5	72.6	76.8	79.1	80.6	81.6	82.4	83.1	83.6	84.1
4	76.0	78.6	80.3	81.5	82.4	83.1	83.8	84.3	84.8	85.1
5	71.8	74.6	76.4	77.7	78.7	79.5	80.2	80.7	81.2	81.6
6	77.4	79.8	81.3	82.4	83.3	83.9	84.5	85.0	85.4	85.7
7	66.1	69.5	71.7	73.3	74.5	75.5	76.3	77.0	77.6	78.2
8	75.2	77.8	79.5	80.7	81.6	82.4	83.0	83.5	84.0	84.4
9	74.6	77.3	79.1	80.3	81.2	82.0	82.6	83.2	83.6	84.0
10	75.5	78.1	79.8	81.0	81.9	82.7	83.3	83.8	84.2	84.6
11	74.1	76.8	78.6	79.8	80.8	81.6	82.2	82.8	83.2	83.6
12	66.4	70.0	72.3	73.9	75.2	76.2	77.0	77.7	78.3	78.9
13	75.9	78.4	80.1	81.3	82.2	82.9	83.5	84.0	84.4	84.8

Table Four

Terminal Temperatures (°C)

cycle/ case	1	2	3	4	5	6	7	8	9	10
1	41.3	44.5	46.2	47.2	47.9	48.5	48.9	49.3	49.6	49.9
2	46.2	46.5	47.3	47.9	48.5	48.9	49.3	49.6	49.8	50.1
3	49.8	48.4	48.4	48.7	49.1	49.4	49.7	49.9	50.1	50.3
4	41.5	--	46.4	--	--	--	--	--	--	50.2
5	40.9	43.9	45.5	46.5	47.2	47.8	48.2	48.6	48.8	49.0
6	41.9	--	46.7	--	--	--	--	--	--	50.3
7	39.8	--	44.5	--	--	--	--	--	--	48.5
8	41.4	44.6	46.2	47.3	48.0	48.6	49.0	49.4	49.7	50.0
9	41.3	44.5	46.1	47.2	47.9	48.5	49.0	49.3	49.6	49.9
11	41.2	44.4	46.0	47.1	47.8	48.3	48.8	49.2	49.5	49.8
12	35.1	39.3	41.5	43.0	43.9	44.7	45.3	45.8	46.2	46.5
13	48.8	--	56.9	--	--	--	--	--	--	63.2

cases for each of the first ten cycles. Table three shows cumulative efficiencies to date, that is, the average efficiency of the system over all cycles up to and including the given cycle. Table four shows terminal temperatures for the different cases for the first ten cycles.

The most striking feature of the results given in the tables is that they do not vary very much. This is especially true of the cumulative storage efficiency after ten years, (and presumably thereafter). Most of the cumulative storage efficiencies are in the range 84±1%. The exceptions are case 7, (thin aquifer, 78.2%) and case 12 (cold aquifer, 78.9%). The best situations are case 6, (thick aquifer, 85.7%) and case 4, (increased pumping rate, 85.1%). It is noted that the ten year cumulative storage efficiencies are essentially identical for the standard and conditioned cases, (cases 1, 2, and 3). Since the annual storage efficiencies are, at the ten year point, slightly higher for the conditioned cases than the standard case long term storage efficiency would be benefited by either of these levels of conditioning.

In general the storage system benefits from changes that result in a storage volume of increasing sphericity, (increasing aquifer thickness, increasing porosity, more leaky aquitards, etc.). Improvement in the surface to volume ratio is also believed to be the explanation for improvements in storage efficiency with increasing pumping rate.

Produced fluid temperature near the end of the outtake phase, particularly for the first few cycles, is generally rather low for all of the unconditioned cases. Conditioning as described above would seem desirable especially for cases involving low aquifer original temperature, (case 12) or high input temperatures, (case 13).

If a storage system were used for a few decades, (say around 40 years) the above evidence indicates cumulative storage efficiencies in the 80 to 90% range. It should be noted that the present model does not take into account regional flow or hydrodynamic dispersion. The presence of these effects would undoubtedly lower storage efficiency.

4.2.4) Size and shape of thermal disturbance

In considering the possible consequences of changes in the environment of the storage aquifer it is useful to consider the size and shape characteristics of the thermal disturbance associated with the standard run. The rationale for this examination is that changes in lithology occuring beyond the thermally active zone are of no consequence to the storage system.

The radial distance of influence within the aquifer depends strongly on the geometry of the aquifer and the pumping rate. This in turn determines the surface area of the aquitards through which vertical energy flows can occur. Since this surface area can be quite large (thousands of m^2), the properties of the immediately adjacent aquitard are clearly of importance. The answer to what constitutes effective adjacency depends on how far, in the vertical sense, thermal changes occur.

In the standard model the thermal front reaches out about 55m radially at the end of the first input phase. Discernable thermal changes (approximately one degree centigrade), occur out to about 75m. After 20 years the radial front has moved out to about 65m at the end of the input phase and discernable thermal changes are observed out to a little over 100m. In these cases the front position has been defined as

the radial distance at which a temperature half way between input temperature and the lower limit of sensible energy temperature is observed.

The movement of the thermal front in the vertical direction is much smaller than movement in the radial direction because of the much smaller hydraulic conductivity in the vertical direction in the aquitard. The movement is greatest in the vicinity of the well since greatest changes in head and longest exposure times to highest temperatures occur there. At a radial distance of 5m from the well the front position, as defined above, is 9, 11, and 12m at 10, 15, and 20 years respectively. Obviously front movement has become very slow and should move only a short distance in the next two decades. The position of the front is somewhat misleading as to the significant depth of penetration of important temperature changes. An indication of the significant depth of penetration is given by the distance that heat drains back into the aquifer from the aquitards during the outtake phase. This drainage distance can be defined by observing which solution nodal points decrease in temperature and those which increase between the end of a given input phase and the end of the associated outtake phase. The thermal drainage distance for a point 5m radially from the well is about 10m after 15 years and about 12m after 20 years. For a point 30m radially from the well the thermal drainage distance is less than 6m and even less a greater radial distances after 20 years. On the basis of the model sensitivity results it seems unlikely that these values would change much for any plausible changes in model parameters. The conclusion that appears to be compelled by these observations is that changes in lithology in the vertical direction are unlikely to be of

significance if they occur at more than around 10m from the vertical bounds of the aquifer, and that changes occurring at more than 20m are certainly unimportant unless the aquitards have very large vertical hydraulic conductivities.

4.2.5) The geothermal gradient and other effects

A model run with vertical extension both above and below the aquifer was made with a superimposed geothermal gradient of 2°C/100m. This necessarily causes the thermal disturbance to be asymmetrically distributed about the aquifer in vertical sense. The inclusion of the geothermal gradient did not significantly alter the model results. There are two reasons why this should be so: firstly the geothermal flux (a few 10's of milliwatts per m^2) is very small in comparison to the amount of energy being cycled through the storage system; and secondly that if the upper and lower aquitards are of similar properties what is gained in one is lost in the other.

The significance of vertical changes in lithology have been considered in the previous section. Similar comments apply to the possibility of surface effects. It was thought unlikely that one would find an acceptable major aquifer starting only a few meters below surface. In light of the generally negative findings on these topics work on them was discontinued.

5) Improvements to mathematical techniques

In the proposal for the present work a section was included for an investigation into improvements to the numerical techniques used in the modelling work. It was known that there were some problems with existing techniques particularly in the area of spatial discretization. It was expected that rather limited improvements would be possible and

that the endeavor would constitute a minor part of the present research project. In actuality the findings have added new dimension to the concept of serendipity.

The general problem of improving the numerical techniques as applied to the solution of PDE's can be divided into four areas. These are time integration, spatial discretization, solver techniques, and error detection and control. The following discussion is so divided. On the presumption that readers of the present report will be primarily interested in the dynamics of aquifer heat storage systems, the mathematics have been strongly summarized. It is hoped that full details can be published in the reasonably near future.

5.1) Improvements to time integration

The standard time integration techniques currently in widespread use are very predominantly two point schemes. That is information from the present time (t_n), and the time at which the solution is desired (t_{n+1}) is used. These solutions can at most be second order accurate (the Crank-Nicolson scheme), and are in general only first order accurate, (classical implicit or non-uniform weighted schemes). The Crank-Nicolson solution, while unconditionally stable for equations of the form of the flow equation, has rather poor stability properties and consequently suffers severe oscillations in the numerical solution whenever shocks are applied to the system. Examples of shocks are initiation or reversal of pumping.

If either better order of accuracy or improved stability is to be obtained data from more than two time points must be used. The Dahlquist theorem establishes that using additional data from the past

cannot yield an unconditionally stable method that is better than second order accurate. Such being the case it becomes necessary to use more data from the future in order to achieve the improvements. This results in general formulations of the type

$$y_{n+1} = a_{11}y_n + a_{13}y_{n+2} + b_{11}y'_n + b_{12}y'_{n+1} + b_{13}y'_{n+2} \qquad (9a)$$

$$y_{n+2} = a_{21}y_n + a_{22}y_{n+1} + b_{21}y'_n + b_{22}y'_{n+1} + b_{23}y'_{n+2} \qquad (9b)$$

These equations can yield up to fourth order accurate solutions. Improvements to stability can be obtained by using less than fourth order accuracy. Specific solutions can be derived by the method of undetermined coefficients.

If equations (9) are semidiscretized, (ie. discretized in space) they can be written in the form

$$\{y_{n+1}\} = [Q]\{y_n\} + [R]\{y_{n+2}\} \qquad (10a)$$

$$\{y_{n+2}\} = [S]\{y_n\} + [T]\{y_{n+1}\} \qquad (10b)$$

It can be shown that the solution will be stable if

$$p([I-TR]^{-1}[S+TQ]) \leq 1$$

where p denotes spectral radius.

It can be shown that it is possible to derive superstable second order accurate formulas and third order formulas that are strongly stable.

Estimates of the truncation error in the formulas (9) can be obtained in the usual way by substituting in a polynomeal of degree one higher than the chosen order of accuracy of the formula. Since the equations (9) form a coupled pair the observed error (block error) will in general depend on both local truncation errors. The block errors can be derived as:

$$BEy_{n+1} = \frac{a_{13}LE_{n+2} + LE_{n+1}}{1 - a_{13}a_{22}} \qquad (11a)$$

and

$$BEy_{n+2} = \frac{a_{22}LE_{n+1} + LE_{n+2}}{1 - a_{13}a_{22}} \qquad (11b)$$

where LE_{n+1} and LE_{n+2} are the local truncation errors of the individual equations.

The coupling of the solution equations implies that both inner and outer iterations would be required for solution. It can be shown that for the standard test equation (the flow equation), there exists a convergence factor (CF) indicative of the rate of convergence of the outer iterations. The convergence factor is given by

$$CF = \frac{a_{13}a_{22} - (a_{13}b_{22} + a_{22}b_{13})\lambda + b_{13}b_{22}\lambda^2}{1 + (b_{12}b_{23})\lambda + b_{12}b_{23}\lambda^2} \qquad (12)$$

Convergence is speeded by choosing formulas that have a very small CF factor.

Note that if less than maximum possible order of accuracy is chosen free parameters are available than can be used to select desired stability, error, and convergence properties. A variety of formulas have been derived that select for various degrees of these different properties. The performance of some examples is discussed below in the section on results.

The above method is known as a solution scheme for ordinary differential equations where it is known as the block implicit method. To the best of the author's knowledge the method has not been reported as having been applied to PDE's. A possible reason for this happenstance is that the block implicit formulation proves to be rather difficult to

solve with conventional solvers. This observation gave stimulus to the search for an already desired fast solver.

5.2) The search for a fast solver

The discretization of a PDE problem normally gives rise to a matrix equation that must be solved to obtain the desired answer. There are basically two ways of doing this, either by using a direct solver or an iterative solver. The direct solvers are essentially variations on Gaussian elimination and as such are inherently sequential processes.

As an example of the type of problem to be solved consider a one dimensional second order elliptic equation. It can, in this case, be shown that for iterative solvers the amount of work required to reduce a norm of the initial error by a factor 10^{-p} is KpN^W where $W > 1$, K is a proportionality constant, and N is the number of nodal points (equations) in the solution. By way of an example W=2 for the optimal successive overrelaxation method, (opt-SOR). Since the work of solution can be expected to be proportional to N if a sequential computer is employed it follows that the number of iterations required by the opt-SOR method is N. The ideal iterative solver was postulated as having the following properties: it should be insensitive to the error of the initial estimate; it should have a very high rate of convergence so that additional accuracy requires few additional iterations; and the number of iterations required for solutions should be very weakly dependent on N, (ideally W=1).

If one examines a series of successive iterates of a Jacobi solution to our test problem it readily becomes apparent that the procedure suffers from a very slow rate of propagation of boundary information

into the body of the problem. In particular it would require 2N iterations to propagate information about one boundary to the other boundary and back again. Better solvers such as Gauss-Seidel, semi-simultaneous, or SOR propagate twice as fast but still retain the linear functional dependence on N.

One way of speeding the long range flow of information is to use block relaxation. This idea comes from the pre-computer era and was apparently due to R.V. Southwell. The observation had been made that areas of same signed residues often occured in solution processes. The sum of the residues in the block could be made zero by adding the appropriate constant value to all function values in the block. The technique was apparently sufficiently successful that texts from the pre-computer era regarded it as a necessary technique for efficient solutions. Unfortunately the technique relied on the human ability to recognize problem areas. As computers have been very inept at pattern recognition the idea disappeared with the advent of computers.

The idea of block relaxation has been revived and systematically implemented in the present work. This has been accomplished in the following way. The entire problem domain is first regarded as a single block and the sum of the residues forced to zero. The domain is then divided into two equal or approximately equal blocks and these blocks iterated once towards solution by a Gauss-Seidel or semisimultaneous method. The two block system is then divided into four blocks etc. up to the single point solution. The amount of effort required for the large blocks (low frequency) is quite small so that the work required to carry out the process for an elliptical problem in one dimension is

equivalent to two Gauss-Seidel iterations. We adopt the terminology of a "cycle" to express the process of iterating through the levels of blocks involved in a given problem.

The above procedure is rather effective. In effect the procedure is able to convert a Gauss-Seidel or semisimultaneous technique to the equivalent of opt-SOR without the use of an acceleration parameter. The process can be accelerated by applying a multiplicative factor to the cumulative correction of a cycle. While no sound theoretical basis has been found for the a priori calculation of the cumulative acceleration factor it has been found to have a much broader minimum than does the opt-SOR acceleration parameter. Thus small errors in selecting the cumulative acceleration factor have little effect on rate of solution. It is also noted that the systematic block relaxation process is rather insensitive to initial estimates of the answer as the lowest frequency shift automatically centers the estimated answer. Results of this methodology are summarized in table five. The results pertain to the solution of the heat flow equation in a non-uniform medium with Dirichlet boundary conditions. A very long time step was used so the solution is essentially to equilibrium.

Table Five

N dependence of no. of iterations
for solution for various solvers

	opt-SOR	Unaccelerated Cyclic	Accelerated Cyclic
one-dimensional	$N^{1.00}$	$N^{1.00}$	$N^{0.62}$
two-dimensional	$N^{0.50}$	$N^{0.50}$	$N^{0.39}$

The improvements are quite noticeable although not dramatic. In practical terms the accelerated cyclic solver might solve about five times as fast as practical SOR for a typical problem. The observed results for coupled equations (arising from the three point time integration schemes above) are rather better.

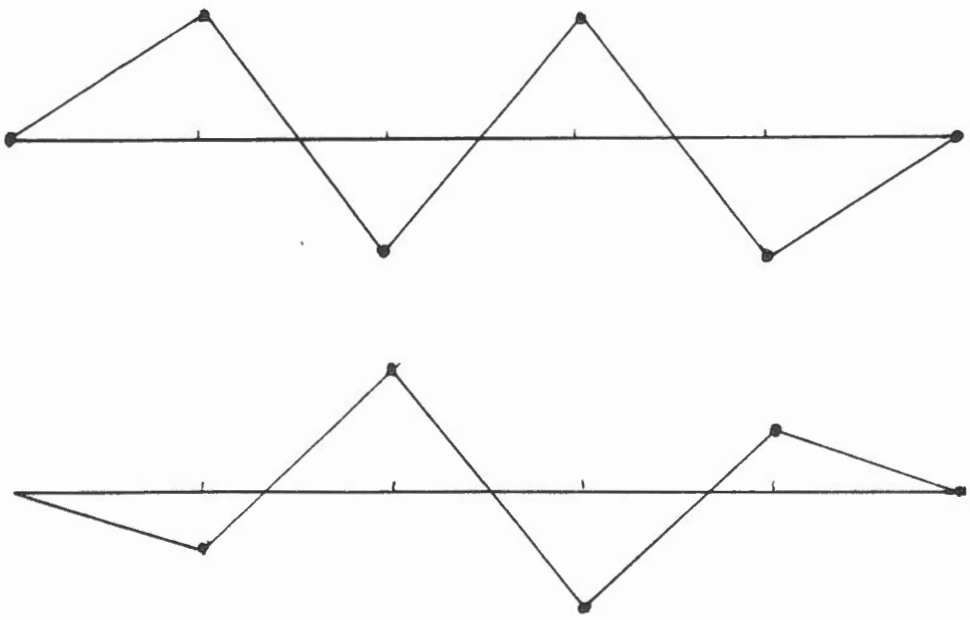
In examining the above block relaxation scheme it was found that the process was afflicted with several apparent defects. The solution of a higher frequency block pattern reintroduces a portion of the lower frequency error. There are rather substantial jumps in the solution between block ends in early iterations. The process tends towards its asymptotic rate of convergence rather slowly. Further work has resulted in reasonable solutions to these problems.

The reintroduction of some low frequency error was found to be a consequence of using a Gauss-Seidel type of solver for the iteration of individual frequencies in the cyclic process. This difficulty can be overcome by simply using a Jacobi based procedure. The problem of jumps between blocks can be overcome by the use of an underacceleration parameter that can be readily calculated. This effect can be pictured in the following way. Suppose the solution at some point is characterized by uniform errors of alternating sign as depicted in figure eight (a). Such error distributions are inherently characteristic of the cyclic block relaxation process. If a normal Jacobi iteration were applied to the distribution of figure eight (a) the result would be as in figure eight (b). Only the outermost solution points have had their error reduced. The inner points have merely changed sign. Further iterations slowly (one point per boundary per iteration) spread the damping into

the solution domain. Collapse to the exact solution, (provided the errors are centered about the true solution) occurs immediately if the estimated corrections for the first iteration are multiplied by the correct damping factor. This damping factor can be shown to be d_i/RAS_i where d_i is the value of the i^{th} diagonal element of the coefficient matrix and RAS_i is the sum of the absolute values of the coefficient of the i^{th} row of the coefficient matrix. Note that the collapse is completely independent of the length of the problem vector and the magnitude of the error.

Figure Eight

Behavior of Jacobi iteration



In actual application the above procedure does not work perfectly. A small misestimate of block centering occurs due to a geometric factor. The result is the formation of a sinusoidal error vector which very rapidly stabilizes to an asymptotic rate of convergence. Application of the above method to a test problem, of an equilibrium heat flow

in a uniform media with Dirichlet boundary conditions, gives an asymptotic error damping rate of approximately 0.45, apparently independently of the number of nodes in the problem! This is an unprecedented claim for an iterative solver.

Substantial further improvement in the solver can easily be obtained. A characteristic of iterative solvers is that they tend to asymptotic rates of convergence. An asymptotic rate of convergence implies that the error in every point in the solution is being damped by the same constant factor for every iteration after the asymptotic rate has effectively been achieved. Thus if e_k and e_{k+1} are respectively the error vectors from the k^{th} and $k+i^{th}$ iterations $e_{k+1}/e_k = \lambda$. The residue vectors R_k and R_{k+1} behave in the same way. It follows that the corrections for all subsequent iterations form a geometric series. On summing the resulting series it can be shown that the total remaining correction is determined by $\frac{1}{1-\lambda} R_k$. This correction can be applied to conventional solvers. There are, however, two major difficulties in application. One is that conventional solvers require many iterations to reach the asymptotic rate of convergence and that the number of iterations required to do so is dependent on N. The second problem is that λ is usually only slightly smaller than one for conventional solvers so that small errors in estimating λ cause huge errors in estimating the total remaining error.

The asymptotic rate of convergence correction can be very successfully applied to the newly developed solver. The combination of block relaxation and damping discussed above leads to very rapid approach to the asymptotic rate of convergence. This rate is quite closely

approached after two cycles. The value of λ for the damped block relaxation scheme is not very close to unity, (in the used example $\lambda \simeq 0.45$) so that very accurate λ values are not necessary. The result is that for the third and subsequent cycles error damping by one to two orders of magnitude is obtainable. Note that the occurance and effect of the asymptotic rate of convergence correction is also independent of the size of the problem, and the magnitude of the error.

The net result of these discoveries is that it is apparently possible to solve problems in four to six cycles independently of the size of the problem. If this finding is born out in more general cases than have so far been explored, (and it is reasonable to believe that it will) it is a major discovery in the field of iterative solvers. It is noted that the proposed solver closely approximates the desirates specified for the ideal iterative solver.

The method is expected to be very effective for coupled problems because of the ability of the low cost low frequency corrections to quickly rough in the required answer. Results obtained to date are very encouraging for coupled problems. It would appear that the new solver will make three point time integration techniques quite computationally feasible. It is also noted that the new solver can apparently be adapted to parallel processing using array processors.

5.3) Improvements to spatial discretization

The standard IFD (integrated finite difference) formulation can be derived from conservation considerations. Thus for the flow equation problem the net flux through the surface of a control element is equated to the time rate of change of the contents of that element. In integral for the equation can be stated as

$$K_H \oint_B \nabla h \cdot \hat{n} dB = S_S \frac{\partial}{\partial t} \int_V h dv$$

The standard IFD approximation for the content integral is to assume that it can be approximated as a mean value and that the nodal value for the element is equal to that value. That is

$$\int_V h dV \simeq h_i \Delta V$$

If we view the solution domain as being conservative then the sum of the content integrals over the whole domain must accurately express the content of the system. This will be true in general only if the content integrals are accurate. One way to proceed is to employ accurate integration techniques. The best way of achieving this from the viewpoint of numerical integration techniques is to locally fit a polynomial and to integrate that polynomial over a subrange of the colocation interval. One could therefore consider expressing the function locally be a linear or quadratic polynomial. This can be done by use of the nodal point in question and its nearest neighbours. The fitting of a higher degree polynomial requires additional points which leads to difficulties at boundaries and in the extension of the technique to higher dimensions.

Using a quadratic local approximation of the function u results in the following:

$$\int U(x) dV \simeq \Delta Y \Delta Z \int_{-\Delta x/2}^{+\Delta x/2} (a_i x^2 + b_i x + c_i) d_x$$

For a uniform grid in one dimension

$$a_i = \frac{U_{i-1} - 2U_i + U_{i+1}}{2(\Delta x)^2}$$

$$b_i = \frac{U_{i+1} - U_{i-1}}{2\Delta x} \quad , \quad c_i = U_i \quad ,$$

so that

$$\int U(x)dV \simeq \Delta Y \Delta Z \left(\frac{\Delta x}{24} U_{i-1} + \frac{11}{12} \Delta x U_i + \frac{\Delta x}{24} U_{i+1} \right)$$

Similar expressions exist for radial coordinates and higher dimensions. In two dimensions the equivalent expression is a nine point biquadratic.

Several advantages accrue from the use of the quadratic elements in addition to the basic improvement in accuracy. Integrals of the product of functions, (as in the second L.H.S. integral of the energy equation) are more realistically represented, tensorally varying functions can be represented, and in higher dimensions the grid orientation problem is alleviated.

The results obtained are substantially better than with the standard approximation. Results for the solution to the flow equation are discussed in the results section below. The two dimensionsl form has been applied to an equilibrium heat flow problem with the surprising finding that the nine point scheme solves faster than the standard five point scheme on the same grid.

5.4) Error detection and control

The quadratic functions described in the preceding section readily lend themselves to a method of detecting and controlling spatial error. local quadratic colocation polynomials would fit perfectly if the function were quadratic. If the function is changing more rapidly the local

approximations will not fit exactly. In this case discontinuities will occur across element boundaries in the predicted value of the function, (first derivatives are, however, continuous in any event). These gaps can be shown to depend on the third spatial derivative and hence form a basis for judging goodness of fit. Error control is achieved by adding nodal points at offending element boundaries if the gaps are too large and removing points if the gaps become very small. This technique has been implemented for the solution of the flow equation with good results.

Means of detecting time integration errors have not been implemented so far. A possible way of doing so is as follows. The size of the local truncation errors of the integration equations depends in general on the particular choice of equation parameter chosen. Suppose that some, say, second order solution is obtained at a given time step. Then

$$\tilde{y}_a = y + k_1 y'''$$

where $\tilde{y}_a$ is the approximation obtained, y is the unknown true answer and $k_1 y'''$ is the truncation error. Alter the integration equation by changing a parameter and reconverge. Then

$$\tilde{y}_b = y + k_2 y'''$$

If the change made is kept small than it is reasonable to suppose that y''' is essentially the same value in both results, hence

$$y''' = \frac{(k_1 - k_2)}{\tilde{y}_a - \tilde{y}_b}$$

Given the rapidity of the new solver and that the changes induced are kept small it is possible that the second approximation could be

reconverged in as little as one solver cycle. Depending on the size of the error indicated by y''' the decision to increase, decrease, or maintain the size of the time step can be made.

5.5) Some numerical results

A comparison of the results obtained for then numerical solution to the one well flow problem in radial coordiantes is given in table six.

Table Six

Some numerical technique results

Function Values (% accuracy)

radial distance	hybrid	2nd order	3rd order
well face	72.28	99.59	99.80
10m	98.67	99.70	99.83
100m	98.51	99.70	99.88
1000m	170.16	99.96	103.48
conservation	99.95	99.96	99.97

Gradient Values (% accuracy)

radial distance	hybrid	2nd order	3rd order
well distance	100.01	100	100
10m	100.00	100.00	100.77
100m	99.95	100.00	99.99
1000m	99.64	99.99	97.30

The results are expressed in terms of percentage accuracy using the Theis solution as the true answer. The results are for essentially the same times. Somewhat different grid systems were used consequently the results for the hybrid well face function values are shown in an unduly bad light. Even with further grid mesh refinement being applied to the hybrid method an error of several percent remains.

The hybrid solution is the standard IFDM started with a fully implicit time integration that is gradually switched over to a Crank-

Nicolson method. The 2nd order solution used a 2nd order superstable three point time integration with quadratic elements on an adaptive grid controlled by gap size. The 3rd order solution used a 3rd order A-stable time integration with quadratic elements on an adaptive grid. The introduction or deletion of nodes in an adaptive grid introduces small shocks into the system. The A-stable method damped these shocks rather slowly as indicated in the errors.

Marked increases in accuracy in the vicinity of the well are noted for the improved methods. In the general solution order of magnitude decreases in error of function estimates are observed for the improved method. The poor ability of the A-stable method to deal with shocks suggests that only strongly stable or superstable methods should be used with adaptive grids. The improved methods have the added advantage of built in indication of accuracy or lack thereof.

In terms of speed the 2nd order results were obtained about six times as fast as the hybrid results and the 3rd order results about ten times as fast. The simple cumulatively accelerated cyclic block solver was used for the improved results.

6) Minor effects

Data and information in the form of tables, graphs and formulas were required on the variation of storage system parameters from diverse sources. The causes of variation were taken to be pressure, temperature, and salinity changes. Functional relationships or sufficient information to permit the deduction of functional relationships was obtained for fluid viscosity, fluid density, thermal conductivities, and specific heats.

It was found that pressure change induced effects were negligibly small over the likely range of pressure changes. It is not clear as to how probable the effects of changes in salinity content would be for aquifer storage systems. As a first approximation it was assumed that fresh water systems were being used. The most dramatic temperature induced changes occur with respect to fluid viscosity. It has been established, both from theoretical considerations and empirical observations in geothermal modelling, that the results of the energy equation are insensitive to changes in hydraulic conductivity, (hence fluid viscosity). There was therefore no reason to include the viscosity variation. The temperature induced changes in the remaining parameters are quite small over the temperature range of interest. In view of the smallness of the changes and the found low sensitivity of the model to most parameter changes it was concluded that the minor effects would likely be smaller than the probable numerical error of the model. Work in this area was therefore discontinued.

7) Other topics

The principle other topic of interest was taken to be that of flows in multiple aquifer situations. The problem was formulated in terms of the University of Regina geothermal well which can be described as have five aquifers within the production zone. This problem was found to be beset by two major difficulties.

The aquifers have in common the well which, aside from minor frictional losses, must be at a single hydraulic potential. This means that the hydraulic potential at the well face calculated for each aquifer must be highly accurate otherwise gross misestimates of flows will

result. This in turn indicates that some suitable method from the new developments be used and that it would be strongly desirable to have both spatial and temporal error control. This would require further numerical developments than have so far been accomplished.

The second difficulty arises in connection with computational difficulty. The work effort required to solve such multiple aquifer systems appears to rise factorially with the number of aquifers involved. It follows that a five aquifer system is about 120 times as computationally difficult to solve as a single aquifer system. On the basis of the observed times required to solve single aquifer systems it was calculated that many days CPU time would be required for solution. As this amount of time was not available the problem had to be shelved until such time as better computational facilities and adequately developed improved numerical techniques became available. These developments have not yet occurred.

8) Conclusions and comments

The conclusions arising from the present work fall into those pertaining to the storage model and those pertaining to the numerical developments. The conclusions have therefore been so divided.

8.1) Conclusions about the aquifer storage model

The major findings about the aquifer thermal storage model are the following:

1. Thermal energy can be stored in suitable aquifers with efficiencies of 80 to 90 percent.
2. The best aquifer-aquitard systems for storage purposes are those which lead to most spherical temperature disturbances.

Thus thick aquifers with high porosity and leaky aquitards are preferable.

3. Problems with the produced fluid temperature are encountered if a simple in out system is used. The fluid temperature history can be usefully improved by conditioning achieved by added input for the initial input phase.
4. Conditioning does not have deleterious effects on storage efficiency in the long term.
5. The model was found to have rather low sensitivity to most system parameter changes.

8.2) Conclusions on the numerical improvements

1. Improvements in accuracy and stability properties of time integration schemes can be achieved by the use of three point block implicit methods.
2. It is possible to construct very fast iterative solvers such that the number of interations required for a given level of convergence is independent of the problem size or very nearly so.
3. Considerable improvement in the quality of approximation of spatial discretization can be achieved by the use of quadratic elements in the IFDM context.
4. The use of quadratic elements leads naturally to a convenient and effective method of spatial error detection and control. Effective error control may also be possible for temporal integration errors.

8.3) Comments

The findings concerning the aquifer thermal storage model are interpreted as indicating that such storage is quite feasible provided thermal energy storage is deemed desirable within the current energy economic milieu. A possible criticism of the present modelling work is that the model built may not be the most desirable model from the engineering point of view. This conclusion stems from the observation that large buildings will need summer time cooling as well as winter time heating. From the findings on the storage of heat it is reasonable to infer that cold fluid could be stored with comparable efficiency. On the assumption that there will likely be at most one suitable aquifer at a given site it would be necessary to utilize the same aquifer for storage of both hot and cold fluid. The modelling of this system would have to be done in three dimensions. Such a model could handle the problem of regional flows and presumably enable calculation of the unbalance needed between input and outtake to compensate for drift effect. Should further research on aquifer heat storage models be contemplated it is strongly recommended that the research examine the three dimensional problem.

The findings in the area of improvements to numerical techniques exceeded the author's wildest expectations. It is, of course, clear that a great deal of additional developmental work is needed before the new techniques can be considered proven. All indications at present are favourable to a positive outcome. If this proves to be the case it is obvious that the numerical developments would be vastly more significant than the actual modelling work.

Appendix I

Parameters defining the standard storage system

A "typical" storage aquifer was taken to be one of reasonably porous sandstone bounded vertically by shale aquitards. The aquifer is assumed to be horizontal, of constant thickness, and to be of effectively infinite horizontal extent. All parameters other than hydraulic conductivities are assumed to be macroscopically constant throughout the individual regions of interest. Hydraulic conductivities are assumed to be anistropic with horizontal conductivities being larger than vertical conductivities.

The "typical" parameter values used for the standard run are as follows.

symbol	parameter	value
P_W	density of water	1000 Kg/m^3
P_{RAF}	density of aquifer rock	2650 Kg/m^3
P_{RAT}	density of aquitard rock	2610 Kg/m^3
N_{AF}	aquifer porosity	0.25 (25%)
N_{AT}	aquitard porosity	0.15 (15%)
b	aquifer thickness	20m
S_{SAF}	specific storativity of aquifer	$1x10^{-5}$
S_{SAT}	specific storativity of aquitard	$1x10^{-6}$
K_{HVAF}	vertical hydraulic conductivity in the aquifer	$1x10^{-7}$m/s
K_{HRAF}	radial hydraulic conductivity in the aquifer	$1x10^{-5}$m/s
K_{HVAT}	vertical hydraulic conductivity in the aquitard	1x10-8m/s

symbol	parameter	value
K_{HRAT}	radial hydraulic conductivity in the aquitard	$1x10^{-7}$m/s
K_{TAFR}	thermal conductivity of aquifer rock	$20x10^{-3}$ cal/cm.sec°C
K_{TATR}	thermal conductivity of aquitard rock	$12x10^{-3}$ cal/cm.sec°C
K_{TW}	thermal conductivity of water	$1.50x10^{-3}$ cal/cm.sec°C
C_{AFR}	specific heat of aquifer rock	0.316 cal/gm°C
C_{ATR}	specific heat of aquitard rock	0.224 cal/gm°C
C_W	specific heat of water	1.00 cal/gm°C
Q	well pumping rate	0.0092 m^3/s