State of the Art Paper on Frost Heave

Government of Canada's Northern Pipeline Preparedness Program

Submitted to: Terrain Sciences Division
Geological Survey of Canada, Natural Resources Canada

June 2004
Project No. 1100051

EBA ENGINEERING CONSULTANTS LTD.

STATE OF THE ART PAPER
ON FROST HEAVE

Project No. 1100051

JUNE 2004

STATE OF THE ART PAPER ON FROST HEAVE

Submitted To:

TERRAIN SCIENCES DIVISION
GEOLOGICAL SURVEY OF CANADA
NATURAL RESOURCES CANADA

Prepared by:

EBA ENGINEERING CONSULTANTS LTD.
EDMONTON, ALBERTA

Project No. 1100051

JUNE 2004

EXECUTIVE SUMMARY

EBA Engineering Consultants Ltd. (EBA) was retained by the Terrain Sciences Division of the Geological Survey of Canada (GSC) to develop a state-of-the-art paper on frost heave.

A technical review of a proposed pipeline in the Mackenzie Valley, NWT, must consider whether frost heave is understood sufficiently and can be controlled or limited to such an extent that safe construction and operation of a large diameter chilled gas pipeline is possible. This state-of-the-art paper on frost heave provides a thorough review of frost heave theory, prediction models, acquisition of input data for prediction models, validation of theory and models by laboratory experiments, and applicability of frost heave theory, prediction models and laboratory experiments to northern pipeline design and operation.

The number of publications on frost heave has increased considerably since the Berger inquiry in the 1970's. Frost heave theory developed by Miller 1978 has been proven in laboratory experiments and is accepted widely. The rapid freezing, associated large thermal gradients, short test duration, and limited sample size have been recognized as limitations of small scale laboratory frost heave tests. Large scale laboratory and field frost heave tests have been used to evaluate scale effects. In addition, scaled frost heave tests in centrifuges have dealt with some of the testing time constraints.

All current frost heave prediction models describe a set of variations of the same equations such as Fourier's law, the heat continuity law, Darcy's law, the mass continuity law, the Clausius-Clapeyron equation, and a frost heave criterion. These sets of equations are described for the frozen zone, the frozen fringe and the unfrozen zone of the freezing soil profile. A complete frost heave prediction model includes a stress analysis that describes elastic and viscous behaviour by Hooke's law and either the Norton-Hoff or Prandtl-Reuss law, respectively. Most researchers and practitioners who proposed frost heave prediction models claimed successful and accurate predictions, validated by small scale and large scale laboratory tests.

Frost heave prediction for buried chilled gas pipelines must consider all elements of the preceding paragraph, moving boundary problems associated with a moving freezing front and frost heave; mechanical behaviour of the buried pipeline, which includes the structural response of the pipeline to soil freezing; and interface conditions at the soil-pipeline interface. The soil must be modelled as a 2D or 3D continuum. A coordinate system and finite element discretization must be used which accommodates the complex coupled heat and water flows and displacements that accompany the ice lenses in a pattern influenced by the cooling from the soil surface and pipeline. The mechanical behaviour of the buried pipeline must be represented as a cylindrical shell which can possess complex non-linear stress-strain-time phenomena. Pipeline-soil interface behaviour must be described by interface phenomena such as frictionless contact, frictional contact or bonded contact and interface constitutive models determined from experiments.

Frost heave prediction models require input data such as climatic data (temperature, precipitation snow cover thickness); soil properties (e.g. soil moisture content, particle size distribution, Atterberg limits, salinity and organic content), thermal soil properties, segregation potential, hydraulic conductivity, and unfrozen water contents. The most critical input parameter for frost heave prediction models is the hydraulic conductivity of the frozen fringe. The hydraulic conductivity of the frozen fringe is difficult to determine. A procedure proposed by Konrad and Morgenstern 1980 to characterize frost susceptible soil in small scale laboratory tests in terms of their segregation potential describing the flow of water through the frozen fringe without knowledge of the hydraulic conductivity is used in many applications. The frost heave prediction model input data can be obtained from weather stations, site investigations along proposed pipeline route; laboratory index testing, ditch wall logging, geophysical programs, instrumentation (including thermistors), small scale laboratory frost heave tests, and laboratory TDR tests.

Frost heave research continues at several institutions and companies in countries such as Canada, USA, UK, Sweden, Finland, Russia, Japan and China. Safe design of buried chilled gas pipelines in cold regions could benefit if some of these research activities were focussed on: a thorough elementary examination of the driving force of frost heave; development of a method to measure the hydraulic conductivity in the frozen fringe; development of an extensive database of frost heave test results; standardization of frost heave test procedures and equipment; development of a method to indirectly measure pressure at the location where a new ice lens forms; development of a three dimensional frost heave prediction model based on the rigid ice model of Miller (1978), the model of Shen and Ladanyi (1987) or the discrete ice lens theory of Nixon (1991), including an extensive stress analysis such as given by Shen and Ladanyi; commercial development of such an advanced practical model; clearer listings of the input parameters required for each frost heave prediction model (access to the computer code is often required to list these input parameters); and an overview of frost heave prediction for buried chilled gas pipelines that includes discussion of pipeline mechanics. These suggested research activities are not necessary requirements for safe pipeline design and serve academic, clarifying and informative purpose.

Design of a chilled buried gas pipeline must consider the following:

- An upper-bound estimate of the differential movement and stresses imposed on a pipeline due to frost heave and other processes;
- An assessment of the level of accuracy in frost heave prediction required to safely construct and operate a buried chilled gas pipeline;
- A thorough review of (differential) frost heave effects mitigation measures;
- A thorough review of pipeline materials and their ability to withstand stresses generated by differential frost heave;
- An evaluation of pipeline construction techniques that can limit differential frost heave stresses; and
- An extensive monitoring program during pipeline operation.

The quality of frost heave prediction models for buried chilled gas pipelines can only be fully assessed based on their performance and accuracy in predicting the stress-strain behaviour of soil and pipelines in large-scale laboratory and field experiments.

TABLE OF CONTENTS

1.0 INTRODUCTION

1.1 Background

The increase of the natural gas price over the last decade, combined with the expiry of oil and gas pipeline construction ban as a result of the Berger inquiry in the mid 1970's, prompted a renewed interest in oil and gas development in the Beaufort Sea/Mackenzie River Delta in the Northwest Territories, Canada. Natural gas producing companies with leases in the Beaufort Sea/Mackenzie River Delta are currently gathering information required for the design of a natural gas pipeline from the Beaufort Sea/Mackenzie River Delta. Construction and operation of a long pipeline is a major project with many technical, environmental, and socio-economic aspects. The pipeline proponents have to comply with current regulations to obtain permits for the pipeline construction and operation. Current legislation includes a federal environmental impact review process in which pipeline proponents need to submit documentation in which the technical, environmental, and socio-economic aspects associated with pipeline construction and operation are addressed, followed by public hearings. Pipelines in which natural gas is transported must be buried for security reasons and for protection of the northern environment (Ladanyi and Lemaire 1984). The Government of Canada has initiated a northern pipeline preparedness program in anticipation of an expected northern natural gas pipeline proposal. The Terrain Sciences Division of the Geological Survey of Canada (GSC), a branch of the Department of Natural Resources Canada, will be involved during the permitting process in the review of technical aspects of pipeline construction and operation.

Thermal degradation in areas with widespread to continuous permafrost conditions is limited by transporting the natural gas as a liquid, by maintaining a high pressure and subzero temperature in pipelines. One of the technical aspects associated with a chilled buried gas pipeline is how differential frost heave affects the integrity of the pipeline. A buried pipeline transporting gas at subzero temperatures generates a frost bulb around the pipeline. Spatial variability of the amount of ice formed in the soil along the pipeline will result in differential frost heaving and differential stresses in the pipeline. Differential frost heave occurs at soil type transition zones or at frozen – non-frozen soil interfaces. A technical review of a proposed Mackenzie Valley pipeline must consider whether frost heave is understood sufficiently and can be controlled or limited to such an extent that safe construction and operation of a large diameter chilled gas pipeline is possible. For such an assessment, the GSC seeks to obtain information on the following aspects of frost heave:

- theory, prediction models, acquisition of input data for prediction models, validation of theory and models by laboratory experiments, and the applicability of frost heave theory, prediction models and laboratory experiments to northern pipeline design and operation;
- stress-strain behaviour of soil and pipe and frozen/freezing soil pipe interaction observations in large/field scale tests and experiments in controlled and natural environments undertaken by industry and government;
- use and applicability of centrifuge testing for frost heave assessment and pipeline design;
- experience in mitigation of frost heave in operation of pipeline and associated facilities in Canada and Alaska; and
- experience in mitigation of frost heave in operation of pipeline and associated facilities elsewhere, e.g. in the former Soviet Union, and their applicability to the design of a large diameter Mackenzie Valley gas pipeline.

The GSC solicited the opinion of a number of experts to acquire this knowledge, as part of the Government of Canada's northern pipeline preparedness program. EBA Engineering Consultants Ltd. (EBA) was asked to address the first aspect.

1.2 Scope of Work

The scope of work for this assignment is to develop a state-of-the-art paper on frost heave. The state-of-the-art paper provides a thorough review of frost heave theory, prediction models, acquisition of input data for prediction models, validation of theory and models by laboratory experiments, and applicability of frost heave theory, prediction models and laboratory experiments to northern pipeline design and operation.

1.3 Methodology

Published state-of-the-art papers on frost heave were firstly reviewed. These state-of-the-art papers were written by several engineering practitioners and scientists during the last few decades: Miller 1980, O'Neill 1983, Smith 1985, Nixon 1987, Kay and Perfect 1988

Ladanyi and Shen 1989, Black and Hardenberg 1991, Konrad 1994, Black 1995, Jones 1995, Kujala 1997, and Henry 2000.

This review was followed by a comprehensive literature search on the development of frost heave theory. Innovations and shortcomings of theories and challenges to the proposed theories are briefly addressed concluding with a description of the current state of frost heave theory and our confidence level in the current frost heave theory.

A comprehensive literature search on laboratory frost heave experiments was also performed. An inventory was made of laboratory frost heave experiments such as those of Penner, Konrad, Akagawa, and many others. A review framework was established that consists of a spreadsheet according to which procedures and results of individual tests could be categorized. Each frost heave experiment is described in terms of experiment details such as: column dimensions, boundary temperatures, freezing rates, temperature control, top down or bottom up freezing, ramped or step freezing, water intake rates and volumes, number of freeze-thaw cycles, and test duration and soil type. Innovations and shortcomings of the laboratory experiments are briefly addressed concluding with a description of the current state of frost heave testing, availability of data, experience, the value of laboratory tests for frost heave prediction, and as support for proposed frost heave theory and models.

A review framework was then established that consists of a spreadsheet according to which aspects of individual frost heave prediction models could be categorized and judged. This review framework is an amended version of the evaluation system described in Schellekens (1997). The aspects used for categorizing the proposed frost heave prediction models include the fundamental natural processes accounted for by each model, the manner that heat transfer, phase transfer, water flow, and resulting soil stress changes are accounted for, frost heave criteria, numerical and computing procedures used, assumptions and data requirements of each frost heave model. A comprehensive literature search on frost heave prediction models was performed. The review framework was used to describe the various aspects of a selection of the published models. Innovations and shortcomings of the models are briefly addressed concluding with a description of the current state of frost heave prediction modelling and our confidence level in the current frost heave models.

Finally, a literature search on the use of frost heave predictions for pipeline design was performed, to evaluate the requirements and procedures for reliable prediction of frost heave effects on chilled buried gas pipelines.

Literature searches included the publications and libraries of the Arctic Science and Technology Information System (ASTIS) of the University of Calgary, the Cold Regions Research and Engineering Laboratories (CRREL) of the US Army Corps of Engineers, and the Scott Polar Research Institute (SPRI) in Cambridge, UK, in addition to a library search at the University of Alberta and WWW-searches. The sources used for this paper are mainly from North America and Northern and Western Europe, with some publications in English by Russian, Chinese and Japanese researchers. The literature used is almost exclusively that, which exists in the public domain. It should be realized that many reports on frost heave around buried chilled gas pipelines are proprietary information of companies such as Canadian Arctic Gas Limited, Polar Gas, Foothills Pipe Lines Ltd., the Gas Arctic/Northwest Project Study Group or Northwest Alaskan Pipeline Company.

The assignment was concluded with a description of the present state of knowledge, the status of ongoing research, deficiencies and gaps in the knowledge base, and additional work required based on the reviews of frost heave theory, prediction models and performed tests.

2.0 FROST HEAVE THEORY

2.1 Historical Review

The theory to explain frost heave in soils has been developed and refined between 1920 and 1990. Frost heave has been studied by soil scientists, soil physicists, chemists, chemical engineers, geotechnical engineers, mathematicians, mechanical engineers, geographers, hydrologists and agricultural engineers. The development of frost heave theory, laboratory and field frost heave tests and frost heave prediction models, occurred in Canada, the USA, Russia, China, Japan, France, UK, Sweden, Norway and Finland. The findings have been published in in-house institutional reports, peer reviewed scientific journals, conferences and symposia.

The scientists and practitioners usually published their findings in journals or presented at conferences and symposia of their own discipline. A lack of communication between practitioners from the various disciplines contributed to some misunderstanding and disagreement on the topic of frost heave between scientists and practitioners, until collaborations between scientists and practitioners from various disciplines started to generate useful results in the 1980’s (e.g. Miller and O’Neill; and Guymon, Hromadka and Berg).

A number of useful state of the art papers on some aspects of frost heave have been presented by: Miller 1980, O’Neill 1983, Smith 1985, Nixon 1987, Kay and Perfect 1988, Ladanyi and Shen 1989, Black and Hardenberg 1991, Konrad 1994, Black 1995, Jones 1995, Kujala 1997 and Henry 2000. A summary of the most significant contributions to the development of the theory of frost heave is given in Table 1.

2.2 Current State

Frost heave is the upward movement of the ground surface or objects on or in the ground caused by the formation of ice in the soil. When a freezing front penetrates into a soil, its pore water freezes and expands, which is called in-situ freezing. Water from the unfrozen soil below is drawn into the freezing soil by a suction induced by the freezing process. This migrated water freezes in ice lenses that segregate the soil particles. This process is known as segregation freezing. Many thick ice lenses form in moist soils that have a hydraulic conductivity that is high enough for water flow to occur, and in which large water potential gradients develop during long periods of slow penetrating subzero temperatures. Such conditions occur in slow freezing of highly frost-susceptible fine-grained soils, in which the resulting segregated ice lenses account for most of the frost heave. An ice lens grows at some distance behind the freezing front (Andersland and Ladanyi 2004). The zone between the base of the warmest ice lens and a freezing front is known as the frozen fringe (Miller 1972). A new ice lens commences to form in the frozen fringe where the effective stress decreases to zero (normal stress equals neutral stress) or in other words where the ice pressure equals the cohesive and overburden pressures (Gilpin 1980). Water and ice in the frozen fringe may or may not be in thermodynamic equilibrium with each other regionally (or everywhere in the frozen fringe), though agreement exists that locally water and ice are in thermodynamic equilibrium.

Table 1
MOST SIGNIFICANT CONTRIBUTIONS TO FROST HEAVE THEORY DEVELOPMENT

Author(s)	Year	Contribution
Bouyoucos	1920	Water in soil does not freeze at one temperature
Taber	1929	Frost heave is generated by growth of ice lenses instead of expansion of freezing water
Casagrande	1931	Frost heave susceptibility depends on the soil particle percentage finer than 20 µm
Beskow	1935	Frost heave can be limited by application of a surcharge on the soil surface
Schofield	1935	A pF scale for the Gibbs' free energy based on measured freezing point depression
Edlefsen and Anderson	1943	Rigorous analysis of the thermodynamics of soil moisture
Gold	1957	Frost heave mechanism based on the surface tension of the ice/water interface below the ice lens
Phillip and De Vries	1957	Water movement is a result of a thermal gradient
Jackson and Chalmers	1958	Heaving mechanism based on the kinetics of solidification
Cass and Miller	1959	The driving force of frost heave is the osmotic potential of diffuse electric double layers on grain surfaces.
Miller et al.	1960	The driving force of water flow to an ice lens is a combination of the osmotic potential gradient of diffuse electric double layers on grain surfaces and the surface tension of the water/ice interface below the ice lens.
Everett	1961	Rigorous analysis of the surface tension model implies an upper limit on the pressure at which an ice lens can grow.
Hoekstra	1966	Ice and water are not in equilibrium with each other during ice lens formation and frost heave.
Miller	1973	A capillary sink mechanism accounts for freezing induced moisture redistribution.
Groenevelt, Raats	1974 1975	Further theoretical development of heat and water transport in freezing soils using irreversible thermodynamics.
Miller et al.	1975	Further theoretical development of heat and water transport in freezing soils using irreversible thermodynamics, using commonly accepted soil functions.
Miller	1977	Ice forms in soils at the freezing front (primary heave) or behind the freezing front (secondary heave); ice in a frozen fringe beneath an ice lens moves by regelation at the same velocity as the ice lens.
Miller; Taylor and Luthin; Gilpin	1978 1980	Introduction of frost heave criteria for initiation of a new ice lens
Forland and Ratkje	1980	Further theoretical development of heat and water transport in freezing soils using irreversible thermodynamics.
Horiguchi	1987	The driving force of water flow to an ice lens is the osmotic potential gradient.
Shen and Ladanyi	1987	Introduction of a description of the stress field in freezing soil.

A soil in which a frost front penetrates experiences transient freezing. The frost heave resulting from the formation of each warmest ice lens and pore ice formation in the frozen fringe during this phase is known as transient frost heave (Konrad and Morgenstern 1982). Transient freezing is followed by stationary freezing, which occurs when the penetrating freezing front reaches a thermal equilibrium state, and will not further penetrate into the soil. During stationary freezing, only the frozen and partially frozen soil zones experience further cooling below 0° C. The frost heave associated with this stationary freezing phase occurs during the formation of the last ice lens, and is known as stationary frost heave (Akagawa 2000). Further freezing of unfrozen soil pore water and unfrozen water films surrounding soil particles behind the warmest ice lens or further growth of ice lenses behind the warmest ice lens due to water redistribution in the frozen soil, is known as long-term frost heave (Goto and Takahashi 1982). In summary, total frost heave consists of heave as a result of in situ freezing of pore water and growth of ice lenses, which both occur during transient, stationary and long term freezing.

Coarse-grained soils with relatively high gravel and coarse sand contents do not experience significant frost heave (Linnell and Kaplar 1959, Chamberlain 1981). In heavy clays, large water potential gradients are developed during freezing. However, their very low hydraulic conductivity limits the water flow to the freezing soil and with that, the amount of frost heave occurring. Soils with a high silt content are the most frost susceptible, due to relatively high suction compared to sands and relatively high hydraulic conductivity compared to clays.

Many soils in the Mackenzie Valley have an abundant water supply for frost heave. Under lakes, rivers and swamps the permafrost table is often lower than under the surrounding terrain, but the soil conditions under lakes, rivers and swamps are very wet. Frost heave may be limited due to low water availability in thick gravel and coarse sands, and in thick stiff, relatively dry clays, especially in regions with low precipitation (arid environments or so called polar deserts).

The osmotic potential gradient is an important part of the total water potential gradient that is the driving force of water movement from the unfrozen soil to the freezing soil zone. Soil pore water salts decreases the osmotic gradients and as a result, frost heave in soils with high soil pore water salinity is less than in soils with relatively low soil pore water salinity.

An increase in external surcharge on a soil limits the amount of frost heave of that soil. However, there is still debate about the existence or non-existence of a soil specific surcharge (or shut off pressure as mentioned by Arvidson and Morgenstern 1977, and Hill and Morgenstern 1977) that terminates frost heave completely.

3.0 SMALL SCALE LABORATORY FROST HEAVE EXPERIMENTS

3.1 Review Framework

A frost heave laboratory experiment review framework was established that consists of a spreadsheet in which individual laboratory frost heave experiments could be compared to each other in terms of their purpose, procedures, methods, equipment and soils used, and in terms of their results.

3.2 Historical Review

Numerous laboratory and field experiments have been conducted to determine the frost heave potential of soils, to acquire input data for frost heave prediction models, and to validate frost heave theory and mathematical models. Most of the reviewed published test data were performed for a scientific purpose. Many frost heave tests performed for the design of foundations, pipelines, railways, road and airport pavements have not been published.

The number of published frost heave experiments increased from the mid 1950's up to 1990. An extensive (though far from complete) list of published frost heave tests is provided in Appendix A. The purpose, procedures, methods and equipment, soils, and the results of selected frost heave experiments are summarized in Table A.1 in Appendix A.

Typical laboratory frost heave tests involved temperature controlled freezing of an instrumented 1 to 30 cm (typically 10 cm) long soil column with a diameter of 6 to 30 cm (typically 10 cm) between a relatively cold and a relatively warm plate. The instrumentation consisted in most cases of 8 to 15 thermistors or thermocouples, a displacement (or frost heave) measuring device, and a water intake measuring device.

Pressure measuring devices such as small porous cups and internal strain measuring devices such as small lead spheres were included in some experiments. X-ray scans were taken to locate the lead spheres or to locate ice lenses during and after the frost heave experiment in some non-standard experiments.

Some experiments were performed in which a closed freezing system existed, i.e. the soil column did not have an external water supply, and did not experience a gain or loss of water. However, most tests were performed using open system freezing conditions, i.e. the soil column had an external water supply, and water could be taken up or could be discharged from the soil. Usually a constant head was maintained on the external source of water.

The experimental set up was placed in a temperature chamber to minimize heat loss and heat gains between the test soil and the surroundings. The temperature of each plate was usually controlled by circulating a fluid from a temperature controlled bath through the plate. Originally, top down freezing experiments were common, while bottom up freezing is applied in most current frost heave experiments, because the cold plate maintains a better contact with the soil during bottom up freezing than in a top down freezing experiment, and the side wall friction forces are reduced. Temperatures of the warm side of the column ranged from –0.2°C to 10°C and temperatures of the cold side ranged from –20°C to –0.1°C. Originally ramped freezing experiments were performed in which the temperature of the cold plate and sometimes the warm plate were lowered at constant rates that ranged from –0.02 to –10°C/hr. Step freezing experiments in which the temperature of the cold side and sometimes the warm side of the soil column were lowered in 1 to 3 steps that ranged from –2 to –15°C have become more common over the last decades.

Most frost heave experiments have been performed with some surcharge or additional overburden on the soil column. This surcharge ranged from 20 to 250 kPa. In most experiments, the soil column was subjected to one freezing cycle, but in some experiments the soil column was subjected to one or more freezing and thawing cycles. The duration of a freezing cycle ranged from 2 to 1000 hrs, and the duration of a frost heave test ranged from 2 to 1000 hours, while the most common frost heave test duration was between 100 and 120 hours.

Prior to and after a frost heave experiment, soil properties such as particle size distribution, wet and dry density, porosity, moisture content and saturation were determined. The measured temperatures, total displacement or frost heave, and water intake were plotted vs. time. Pressures and displacements within the soil column were plotted vs. time in experiments in which these parameters were measured. Additional parameters such as thermal gradients, water intake rates and the segregation potential, were derived from these measurements.

Remoulded and undisturbed samples of a variety of soils ranging from clay and silts to granular soils such as sands and crushed limestone were used in frost heave tests. The most frequent tested soils were high frost susceptible clayey silts.

The most significant observations from frost heave experiments are listed in Table 2.

3.3 Current State

Strict international or national standards for equipment, procedures and methods to perform frost heave tests, are currently not available despite occasional calls for such standards. A standard procedure was developed to perform frost heave tests at the National Research Council of Canada (Penner and Eldred 1985). Another frost heave test procedure, ASTM procedure (D5918-96) to determine the frost susceptibility of a soil was developed based on experimental work performed by Chamberlain et al. 1984 and Johnson et al 1986 at the United States Army Corps of Civil Engineers Cold Regions Research and Engineering Laboratories (CRREL) in Hanover, New Hampshire. This procedure is rarely used for tests to derive parameters for frost heave prediction.

Soil samples in most small scale or bench type frost heave tests in North America are frozen in a five day long open system bottom up freezing of remoulded or undisturbed soil samples that are typically 5 to 20 cm long and 6 to 15 cm in diameter in an instrumented frost heave cell. The instrumentation consists in most cases of 8 to 15 thermistors, a displacement (or frost heave) measuring device, and a water intake measuring device.

Table 2
SIGNIFICANT OBSERVATIONS FROM FROST HEAVE EXPERIMENTS

Observations	Source
Rhythmic ice lens banding	Taber 1929, 1930, Martin 1959
Water migrates at sub-zero temperatures	Dirksen 1964, Hoekstra 1966
Heaving pressures are much higher than predicted by capillary driving force theory	Penner 1967, Hoekstra 1969 water movement paper, Sutherland and Gaskin 1973
Water flows through frozen soil	Hoekstra 1969, Xu at al. 1985
Hydraulic conductivity decreases with temperature below 0°C	Burt and Williams 1976
During initial rapid freezing water is expelled from the freezing soil into unfrozen soil	Penner and Ueda 1977, Loch and Kay 1978
Thermodynamic equilibrium exists at ice-water contact in frozen fringe	Vignes and Dijkema 1974, Biermans et al 1978
Frozen fringe exists	Loch and Kay 1978, Loch 1979
Water migrates within the frozen fringe	Penner and Walton 1978, Mageau and Morgenstern 1980
In one step frost heave tests the frost heave process occurs in three phases: 1. constant water intake velocity; 2. water intake velocity decreases continuously with time and 3. frost heave rate decreases monotonically with time (growth of final ice lens)	Konrad and Morgenstern 1980, Akagawa 1988
During formation of the last ice lens, the water intake is proportional to the temperature gradient	Konrad and Morgenstern 1980
Ice lenses can develop in frozen soil beyond the warmest ice lens	Penner and Goodrich 1980, Ohrai and Yamamoto 1985
Regelation occurs in freezing and frozen soil	Ohrai and Yamamoto 1985
Long term heave occurs	Caen experiment, Goto and Takahashi 1982
Transient frost heave accounts for 25% of the total frost heave, stationary heaving accounts for 70% and long-term heaving accounts for 5% of total frost heave in 400 hour long laboratory frost heave test	Caen experiment, Goto and Takahashi 1982

The warm side temperature is typically 0 to 5°C; the cold side temperature is typically -3 to –12°C, and the boundary temperature(s) are decreased in one to three temperature steps. The experimental setup is placed in a temperature chamber to minimize heat loss and heat gains between the test soil and the surroundings. The temperature of each plate is usually controlled by circulating a fluid from a temperature controlled bath through the plate. A constant head is maintained on the external source of water.

Most frost heave experiments are performed with some surcharge or additional overburden on the soil column. This surcharge ranges from 20 to 250 kPa, which is equivalent to 1 to 15 m of soil on top of the heaving soil sample. The soil column is typically subjected to one freezing cycle.

Prior to and after a frost heave experiment, soil properties such as particle size distribution, wet and dry density, porosity, moisture content and saturation are determined. The measured temperatures, total displacement or frost heave, and water intake are plotted vs. time. Additional parameters such as thermal gradients, water intake rates and the segregation potential, are derived from these measurements.

The following has been proven from frost heave tests:

- Ice forms in pores and in segregated ice lenses when a freezing front penetrates a soil containing water;
- The further the freezing front penetrates in the soil, the thicker the ice lenses and the further the lenses are spaced apart from each other;
- The ice lenses form in a direction perpendicular to the heat extraction, behind the freezing front;
- Water movement occurs from the unfrozen soil to the freezing soil as a result of a thermal gradient in a freezing soil;
- Slow freezing and associated small temperature gradients cause thicker ice lenses and more frost heave than fast freezing and associated large temperature gradients;
- Bottom up freezing of a soil column in a frost heave test results in a higher frost heave than that of a soil column subjected to top down freezing;
- Water movement occurs within the frozen fringe and within the frozen soil;.
- Water redistribution during freezing occurs by water flow through unfrozen water films and movement of ice by regelation;

- Ice lenses continue to grow after new ice lenses have been formed at warmer locations within the soil;
- The water intake during the formation of the final ice lens in a laboratory frost heave test is proportional to the temperature gradient; the proportionality parameter is the segregation potential of the soil;
- The segregation potential is a function of the suction, thermal gradient and surcharge;
- Salt concentrations in the soil pore water decrease the amount of frost heave of the soil; and
- The amount of frost heave decreases with an increase in surcharge on the heaving soil.

The following is not proven, or requires further study:

- An extensive database of frost heave test results does not exist; and
- A method to (indirectly) measure pressure at the location where a new ice lens is forming, should be developed.

The following laboratories in Canada have frost heave test capabilities that have been used recently: AMEC Earth & Environmental Ltd., Calgary; Carleton University, Ottawa; EBA Engineering Consultants Ltd., Edmonton; Golder Associates, Calgary; and Laval University, Quebec City. C-Core in St. John's has the capability to perform centrifuge frost heave experiments.

4.0 FROST HEAVE PREDICTION MODELS

4.1 Modelling of a Natural Process

A natural process or system may be represented in a model, once knowledge of a process is obtained from experiments and observations in the field and laboratory. The more is known about the process involved, the better reality can be represented in the model. Models are used to predict the result of a process, and are used to develop a better understanding of the process.

The two main categories of models are: physical models (scale models and analog models) and theoretical models. A physical model may be used to visualize a problem.

An example of a scale model of the relief of the terrain is a 3D model made of gypsum or plastic. An example of an analog model is the representation of water flow by an electrical circuit. Theoretical models can be subdivided into: statistical models (random/ phenomenological, probabilistic and stochastic models) and deterministic models (mechanistic or non-mechanistic models), or a mixture of these (e.g. stochastic-deterministic models). In a statistical model or process, probabilities or chances of a process happening, or a process variable taking a certain value, are calculated. The statistical model is called a random or phenomenological model, if the chances are based on correlation and not determined by any physical laws. The statistical model is called a probabilistic model, if the chances to get certain outcomes are larger than others. The probabilistic models are called stochastic models in the special cases that chances depend on time. In statistical models, the emphasis is on finding correlation between variables that might be important in the process, without knowing exactly how the process works. Therefore, these models are very useful in the exploratory phase of research into a process.

It may be possible to work towards a deterministic model when more details are known about the process. In a deterministic mode, the relation between cause and effect is mathematically described. A mechanistic model of a process in nature aims at a detailed physics-based description of all known sub-processes and interactions of variables involved in the process. Certain simplifying assumptions to omit complex or speculative relationships are used in a non-mechanistic model.

The process or system in reality may be represented or simulated by any of the models mentioned above. The most desirable description of reality is a mechanistic deterministic model. An exact solution of the problem may be obtained analytically, or an approximation of the exact solution may be found numerically once the problem is formulated mathematically in a mechanistic deterministic model. Sometimes numerical procedures lead to the exact solution. The main advantage of using numerical methods is that they can be written as a computer program, and a computer may perform the calculations.

A practical model is a model that delivers predictions that are as accurate as possible, while the model is kept simple or otherwise user friendly, computer requirements such as required software and hardware and required computing time are kept within reasonable limits, and for which the input parameters are relatively easy to determine.

Generalization, such as simplifying the physical base of the process by omitting certain details, or a decrease of the amount of spatial or temporal steps in the numerical approximation, may be necessary to reduce the computing time in order to make the model practical and to obtain an optimal efficiency. The errors generated by these actions should remain within a limited range. Whether these errors are allowable depends on the purpose of the model. Some degree of generalization or simplification, usually required to make scientific mechanistic deterministic models practical, results in non-mechanistic deterministic models being used in practice.

The input data required for the model should be evaluated. Input data which can be measured, and input data which cannot be measured should be estimated. Generalizations and approximations in the determination of the input data may be necessary and possible; however, the quality of the output of a model will only be as good as the quality of the model and its input data. It is important to know which parameters are crucial in the determination of the output. A sensitivity analysis can be performed to evaluate the sensitivity of the output to each of the input data parameters.
The conditions under which the model is valid must be outlined. This depends on the assumptions and the validity of the physical laws used in the model.

The model has to be tested. Calculated results can be compared to measurements and observations of the calculated variables in reality (in natural processes those measurements may be field or laboratory measurements), or the calculated results may be compared with those calculated with other models. The latter method is less accurate than the former, because it is often not sure if the model with which the new model is compared is a close approximation of reality. The importance of each detail in a model can be evaluated by comparing a version of the model that incorporates that detail with a version that does not incorporate that particular detail.

4.2 Review Framework

A review framework that consists of a spreadsheet in which various aspects of frost heave prediction models are listed has been developed to provide a means to describe and compare the models. These aspects include the fundamental natural processes accounted for by each model, the manner that heat transfer, phase transition, water flow, heat and water flow coupling and resulting soil stress changes are accounted for, and frost heave criteria. This review framework is an amended version of the frost heave model evaluation system described by Schellekens (1997). A classification system is used to categorize the models, based on the natural processes model types mentioned in the previous section and the most significant features included in each model. The following categories of frost heave prediction models have been used:

1. Frost heave driving force models
    a. Capillary driving force models
    b. Adsorption driving force models

2. Frost heave prediction models
    a. Statistical frost heave prediction models
    b. Deterministic frost heave prediction models

Deterministic frost heave prediction models can be further categorized depending on whether they do include or do not include a frozen fringe (F), thermodynamic equilibrium (T), a frost heave criterion (C), regelation (R), and stress analysis (S). Assumptions determine if the deterministic model is mechanistic (M) or non-mechanistic (N). For example, the discrete ice lens model of Nixon (1991) is categorized as 2bFTCM.

4.3 Historical Review

The first frost heave modelling attempts such as the work of Gold (1957), Cass and Miller (1959) and Everett (1961), focussed on the modelling of the driving force of water flow toward a freezing soil zone. The first frost heave prediction models that included a mathematical model of the frost heave process, numerical approximations of the analytical formulations, a computer code to perform the calculations, and an ultimate frost heave prediction were proposed in the 1970's. From the 1970's until the mid 1990's many models were proposed, while frost heave prediction modelling activity has decreased over the last decade. A listing of almost one hundred published frost heave models is given in Appendix B. A selection of the models listed was examined in detail using the review framework. The review is summarized in Table B.1 in Appendix B.

The most significant steps in the development of frost heave prediction modelling are listed in Table 3.

Table 3
MOST SIGNIFICANT DEVELOPMENTS IN FROST HEAVE PREDICTION MODELLING

Development	Source
Model of the driving force of frost heave	Gold 1957
First frost heave prediction model using a coupled heat and water flow	Harlan 1973
First frost heave prediction model, a frost heave criterion and an ice phase moving by regelation	Miller 1978
Frost heave prediction model that used a slightly different frost heave criterion	Gilpin 1980
First detailed procedure to obtain frost heave prediction model input parameters and suggestions for their use in frost heave prediction	Konrad and Morgenstern 1980, 1981, 1982
First mechanistic solution of the Miller 1978 model	O'Neill and Miller 1982, 1985
First simplification (non-mechanistic solution) of O'Neill and Miller's 1982 solution of the Miller 1978 model	Holden and Jones 1985
A mechanistic model that considered the osmotic potential gradient as the driving force of frost heave	Horiguchi 1987
First frost heave prediction model that considered detailed stress development in the freezing soil	Shen and Ladanyi 1987
Redefinition of the mechanistic solution of the Miller 1978 model by O'Neill and Miller 1985, using the input parameter procedures proposed by Konrad and Morgenstern 1980, and addition of a detailed stress analysis	Shah 1990
Mechanistic frost heave prediction model that included solute transport	Padilla and Villeneuve 1990, 1992
Mechanistic two dimensional frost heave prediction model using a coupled heat and water flow, a frozen fringe, a frost heave criterion and stress analysis	Frémond and Mikkola 1991
Development of a practical standard frost heave prediction model by U.S. Army CRREL	Guymon et al. 1993
Development of a practical standard frost heave prediction model for frost heave prediction under Finnish roads	Saarelainen 1992
Development of a practical standard frost heave prediction model for frost heave prediction under Swedish roads	Sheng et al. 1995
Further simplification of the O'Neill and Miller's 1982 solution of the Miller 1978 model	Fowler and Noon 1993, Gorelik et al. 1998, Fowler 2003

4.4 Current State

The current practical frost heave prediction models are non-mechanistic deterministic models, in which the soil subjected to freezing is divided into three zones: the frozen zone, the frozen fringe and the unfrozen zone. Water and heat flow, and deformations as a result of stresses and pressures, are treated separately for each zone. The frozen fringe is a very thin zone with a thickness depending on soil type and freezing rate, varying mainly from a fraction of 1 mm to l0 mm, and up to 100 mm where very small thermal gradients exist. The latent heat release in this thin zone originally generated many computational problems. Continuum mechanics procedures gained popularity since the early 90's. In these procedures, the frozen fringe is reduced to a boundary over which parameters jump in value. This treatment has had considerable success.

Heat transfer and phase transition in each zone of the freezing soil are described by the Fourier equation and mass energy continuity equation, often combined in the general heat transfer equation. Current frost heave prediction models consider a water potential gradient induced by the thermal gradient in a freezing soil as the driving force for water flow from the unfrozen soil to the freezing soil. The Darcy and mass continuity equations, often combined in the general moisture transfer equation, are used to describe water flow in the unfrozen zone, frozen fringe and frozen zone. The heat and water transfer are coupled in a form of the Clausius-Clapeyron equation, which is valid for thermodynamic equilibrium in the frozen fringe, especially at the base of the ice lens.

In the frost heave prediction model, a relationship or frost heave criterion is included that prescribes the location of the formation of the new (warmest) ice lens. Examples of such a criterion are: 85% ice content in soil pores (Taylor and Luthin 1978), zero effective stress (Miller 1978), or a separation pressure equalling overburden pressure and cohesion (Gilpin 1980). Some models include a detailed stress analysis, describing elastic and viscous behaviour using Hooke's law and either the Prandtl-Reuss or Norton-Hoff law. The frost heave calculation includes freezing of pore water in situ and freezing of water in segregated ice lenses.

The analytical formulation of the coupled heat and water flow equations is usually approximated by a Galerkin weighted residual finite element method in the space domain and a finite difference method in the temporal domain. The analytical formulation of the stress analysis is usually approximated by a Ritz finite element method in the space domain and a finite difference method in the temporal domain.

Published frost heave prediction models currently used in practise are given with the regions in which they are used in Table 4:

Table 4
PUBLISHED FROST HEAVE PREDICTION MODELS CURRENTLY USED

Model Developer	Institute or Company	Region of Use
Nixon 1991	Nixon Geotech	Canada, Alaska
Konrad and Morgenstern 1980	Laval University	Canada, Japan
Guymon et al. 1993	Cold Regions Research & Engineering Laboratory (CRREL)	USA
N/A	Transport and Road Research Laboratory (TRRL), Department of the Environment	UK
Frémond and Mikkola 1991, 1993	Laboratoire Central des Ponts et Chaussées (LCPC)	France
Sheng et al 1995	University of Technology Lulea	Sweden
Saarelainen 1992 (SSR)	Valtion Teknillinen Tutkimuskeskus (VTT, Technical Research Centre)	Finland

Successful predictions have been claimed by the authors of all the models mentioned in Table 4.

Description of the heat flow requires knowledge about the thermal properties of the soil, and the local cyclical climatic input data, including climatic trends. Darcy flow through the frozen fringe requires the hydraulic conductivity of the frozen fringe. This hydraulic conductivity of the frozen fringe is the most significant input parameter required in frost heave prediction models because it varies with temperature below 0°C by several orders of magnitude. Nixon (1991) presents a compilation of measured subzero hydraulic conductivities. Konrad and Morgenstern (1980, 1981) proposed a water flow to the warmest ice lens determination method that avoids determination of the hydraulic conductivity of the freezing fringe. This method uses the temperature gradient and a mapped soil specific segregation potential surface, and has been successfully applied in many engineering design projects. Nixon's compilation could be extended and used concurrently with Konrad and Morgenstern's SP-method to determine the water flux density in the frozen fringe.

Other frost heave prediction model input parameters that are difficult to determine are the frost heave criteria such as the stress partitioning factor of Miller 1978 or the separation pressure of Gilpin 1980, although some researchers and practitioners who developed solutions of these two conceptual models claimed good agreement of predicted and measured frost heave in laboratory experiments after estimating these parameters.

A few researchers continue with efforts to improve existing frost heave models (e.g. a version of the Miller 1978 model by Fowler 2003), although currently not much effort is put towards the further development of the existing frost heave models. A 3-dimensional frost heave prediction model has not been published.

The frost heave prediction models that show the most promise for further development are those of Shen and Ladanyi (1987), Frémond and Mikkola (1991), and Shah (1990). Advances have been made to simplify and improve Frémond and Mikkola's model. Shah's model could be simplified. A first attempt of simplification by Wang (1994) was not successful, because critical elements such as the frost heave criterion were omitted. Currently most of the modelling effort focuses on the development of frost heave models for four specific applied purposes:

- Modelling of (differential) frost heave along buried chilled gas pipelines;
- Modelling of (differential) frost heave along (oil) pipelines in cold environments;
- Modelling of (differential) frost heave under highway and airport pavements and railroad tracks; and
- Modelling of (differential) frost heave under and around foundations, piles and retaining walls

These four fields of frost heave prediction application have their own specific issues. The first category of frost heave models is further discussed in the following section.

5.0 FROST HEAVE PREDICTION FOR BURIED CHILLED GAS PIPELINES

5.1 Problem Statement

The prediction of frost heave around buried chilled gas pipelines requires the following to be considered:

- a frost heave prediction model as presented in Section 4.4;
- the complex geometry due to a planar heat source or heat sink at the soil surface, and a cylindrical heat source or heat sink formed by the pipeline;
- the seasonal variability of the ground surface thermal boundary;
- the response of the pipeline to the frost heave process; and
- the soil-pipeline interaction.

Buried chilled gas pipelines generate freezing and frost heave of the surrounding soil in addition to the natural occurring freezing and frost heave. In the summer, the soil surrounding the chilled gas pipeline is warmer than the pipeline. Heat flow occurs from the soil surface downward, and from the soil surrounding the pipeline towards the pipe. The pipeline itself is the source of cooling, freezing and frost heave. In the winter the soil surrounding the pipeline is colder than the pipeline and heat flows from the pipeline towards the surrounding soil. A 2-D or 3-D frost heave prediction model is required for a proper modelling of soil freezing and frost heave around a buried chilled gas pipeline.

5.2 Historical Review

Frost heave prediction models for buried chilled gas pipelines in North America have been developed since northern gas pipelines were proposed in the early 1970's. The number of publications and the advances in this field of study have increased substantially over the last 15 years, however, a state of the art review of frost heave prediction for buried gas pipelines does not exist. A listing of publications on frost heave prediction models for buried gas pipelines is presented in Appendix C. A selection of publications was examined and is summarized in Table C.1 in Appendix C. A summary of the most significant steps in the development of frost heave prediction modelling for buried chilled gas pipelines is presented in Table 5.

Table 5
SIGNIFICANT DEVELOPMENTS IN FROST HEAVE PREDICTION FOR BURIED CHILLED GAS PIPELINES

Author	Year	Development
Hwang	1977	Model predicting upper-bound frost heave under buried chilled gas pipeline (model includes capillary suction, shut-off pressure, and does not include frozen fringe and detailed stress analysis).
Several authors	Up to 1980	Soil response to frost heave is represented as discrete 1D spring or Winkler elements.
Sharma and Pralong	1982	Improved Stefan/Neumann approach to soil freezing using internal energy, enthalpy and heat flux; no water flow, ice lens formation or stress considerations.
Nixon et al.	1983	Soil mass is treated as an elastic or viscous continuum. Solution using STARDYNE stress analysis package and HERMAN non-linear finite element stress analysis program. Heave computed using SP.
Konrad and Morgenstern	1984	Calculation of thermal field around the pipeline using cylindrical polar coordinate system; incremental total frost heave calculation, using SP to calculate water flow density in frozen fringe.
Shen and Ladanyi	1987	2D hydrodynamic frost heave prediction model with detailed stress analysis, frost heave criterion of Taylor and Luthin 1978.
Fremond and Mikkola	1991	Use of continuum mechanics to model frost heave around a buried chilled gas pipeline.
Selvadurai	1992	Description of the 6 required elements of frost heave prediction around chilled buried gas pipelines.
Nixon	1992	Discrete ice lens model; use of quasi static 2D method of Hwang to determine temperature gradients beneath buried pipeline; use of these gradients in the discrete ice lens model; no detailed soil and pipeline stress analysis.
Shah and Razaqpur	1993	2D rigid ice model predicting frost heave around pipelines using Galerkin finite element solution method in space.
Selvadurai et al.	1999	3D hydrodynamic frost heave prediction model based on Shen and Ladanyi 1987, includes stress analysis and frost heave criterion of Taylor and Luthin 1978. Achieved better agreement with measured data than Shen and Ladanyi 1987.

Comprehensive analysis of a soil-pipeline interaction problem that describes soil freezing and frost heave around a buried chilled gas pipeline should take into account:

1. Coupled heat flow and moisture transport within the frozen and unfrozen soils;
2. The mechanical behaviour of the unfrozen soil in particular the soil response to freezing and frost heave;
3. Moving boundary problems associated with a moving freezing front and frost heave;
4. Growth of pore ice and ice lenses;
5. Mechanical behaviour of the buried pipeline, which includes the structural response of the pipeline; and
6. Interface conditions at the soil-pipeline interface

The models used for frost heave prediction around buried chilled gas pipelines have become more sophisticated with time. The modelling efforts commenced with very elementary Stefan/Neumann approaches, followed by elementary SP application, and eventually to a 2D rigid ice model and a 3D form of the Shen and Ladanyi (1987) model.

Similar trends are observed for the modelling of the response of the soil to heaving. Originally the soil was represented as a discrete 1D spring or Winkler element, and later as a 2D or 3D continuum which included viscous and elastic behaviour. Plastic behaviour of the frozen soil as a result of pipeline uplift has also been modelled.

Shen and Ladanyi (1989) and others have considered the moving boundary problems and optimizing numerical efficiencies required for pipeline problems in freezing ground. Mechanical behaviour of the pipeline has been modelled simply as a flexible beam which possesses flexural, axial shear and torsional stiffness, and more complex as a cylindrical shell which can possess complex non-linear stress-strain-time phenomena. The gas pressure in a pipeline decreases with distance from the last compressor and chilling station. The gas temperature decreases with the pressure decrease, according to the Joule-Thompson effect, and the pipeline temperature reaches a minimum when the gas arrives at a compressor station, where it is subsequently chilled and compressed. The pipeline temperature decrease with distance from a compressor station has to be taken into account in frost heave modelling for buried chilled gas pipelines.

Interface behaviour of the soil and the pipeline has been simply described by either continuity or separation in the displacements between the pipeline and the one dimensional soil models. This interface behaviour can be described by interface phenomena such as frictionless contact, frictional contact or bonded contact and interface constitutive models determined from experiments, although presently it has almost exclusively been described by bonded contact.

5.3 Current State

The state of the art of the modelling of the six elements of the frost heave prediction around buried chilled pipelines is described below.

1. Coupled heat flow and moisture transport within the frozen and unfrozen soils is addressed in Section 4.4.
2. The mechanical behaviour of the unfrozen soil, in particular, the soils response to the freezing soil is modelled as a 2D or 3D continuum.
3. A finite element discretization is used which accommodates the complex coupled heat and water flows and displacements that accompany the between horizontal and pipeline concentric pattern of ice lens growth.
4. Growth of pore ice and ice lenses is addressed in Section 4.4.
5. The buried pipeline is represented as a cylindrical shell which can posses complex non-linear stress-strain-time phenomena.
6. The soil - pipeline interface behaviour is described by interface phenomena such as frictionless contact, frictional contact or bonded contact and interface constitutive models.

The manner in which the models are applied varies from simple to complex. The most simple application of frost heave prediction around buried chilled pipelines is a repetition of the application of a simple one dimensional frost heave prediction model along the pipeline using the location specific thermal regime and soil properties. The frost heave under the pipeline is predicted, which is used to calculate the resulting stress in the pipeline.

The most complex application of frost heave prediction around buried chilled pipelines uses a two or three dimensional frost heave prediction model along the pipeline using location specific thermal and hydraulic soil properties defined at nodes of a mesh around the pipeline. Frost heave effects including the strains at every node around the pipeline are predicted, and the calculated strains around the pipeline are used to calculate the resulting stress in the pipeline. The response of the soil to pipeline uplift is also predicted.

5.4 Input Data for Models Predicting Frost Heave Around Buried Chilled Gas Pipelines

Table 6 provides the input data required for the prediction of frost heave around buried chilled gas pipelines, and the source of these input data:

Table 6
INPUT DATA AND DATA SOURCES REQUIRED FOR FROST HEAVE PREDICTION AROUND BURIED CHILLED GAS PIPELINES

Data	Source
Climatic data (temperature, precipitation, snow cover thickness)	Weather stations
Soil properties (e.g. soil moisture content, particle size distribution, Atterberg limits, salinity and organic content)	Site investigations along proposed pipeline route; laboratory index testing; ditch wall logging, geophysical programs
Thermal soil conditions	Instrumentation, geophysical program
Segregation potential	Small scale laboratory frost heave tests
Hydraulic conductivity	Small scale laboratory frost heave tests
Unfrozen water contents	Laboratory TDR tests

6.0 STATUS OF ONGOING RESEARCH

Most of the ongoing frost heave research is currently focused on application of frost heave prediction for specific applications such as buried gas chilled pipelines, road and airport pavements, and building foundations. Experimental frost heave research has decreased from the high research intensity in the 1970's and 1980's, especially in North America, over the last decade. Further development of frost heave prediction models still occurs in the UK, Sweden, Finland, China and Russia.

Centres where researchers or practitioners have worked on some aspects of frost heave theory, testing and/or modelling are listed in Appendix D. Frost heave research is ongoing at institutions and companies listed in Table 7.

Table 7
INSTITUTIONS AND COMPANIES WITH ON-GOING FROST HEAVE RESEARCH

Institution or Company	City	Country
Carleton University	Ottawa	Canada
Laval University	Laval	Canada
C-Core	St.John's	Canada
Nixon Geotech	Calgary	Canada
McGill University	Montreal	Canada
CRREL	Hanover	USA
University of Alaska	Fairbanks	USA
Northern Engineering & Scientific	Anchorage	USA
Exxon Production Research Company	Houston	USA
University of Colorado	Boulder	USA
University of Nottingham, British Drilling & Freezing Co.	Nottingham	UK
Oxford University	Oxford	UK
University of Aston	Birmingham	UK
LCPC	Paris	France
Technical University of Lulea	Lulea	Sweden
VTT	Espoo	Finland
Helsinki University of Technology	Helsinki	Finland
University of Oulu	Oulu	Finland
Moscow State University	Moscow	Russia
Earth Cryosphere Institute SB RAS	Tyumen	Russia
Hokkaido University	Sapporo	Japan
Lanzhou Institute of Glaciology and cryopedology	Lanzhou	China

A more accurate account of current and past activity on frost heave theory, testing and modelling can be obtained by mailing a questionnaire to the centres listed in Appendix D.

7.0 FROST HEAVE EVALUATION SUMMARY

The number of publications on frost heave has increased considerably since the Berger inquiry in the 1970's. Frost heave theory developed by Miller 1978 has been proven in laboratory experiments and is accepted widely. A procedure proposed by Konrad and Morgenstern 1980 to characterize frost susceptible soil in small scale laboratory tests in terms of their segregation potential is used in many applications. The rapid freezing, associated large thermal gradients, short test duration, and limited sample size have been recognized as limitations of small scale laboratory frost heave tests. Large scale laboratory and field frost heave tests have been used to evaluate scale effects. In addition, scaled frost heave tests in centrifuges have dealt with some of the testing time constraints.

All current frost heave prediction models describe a set of variations of the same equations such as Fourier's law, the heat continuity law, Darcy's law, the mass continuity law, the Clausius-Clapeyron equation, and include a frost heave criterion for the frozen zone the frozen fringe and the unfrozen zone of the freezing soil profile. The models that include a stress analysis describe elastic and viscous behaviour by Hooke's law and either the Norton-Hoff or Prandtl-Reuss law respectively. Most researchers and practitioners who proposed frost heave prediction models claimed successful and accurate predictions, validated by tests. Models that have been applied for frost heave prediction are provided in Table 4.

Research efforts in frost heave prediction for buried chilled pipelines have increased since 1980. This research has benefited from large scale laboratory and field test programs such as the experiments in Fairbanks, Calgary and Caen. Computed pipe stresses caused by differential heave at a sand-silt boundary in the Caen experiment give a good indication of the maximum differential stresses that can be expected in the pipeline. Validation of the frost heave predictions around the buried pipelines in these large scale facilities is beyond the scope of this paper.

Accurate frost heave prediction is considered to be in an advanced research state. Presently, frost heave predictions are not routinely carried out in industry and commercial software is not available. However, conservative estimates of frost heave upper bounds enable a safe design of chilled gas pipelines. The current trend in modelling for frost heave design of chilled pipelines is to make these upper bound estimates less conservative.

The most difficult aspect of frost heave prediction is the acquisition of the input data for frost heave prediction models. Frost heave prediction for a buried chilled gas pipeline adds another degree of complexity as the soil properties along the pipeline can vary significantly. It is not possible to characterize soil properties along an entire pipeline. A sensitivity or probability approach must be carried out to evaluate the influence of soil properties on the frost heave prediction. The spatial variability of the soil properties along the pipeline routing must be assessed. Conservatism must be used in the pipeline design to accommodate for uncertainties in and variability of the input parameters in the frost heave prediction model, and for generalizations in the prediction model itself.

Most frost heave predictions resulted in over-prediction of long-term frost heave (e.g. Konrad and Morgenstern 1984, Shen and Ladanyi 1987, and Nixon 1991), and representation of the soil by Winkler elements over-predicts the pipeline stresses generated by the maximum differential frost heave. Therefore, the simpler and older methods that predict differential frost heave around buried chilled gas pipelines provide conservative upper-bound estimates of frost heave and frost heave generated pipeline stresses, required for safe pipeline design.

8.0 FURTHER REVIEW AND RECOMMENDATIONS

Frost heave knowledge could benefit from the following research activities:

- A thorough elementary examination of the driving force of frost heave;
- Development of a method to measure the hydraulic conductivity in the frozen fringe;
- Development of an extensive database of frost heave test results;
- Standardization of frost heave test procedures and equipment;
- Development of a method to indirectly measure pressure at the location where a new ice lens forms;
- A three dimensional frost heave prediction model based on Miller's rigid ice model, Shen and Ladanyi's model or Nixon's discrete ice lens theory, including an extensive stress analysis such as given by Shen and Ladanyi;
- Commercial development of such an advanced practical model;
- Clearer statements of the input parameters required for each frost heave prediction model. Access to the computer code is often required to list these input parameters; and
- An overview of frost heave prediction for buried chilled gas pipelines that includes discussion of pipeline mechanics.

The research activities suggested above are not necessary requirements for safe pipeline design, but merely of academic, clarifying and informative value.

Design of a chilled buried gas pipeline must consider the following:

- An upper-bound estimate of the differential movement and stresses imposed on a pipeline due to frost heave and other processes;
- An assessment of the level of accuracy in frost heave prediction required to safely construct and operate a buried chilled gas pipeline;
- A thorough review of (differential) frost heave effects mitigation measures;
- A thorough review of pipeline materials and their ability to withstand stresses generated by differential frost heave;
- An evaluation of pipeline construction techniques that can limit differential frost heave stresses; and
- An extensive monitoring program during pipeline operation.

The quality of frost heave prediction models for buried chilled gas pipelines can only be fully assessed based on their performance and accuracy in predicting the stress-strain behaviour of soil and pipelines in large scale laboratory and field experiments. It is understood that the GSC has commissioned another report to review this aspect.

9.0 CLOSURE

EBA is pleased to have provided this state of the art paper on frost heave. Please do not hesitate to contact the undersigned should you have any questions or comments regarding this paper.

Yours truly,
EBA Engineering Consultants Ltd.

F. Schellekens

Fons Schellekens, Ph.D.
Geotechnical Specialist
Circumpolar Regions
(Direct Line: (780) 451-2130 ext. 272)
(e-mail: fschellekens@eba.ca)

Reviewed by:

W.T. Horne, P.Eng.
Senior Project Engineer
Circumpolar Regions
(Direct Line: (780) 451-2130, ext. 276)
(e-mail: bhorne@eba.ca)

FS:ln

REFERENCES

Akagawa, S., 1988. Experimental Study of frozen fringe characteristics. Cold Regions Science and Technology, Vol.15, pp.209-223.

Akagawa, S., 2000. A method for controlling stationary frost heaving. Ground Freezing 2000—Frost Action in Soils. Proceedings of the International Symposium on Ground Freezing, Leuven, J.F. Thimus ed., A.A. Balkema, Rotterdam, pp.63-68.

Andersland, O.B., and Ladanyi, B., 2004. Frozen Ground Engineering. ASCE, John Wiley & Sons, Hoboken, NJ, 363p.

Arvidson, W.D., and Morgenstern, N.R., 1977. Water flow induced by soil freezing. Canadian Geotechnical Journal, Vol.14, pp.237-245.

Beskow, G., 1935. Soil Freezing and Frost heaving with special application to roads and railroads. The Swedish Geological Society, C, no.375, Year Book no. 3 (Translated by J.O. Osterberg). Technological Institute, Northwestern University.

Biermans, M., Dijkema, K., and de Vries, D.A., 1978. Water movement in porous media towards an ice front. Journal of Hydrology, Vol.37, pp.137-148.

Black, P.B., 1995. RIGIDICE Model of secondary frost heave. CRREL Report 95-12, 32p.

Black, P.B., and Hardenberg, M.J., 1991. Historical perspectives in frost heave research. The early works of S. Taber and G. Beskow. CRREL Special Report 91-23, 169 p.

Bouyoucos, G.J., 1920. Degree of temperature to which soils can be cooled without freezing. Journal of Agricultural Research., Vol. 20, pp.267-269.

Burt, T. and Williams, P.J., 1976. Hydraulic conductivity in frozen soils. Earth Surface Processes, Vol.1, no.3, pp.349-360.

Casagrande, A., 1931. Discussion of frost heaving. Highway Research Board Proceedings, Vol.11, pp.168-172.

Cass, L., and Miller, R.D., 1959. Role of the electric double layer in the mechanism of frost heaving. USA Snow, ice and permafrost establishment, Research Report 49.

Chamberlain, E.J., 1981. Frost susceptibility of soil: review of index tests. CRREL Monograph 81-2, 110p.

Dirksen, C., 1964. Water movement and frost heaving in unsaturated soil without an external source of water. Ph.D.thesis, Cornell University, Ithaca, New York.

Edlefsen, N.E., and Anderson, A.B.C., 1943. Thermodynamics of soil moisture. Hilgardia, Vol.15, No.2, pp.31-298.

Everett, D.H., 1961. The thermodynamics of frost damage to porous solids. Transactions of the Faraday Society, Vol.57, pp.1541-1551.

Forland, T., and Ratkje, S., 1980. On the theory of frost heave. Frost i jord, Vol.21, pp.45-48.

Fowler, A.C., 2003. A mathematical model of differential frost heave. Permafrost, Eight International Conference, Proceedings, Zurich, M. Philips, S.M. Springman and L. Arenson, eds., A.A. Balkema, Rotterdam, Vol.1, pp. 249-252.

Fowler, A.C., and Noon, C.G., 1993. A simplified numerical solution of the Miller model of secondary frost heave. Cold Regions Science and Technology, Vol. 21, pp.327-336.

Frémond, M., and Mikkola, M., 1991. Thermomechanical modelling of freezing soil. Ground Freezing 91, Sixth International Symposium on Ground Freezing, Beijing, X. Yu, and C. Wang eds., A.A. Balkema, Rotterdam, pp.17-24.

Frémond, M., and Mikkola, M., 1993. Thermomechanical modelling of freezing soil. Gas pipelines, oil pipelines and civil engineering in arctic climates, Proceedings of a seminar held in Caen and Paris, France, P.J. Williams ed., Geotechnical Science Laboratories, Carleton University, Ottawa, Canada, pp.48-60.

Gilpin, R.R., 1979. A model of the liquid-like layer between ice and a substrate with application to wire regelation and particle migration. Journal of Colloid and Interface Science, Vol.68, no.2, pp.235-251.

Gilpin, R.R., 1980. A model for the prediction of ice lensing and frost heave in soils. Water Resources Research, Vol.16, pp.918-930.

Gold, L.H., 1957. A possible force mechanism associated with the freezing of liquids. Highway Research Board Bulletin 168, pp.65-72.

Gorelik, J.B., Kolunin, V.S., and Reshetnikov, A.K., 1998. Rigid-ice model and stationary growth of ice. Permafrost. Seventh International Conference, Proceedings, Yellowknife, Canada, A.G. Lewkowicz and M. Allard eds., Collection Nordicana No. 57, Université Laval, Laval, pp.327-333.

Goto, S., and Takahashi, Y., 1982. Frost heave characteristics of soil under extremely low frost penetration rate. Ground Freezing 82, Third International Symposium on Ground Freezing, Hanover, NH, USA, pp.261-268.

Groenevelt, P.H., 1974. On thermo-osmosis and thermo-filtration in porous media. 10th International Congress of Soil Scientists, Transactions, Vol.1, pp.47-55.

Guymon, G.L., Berg, R.L., and Hromadka, T.V., 1993. Mathematical model of frost heave and thaw settlement in pavements. CRREL Report 93-2. 126p.

Harlan, R.L., 1973. An analysis of coupled heat-fluid transport in partially frozen soil. Water Resources Research, Vol.9, pp.1314-1323.

Henry, K., 2000. A review of the thermodynamics of frost heave. Technical Report ERDC/CCREL TR-0016, 19p.

Hill, D.W., and Morgenstern, N.R., 1977. Influence of load and heat extraction on moisture transfer in freezing soils. International Symposium on Frost Action in Soils, Lulea, Vol.1, pp.76-91.

Hoekstra, P., 1966. Moisture movement in soils under temperature gradients with the cold-side temperature below freezing. Water Resources Research, Vol.2, No.2, pp.241-250.

Hoekstra, P., 1969. Water movement and freezing pressures. Soil Science Society of America Proceedings, Vol.33, pp.512-518.

Holden, J.T., and Jones, R.H., 1985. Some developments in rigid ice model frost heave. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, Vol.1, pp.93-99.

Horiguchi, K., 1987. An osmotic model for soil freezing. Cold Regions Science and Technology, Vol.14, p.13-22.

Hwang, C.T., 1977. Frost heave design of a chilled gas pipeline. 30th Canadian Geotechnical Conference, Saskatoon, pp.V-59 – V-88.

Jackson, K.A., and Chalmers, B., 1958. Freezing liquids in porous media with reference to frost heaving in soils. Journal of Applied Physics, Vol. 29, pp.1178-1181.

Jones, R.H., 1995. Chapter 3 Properties of freezing, frozen and thawed soils. pp.26-69. In: Ground freezing in practice. J.S. Harris ed., Thomas Telford, London, 264p.

Kay, B.D. and Perfect, E. 1988. State of the art: Heat and mass transfer in freezing soils. Ground Freezing 88, Fifth International Symposium on Ground Freezing, Nottingham, R.H. Jones and J.T. Holden eds., A.A. Balkema, Rotterdam, Vol.1, pp.3-21.

Konrad, J.-M., 1994. Sixteenth Canadian Geotechnical colloquium: Frost heave in soils: concepts and engineering. Canadian Geotechnical Journal, Vol.31, pp.223-245.

Konrad, J.-M., and Morgenstern, N.R., 1980. A mechanistic theory of ice lens formation in fine-grained soils. Canadian Geotechnical Journal, Vol.17, pp.473-486.

Konrad, J.-M., and Morgenstern, N.R., 1981. The segregation potential of a freezing soil. Canadian Geotechnical Journal, Vol.18, pp.482-491.

Konrad, J.-M., and Morgenstern, N.R., 1982. Effect of applied pressure on freezing soils. Canadian Geotechnical Journal, Vol.19, pp.494-505.

Konrad, J.-M., and Morgenstern, N.R., 1984. Frost heave prediction of chilled pipelines buried in unfrozen soils, Canadian Geotechnical Journal, Vol.21, pp.100-115.

Kujala, K., 1997. Estimation of frost heave and thaw weakening by statistical analyses and physical models. Ground freezing 97—Frost Action in Soils, Eight International Symposium on Ground Freezing, Lulea, S. Knutsson ed., A.A. Balkema, Rotterdam, pp.31-41.

Ladanyi, B., and Lemaire, G., 1984. Behaviour of a buried pipeline under differential frost heave conditions. Proceedings of the Canadian Society of Civil Engineers Cold Regions Engineering Specialty Conference, Montreal, pp. 161-176.

Ladanyi, B., and Shen, M., 1989. Mechanics of freezing and thawing in soils. Proceedings of an International Symposium on Frost in Geotechnical Engineering, Saariselka, Finland, VTT Symposium no.95, H. Rathmayer ed., Technical Research Centre of Finland, Espoo, Vol.1, pp.73-103.

Linnell, K.A., and Kaplar, G.W., 1959. The factor of soil and material type in frost action. Highway Research Board Bulletin 225, pp.81-128.

Loch, J.P.G., 1979. Influence of the heat extraction rate on the ice segregation rate of soils. Frost I Jord, Vol.20, pp.19-30.

Loch, J.P.G., and Kay, B.D., 1978. Water redistribution in partially frozen, unsaturated soil under several temperature gradients and overburden loads. Soil Science Society of America Journal, Vol.42, pp.400-406.

Mageau, D.W., and Morgenstern, N.R., 1980. Observations of moisture migration in frozen soils. Canadian Geotechnical Journal, Vol.17, no.1, pp.54-60.

Martin, R.T., 1959. Rhythmic ice banding in soil. Highway Research Board Bulletin 218, NAS-NRC, Washington, pp.11-23.

Miller, R.D., 1972. Freezing and heaving of saturated and unsaturated soils. US Highway Research Record 393, pp.1-11.

Miller, R.D., 1973. Soil freezing in relation to pore water pressure and temperature. Permafrost, The North American Contribution to the 2nd International Conference on Permafrost, Yakutsk, National Academy of Sciences, Washington D.C., pp.344-352.

Miller, R.D., 1977. Lens initiation in secondary heaving. Proceedings of the International Symposium on Frost Action in Soils, Vol.2, pp.68-74.

Miller, R.D., 1978. Frost heaving in non-colloidal soils. Permafrost, Third International Conference, Proceedings, Edmonton, National Research Council of Canada, Ottawa, pp.708-713.

Miller, R.D., 1980. Freezing phenomena in soils. In: Applications of soil physics, D. Hillel ed., Academic Press, New York, pp.254-299.

Miller, R.D., Baker, J.H., and Kolaian, J.H., 1960. Particle size, overburden pressure, pore water pressure and freezing temperature of ice lenses in soil. 7th International Congress of Soil Scientists, Transactions, Vol.1, pp.122-129.

Miller, R.D., Loch, J.P.G., and Bresler, E., 1975. Transport of water and heat in a frozen permeameter. Soil Science Society of America, Proceedings, Vol.39, No.6, pp.1029-1036.

Nixon, J.F., 1987. Ground freezing and frost heave – a review. Northern Engineer, Vol. 19, no3/4, pp.8-18.

Nixon, J.F., 1991. Discrete ice lens theory for frost in soils. Canadian Geotechnical Journal, Vol.28, pp.843-859.

Nixon, J.F., 1992. New frost heave prediction model for design of northern pipelines. Proceedings of the Second (1992) International Offshore and Polar Engineering Conference, San Francisco, ISOPE, Vol.II, pp.32-39.

Nixon, J.F., Morgenstern, N.R., and Reesor, S.N., 1983. Frost heave – pipeline interaction using continuum mechanics. Canadian Geotechnical Journal, Vol.20, pp.251-261.

Ohrai, T., and Yamamoto, H., 1985. Growth and migration of ice lenses in partially frozen soil. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, Vol.1, pp.79-84.

O’Neill, K., 1983. The physics of mathematical frost heave models: A review. Cold Regions Science and Technology, Vol.6, pp.275-291.

O’Neill, K., and Miller, R.D., 1982. Numerical solutions for a rigid ice model of secondary frost heave. U.S. Army CRREL Report 82-13, Hanover, NH, 11p.

O’Neill, K., and Miller, R.D., 1985. Exploration of a rigid ice model of frost heave. Water Resources Research, Vol.21, no.3,pp.281-296.

Padilla, F., and Villeneuve, J.P., 1990. Modelling the movement of water, heat and solutes in frost susceptible soils. Proceedings of the Fifth Canadian Permafrost Conference, Quebec. Collection Nordicana 54, pp.43-49.

Padilla, F., and Villeneuve, J.P., 1992. Modelling and experimental studies of frost heave including solute effects. Cold Regions Science and Technology, Vol.20, n.2, pp.183-194.

Penner, E., 1967. Heaving pressures in soils during unidirectional freezing. Canadian Geotechnical Journal, Vol.4, pp.398-408.

Penner, E., and Eldred, D., 1985 Equipment and methods for soil frost action studies. National Research Council of Canada, Division of Building Research, DBR Internal Report No. 503, 8 p.

Penner, E., and Goodrich, L., 1980. Location of segregated ice in frost susceptible soil. Second International Symposium on Ground Freezing, Trondheim, Norwegian Institute of Technology, pp.626-639.

Penner, E., and Ueda, T., 1977. The dependence of frost heaving on load application – preliminary results. Proceedings of the International Symposium on Frost Action in Soils, Lulea, University of Lulea, Vol.1, pp.92-101.

Penner, E., and Walton, T., 1978. Effects of temperature and pressure on frost heaving. Engineering Geology, Vol.13, no.1-4, pp.29-39.

Phillip and DeVries 1957. Moisture movement in porous material under temperature gradients. American Geophysical Union, Transactions, Vol. 38, pp.222-232.

Raats, P.A.C., 1975. Transformation of fluxes and forces describing the simultaneous transport of water and heat in unsaturated porous media. Water Resources Research, Vol.11, pp.938-942.

Saarelainen, S., 1992. Modelling frost heaving and frost penetration in soils at some observation sites in Finland. The SSR model. VTT Publication 95. Technical Research Centre of Finland, Espoo, 119 p.

Schellekens, F.J., 1997. Fundamentals, accuracy and input parameters of frost heave prediction models. Unpublished Ph.D.-thesis, Carleton University, Ottawa, 304p.

Schofield, R.K., 1935. The pF of water in soil. International Congress on Soil Science, Transactions, Third Edition, Vol.2, Oxford, UK, pp.37-48.

Selvadurai, A.P.S., 1992. Soil – pipeline interaction at a frost heave zone. Proceedings of the 11th International Conference on Offshore Mechanics and Arctic Engineering (OMAE), Calgary, ASME, New York, Vol.V-B, pp.337-348.

Selvadurai, A.P.S., Hu, J., and Konuk, I., 1999. Computational modelling of frost heave induced soil-pipeline interaction I. Modelling of frost heave. Cold Regions Science and Technology, Vol.29, pp.215-228.

Shah, K., 1990. Deformations and stresses in pipelines buried in freezing ground. M.Eng.-thesis, Carleton University, Ottawa, 150p.

Shah, K., and Razaqpur, A.G., 1993. A two-dimensional frost-heave model for buried pipelines. International journal for numerical methods in engineering, Vol.36, pp.2545-2566.

Sharma, D., and Pralong, P.-J., 1982. Transient freezing and thawing of soils around buried pipelines. Numerical methods in geomechanics, International symposium on numerical models in geomechanics, Zurich, R. Dungar, G.N. Pande, and J.A. Studer eds., A.A. Balkema, Rotterdam, pp.513-524.

Shen, M., and Ladanyi, B., 1987. Modelling of coupled heat, moisture and stress field in freezing soil. Cold Regions Science and Technology, Vol.14, pp.237-246.

Shen, M., and Ladanyi, B., 1989. Numerical solutions for freezing and thawing soils using boundary-conforming curvilinear coordinate systems. Proceedings of an International Symposium on frost in geotechnical engineering, Saariselka, Finland, VTT Symposium No.95, H. Rathmayer ed. Technical Research Centre of Finland, Espoo, Vol.1, pp.391-400.

Sheng, D., Axelsson, K., and Knutsson, S., 1995. Frost heave due to ice lens formation in freezing soils. Theory and verification. Nordic Hydrology, Vol.26, no.2, pp.125-146.

Smith, M.W., 1985. Models of soil freezing. In Field and theory, M.Church and O. Slaymaker eds., University of British Columbia Press, Vancouver, pp.97-120.

Sutherland, H.B., and Gaskin, P.N., 1973. Pore water and heaving pressures developed in partially frozen soils. Permafrost, Second International Conference, Proceedings, Yakutsk, North American contributions, National Academy of Sciences, Washington DC, pp.409-419.

Taber, S., 1929. Frost heaving. Journal of Geology, Vol.37, pp.428-461.

Taber, S., 1930. The mechanics of frost heaving. Journal of Geology, Vol.38, pp.303-317.

Taylor, G.S., and Luthin, J.N., 1978. A model for coupled heat and moisture transfer during soil freezing. Canadian Geotechnical Journal, Vol.15, pp.548-555.

Vignes, M., and Dijkema, K., 1974. A model for the freezing of water in a dispersed medium. Journal of Colloid Interface Science, Vol.49, pp.165-172.

Wang, D., 1994. A coupled thermo-mechanical analysis of pipelines buried in freezing ground. unpublished M.Eng.thesis, Carleton University, Ottawa, 210p.

Xu, X. Z., Oliphant, J.L., and Tice, A.R., 1985. Prediction of unfrozen water contents in frozen soils by a two points or one point method. Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, Vol.2, pp.83-87.

APPENDIX A

SMALL SCALE FROST HEAVE EXPERIMENTS LISTING AND COMPARISON

FROST HEAVE EXPERIMENTS REFERENCE LIST

(the publications printed in bold are summarized in Table A.1)

A1. Akagawa, S., 1988a. Evaluation of the X-ray radiography efficiency for heaving and consolidation observation. Ground Freezing 88, Fifth International Symposium on Ground Freezing, Nottingham, R.H. Jones and J.T. Holden eds., A.A. Balkema, Rotterdam, pp. 23-28.

A2. Akagawa, S., 1988b. Experimental study of frozen fringe characteristics. Cold Regions Science and Technology, Vol. 15, pp. 209-223.

A3. Akagawa, S., 2000. A method for controlling stationary frost heaving. Ground Freezing 2000 – Frost Action in Soils, Ninth International Symposium on Ground Freezing, Leuven, J.F. Thimus ed., A.A. Balkema, Rotterdam, pp. 63-68.

A4. Akagawa, S., and Fukuda, M., 1991. Frost heave mechanism in welded tuff. Permafrost and Periglacial Processes, Vol. 2, pp. 301-309.

A5. Akagawa, S., Yamamoto, Y., and Hashimoto, S., 1985. Frost heave characteristics and scale effect of stationary frost heave. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, pp. 37-143.

A6. ASTM Designation D5918-96. Standard Test Methods for Frost Heave and Thaw Weakening Susceptibility of Soils, pp. 795-804.

A7. Berg, R.L., Ingersoll, J., and Guymon, G.L., 1980. Frost heave in an instrumented soil column. Cold Regions Science and Technology, Vol.3, pp. 211-221.

A8. Goto, S., and Takahashi, Y., 1982. Frost heave characteristics of soil under extremely low frost penetration rate. Proceedings of the Third International Symposium on Ground Freezing, Hanover, NH, US Army Corps of Engineers, pp. 261-268.

A9. Hazen, B., Nixon, J.F., Heuer, C.E., Caldwell, J.B., and Brudie, E.L., 1993. Frost heave predictions for Alaskan soils. Permafrost, Sixth International Conference, Proceedings, Beijing, J. Brown, H.M. French, N.A. Grave, G. Cheng, L. King, and E.A. Koster eds., South China University of Technology Press, Wuhan, pp. 244-249.

A10. Hoekstra, P., 1969. Water movement and freezing pressures. Soil Science Society of America Proceedings, Vol. 33, n. 4, pp. 512-518.

A11. Ishizaki, T., and Nishio, N., 1988. Experimental study of frost heaving of a saturated soil. Ground Freezing 88, Fifth International Symposium on Ground Freezing, Nottingham, R.H. Jones and J.T. Holden eds., A.A. Balkema, Rotterdam, pp. 65-72.

A12. Ito, Y., Vinson, T.S., Nixon, J.F., and Stewart D., 1998. An improved step freezing test to determine segregation potential. Permafrost, Seventh International Conference, Proceedings, Yellowknife, Lewkowicz and Allard eds., Collection Nordicana No.57, Université Laval, Laval, pp. 509-516.

A13. Jessberger, H.L., and Jagow, R., 1989. Determination of frost susceptibility of soils. Frost in Geotechnical Engineering, International Symposium Saariselkä, VTT symposium 95, H. Rathmayer ed., Technical Research Centre of Finland, Espoo, Vol.2, pp. 449-469.

A14. Knutsson, S., Domaschuk, L., and Chandler, N., 1985. Analysis of large scale laboratory and in situ frost heave tests. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, pp. 65-70.

A15. Konrad, J.M., 1987. Procedure for determining the segregation potential of freezing soils. Geotechnical Testing Journal, Vol. 10, No.2, pp. 51-58.

A16. Konrad, J.M., 1989. Pore water pressure at an ice lens: its measurement and interpretation. Cold Regions Science and Technology, Vol. 16, pp. 63-74.

A17. Konrad, J.M., and Nixon, J.F., 1994. Frost heave characteristics of a clayey silt subjected to small temperature gradients. Cold Regions Science and Technology, Vol.22, pp. 299-310.

A18. Kujala, K., and Ravaska, O., 1989. Influence of test conditions and equipment on the frost heave test. Frost in Geotechnical Engineering, International Symposium Saariselkä, VTT symposium 95, H. Rathmayer ed., Technical Research Centre of Finland, Espoo, Vol.2, pp. 931-944.

A19. McGaw, R., 1972. Frost heaving versus depth to water table. Highway Research Record, Vol. 393, pp. 45-55.

A20. Nakano, Y., and Horiguchi, K., 1985. Role of phase equilibrium in frost heave of fine grained soil under negligible overburden pressure. Advances in Water Resources, Vol. 8, pp. 50-68.

A21. Nishimura, T., Ogawa, S., and Fukuda, M., 1994. Effective stress in unsaturated soils after freezing and thawing. Ground Freezing 94, Seventh International Symposium on Ground Freezing, Nancy, M. Frémond ed., A.A. Balkema, Rotterdam, Vol.1, pp. 121-128.

A22. Nixon, J.F., 1982. Field frost heave predictions using the segregation potential concept. Canadian Geotechnical Journal, Vol. 19, pp. 526-529.

A23. Ohrai, T. and Yamamoto, H.,1985. Growth and migration of ice lenses in partially frozen soil. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, pp. 79-84.

A24. Penner, E., 1957. Soil moisture tension and ice segregation. Highway Research Board Bulletin 168, pp. 50-64.

A25. Penner, E., 1986. Aspects of ice lens growth in soils. Cold Regions Science and Technology, Vol. 13, pp. 91-100.

A26. Penner, E., and Eldred, D., 1985. Equipment and methods for soil frost action studies. National Research Council of Canada, Division of Building Research, DBR Internal Report No. 503, 8 p.

A27. Penner, E., and Goodrich, L.E., 1980. Location of segregated ice in frost susceptible soil. Second International Symposium on Ground Freezing, Trondheim, P.E Frivik, N. Janbu, R. Saetersdal and L.I. Finborud eds., Norwegian Institute of Technology, pp. 626-639.

A28. Penner, E., and Ueda, T., 1978. A soil frost-susceptibility test and a basis for interpreting heaving rates. Permafrost, Third International Conference, Proceedings, Edmonton, National Research Council of Canada, Ottawa, Vol. 1, pp. 721-727.

A29. Penner, E., and Walton, T., 1979. Effects of temperature and pressure on frost heaving. Engineering Geology, Vol. 13, pp. 29-39.

A30. Radd, F.J., and Oertle, D.H., 1973. Experimental pressure studies of frost heave mechanisms and the growth-fusion behaviour of ice. Permafrost, Second International Conference, Proceedings, Yakutsk, Vol. 1 North American contribution, National Academy of Sciences, Washington DC, pp. 377-384.

A31. Ryokai, K., 1985. Frost heave theory of saturated soil coupling water/heat flow and its application. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, pp. 101-108.

A32. Seto, J.T.C., and Konrad, J.M., 1994. Pore pressure measurements during freezing of an over consolidated clayey silt. Cold Regions Science and Technology, Vol. 22, pp. 319-338.

A33. Sheng, D., Axelsson, K., and Knutsson, S., 1995. Estimation of frost heave for stratified soil profile. Ground Freezing 94, Seventh International Symposium on Ground Freezing, Nancy M. Fremond ed., A.A. Balkema, Rotterdam, pp. 129-142.

A34. Sheng, Y., 1994. Calculation of frost heave in saturated soil under constant surcharge. Seventh International Cold Regions Engineering Specialty Conference, Edmonton, ASCE/CSCE, pp. 729-735.

A35. Sutherland, H.B., and Gaskin, P.N., 1973. Pore water and heaving pressures developed in partially frozen soils. Permafrost, Second International Conference, Proceedings, Yakutsk, USSR. North American Contribution, pp. 409-419.

A36. Svec, O.J., 1981. Frost heave control of a chilled gas pipeline. Cold Regions Science and Technology, Vol. 4, pp. 215-225.

A37. Svec, O.J., 1989. A new concept of frost heave characteristics of soils. Cold Regions Science and Technology, Vol. 16, pp. 271-279.

A38. Takashi, T., Ohrai, T., Yamamoto, H., and Okamoto, J., 1980. Upper limit of heaving pressure derived by pore water pressure measurements of partially frozen soil. The Second International Symposium on Ground Freezing, Trondheim, Norwegian Institute of Technology, pp. 713-724.

A39. Takeda, K., 1988. Experimental study on ice segregation during soil freezing. Thesis, Hokkaido University, Sapporo, Japan, 78 p.

A40. Takeda, K., and Nakano, Y., 1990. Quasi-steady problems in freezing soils: II Experiment on the steady growth of an ice layer. Cold Regions Science and Technology, Vol. 18, pp. 225-247.

A41. Williams, P.J., Riseborough, D.W., and Smith, M.W., 1992. The France-Canada joint study of deformation of an experimental pipeline by differential frost heave. Proceedings of the Second International Off-shore and Polar Engineering Conference, San Francisco, ISOPE, Vol. II, pp. 40-45.

A42. Williams, P.J., and Wood, J.A., 1985. Internal stresses in frozen ground. Canadian Geotechnical Journal, Vol. 22, pp. 413-416.

A43. Wood, J.A., and Williams, P.J., 1985. Stress distribution in soils. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, pp. 165-171.

A44. Yanagisawa, E., and Yao, Y.L., 1985. Moisture movement in freezing soils under constant temperature condition. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, pp. 85-91.

TABLE A.1 SMALL SCALE FROST HEAVE EXPERIMENTS COMPARISON (PAGE 1 OF 3)

Frost heave experiments reference nr.	A1	A2	A3	A4
author(s)	Akagawa 1988a	Akagawa 1988b	Akagawa 2000	Akagawa and Fukuda 1991
purpose	strain distribution profiles	strain distribution profiles		to show frost heave in tuff
length of column (m)	0.095	0.097	0.06	0.25 and 0.15
diameter of column (m)	0.06	0.06	0.06	0.29 and 0.05
number of layers	?	?		
dry density (kg/m3)				
wet density (kg/m3)	1450	1450		
water content				0.3
porosity				
saturation (%)				92, 100
temperature top (oC)	warm: 2.2 and 3.0	warm: 3.0	warm: 0.2	cold: -14 and -5
number of temperature sensors	15	15		10
temperature bottom (^{o}C)	cold: -5.9 and -5.5	cold: -5.5	cold: -5.5	warm: 4.5 and 2
open system	yes	yes	yes	yes
closed system	no	no	no	no
intitializing ice nucleation				
ramped freezing	no	no	no	no
ramp rate (^{o}C/hr)	N/A	N/A	N/A	N/A
step freezing	1 step	1 step	1 step	1 step
temperature step (^{o}C)	?	?	?	?
duration step (hr)				
temperature control bath (^{o}C)	yes	yes	yes	yes
temperature control chamber (^{o}C)	?	0	0	
freeze-thaw cycles	1 freezing	1 freezing	1 freezing	1 freezing
test duration (hr)	1000 and 650	700	450	1000 and 250
water supply	yes	yes	yes	yes
constant head	yes	yes	yes, 0 kPa	yes
pore water pressure measurement	no	no		during second series at 0.03, 0.05 and 0.07 m from top
surcharge (kPa)	60 and 110	110	60	just to maintain contact
displacement measurement	yes	yes	yes	yes
SP vs. time (K and M or Nixon method)			no, but possible	
temperature profile at time t	yes	yes	yes	
frost heave vs. time	yes	yes	yes	
frost penetration vs. time	yes	yes	yes	
water intake vs. time		yes	yes	
final water content profile		yes	yes	
frost susceptibility classification				
lens formation	yes	yes	no	
segregation temperature	yes	yes	no	
transient/stationary/long term	t,s and l	t,s and l	t,s and l	
soils	clay and silt with some sand	clay and silt with some sand	clay and silt, trace sand	Ohya Tuff
undisturbed/remoulded	undisturbed	undisturbed	undisturbed	?
lead spheres with thermocouples	yes, 15	yes, 14	no	
x-rays	yes, 136	yes, 74	no	
comments				

TABLE A.1 SMALL SCALE FROST HEAVE EXPERIMENTS COMPARISON (PAGE 2 OF 3)

Frost heave experiments reference nr.	A5	A6	A9	A11
author(s)	Akagawa et al. 1985	ASTM D5918-96	Hazen et al. 1993	Ishizaki and Nishio 1988
purpose	empirical formula for stationary heave	frost susceptibility determination	frost heave prediction	validation frost heave prediction model of Miller 1978
length of column (m)	0.015-0.25	0.1651	?	0.054
diameter of column (m)	0.06	0.146	?	0.125
number of layers	?	6		
dry density (kg/m^3)		initial and final		1500
wet density (kg/m^3)				1990
water content		initial and final		
porosity		initial and final		0.487, 0.348
saturation (%)		field condition		
temperature top (°C)	warm: 0.2-5	cold: 3, -3, -12,12,3,-3,-12,12,3		warm, T_0 is 1, T_0 is 2.5 to 10
number of temperature sensors	?	8		11
temperature bottom (°C)	cold: -0.8 to -20	warm: 3,3,0,3,3,3,0,3,3		cold, T_0 is 1
open system	yes	when low water table field conditions exist		yes
closed system	no	when low water table field conditions exist		no
intitializing ice nucleation				no
ramped freezing	no	no		yes
ramp rate (°C/hr)	N/A	N/A		warm end and cold end -0.025, -0.05,-0.1,-0.2 and -0.4
step freezing	1 step	2 steps		no
temperature step (°C)	?	from 3 to -3 to -12		N/A
duration step (hr)		resp 24, 8 and 18		N/A
temperature control bath (°C)	yes	yes		
temperature control chamber (°C)	?	2		1
freeze-thaw cycles	1 freezing	24 lead time, 24 freezing, 24 thawing, 24 freezing, 24 thawing		1 freezing
test duration (hr)	100-1000	120		100
water supply	yes	yes		
constant head	yes	yes	0.10 m on top of the soil	
pore water pressure measurement	no	no		
surcharge (kPa)	60, 110 and 160	0.7	9	170 + 50 back pressure
displacement measurement	yes			
SP vs. time (K and M or Nixon method)		no		
temperature profile at time t	yes	possible, but not required		yes
frost heave vs. time	yes	yes		yes
frost penetration vs. time		yes		yes
water intake vs. time		possible, but not required		yes
final water content profile		yes		
frost susceptibility classification		yes		
lens formation		no		
segregation temperature		no		
transient/stationary/long term	t,s and l			
soils	clay and silt, trace sand			clay and silt
undisturbed/remoulded	undisturbed			undisturbed
lead spheres with thermocouples				yes, 8 without thermocouples
x-rays	no			yes
comments		should not be used for frost heave prediction		

TABLE A.1 SMALL SCALE FROST HEAVE EXPERIMENTS COMPARISON (PAGE 3 OF 3)

Frost heave experiments reference nr.	A12	A15	A20	A26
author(s)	Ito et al. 1998	Konrad 1987	Nakano and Horiguchi 1985	Penner and Eldred 1985
purpose	segregation potential determination	segregation potential determination	validation of phase equilibrium	DBR/IRC standard
length of column (m)	0.08 to 0.1	Penner 1986	0.015	
diameter of column (m)		Penner 1986	0.1025	
number of layers				
dry density (kg/m^3)			1490 and 1130	
wet density (kg/m^3)				
water content				
porosity				
saturation (%)	1.5 to 2 times liquid limit			1 or 2 % above liquid limit
temperature top (^{o}C)	warm, T_0 is 0.5 to 1.5, 1.6, 0.8 and 0.4	warm, T_0=5, -0.16^{o}C/hr first 24 hrs, 0.008^{o}C/hr after; also -0.024^{o}C/hr first 32 hrs, -0.014^{o}C/hr after	cold T?	warm, T_0 optional eg. 0.55
number of temperature sensors	14	10		10
temperature bottom (^{o}C)	cold, pretest 0.5 to 1.5, t_0 is -10 or -3.7, -2.4 and -1.3	cold, T_0=-0.2, -0.146^{o}C/hr first 24 hrs, -0.008^{o}C/hr after; also T_0=-0.2, 0.025^{o}C/hr first 32 hrs, -0.014^{o}C/hr after	warm, 0.1	cold T_0 optional eg. -0.35
open system			yes	
closed system				
intitializing ice nucleation				cold side -10
ramped freezing	no	see T bottom and T top	? Constant heat removal	yes
ramp rate (^{o}C/hr)	no	see T bottom and T top		optional e.g. -0.00083
step freezing	yes	1 freezing	No	no
temperature step (^{o}C)	1 step	warm end 2, cold end -4		N/A
duration step (hr)		22		N/A
temperature control bath (^{o}C)			yes	
temperature control chamber (^{o}C)			yes	0.1
freeze-thaw cycles	1 freezing			1, only freezing
test duration (hr)	50	100-120	2 to 2.5	267 up to 800
water supply		yes	yes	from top
constant head			yes	
pore water pressure measurement		no	no	
surcharge (kPa)	45	50	no	
displacement measurement		yes	yes	X-rays used to locate ice growth
SP vs. time (K and M or Nixon method)	Yes, K and M	yes		no
temperature profile at time t	yes	yes		yes
frost heave vs. time	yes	yes	yes	yes
frost penetration vs. time	yes	yes		yes
water intake vs. time	yes	yes		yes
final water content profile	yes	yes		yes
frost susceptibility classification	possible	possible		possible
lens formation	no	yes		yes
segregation temperature	yes	yes		
transient/stationary/long term				
soils			Morin clay and Fox tunnel silt	
undisturbed/remoulded				
lead spheres with thermocouples				
x-rays				
comments				

APPENDIX B

FROST HEAVE PREDICTION MODELS LISTING AND COMPARISON

FROST HEAVE PREDICTION MODEL REFERENCE LIST

(the publications printed in bold are summarized in Table B.1)

B1. Arakawa, K., 1966. Theoretical studies of ice segregation in soils. Journal of Glaciology, Vol.6, No. 44, pp. 255-260.

B2. Berg, R.L., Gartner, K.E., and Guymon, G.L., 1977. A mathematical model to predict frost heave. International Symposium on Frost Action in Soils, Lulea, Vol. 2, pp. 92-109.

B3 Black, P.B., 1995. RIGIDICE Model of secondary heave. CRREL Report 95-12, 32p.

B4. Black, P.B., and Miller, R.D., 1985. A continuum approach to modelling of frost heaving. In: Freezing and thawing of soil-water systems, American Society of Civil Engineers, Technical Council on Cold Regions Monograph, ed. D.M. Anderson and P.J. Williams, pp. 36-45.

B5. Blanchard, D., and Frémond, M., 1985. Soils frost heaving and thaw settlement. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, pp. 209-216.

B6. Cary, J.W., and Mayland, H.F., 1987. A new method for calculating frost heave including solute effects. Water Resources Research, Vol. 23, No. 8, pp. 1620-1624.

B7. Chang, Y., 1993. An integrated experimental numerical frost heave prediction model. Unpublished M.Eng. thesis, Carleton University, Ottawa, 119 p.

B8. Derjaguin, B.D., and Churaev, N.V., 1978. The theory of frost heaving. Journal of Colloid and Interface Science, Vol. 67, No. 3, pp. 391-396.

B9. Derjaguin, B.D., and Churaev, N.V., 1986. Flow of non-freezing water interlayers and frost heaving. Cold Regions Science and Technology, Vol. 12, pp. 57-66.

B10. Dudek, S.J.M., and Holden, J.T., 1979. A theoretical model for frost heave. Proceedings of the First International Conference on Numerical Methods in Thermal Problems, Swansea, pp. 216-229.

B11. Duquennoi, C., Frémond, M., and Levy, M., 1989. Modelling of thermal soil behaviour. VTT Symposium 95, Saariselka, H. Rathmayer ed., Technical Research Centre of Finland, Espoo, pp. 895-915.

B12. Everett, D.H., 1961. The thermodynamics of frost heave damage to porous solids. Transactions of the Faraday Society, Vol. 57, pp. 1541-1551.

B13. Forland, T., and Ratkje, S.K., 1980. On the theory of frost heave. Frost I Jord nr. 21, pp. 45-48.

B14. Forland, T., 1980. Irreversible thermodynamic treatment of frost heave. The 2nd international symposium on ground freezing, Trondheim, NTI, pp. 611-617.

B15. Fowler, A.C., 1989. Secondary frost heave in freezing soils. Society for Industrial and Applied Mathematics Journal of Applied Mathematics, Vol. 49, pp. 991-1008.

B16. Fowler, A.C., and Noon, C.G., 1993. A simplified numerical solution of the Miller model of secondary frost heave. Cold Regions Science and Technology, pp. 327-336.

B17. Fowler, A.C., and Krantz, W.B., 1994. A generalized secondary frost heave model. Society for Industrial and Applied Mathematics Journal of Applied Mathematics, Vol. 54, No. 6, pp. 1650-1675.

B18. Frémond, M., and Mikkola, M., 1993. Thermo-mechanical model of freezing soil. In: Gas Pipelines, Oil pipelines and Civil Engineering and Arctic Climates, Proceedings of a Seminar held in Caen and Paris, France. Geotechnical Science Laboratories, Ottawa, pp. 48-60.

B19. Fukuda, M., and Nakagawa, S., 1985. Numerical analysis of frost heaving based upon the coupled heat and water flow model. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, pp. 109-117.

B20. Gilpin, R.R., 1980. A model for the prediction of ice lensing and frost heave in soils. Water Resources Research, Vol. 16, No. 5, pp. 918-930.

B21. Gold, L.W., 1957. A possible force mechanism associated with the freezing of water in porous materials. Highway Research Board Bulletin No. 168, pp. 65-73.

B22. Gorelik, J.B., and Kolunin, V.S., 2000. Generalized model of heat and mass transfer for freezing and thawing soils under load. Ground Freezing 2000, Proceedings of international symposium on ground freezing, Leuven, Thimus ed., A.A. Balkema, Rotterdam, pp. 37-42.

B23. Guymon, G.L. and Berg, R.L., 1976. Galerkin finite element analog of frost heave. Proceedings of the 2nd conference on soil water problems in cold regions, Edmonton, Hydrology Division, American Geophysical Union, Washington DC, pp. 111-113.

B24. Guymon, G.L., Berg, R.L. and Chamberlain, E., 1986. A predictive model of frost heave and thaw weakening. Transportation Research Board Pre-print, 17 p.

B25. Guymon, G.L., Berg, R.L., and Hromadka, T.V., 1993. Mathematical model of frost heave and thaw settlement in pavements. CRREL Report 93-2, 126 p.

B26. Guymon, G.L., Berg, R.L., Johnson, T.C., and Hromadka, T.V., 1981. Results from a mathematical model of frost heave. Transportation Research Record 809, pp. 2-6.

B27. Guymon, G.L., Harr, M.E., Berg, R.L., and Hromadka, T.V., 1981. A probabilistic-deterministic analysis of one-dimensional ice segregation in a freezing soil column. Cold Regions Science and Technology Vol. 5, pp. 127-140.

B28. Guymon, G.L., Hromadka T.V, and Berg, R.L., 1980. A one dimensional frost heave model based upon simulation of simultaneous heat and water flux. Cold Regions Science and Technology, Vol. 3, No. 2 and 3, pp. 253-263.

B29. Guymon, G.L., Hromadka T.V, and Berg, R.L., 1984. Two dimensional model of coupled heat and moisture transport in frost heaving soils. Journal of Energy Resources Technology, ASME, Vol. 106, pp. 336-343.

B30. Guymon, G.L., and Luthin, 1974. A coupled heat and moisture transport model for arctic soils. Water Resources Research , Vol. 10, No. 5, pp. 995-1003.

B31. Harlan, R.L., 1973. Analysis of coupled heat-fluid transport in partially frozen soil. Water Resources Research, Vol. 9, No. 5, pp. 1314-1322.

B32. Hayhoe, H.N., 1978. Study of the relative efficiency of finite difference and Galerkin techniques for modelling soil-water transfer. Water Resources Research, Vol. 14, pp. 97-102.

B33. Henry, K., 1988. Chemical aspects of soil freezing. CRREL Report 88-7, 15 p.

B34. Holden, J.T., 1983. Approximate solutions for Miller's theory of secondary heave. Permafrost, Fourth International Conference, Proceedings, Fairbanks, AL, USA, National Academy Press, Washington DC, pp. 498-503.

B35. Holden, J.T., 1991. Towards a three-dimensional model of frost heave. Ground Freezing 91, Sixth International Symposium on Ground Freezing, Beijing, X. Yu and C. Wang eds., A.A. Balkema, Rotterdam, pp. 231-235.

B36. Holden, J.T., Jones, R.H., and Dudek, S.J.M., 1981. Heat and mass flow associated with a freezing front. Engineering Geology, Vol. 18, pp. 153-164.

B37. Holden, J.T., Piper, D., and Jones, R.H., 1985. Some developments of a rigid ice model of frost heave. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, Vol. 1, pp. 93-99.

B38. Hopke, S.W., 1980. A model for frost heave including overburden. Cold Regions Science and Technology, Vol. 3, No. 2, pp. 111-127.

B39. Horiguchi, K., 1987. An osmotic model for soil freezing. Cold Regions Science and Technology, Vol. 14, pp. 13-22.

B40. Horiguchi, K., and Nakano, Y., 1993. An osmotic model for soil freezing. Proceedings of the Fourth International Symposium on Thermal Engineering and Science for Cold Regions, Hanover, CRREL Special Report 93-22, pp. 17-23.

B41. Hromadka, T.V., Guymon, G.L., and Berg, R.L., 1981. Some approaches to modelling phase change in freezing soils. Cold Regions Science and Technology, Vol. 4, pp. 137-145.

B42. Hromadka, T.V., Guymon, G.L., and Berg, R.L., 1982. Sensitivity of a frost heave model to the method of numerical simulation. Cold Regions Science and Technology, Vol. 6, pp. 1-10.

B43. Jame, Y.W and Norum, D.I., 1976. Heat and Mass transfer in freezing unsaturated soil in a closed system. Proceedings of the 2nd conference on soil water problems in cold regions, Edmonton, Hydrology Division, American Geophysical Union, Washington, DC, pp. 46-62.

B44. Jame, Y.W., and Norum, D.I., 1977. Numerical simulation of coupled heat and mass transfer in freezing unsaturated soil. AGU annual meeting, San Francisco.

B45. Jame, Y.W., and Norum, D.I., 1980. Heat and mass transfer in a freezing unsaturated porous medium. Water Resources Research, Vol. 16, No. 4, pp. 811-819.

B46. Kay, B.D., Sheppard, M.I., and Loch, J.P.G., 1977. A preliminary comparison of simulated and observed water redistribution in soils freezing under laboratory and field conditions. Proceedings of an International Symposium on Frost Action in Soils, Lulea, pp. 29-40.

B47. Konrad J.M., 1989. Physical processes during freeze-thaw cycles in clayey silts. Cold Regions Science and Technology, Vol. 16, No. 3, pp. 291-303.

B48. Konrad J.M., and Morgenstern, N.R., 1980. A mechanistic theory of ice lens formation in fine grained soils. Canadian Geotechnical Journal, Vol. 17, pp. 473-486.

B49. Konrad J.M., and Morgenstern, N.R., 1981. The segregation potential of a freezing soil. Canadian Geotechnical Journal, Vol. 18, pp. 482-491.

B50. Konrad J.M., and Morgenstern, N.R., 1982. Prediction of frost heave in the laboratory during transient freezing. Canadian Geotechnical Journal, Vol. 19, pp. 250-259.

B51. Kuroda, T., 1985. Theoretical study of frost heaving - kinetic process at water layer between ice lens and soil particles. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, pp. 39-45.

B52. Lewis, R.L. and Sze, W.K., 1988. A finite element simulation of frost heave in soils. Ground Freezing 88, Fifth International Symposium on Ground Freezing, Nottingham, R.H. Jones and J.T. Holden eds., A.A. Balkema, Rotterdam, pp. 73-80.

B53. Lundin, L.C., 1989. Ice pressure and maximum heaving pressure in a dynamic frozen soil system. Cold Regions Science and Technology, pp. 1-20.

B54. Michalowski, R.L., 1993. Phenomenological modelling of frost-susceptible soils. Permafrost, Sixth International Conference, Proceedings, Beijing, J. Brown, H.M. French, N.A. Grave, G Cheng, L. King, and E.A. Koster, South China University of Technology Press, Wuhan, Guangzhou, pp. 465-470.

B55. Miller, R.D., 1972. Freezing and heaving of saturated and unsaturated soils. Highway Research Record, Vol. 393, pp. 1-11.

B56. Miller, R.D., 1978. Frost heaving in non-colloidal soils. Permafrost, Third International Conference, Proceedings, Edmonton, pp. 708-713.

B57. Miller, R.D., 1980. The adsorbed film controversy. Cold Regions Science and Technology, Vol. 3, pp. 83-86.

B58. Miller, R.D., 1980. Freezing phenomena in soils. Applications of soil physics, Hillel, D., ed. Academic Press, New York, pp. 254-299.

B59. Miyata, Y., 1988. A frost heave mechanism model based on energy equilibrium. Ground Freezing 88, Fifth International Symposium on Ground Freezing, Nottingham, R.H. Jones and J.T. Holden eds., A.A. Balkema, Rotterdam, pp. 91-98.

B60. Nakano, Y., 1986. On the stable growth of segregated ice in freezing soil under negligible overburden pressure. Advances in Water Resources, Vol. 9, No. 4, pp. 223-235.

B61. Nakano, Y., and Horiguchi, K., 1984. Role of heat and water transport in frost heaving of fine grained porous media under negligible overburden pressure. Water Resources Research, Vol. 7, pp. 93-102.

B62. Nakano, Y., and Horiguchi, K., 1985. Role of phase equilibrium in frost heave of fine grained soil under negligible overburden pressure. Water Resources Research, Vol. 8, pp. 50-68.

B63. Nakano, Y., and Takeda, K., 1990. Evaluation of existing hypotheses used in the mathematical description of ice segregation in freezing soils. Fifth international colloquium on free boundary problems, Montreal.

B64. Nixon, J.F., 1982. Field frost heave predictions using the segregation potential concept. Canadian Geotechnical Journal, Vol. 19, pp. 526-529.

B65. Nixon, J.F., 1991. Discrete ice lens theory for frost heave in soils. Canadian Geotechnical Journal, Vol. 28, pp. 843-859.

B66. O'Neill, K., and Miller, R.D., 1980. Numerical solutions for rigid-ice model of secondary frost heave. Proceedings of the Second International Symposium on Ground Freezing, Trondheim, Norwegian Institute of Technology, pp. 656-669.

B67. O'Neill, K., and Miller, R.D., 1982. Numerical solutions for rigid-ice model of secondary frost heave, CRREL Report 82-13, 10 p.

B68. O'Neill, K., and Miller, R.D., 1985. Exploration of a rigid ice model of frost heave. Water Resources Research, Vol. 3, pp. 281-296.

B69. Outcalt, S., 1976. A numerical model of ice lensing in freezing soils. Proceedings of the 2nd conference on soil water problems in cold regions, Edmonton, Hydrology Division, American Geophysical Union, Washington DC, pp. 63-74.

B70. Outcalt, S., 1979. The effect of iteration frequency on a numerical model of near surface ice segregation. Engineering Geology, Vol. 13, pp. 111-124.

B71. Outcalt, S., 1980a. A step function model of ice segregation. Proceedings of the Second International Symposium on Ground Freezing, Trondheim, Norwegian Institute of Technology, pp. 515-524.

B72. Outcalt, S., 1980b. A simple energy balance model of ice segregation. Cold Regions Science and Technology, Vol. 3, pp. 145-151.

B73. Padilla, F., and Villeneuve, J.P., 1990. Modelling the movement of water, heat and solutes in frost susceptible soils. Proceedings of the Fifth Canadian Permafrost Conference, Quebec, Collection Nordicana 54, pp. 43-49.

B74. Padilla, F., and Villeneuve, J.P., 1992. Modelling and experimental studies of frost heave including solute effects. Cold Regions Science and Technology, Vol. 20, No. 2, pp. 183-194.

B75. Padilla, F., Villeneuve, J.P., and Leclerc, M., 1988. Finite element analysis of the transport of water, heat and solutes in frozen saturated-unsaturated soils with self-imposed boundary conditions. Proceedings of the VIIth international conference on computational methods in water resources, MIT, Vol. 35, pp. 121-126.

B76. Penner, E., 1956. Soil moisture tension and ice segregation. Highway Research Board Bulletin No. 135, pp. 50-64.

B77. Penner, E., 1959. The mechanism of frost heaving in soils. Highway Research Board Bulletin No. 225, pp. 1-22.

B78. Penner, E., 1963. Frost heaving in soils. Permafrost, Proceedings of an International Conference, Lafayette, IN, USA, N.A.S.-N.R.C., Washington DC. pp. 197-202.

B79. Piper, D., Holden, J.T., and Jones, R.H., 1988. A mathematical model of frost heave in granular materials. Proceedings of the Fifth International Conference on Permafrost, Trondheim, Tapir Publishers, Vol. 1, pp. 370-376.

B80. Piper, D., Holden, J.T., and Jones, R.H., 1988. A mathematical model of frost heave in granular materials. Ground Freezing 88, Fifth International Symposium on Ground Freezing, Nottingham, R.H. Jones and J.T. Holden eds., A.A. Balkema, Rotterdam, Vol. 2, pp. 569-570.

B81. Ryokai, K., 1985. Frost heave theory of saturated soil coupling water/heat flow and its application. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, S. Kinosita and M. Fukuda eds., A.A. Balkema, Rotterdam, pp. 101-108.

B82. Shen, M., and Ladanyi, B., 1987. Modeling of coupled heat, moisture and stress field in freezing soil. Cold Regions Science and Technology, Vol. 14, pp. 237-246.

B83. Shen, M., and Ladanyi, B., 1988. Calculation of the stress field in soils during freezing. Ground Freezing 88, Fifth International Symposium on Ground Freezing, Nottingham, R.H. Jones and J.T. Holden eds., A.A. Balkema, Rotterdam, pp. 121-127.

B84. Sheng, D., Axelsson, K., and Knutsson, S., 1993. Estimation of frost heave for stratified soil profile. Ground Freezing 94, Seventh International Symposium on Ground Freezing, Nancy, M. Frémond, ed., A.A. Balkema, Rotterdam, pp. 129-141.

B85. Sheng, D., and Knutsson, S., 1993. Sensitivity analysis of frost heave - a theoretical study. Frost in Geotechnical Engineering, Proceedings of the Second International Symposium on Frost in Geotechnical Engineering, Anchorage. A. Phukan ed. A.A. Balkema, Rotterdam, pp. 3-16.

B86. Sheppard, M.I., Kay, B.D., and Loch, J.P.G., 1978. Development and testing of a computer model for heat and mass flow in freezing soils. Permafrost, Third International Conference, Proceedings, Edmonton, National Research Council, Ottawa, pp. 76-81.

B87. Svec, O., and Chang, Y., 1990. Critical segregation potential frost-heave upper limit. Permafrost, Sixth International Conference Proceedings, Beijing, J. Brown, H.M. French, N.A. Grave, G. Cheng, L. King, and E.A. Koster, eds., South China University of Technology Press, Wuhan, Guangzhou, pp. 596-602.

B88. Taber, S., 1930. The mechanics of frost heaving. Journal of Geology, Vol. 38, pp. 309-317.

B89. Takagi, S., 1963. Fundamentals of the theory of frost-heaving. Permafrost, First International Conference, Lafayette, IN, USA, N.A.S.-N.R.C., Washington, D.C., pp.200-205.

B90. Takagi, S., 1965. Principles of frost heaving. CRREL Research Report 140, 9 p.

B91. Takagi, S., 1970. An analysis of ice lens formation. Water Resources Research, Vol. 6, No. 3, pp. 736-749.

B92. Takagi, S., 1977. Segregation-freezing temperature as the cause of suction force. Proceedings of an international symposium on frost action in soils, Lulea, Vol. 1, pp. 59-66.

B93. Takagi, S., 1978. Segregation-freezing temperature as the cause of suction force for ice lens formation, CRREL Report 78-6, 13 p.

B94. Takagi, S., 1979. Segregation freezing as the cause of suction force for ice lens formation. Engineering Geology, Vol. 13, pp. 93-100.

B95. Takagi, S., 1980. The adsorption force theory of frost heaving. Cold Regions Science and Technology, Vol. 3, No. 1, pp. 57-81.

B96. Taylor, G.S. and Luthin, J.N., 1976. Numeric results of coupled heat-mass flow during freezing. Proceedings of the 2nd conference on soil water problems in cold regions, Edmonton, Hydrology Division, American Geophysical Union, Washington, DC, pp. 155-172.

B97. Taylor, G.S., and Luthin, J.N., 1978. A model for coupled heat and moisture transfer during soil freezing. Canadian Geotechnical Journal, Vol. 15, pp. 548-555.

B98. Vignes, M., and Dijkema, K.H., 1974. A model for the freezing of water in a dispersed medium. Journal of Colloid and Interface Science, Vol. 49, No. 2, pp. 165-172.

TABLE B.1 FROST HEAVE PREDICTION MODEL COMPARISON (PAGE 1 OF 6)

Frost heave prediction model reference nr	B1	B3	B5	B6	B10	B11
Author(s)	Arakawa 1966	Black 1995	Blanchard and Fremond 1985	Cary and Mayland 1987	Dudek and Holden 1979	Duquennoi et al. 1989
Ice formation	Yes	Yes	Yes	Yes	Yes	Yes
Pore ice	No			Yes	Yes	Yes
Ice lens	Yes	Yes	No	Yes		No
Rhytmic Banding	No	Yes	No	No		No
Freezing fringe	No	Yes	No	No	No	No
Thermal gradient	Yes	Yes	Yes	Yes		
Fourier equation	Yes	Yes	Yes	Yes	Yes	Yes
Energy continuity equation	Yes	Yes	Yes	Yes	Yes	Yes
General heat transfer equation	Yes		Yes		Yes	
Diffusion form of general heat transfer equation	No					
Moisture content gradient	Yes	Yes	Yes			
Matrix potential gradient			No	Yes		
Elevation potential gradient			No	No		
Vapor pressure	No	No		Yes		No
Osmotic potential gradient	No	No	No	Yes	No	No
Darcy equation		Yes	Yes	Yes	Yes	No
Mass continuity equation		Yes	Yes	Yes	Yes	Yes
General water transport equation			Yes	Yes	Yes	
Diffusion form of general water transport equation	Yes					
Capillary pressure	No	Yes	No	Yes	Yes	No
Surface tension	No	No	No	Yes	Yes	No
Thermodynamic equilibrium		Yes	sort of	No		Yes?
Clausius-Clapeyron equation	No	Yes	No	No	Yes	sort of
Irreversible thermodynamics	No	No	No	No		No
Criterion for a new ice lens	No	Yes	No	No	No	No
Stress development	No	No	Yes	No	No	No
Validation	No	No	No	Yes	Yes, by own exp., not great	No
Class	2bN	2bFTCRM	2bSM	2bM	2bM	2bSM

CLASSIFICATION LEGEND	
1 Frost Heave Driving Force Models 1a Capillary driving force models 1b Adsorption driving force models	2 Frost Heave Prediction Models 2a Statistical Frost Heave Models 2b Deterministic Frost Heave Models F=Frozen fringe; T=Thermodynamic equilibrium; C=Frost heave criterion; R=Regelation; S=Stress analysis; M=Mechanistic; N=Non-mechanistic

TABLE B.1 FROST HEAVE PREDICTION MODEL COMPARISON (PAGE 2 OF 6)

Frost heave prediction model reference nr	B12	B13	B14	B16	B17	B19
Author(s)	Everett 1961	Forland and Ratkje 1980	Forland 1980	Fowler and Noon 1993	Fowler and Krantz 1994	Fukuda and Nakagawa 1985
Ice formation	Yes	Yes	Yes	Yes	Yes	Yes
Pore ice		No	No	Yes	Yes	No
Ice lens	Yes	No	No	Yes	Yes	No
Rhytmic Banding	No	No	No	Yes	Yes	No
Freezing fringe	No	No	No	Yes	Yes	No
Thermal gradient		Yes	Yes	Yes	Yes	Yes
Fourier equation	No	No	No	Yes	Yes	Yes
Energy continuity equation	No	No	No	Yes	Yes	Yes
General heat transfer equation	No	No	No	Yes	Yes	Yes
Diffusion form of general heat transfer equation	No	No	No	No	No	
Moisture content gradient		No	No			
Matrix potential gradient		No	No	Yes	Yes	Yes
Elevation potential gradient		No	No	Yes	Yes	
Vapor pressure	No	No	No	No	No	No
Osmotic potential gradient	No	No	No	No	No	No
Darcy equation	No	No	No	Yes	Yes	Yes
Mass continuity equation	No	No	No	Yes	Yes	Yes
General water transport equation	No	No	No	Yes	Yes	No
Diffusion form of general water transport equation	No	No	No			Yes
Capillary pressure	Yes	No	No	Yes	Yes	No
Surface tension	Yes	No	No	No	No	No
Thermodynamic equilibrium	Yes	No	No	Yes	Yes	No
Clausius-Clapeyron equation	sort of	No	No	Yes	Yes	No
Irreversible thermodynamics	No	Yes	Yes	No	No	No
Criterion for a new ice lens	No	No	No	Yes	Yes	No
Stress development	No	No	No	No	No	No
Validation	some, Penner 1958,1959	No	No	No	No	Yes, but an adjustment is included
Class	1a	Not a working frost heave model, leading up to	Not a working frost heave model, leading up to	2bFTCRN	2bFTCRN	2bM

CLASSIFICATION LEGEND	
1 Frost Heave Driving Force Models	2 Frost Heave Prediction Models
1a Capillary driving force models	2a Statistical Frost Heave Models
1b Adsorption driving force models	2b Deterministic Frost Heave Models
	F=Frozen fringe; T=Thermodynamic equilibrium; C=Frost heave criterion; R=Regelation; S=Stress analysis; M=Mechanistic; N=Non-mechanistic

TABLE B.1 FROST HEAVE PREDICTION MODEL COMPARISON (PAGE 3 OF 6)

Frost heave prediction model reference nr	B20	B21	B22	B25	B26	B27
Author(s)	Gilpin 1980	Gold 1957	Gorelik and Kolunin 2000	Guymon et al. 1993	Guymon et al. 1981	Guymon et al. 1981
Ice formation	Yes	Yes	Yes	Yes	Yes	Yes
Pore ice	Yes	Yes	Yes			
Ice lens	Yes	Yes	Yes			
Rhytmic Banding	Yes	No	No			
Freezing fringe	Yes	No	Yes			
Thermal gradient	Yes		Yes	Yes	Yes	Yes
Fourier equation	Yes	No	Yes	Yes	Yes	Yes
Energy continuity equation	Yes	No	Yes	Yes	Yes	Yes
General heat transfer equation	Yes	No	Yes	Yes	Yes	Yes
Diffusion form of general heat transfer equation		No				
Moisture content gradient		No				
Matrix potential gradient	Yes	No				
Elevation potential gradient	Yes	No				
Vapor pressure	No	No	No	No		
Osmotic potential gradient	No	No	No	No		
Darcy equation	sort of	No	Yes	Yes	Yes	Yes
Mass continuity equation	Yes	No	Yes	Yes	Yes	Yes
General water transport equation		No	Yes	Yes	Yes	Yes
Diffusion form of general water transport equation		No	No			
Capillary pressure	Yes	Yes	No	No		
Surface tension	Yes	Yes	No	No		
Thermodynamic equilibrium	Yes	Yes	Yes	No	Yes	Yes
Clausius-Clapeyron equation	sort of	sort of	Yes	No	Yes	Yes
Irreversible thermodynamics	No	No	No	No		
Criterion for a new ice lens	Yes	No	No	No		
Stress development	No	No	some	some		
Validation	No	No	No	Yes, lab CRREL, Kinosita (1978), field	Yes, fudging was necessary	
Class	2bFTCM	1a	2bFTRN	2bM	2bN	2a/2bM

CLASSIFICATION LEGEND	
1 Frost Heave Driving Force Models 1a Capillary driving force models 1b Adsorption driving force models	2 Frost Heave Prediction Models 2a Statistical Frost Heave Models 2b Deterministic Frost Heave Models F=Frozen fringe; T=Thermodynamic equilibrium; C=Frost heave criterion; R=Regelation; S=Stress analysis; M=Mechanistic; N=Non-mechanistic

TABLE B.1 FROST HEAVE PREDICTION MODEL COMPARISON (PAGE 4 OF 6)

Frost heave prediction model reference nr	B30	B31	B34	B35	B36	B37
Author(s)	Guymon and Luthin 1974	Harlan 1973	Holden 1983	Holden 1991	Holen et al. 1981	Holden et al. 1985
Ice formation	Yes	Yes	Yes	Yes	Yes	Yes
Pore ice		Yes	Yes	Yes	Yes	Yes
Ice lens	No		Yes	Yes	Yes	Yes
Rhytmic Banding	No	No	Yes	Yes	Yes, depending on time and space steps?	Yes
Freezing fringe	No	No	Yes	Yes	No	Yes
Thermal gradient	Yes	Yes	Yes	Yes	Yes	Yes
Fourier equation	Yes	Yes	Yes	Yes	Yes	Yes
Energy continuity equation	Yes	Yes	Yes	Yes	Yes	Yes
General heat transfer equation	Yes	Yes	Yes	Yes	Yes	Yes
Diffusion form of general heat transfer equation						
Moisture content gradient						
Matrix potential gradient		Yes				
Elevation potential gradient		Yes				
Vapor pressure	No	Yes	No	No	No	No
Osmotic potential gradient	No	No	No	No	No	No
Darcy equation	Yes	Yes	Yes	Yes	Yes	Yes
Mass continuity equation	Yes	Yes	Yes	Yes	Yes	Yes
General water transport equation	Yes	Yes	Yes	Yes		Yes
Diffusion form of general water transport equation					Yes	
Capillary pressure			Yes	Yes	Yes	Yes
Surface tension		No	Yes	Yes	Yes	Yes
Thermodynamic equilibrium	Yes	sort of	Yes	Yes	Yes	Yes
Clausius-Clapeyron equation	Yes	no ice term	Yes	Yes	Yes	Yes
Irreversible thermodynamics	No	No	No	No	No	No
Criterion for a new ice lens	No	No	Yes	Yes	Yes, a constant seg. temperature	Yes
Stress development	No	No	No	No	No	No
Validation	No	No heave predicted	No	No	Yes, overpredictio n	No
Class	2bM	2bM	2bFTRN	2bFTRN	2bN	2bFTRN

CLASSIFICATION LEGEND	
1 Frost Heave Driving Force Models 1a Capillary driving force models 1b Adsorption driving force models	2 Frost Heave Prediction Models 2a Statistical Frost Heave Models 2b Deterministic Frost Heave Models F=Frozen fringe; T=Thermodynamic equilibrium; C=Frost heave criterion; R=Regelation; S=Stress analysis; M=Mechanistic; N=Non-mechanistic

TABLE B.1 FROST HEAVE PREDICTION MODEL COMPARISON (PAGE 5 OF 6)

Frost heave prediction model reference nr	B38	B39	B64	B65	B73	B82
Author(s)	Hopke 1980	Horiguchi 1987	Nixon 1991	Nixon 1991	Padilla and Villeneuve 1990	Shen and Ladanyi 1987
Ice formation	Yes	Yes	Yes	Yes	Yes	Yes
Pore ice		No		Yes	Yes	
Ice lens	No	Yes		Yes	Yes	
Rhytmic Banding	No			Yes	Yes	Yes, but not true to reality
Freezing fringe	No	Yes		Yes	Yes	Yes
Thermal gradient	Yes	Yes	Yes	Yes	Yes	Yes
Fourier equation	Yes	Yes		Yes	Yes	Yes
Energy continuity equation	Yes	Yes		Yes	Yes	Yes
General heat transfer equation	Yes				Yes	Yes
Diffusion form of general heat transfer equation						
Moisture content gradient						
Matrix potential gradient						Yes
Elevation potential gradient						Yes
Vapor pressure	No	Yes		No	No	No
Osmotic potential gradient	No	Yes		No	Yes	No
Darcy equation	Yes	Yes		Yes	Yes	Yes
Mass continuity equation	Yes	Yes		Yes	Yes	Yes
General water transport equation	Yes			Yes	Yes	Yes
Diffusion form of general water transport equation				No		
Capillary pressure	Yes	Yes		No		
Surface tension	Yes	No		No	No	
Thermodynamic equilibrium	Yes	Yes		Yes	Yes	Yes
Clausius-Clapeyron equation	Yes	Yes		Yes	Yes	Yes
Irreversible thermodynamics	No	No	No	No	No	No
Criterion for a new ice lens	No	No	No	Yes, P_{sep} which is adjusted after test results	Yes	Yes, ice content 85% of porosity
Stress development	No	No	No	No	No	Yes
Validation	Yes, Penner and Ueda (1978)	No	Yes, overpredictio n 20-30%	Yes, Penner (1986), Konrad (1988), Ishizaki and Nishio (1988), Konrad (1989)	Yes, field (Quebec City pavement)	Yes, Penner 1986
Class	2bN	2bFTM	2a	2bFTCM	2bFTM	2bFTSN

CLASSIFICATION LEGEND	
1 Frost Heave Driving Force Models 1a Capillary driving force models 1b Adsorption driving force models	2 Frost Heave Prediction Models 2a Statistical Frost Heave Models 2b Deterministic Frost Heave Models F=Frozen fringe; T=Thermodynamic equilibrium; C=Frost heave criterion; R=Regelation; S=Stress analysis; M=Mechanistic; N=Non-mechanistic

TABLE B.1 FROST HEAVE PREDICTION MODEL COMPARISON (PAGE 6 OF 6)

Frost heave prediction model reference nr	B83	B84	B85			
Author(s)	Shen and Ladanyi 1988	Sheng et al. 1993	Sheng and Knutsson 1993			
Ice formation	Yes	Yes	Yes			
Pore ice		Yes	Yes			
Ice lens		Yes	Yes			
Rhytmic Banding	Yes, but not true to reality	Yes	Yes			
Freezing fringe		Yes	Yes			
Thermal gradient	Yes	Yes	Yes			
Fourier equation	Yes	Yes	Yes			
Energy continuity equation	Yes	Yes	Yes			
General heat transfer equation	Yes	Yes	Yes			
Diffusion form of general heat transfer equation						
Moisture content gradient						
Matrix potential gradient	Yes	Yes	Yes			
Elevation potential gradient	Yes	Yes	Yes			
Vapor pressure	No	No	No			
Osmotic potential gradient	No	No	No			
Darcy equation	Yes	Yes	Yes			
Mass continuity equation	Yes	Yes	Yes			
General water transport equation	Yes					
Diffusion form of general water transport equation						
Capillary pressure		No	No			
Surface tension		No	No			
Thermodynamic equilibrium	Yes	Yes	Yes			
Clausius-Clapeyron equation	Yes	Yes	Yes			
Irreversible thermodynamics	No	No	No			
Criterion for a new ice lens	Yes, ice content 85% of porosity	Yes	Yes			
Stress development	Yes	No	No			
Validation	Yes, Penner 1986	Yes, field	Yes, Takeda and Nakano 1990, Penner and Ueda 1977, Konrad and Morgenstern			
Class	2bFTCSN	2bFTCM	2bFTCM			

CLASSIFICATION LEGEND	
1 Frost Heave Driving Force Models 1a Capillary driving force models 1b Adsorption driving force models	2 Frost Heave Prediction Models 2a Statistical Frost Heave Models 2b Deterministic Frost Heave Models F=Frozen fringe; T=Thermodynamic equilibrium; C=Frost heave criterion; R=Regelation; S=Stress analysis; M=Mechanistic; N=Non-mechanistic

APPENDIX C

FROST HEAVE PREDICTION FOR BURIED CHILLED GAS PIPELINES LISTING AND COMPARISON

FROST HEAVE PREDICTION FOR BURIED CHILLED GAS PIPELINES
REFERENCE LIST
(the publications printed in bold are summarized in Table C.1)

C1. Bowes, W.H., 1984. Bending stresses in pipe due to frost heave. In: Pipelines and Frost Heave, Proceedings of a seminar at Caen, France, S.R. Dallimore, and P.J. Williams eds., pp. 31-33.

C2. Carlson, L.E., 1984. Frost heave and thaw settlement test facilities. In: Pipelines and Frost heave, Proceedings of a seminar at Caen, France, S.R. Dallimore, and P.J. Williams eds., pp. 43-47.

C3. Carlson, L.E., Elwood, J.R., Nixon, J.F., and Slusarchuk, W.A., 1982. Field test results of operating a chilled, buried pipeline in unfrozen ground. Proceedings of the Fourth Canadian Permafrost Conference, Edmonton, pp. 475-480.

C4. Carlson, L.E., and Nixon, J.F., 1987. Subsoil investigation of ice lensing at the Calgary, Canada, frost heave test facility. Canadian Geotechnical Journal, Vol. 25, pp. 307-319.

C5. Chen, X., Schofield, A.N., and Smith, C.C., 1994. Centrifuge modelling of frost heave of pipelines. Ground Freezing 94, Seventh International Symposium on Ground Freezing, Nancy, M. Frémond ed., A.A. Balkema, Rotterdam, Vol. 1, pp. 91-96.

C6. Foriero, A., and Ladanyi, B., 1994. Pipe uplift resistance in frozen soil and comparison with measurements. Journal of Cold Regions Engineering, Vol. 8, No. 3, pp. 93-111.

C7. Greene, D.P., and Kettle, R.J., 1993. Soil-pipeline interaction associated with a large diameter chilled pipeline in temperate climates. In: Gas pipelines, oil pipelines and civil engineering in arctic climates, Proceedings of a seminar held in Caen and Paris, France, Carleton University, Ottawa, pp. 25-33.

C8. Greene, D.P., Kettle, R.J., and Middleton, E., 1995. Instrumentation and monitoring of large-diameter natural gas pipelines operating at sub-zero temperatures in the United Kingdom. Proceedings of the Fifth International Offshore and Polar Engineering Conference, The Hague, ISOPE, Vol. II, pp. 41-46.

C9. Huang, S., Akagawa, S., Tanaka, T., Ono, T., Nasu, Y., O'Hashi, K., and Fukuda, M., 2002. Ground temperature variation induced by a buried chilled gas pipeline. Cold Regions Engineering, 11th International Specialty Conference, Anchorage, K.S. Merrill ed., ASCE Publications, Reston, VA, pp. 158-169.

C10. Hwang, C.T., 1977A. On quasi-static solutions for buried pipes in permafrost. Canadian Geotechnical Journal, Vol. 14, pp. 180-192.

C11. Hwang, C.T., 1977B. Frost heave design of a chilled gas pipeline. 30th Canadian Geotechnical Conference, Saskatoon, pp. V-59-V-88.

C12. Kettle, R.J., 1984. Soil – pipeline interaction: A review of the problem. In: Pipelines and frost heave, Proceedings of a seminar at Caen, France, S.R. Dallimore and P.J. Williams eds., Geotechnical Science Laboratories, Carleton University, Ottawa, pp. 35-37.

C13. Konrad, J.M., and Morgenstern, N.R., 1984. Frost heave prediction of chilled pipelines buried in unfrozen soils. Canadian Geotechnical Journal, Vol. 21, pp. 100-115.

C14. Ladanyi, B., and Lemaire, G., 1984. Behaviour of a buried pipeline under differential frost heave conditions. Proceedings of the Canadian Society of Civil Engineers Cold Regions Engineering Specialty Conference, Montreal, pp. 161-176.

C15. Ladanyi, B., and Shen, M., 1993. Freezing pressure development on a buried chilled pipeline. Frost in Geotechnical Engineering, A. Phukan ed., A.A. Balkema, Rotterdam, pp. 23-33.

C16. Nixon, J.F., 1984. A method for predicting frost heave of buried chilled pipelines. Proceedings of a seminar on pipelines and frost heave, Caen, S.R. Dallimore, and P.J. Williams eds., Carleton University, Ottawa, pp. 55-60.

C17. Nixon, J.F., 1986. Pipeline frost heave prediction using a 2-D thermal model. In: Research on Transportation Facilities in Cold Regions, O.B. Andersland, and F.H. Sayles eds., ASCE, New York, pp. 67-82.

C18. Nixon, J.F., 1987a. Pipeline frost heave prediction using the segregation potential frost heave method. Proceedings of the Offshore Mechanics and Arctic Engineering (OMAE) Conference, Houston, pp. 1-6.

C19. Nixon, J.F., 1987b. Thermally induced frost heave beneath chilled pipelines in frozen ground, Canadian Geotechnical Journal, Vol. 24, pp. 260-266.

C20. Nixon, J.F., 1992a. New frost heave prediction model for design of northern pipelines. Proceedings of the Second (1992) International Offshore and Polar Engineering Conference, San Francisco, ISOPE, Vol. II, pp. 32-39.

C21. Nixon, J.F., 1992b. Discrete ice lens theory for frost heave beneath pipelines. Canadian Geotechnical Journal, Vol. 29, pp. 487-497.

C22. Nixon, J.F., 1994. Role of frost heave pressure dependency and soil creep in stress analysis for pipeline frost heave. Proceedings of the 7th International Cold Regions Engineering Specialty Conference, Edmonton, pp. 397-412.

C23. Nixon, J.F., and Hazen, B., 1993. Uplift resistance of pipelines buried in frozen ground. Permafrost, Sixth International Conference, Proceedings, Beijing, Vol. 1, pp. 494-499.

C24. Nixon, J.F., and MacInnes, K., 1996. Application of pipe temperature simulator for Norman Wells oil pipeline. Canadian Geotechnical Journal, Vol. 33, pp. 140-149.

C25. Nixon, J.F., Morgenstern, N.R., and Reesor, S.N., 1983. Frost heave - pipeline interaction using continuum mechanics, Canadian Geotechnical Journal, Vol. 20, pp. 251-261.

C26. Nixon, J.F., Sortland, K.A., and James, D.A., 1990. Geotechnical aspects of northern gas pipeline design. Proceedings of the Fifth Canadian Permafrost Conference, Quebec, Collection Nordicana 54, pp. 299-307.

C27. Nixon, J.F., Stuchly, J., and Pick, A.R., 1984. Design of Norman Wells pipeline for frost heave and thaw settlement. Proceedings of the Third International Offshore and Arctic Engineering Symposium, New Orleans, ASME, 8 p.

C28. Northern Engineering Services Company Limited, 1975. Mechanical Stress Analysis of Buried Pipeline. Report prepared for Canadian Arctic Gas Study Limited, Calgary, 269 p.

C29. Nyman, K.J., and Lara, P., 1986. Structural monitoring concepts for arctic pipelines. In: Research on Transportation Facilities in Cold Regions, O.B. Andersland, and F.H. Sayles eds., ASCE, New York, pp. 47-66.

C30. Rajani, B., and Morgenstern, N.R., 1993. Stress history and vertical displacement matching for the pipeline at Caen, France, subjected to frost heave. In: Gas pipelines, oil pipelines and civil engineering in arctic climates, proceedings of a seminar held in Caen and Paris, France, Carleton University, Ottawa, pp. 34-47.

C31. Rajani, B., and Morgenstern, N.R., 1994. Comparison of predicted and observed responses of pipeline to differential frost heave. Canadian Geotechnical Journal, Vol. 31, pp. 803-816.

C32. Razaqpur, A.G., and Wang, D., 1996. Frost induced deformations and stresses in pipelines, Journal of Pressure Vessels and Piping, Vol. 69, No. 2, pp. 105-118.

C33. Riseborough, D.W., Williams, P.J., and Smith, 1993. Pipelines buried in freezing soil: A comparison of two ground-thermal conditions. Proceedings of the Offshore Mechanics and Arctic Engineering (OMAE) Conference, Vol. V, pp. 183-193.

C34. Selvadurai, A.P.S., 1992a. Soil - pipeline interaction at a frost heave zone. Proceedings of the 11th International Conference on Offshore Mechanics and Arctic Engineering (OMAE), Calgary, ASME, New York, Vol. V-B, pp. 337-348.

C35. Selvadurai, A.P.S., 1992b. The uplift behaviour of a rigid pipe embedded in a creep susceptible frozen soil. Proceedings of the 11th International Conference on Offshore Mechanics and Arctic Engineering (OMAE), Calgary, ASME, New York, Vol. V-B, pp. 349-357.

C36. Selvadurai, A.P.S., Hu, J. and Konuk, I.,1999a. Computational modelling of frost heave induced soil-pipeline interaction I. Modelling of frost heave, Cold Regions Science and Technology, Vol. 29, pp. 215-228.

C37. Selvadurai, A.P.S., Hu, J. and Konuk, I., 1999b. Computational modelling of frost heave induced soil-pipeline interaction II. Modelling of experiments at the Caen test facility, Cold Regions Science and Technology, Vol. 29, pp. 229-257.

C38. Selvadurai, A.P.S., and Sepehr, K., 1997. Discrete element modelling of pipe uplift in frozen ground regimes. Ground Freezing 97 – Frost Action in Soils, Eighth International Symposium on Ground Freezing, Lulea, Sweden, S. Knutsson ed. A.A. Balkema, Rotterdam, pp. 345-358.

C39. Selvadurai, A.P.S., and Shinde, S.B., 1993. Frost heave induced mechanics of buried pipelines, Journal of Geotechnical Engineering, ASCE, Vol. 119, No.12, 1929-1951.

C40. Shah, K., 1990. Deformations and stresses in pipelines buried in freezing ground. Unpublished M.Eng. thesis, Carleton University, Ottawa, 149 p.

C41. Shah, K., and Razaqpur, A.G., 1993. A two dimensional frost heave model for buried pipelines. International Journal for Numerical Methods in Engineering, Vol. 36, pp. 2545-2566.

C42. Sharma, D., and Pralong, P.-J., 1982. Transient freezing and thawing of soils around buried pipelines. Numerical Models in Geomechanics, International Symposium on Numerical Models in Geomechanics, Zurich, R. Dungar, G.N. Pande, and J.A. Studer eds., A.A. Balkema, Rotterdam, pp. 513-524.

C43. Shen, M., and Ladanyi, B., 1991. Soil-pipeline interaction during frost heave around a buried chilled pipeline. Cold Regions Engineering, ASCE 6th Int. Special Conf., ASCE Publications New York, pp. 11-21.

C44. Slusarchuk, W.A., Clark, J.I., Nixon, J.F., Morgenstern, N.R., and Gaskin, P.N., 1978. Field test results of a chilled pipeline buried in unfrozen ground. Permafrost, Third International Conference, Proceedings, Edmonton, NRC Canada, pp. 878-883.

C45. Smith, M.W., Dallimore, S.R., and Kettle, R.J., 1985. Observations and prediction of frost heave of an experimental pipeline. Ground Freezing 85, Fourth International Symposium on Ground Freezing, Sapporo, Japan, S. Kinosita and M. Fukuda eds., Balkema, Rotterdam, pp. 297-304.

C46. Smith, S.L., and Williams, P.J., 1990. Ice lens orientation around a chilled buried pipe. Proceedings of the Fifth Canadian Permafrost Conference, Collection Nordicana No. 54, Laval University, Laval, pp. 83-87.

C47. Smith, S.L., and Williams, P.J., 1994. Ice lens formation at a silt-sand interface. Canadian Geotechnical Journal, Vol. 32, pp. 488-495.

C48. Svec, O., 1981. Frost heave control of a chilled gas pipeline. Cold Regions Science and Technology, Vol. 4, pp. 215-225.

C49. Wang, D., 1994. A coupled thermo-mechanical analysis of pipelines buried in freezing ground. Unpublished M.Eng.-thesis, Carleton University, Ottawa, 210p.

C50. Williams, P.J., 1980. Design considerations for large-diameter pipelines in cold regions. Ground Freezing 80, Second International Symposium on Ground Freezing, Trondheim, Norwegian Institute of Technology, pp. 1068-1075.

TABLE C.1 FROST HEAVE PREDICTION FOR BURIED PIPELINES COMPARISON (PAGE 1 OF 4)

frost heave prediction for buried pipelines reference nr	C1	C6	C7	C11	C13
author(s)	Bowes 1984	Foriero and Ladanyi 1994	Greene and Kettle 1993	Hwang 1977B	Konrad and Morgenstern 1984
model/no model	no model	no model	no model	model	model
focus	bending stresses in pipeline and strain measurements	pipe uplift resistance	ground cracking above buried pipeline		
frost heave prediction model used	no	no	no		SP
soil response to frost heave		yes	no		
discrete spring(s) or continuum			no	discrete springs	discrete springs
rheology of the surrounding soil		creep and plastic	no		elastic?
2D/3D	no	2D	no	2D	3D
cartesian/polar coordinates	no	polar	no		polar
numerical methods	no		no		
pipe: rigid, complex, free moving	rigid		no		free moving
soil-pipeline interaction	no	bonded and frictionless	no		no

TABLE C.1 FROST HEAVE PREDICTION FOR BURIED PIPELINES COMPARISON (PAGE 2 OF 4)

frost heave prediction for buried pipelines reference nr	C14	C15	C16	C20	C21
author(s)	Ladanyi and Lemaire 1984	Ladanyi and Shen 1993	Nixon 1984	Nixon 1992a	Nixon 1992b
model/no model	model	model		model	model
focus					
frost heave prediction model used	vertical pile model		SP	Nixon 1991	Nixon 1991
soil response to frost heave		independent linear spring			
discrete spring(s) or continuum				discrete spring	discrete spring
rheology of the surrounding soil					
1D/2D/3D				2D	2D
cartesian/polar coordinates				polar	polar
numerical methods				fin diff	fin diff
pipe: rigid, complex, free moving		rigid, free floating			
soil-pipeline interaction				no	no

TABLE C.1 FROST HEAVE PREDICTION FOR BURIED PIPELINES COMPARISON (PAGE 3 OF 4)

frost heave prediction for buried pipelines reference nr	C25	C26	C33	C34	C35
author(s)	Nixon et al. 1983	Nixon et al. 1990	Riseborough et al. 1993	Selvadurai 1992a	Selvadurai 1992b
model/no model	model	no model	no model	model	
focus		geotechnical aspects of northern gas pipeline design	description Caen experiments 1982-1992	maximum bending moment along pipeline over time at a frozen-non-frozen interface	
frost heave prediction model used	SP	SP		Nixon 1987	
soil response to frost heave	yes	yes			
discrete spring(s) or continuum	continuum	discrete springs		continuum	continuum
rheology of the surrounding soil	elastic or non-linear viscous			vol. strain, elastic, creep	vol.strain, elastic, creep, creep damage
2D/3D		1D/2D		2D	
cartesian/polar coordinates					
numerical methods					
pipe: rigid, complex, free moving		non-linear, elastic-plastic, viscous		1D Bernouilli-Euler beam	
soil-pipeline interaction				bonded	bonded

TABLE C.1 FROST HEAVE PREDICTION FOR BURIED PIPELINES COMPARISON (PAGE 4 OF 4)

frost heave prediction for buried pipelines reference nr	C36	C41	C42	C43	C45
author(s)	Selvadurai et al. 1999a	Shah and Razaqpur 1993	Sharma and Pralong 1982	Shen and Ladanyi 1991	Smith et al.
model/no model	model	model	model	model	model
focus	3D formulation of Shen and Ladanyi 1987 and experimental model calibration	2D formulation of rigid ice model; application to frost heave prediction for pipelines	freezing around a chilled gas pipeline		observations and frost heave prediction using SP
frost heave prediction model used	Shen and Ladanyi 1987	O'Neill and Miller 1985	heat treatment only	Shen and Ladanyi 1987	SP
soil response to frost heave	no	no	no		no
discrete spring(s) or continuum	continuum				discrete spring
rheology of the surrounding soil	no	no		vol. strain, elastic, creep	no
2D/3D	3D	2D			1D
cartesian/polar coordinates	cartesian	cartesian			cartesian
numerical methods	finite elements	finite elements		finite elements	analytical
pipe: rigid, complex, free moving		N/A		rigid, free floating	free moving
soil-pipeline interaction		no	no		no

APPENDIX D

INSTITUTIONS AND COMPANIES ACTIVE IN FROST HEAVE RESEARCH IN PAST OR PRESENT

CENTRES OF FROST HEAVE KNOWLEDGE

Institution or Company	Location	Practitioners/Researchers
Canada:		
Agriculture Canada	Ottawa	Balchin, Hayhoe
Agriculture Canada	Swift Current	Jame
Carleton University	Ottawa	Burt, Chang, Jetchik, Razaqpur, Riseborough, Schellekens, Selvadurai, Shah, Smith M.W., Smith S., Wang, Williams, Wood
C-CORE	St. John's	Clark, Kenny, Morgan
EBA Engineering Consultants Ltd.	Edmonton	Hayley, Horne, Hwang, Jones K., Schellekens, Seto
Environment Canada	Ottawa	Harlan
Foothills Pipe Lines Ltd.	Calgary	Carlson, Ellwood
Geo-engineering Ltd.	Calgary	Saunders
Geological Survey of Canada	Ottawa	Burgess, Dallimore, Konuk, Lawrence, Smith S.
Golder Associates	Calgary	Crooks
R.M. Hardy and Associates/Hardy BBT Ltd./AGRA Earth & Environmental Ltd./AMEC Earth & Environmental Ltd.	Calgary/ Edmonton	Barnes, McRoberts, Oswell, Hanna, Slusarchuk, Van Gassen, Reesor, Nixon
Lakehead University	Thunder Bay	Eigenbrod
L.E.C. Engineering Ltd.	Calgary	Carlson
McGill University	Montreal	Hu, Selvadurai
National Energy Board	Ottawa	Walton

CENTRES OF FROST HEAVE KNOWLEDGE

Institution or Company	Location	Practitioners/Researchers
National Research Council	Ottawa	Gold, Goodrich, Penner, Rajani Svec, Ueda, Williams
Nixon Geotech Ltd.	Calgary	Nixon
Queen’s University	Kingston	Gaskin
Saint Mary’s University	Halifax	Tarnawski
Université de Montréal	Montreal	Foriero, Ladanyi, Shen,
Université du Québec	Sainte-Foy	Padilla, Villeneuve
Université Laval	Quebec City	Konrad, Seto, Shen
University of Alberta	Edmonton	Biggar, Gilpin, Horne, Konrad, Mageau, McRoberts, Morgenstern, Murray, Nixon, Rajani, Sego, Van Gassen
University of Guelph	Guelph	Groenevelt, Kay, Perfect
University of Manitoba	Winnipeg	Chandler, Domaschuk
University of Ottawa	Ottawa	Evgin
University of Saskatchewan	Saskatoon	Norum
U.S.A.:		
Battelle Pacific Northwest Laboratory	Richland, WA	Cary
Continental Oil Co.	Ponca City, OK	Radd, Oertle
Cornell University	Ithaca, NY	Black, Bresler, Cass, Dirksen, Hoekstra, Koopmans, Koslow, Miller, Romkens
Dames & Moore	Golden, CO	Sharma, Pralong
Exxon Production Research Company	Houston, TX	Heuer, Hopke, Jahns, Miller T.W., Power, Rickey, Taylor, Wheeler

CENTRES OF FROST HEAVE KNOWLEDGE

Institution or Company	Location	Practitioners/Researchers
Johns Hopkins University	Baltimore, MD	Michalowski
Northern Engineering & Scientific	Anchorage, AK	Hazen
Ohio State University	Columbus, OH	Taylor
Oregon State University	Corvallis, OR	Vinson
Purdue University	West-Lafayette, IN	Harr, Leonards Low,
Texas A & M University	College Station, TX	Anderson D.M.
U.S. Army Cold Regions Research and Engineering Laboratories	Hanover, NH	Anderson, Berg, Bigl, Black, Chamberlain, Gow, Henry, Hoekstra, Ingersoll, Johnson T.C., Ketcham, McGaw, Nakano, O'Neill, Shoop, Takagi
U.S. Department of Agriculture	Kimberly, ID	Cary
U.S. Geological Survey	Menlo Park, CA	Ferrians, Kachadoorian
University of Alaska	Fairbank, AK	Goering, Guymon, Phukan, Shur, Zarling
University of California	Davis, CA	Berggren, Luthin
University of California	Fullerton, CA	Hromadka
University of California	Irvine, CA	Guymon, Hromadka
University of Colorado	Boulder, CO	Krantz, Peterson
University of Michigan	Ann Arbor, MI	Outcalt
U.K.:		
Aston University	Birmingham	Kettle, Johnson B.D.
British Drilling & Freezing Co. (formerly Foraky)	Nottingham	Harris J.S.

CENTRES OF FROST HEAVE KNOWLEDGE

Institution or Company	Location	Practitioners/Researchers
British Gas	Solihull	Piper
Transport Research Laboratory	Crowthorne	
University of Bristol	Bristol	Everett
University of Glasgow	Glasgow	Sutherland
University of Leeds	Leeds	Stewart
University of Nottingham	Nottingham	Baba, Dudek, Harris J.S., Holden, Jones R.H., Piper
University of Oxford	Oxford	Fowler, Noon
University of Wales	Swansea	Lewis, Sze
WS Atkins Engineering Sciences	Epsom, Surrey	Piper
France:		
Centre National de la Recherche Scientific (CNRS)	Caen	Van Vliet-Lanoe
Laboratoire Central des Ponts et Chaussées (LCPC)	Paris	Frémond, Blanchard, Levy, Duquennoi, Livet, Dupas
Germany:		
Ruhr University	Bochum	Ebel, Jagow, Jessberger, Jordan
Netherlands:		
Eindhoven University of Technology	Eindhoven	Biermans, De Vries, Dijkema, Vignes

CENTRES OF FROST HEAVE KNOWLEDGE

Institution or Company	Location	Practitioners/Researchers
Sweden:		
Swedish Geotechnical Institute	Linkoping	Bergau
Technical University of Lulea	Lulea	Axelsson, Knutsson, Pusch, Sheng, Viklander
Uppsala University	Uppsala	Lundin
Norway:		
Norwegian Road Research Laboratory	Oslo	Saetersdal
Norwegian University of Science and Technology	Trondheim	Forland, Frivik, Johansen, Ratkje
Finland:		
Helsinki University of Technology	Helsinki	Aalto, Gustavsson, Hartikainen, Mikkola, Ravaska
Tampere University of Technology	Tampere	Saarelainen
Technical Research Centre of Finland (VTT)	Espoo	Saarelainen, Rathmayer
University of Oulu	Oulu	Kujala, Ravaska
Russia:		
All Union Research Institute for Hydrogeology and Engineering Geology	Moscow	Grechishchev, Pavlov, Ponomarev
Earth Cryosphere Istitute SB RAS	Tyumen	Gorelik, Kolunin, Reshetnikov

CENTRES OF FROST HEAVE KNOWLEDGE

Institution or Company	Location	Practitioners/Researchers
Moscow State University	Moscow	Cheverev, Chuvilin, Ershov, Grigorjan, Guseva, Krass, Magomedgadzhieva, Vidyapin
Japan:		
Ashikaga Institute of Technology	Tochigi	Nishimura
Central Research Institute of Electric Power Industry	Tokyo and Chiba	Ogata, Kataoka
Hokkaido University	Sapporo	Akagawa, Arakawa, Fukuda (Masami), Horiguchi, Ishizaki, Kuroda,
Kitami Institute of Technology	Kitami	Liu, Sawada, Suzuki
Konoike Construction Co.	Osaka	Takeda
Nagaoka University of Technology	Nagaoka	Aoyama, Ogawa
Nippon Koukan Co.	Kawasaki	Nakagawa
Odakyu Construction Co.	Tokyo	Komiya
Seiken Co.	Osaka	Ohrai, Okamoto, Takashi, Yamamoto
Setsunan University	Osaka	Ito
Shimizu Construction Co.	Tokyo	Akagawa, Ryokai
Sumitomo Mitsui Construction Co.	Tokyo	Fukuda
Tokyo Electric Power Co.	Tokyo	Hashimoto, Yamamoto

CENTRES OF FROST HEAVE KNOWLEDGE

Institution or Company	Location	Practitioners/Researchers
Tokyo Gas Co.	Tokyo	Ishizaki, Minami, Miyata, Nishio
China:		
Cold Region Development Institute	Harbin	
Heilongjiang Cold Regions Construction Research Institute	Harbin	
Lanzhou Institute of Glaciology and Cryopedology, Academia Sinica	Lanzhou	Ding, Xu