MULTIPLICATIVE AND PRODUCT MODEL CONSTRAINTS UPON SPECKLE FILTERING OF SAR IMAGES

R. Touzi

Canada Centre for Remote Sensing

588 Booth Street, Ottawa

Ontario, Canada K1A OY7

Abstract— **Speckle filter performances depend strongly on the speckle and scene models used as the basis for filter development. These models that incorporate implicitly certain assumptions on speckle, scene and observed signals, were generally adopted and used without any justification. In this study, the multiplicative and the product speckle models, which have been used as the basis for the development of the most well known filters, are analyzed. Their implicits assumptions are discussed with regards to the stationarity-nonstationarity of speckle, observed and scene signals. Two categories of speckle filters are distinguished as a function of the stationarity-nonstationary assumption on speckle random variations. The various approximate models used for the multiplicative speckle noise model are then assessed as functions of speckle and scene characteristics. The Madsen method [6] was extended to the various models to derive the requirements on scene signal variations for the validity of the multiplicative stationary speckle model, and the product model which forces speckle to be a nonstationary process.**

I. SAR SYSTEM MODEL

For this study, the SAR system model of [8] is used. SAR is modeled as a two-dimensional (range, azimuth) linear system. Fully developed speckle is modeled as a white zero mean complex Gaussian process which modules the scene complex reflectivity $r(t)$ (at the spatial position t) to form, under "the complex multiplicative speckle model" assumption, the input signal $f(t)$ to the linear SAR system: $f(t) = r(t) \cdot n(t)$. The input signal f, which is quadratically phase modulated and amplitude weighted by the prefilter w, then compressed by the processor filter h, gives the following complex voltage at the output: $g(t) = f(t) * q(t) + b(t) * h(t)$, where q is the system impulse response ($q = w * h$ with * denoting convolution), and b is the receiver noise complex signal. The latter noise term can be ignored and the detected power is given by:

$$I(t) = |f(t) \cdot n(t) * q(t)|^2 . \tag{1}$$

II. SPECKLE-SCENE MODELS

A. The multiplicative speckle noise model

In order to retrieve the "unspeckled" scene radar backscatter from the observed image sample (pixel), a model that relates the two entities, at each pixel, as a function of speckle noise is used. The most commonly used model is the multiplicative speckle noise model, which expresses the observed intensity as the product of the scene radar backscattering and the speckle noise intensities:

$$I(t) = S(t)u(t), \tag{2}$$

where $I(t)$ is the observed intensity of the pixel located at t=(x,y), $S(t)$ is the terrain reflectivity ($S(t) = |r(t)|^2$), and $u(t)$ is the intensity of fully developed speckle noise which is unit mean Gamma distributed. The approximate intensity expression (2) might be deduced from the exact intensity expression of (1) in various ways, leading to various expressions for the named "multiplicative speckle model" [9], [3], [1], [12], [2]. These models incorporate implicitly certain assumptions on speckle, scene and observed signals. Few of them assume that the multiplicative speckle noise intensity u is white noise [3], [1]. Others assume that u is correlated noise [9], [12]. Scene reflectivity S might be presented as an entity free from the system impulse response q [9], or related to the system characteristics [2].

B. The product model

Under the assumption that the multiplicative speckle model of equation (2) is satisfied at each pixel position t, the unconditional pdf of the observed intensity is given by:

$$P(I(t)) = \int_0^{+\infty} P_{|n(t)|^2}(I(t) \mid S(t))P_S(S(t))dS(t) , \tag{3}$$

where the fully developed speckle of χ^2 pdf is assumed to be nonstationary in intensity mean, with spatially varying mean $E[|n(t)|^2(t)] = S(t)$, and P_S is the spatial distribution of the speckle mean $S(t)$. The spatial averaging of the conditional speckle distribution leads to the unconditional distribution of stationary mean $\bar{S} = < E(I(t) \mid S(t) >_{S(t)} = < S(t) >_t$. This supposes that the limit $\bar{S}$ exists and that the speckle mean variation process $S(t)$ is ergodic and stationary such that its spatial average converges to its ensemble average: $E(S(t)) = < S(t) >_t = \bar{S}$ [10], [11].

III. EXPLICIT FILTER MODEL ASSUMPTIONS ON THE STATIONARY-NONSTATIONARY NATURE OF SPECKLE AND SPECKLE NOISE

All (scalar) speckle filters of one channel polarization SAR images assume (via equation (2)) that speckle noise is a multiplicative unit mean wide-sense stationary process. Such an assumption significantly simplifies filter processing, as speckle statistics that are constant on the whole scene need to be estimated once. However, even though

these filters assume that speckle noise is a stationary random process, speckle might be considered as a stationary or nonstationary process. Two categories of speckle filters might be distinguished as a function of the implicit model assumptions on stationarity-nonstationarity nature of the speckle random process.

1. Multiplicative Stationary Speckle Model Filters (MSSM filters): they assume that the speckle random process is stationary over the whole image. The most well known filters, such as the Lee [3] and the Frost [1] filters, belong to this category.

2. The Product Model Filters: they assume that speckle is not "locally" stationary within the moving processing window. This is for example the case of the filters based on the product speckle scene model of (3), such as the Kuan filter [2] and the Gamma filter [4] which force the speckle to be nonstationary in mean, with an intensity mean Gaussian [2] or Gamma [4] distributed. Theses filters might also be named the Multiplicative NonStationary Speckle Model Filters (MNSSM filters).

Both of the categories above assume that the multiplicative speckle noise model of equation (2)) is satisfied at each pixel. In the following, the various approximate expressions of the "multiplicative speckle noise model" are considered, and assessed with reference to the exact expression of (1). The Madsen method [6] is extended to the various models to determine the multiplicative nonstationary speckle models constraints upon variations of nonstationary scene signals (i.e. for the Product Model (MNSSM) Filters).

IV. ASSESSMENT OF THE VARIOUS MULTIPLICATIVE SPECKLE MODELS AND THE RELATED CONSTRAINTS

A. Exact first and second order satistics of the image intensity

Under the assumption that the speckle random process n at the input of the linear SAR system is white, the mean of the detected intensity can be derived from (1):

$$E(I(t)) = E(|r(t)|^2) * |q(t)|^2 \ , \qquad (4)$$

where $|r(t)|^2 = S(t)$. This means that the average transfer function of an optical system for a coherently illuminated diffuse object is the incoherent transfer function of the optical system and not the coherent transfer function, as shown in [5], [8]. For a wide stationary scene reflectivity r, the following expression is obtained for the observed intensity autocorrelation:

$$R_I(\tau) = \int_{-\infty}^{+\infty} R_{|r|^2}(y - x + \tau)|q(x)|^2|q(y)|^2 dx dy$$
$$+ \int_{-\infty}^{+\infty} R_{|r|^2}(y - x)q(x)q^*(x - \tau)q^*(y)q(y - \tau)dx dy \quad (5)$$

The first term of the equation (5) can be written in the form $R_{Sq}(\tau)$, where $Sq = |r|^2 * |q|^2$. This term, which corresponds to the scene signal autocorrelation is the autocorrelation of the incoherent image of the original image (i.e. the scene viewed through an optical system whose impulse response is the incoherent transfer function $|q|^2$). The second term of (5) is the space speckle autocorrelation

function, which is a measure of the average speckle size [5].

For nonstationary scenes, the space averaged autocorrelations should be involved in order to transform a nonstationary correlation function to a stationary correlation measurement provided by the space averaged entity [7]. This supposes that the averaging leads to finite limit (i.e. the space averaged autocorrelation function exists). Under a such condition, the space averaged intensity autocorrelation $\bar{R}_I(\tau)$ can be derived, as done in [5], [6]. The expression of $\bar{R}_I(\tau)$ obtained is equivalent to (5) with $R_{|r|^2}(u)$ being replaced by the space averaged autocorrelation $\bar{R}_{|r|^2}(u)$, as shown in [6].

B. Multiplicative speckle model with correlated speckle noise and uncorrelated scene signal

The Saleh model equation is given by [9]:

$$I_m(t) = |r(t)|^2 R_q(0) \cdot u(t) \ , \qquad (6)$$

where the normalized multiplicative speckle noise $u(t)$: $u(t) = [|n(t) * q(t)|^2]/R_q(0)$ is a correlated process distributed along a unit mean Gamma. Under the assumption that the speckle process n at the input of the system is white, it can be shown that the intensity mean $E(I_m(t))$ is identical to equation (4) provided that $E(|r(t)|^2)$ is slowly varying within the width of the system impulse response $q(x)$. Such a condition is not needed if the model equation above is replaced by the following one adopted in [12]:

$$I_m(t) = [E(|r(t)|^2) * |q(t)|^2] \cdot [|r(t)|^2/E(|r(t)|^2)] \cdot |n(t) * q(t)|^2 \ . \ (7)$$

The intensity autocorrelation function is derived. The expression obtained is identical to the exact one of equation (5), provided that the scene reflectivity autocorrelation function $R_{|r|^2}(\tau)$ is slowly varying compared to the system impulse response q. For nonstationary scene signals, the multiplicative model is valid provided that the space averaged autocorrelation is slowly varying compared to the system impulse response q, as shown in [6].

C. Multiplicative speckle model with correlated speckle noise and correlated scene signal

The model that was adopted in [2] might be expressed in the following explicit form:

$$I_m(t) = \left[|r(t)|^2 * |q(t)|^2 \right] \cdot u(t) \ , \qquad (8)$$

where $u(t)$ is the normalized correlated noise distributed along a unit mean Gamma. The intensity autocorrelation might be derived under the condition that n is a white circular Gaussian process:

$$R_{Im}(\tau) = R_{Sq}(\tau)[1 + |R_q(\tau)|^2/|R_q(0)|^2] \ . \qquad (9)$$

Compared to the previous model of equations (6, 7), the use of the incoherent convolution of the scene in the first term in equation (8) leads to exact expressions for both the intensity mean in equation (1), and scene autocorrelation (first term in equation (5)). However, the second term in equation (9) remains different from the one in equation (5). The two expressions are identical provided that the reflectivity autocorrelation function is slowly varying within the

impulse system width $q(x)$. The same expression might be extended to nonstationary signals using the space averaged entities. The multiplicative model remains valid provided that the reflectivity space autocorrelation function varies slowly within the impulse system width.

D. Frost model with white speckle noise

The Frost model introduced in [1] might be better adapted to SAR systems using the following expression:

$$I_m(t) = \left[|r(t)|^2 \cdot u(t) \right] * |q(t)|^2, \qquad (10)$$

where $u(t) = |n(t)|^2 / E(|n(t)|^2)$ is a unit mean Gamma distributed white process. It can be shown that the intensity mean $E(I_m(t))$ is identical to the exact expression (4). The intensity autocorrelation function is derived. The first term of the autocorrelation is identical to that of the exact scene autocorrelation R_{Sq} of (5). Under the condition that the reflectivity autocorrelation function ($R_{|r|^2}(\tau)$ is slowly varying within the impulse system width $q(x)$, the following expression is obtained for the intensity autocorrelation, as a function of $\sigma^0 = R_{|r|^2}(0)$: $R_{Im}(\tau) = (\sigma^0)^2[|R_q(0)|^2 + R_{|q|^2}(\tau)]$. This equation is identical to equation (5) if the system impulse function satisfies, in addition to the conditions above, the following relationship:

$$|R_q(\tau)|^2 = R_{|q|^2}(\tau) \qquad (11)$$

Such a restrictive condition might be satisfied under certain circumstances. Using the SAR Gaussian model of [8], it can be shown that this relation is satisfied (modulo a multiplicative constant) for a perfectly matched system. The results above concerning the intensity autocorrelation function can be extended to the space averaged autocorrelation for nonstationary scene signals.

E. Multiplicative speckle model with uncorrelated speckle noise and uncorrelated scene signal

Most of the existing filters ignore the system and scene correlation. Under the assumption that the terrain reflectivity $r(t)$ is slowly varying within the resolution cell (i.e. locally stationary within the resolution cell), the most commonly used multiplicative model is given by [3], [2]: $I(t) = |r(t)|^2 * u'(t)$, where $u'(t) = |n(t)|^2 / E(|n(t)|^2)$ is a white unit mean Gamma distributed process. Under the condition that the scene signal is slowly varying within the system pulse width, the statistics of the model above are only identical to the exact solutions of equations (4) and (5) provided that the image pixels are uncorrelated. This might be achieved by under-sampling the image with the risk of information corruption for not respecting the Shannon sampling theorem [2].

V. Models constraint upon scene signal variations

A. Multiplicative Stationary Speckle Model (MSSM) Filters

Speckle and scene signals are assumed to be stationary, as discussed in Section III for the MSSM filters. All the multiplicative speckle noise models yield approximate expressions for the observed intensity whose probability density function (pdf) first and second order expressions are similar to the exact ones obtained from the multiplicative complex speckle model provided that scene reflectivity signal and its autocorrelation function are slowly varying within the impulse system width $q(x)$. The model in equation (8), which involves the incoherent convolution of the scene, looks to be the least restrictive one. It leads to exact expressions of the intensity mean, and exact scene autocorrelation without imposing any condition on the scene signal (with the exception of signal wide sense stationarity). The same remark might be extended to the Frost model (equation(10)) provided that the SAR system satisfies the conditions given by equation (11).

B. Multiplicative NonStationary Speckle Model (MNSSM) Filters

The product model assumes that the multiplicative speckle noise model is valid at each pixel. This condition is satisfied provided that the scene signal is slowly varying within the system impulse width. Besides, the MNSSM model assumes that speckle is nonstationary in mean, and that the speckle mean process $S(t)$ is stationary (and ergodic) in mean. According to Section IV, the multiplicative speckle noise model remains valid provided that the averaged scene autocorrellation function (i.e. the averaged nonstationary speckle autocorrelation) varies slowly within the system impulse width. This should limit the degree of variations of the scene signal within the processing window used for speckle filtering.

References

[1] V.S. Frost, J.A. Stiles, K.S. Shanmugan, and J.C. Holtzman. A model for radar images and its application to adaptive digital filtering of multiplicative noise. *IEEE Trans. Pat. Anal. Mach. Intell.*, 4:157–166, 1982.

[2] D.T. Kuan, A.A. Sawchuk, T.C. Strand, and P. Chavel. Adaptive restoration of images with speckle. *IEEE Trans. on Acous. Speech and Sig. Proc.*, 35:373–383, 1987.

[3] J.S. Lee. Digital image enhancement and noise filtering by use of local statistics. *IEEE Trans. Pat. Anal. Mach. Intell.*, 2:165–168, 1980.

[4] A. Lopes, E. Nezry, R. Touzi, and H. Laur. Structure detection and statistical adaptive speckle filtering in sar images. *Int. J. Rem. Sens.*, 14:1735–1758, 1993.

[5] S. Lowental and H. Arsenault. Image formation for coherent diffuse objects: statistical properties. *Journal of the Optical Society of America*, 60:1478–1483, 1970.

[6] S.N. Madsen. Spectral properties of homogeneous and nonhomgeneous radar images. *IEEE Trans. Aerospace Elect. Syst.*, AES-23(4):583–588, 1987.

[7] A. Papoulis. *Probability, Random Variables and Stochastic Processes*. McGraw-Hill, New York, 1965.

[8] R. K. Raney. SAR response to partially coherent phenomena. *IEEE Trans. Antenna Propagat.*, AP–26(6):777–787, 1980.

[9] B.E.A. Saleh and M. Rabbani. Linear filtering of speckled images. *Optics Communications*, 35:327–331, 1980.

[10] R. Touzi. A protocol for speckle filtering of SAR images . In *Proc. of the CEOS SAR workshop, ESA SP-450, http://www.estec.esa.nl/CONFANNOUN/99b02*, March, 2000.

[11] R. Touzi. A protocol for speckle filtering of SAR images . *submitted to IEEE TGRS*, November 2000.

[12] F.T. Ulaby, F. Kouyate, and B. Brisco. Textural information in sar images. *IEEE Trans. Geo. Rem. Sens.*, 24:235–245, 1986.