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CONDITIONS OF SIMILITUDE FOR MODEL STUDIES OF GROUNDWATER FLOW SYSTEM

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In this report attempts will be made to determine the conditions of similitude for representing the groundwater flow system with a free surface by a laboratory model.

The concept of pressure in an incompressible fluid, such as groundwater where the density is not dependent on pressure, presents certain difficulties to the physical understanding. The pressure is considered a variable of state, but is denied any influence upon the way in which the fluid occupies the space. The fluid has a free surface when it is bounded by vacuum or air. The condition for the free surface (Sommerfeld, 1950, p. 169) is

$$p = 0$$

i.e., the atmospheric pressure is taken as zero.

If the hydraulic head h is taken as the height of any point with respect to a reference level, the velocity at that point along a streamline is given by (Scheidegger, 1960; Muskat, 1949)

$$\vec{v} = -\frac{K}{P} \text{ grad } h \qquad \ldots (1)$$

where K is the hydraulic conductivity and P, the porosity of the soil, a dimensionless quantity. The hydraulic conductivity K is equal to $k\rho g/\mu$ where k is a property of the soil, called permeability and expressed as sq. cm., ρ the density of the fluid, g the acceleration due to gravity and μ the viscosity of the fluid. K has the dimension of length upon time.

The free surface is a streamline under steady-state conditions. Hence the velocity of the free surface is given by

$$\vartheta_f = - \frac{K}{P} \text{ grad }_n h \qquad \ldots(2)$$

where grad $_n h$ denotes the normal component of the gradient of the hydraulic head at the free surface.

Let the length and time parameters be denoted by

Length = $l_o L$

and Time = $t_o T$

where the unit quantities of length and time are l_o and t_o, whereas the dimensionless measure numbers of the parameters are L and T.

If we assume K_o/P_o as the unit quantity of the parameter K/P, we have

$$\frac{K}{P} = \frac{K_o}{P_o} \left(\frac{\overline{K}}{\overline{P}}\right) \qquad \ldots(3)$$

where $(\frac{\overline{K}}{\overline{P}})$ is the dimensionless measure number of the variable (K/P).

Equation (2) may, therefore, be written as

$$\overline{\vartheta}_f = - \frac{t_o}{l_o} \cdot \frac{K_o}{P_o} \left(\frac{\overline{K}}{\overline{P}}\right) \text{ grad }_n h \qquad \ldots(4)$$

where $\overline{\vartheta}_f$ is the measure number of the velocity. The expression grad $_n h$ will remain unchanged because this is a dimensionless quantity.

In order that the flow pattern of the groundwater in the field remains similar to that in the model, it is necessary and sufficient, subject to the restrictions imposed on the validity of (1), that the coefficient $(t_o K_o / l_o P_o)$ be identical in both the systems. In other words, the characteristic parameter C in

$$\frac{t_o}{l_o} \cdot \frac{K_o}{P_o} = C \qquad \ldots(5)$$

be invariant to a change of scale.

If we reduce the length parameter by a factor of 1000, say, we have

$$\left(\frac{t_o K_o}{P_o}\right)_f = 1000 \left(\frac{t_o K_o}{P_o}\right)_m$$

where the subscripts f and m denote the values of the parameters in the field and in the model respectively. Furthermore, if the soil and fluid properties remain unchanged,

$$(t_o)_f = 1000\,(t_o)_m\,,$$

i.e., any phenomenon occurring in the model will take place one thousand times faster than in the field. Thus, the time scale is reduced by the scaling factor of length provided that the soil and fluid properties remain fixed.

Now if, $l_f/l_m = \gamma_l$ and

$$\frac{(K_o/P_o)_m}{(K_o/P_o)_f} = \gamma_K\,,$$ we have

$$(t_o)_f = \gamma_l\,\gamma_K\,(t_o)_m. \qquad \ldots (6)$$

Utilizing (6), we may be able to achieve a considerable reduction in the ratio of the time scales in the model and in the field.

This is a pleasure to record grateful thanks to Dr. P. Meyboom and Mr. L.V. Brandon for valuable discussions.

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