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New interpretive tools for three-dimensional structural geological modelling: Bézier-based curves, ribbons, and skeletons

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Abstract: Interpreting the geometry of geological objects is a standard activity of field-based geologists. We present new graphics tools that will aid in extending standard methodologies for two-dimensional (2D) geological mapping into a three-dimensional (3D) environment. Much of the existing 3D geological modelling software supports the construction of objects with the input of dense control data. However, for regional mapping and mining exploration work, sparse data is the norm. Tools are required, therefore, that give the expert interpreter full control of the graphics objects, while at the same time being constrained to specific control data from field observations. We present the initial results of a software design and programming project for the visualization of complex regional-scale geological objects using Bézier-based graphics. In addition, we introduce the concept of a structural ribbon, which is an extended map trace, along with methods for the optimization of surface construction using graphical *skeletons*.

Résumé : L'interprétation de la géométrie des structures géologiques est une activité courante chez les géologues de terrain. Nous présentons de nouveaux outils graphiques qui permettront l'application de méthodes normalisées utilisées en cartographie géologique bidimensionnelle à un milieu tridimensionnel. Les logiciels actuels de modélisation géologique tridimensionnelle permettent de reconstituer des structures en entrant de nombreuses données de contrôle. Toutefois, les données utilisées pour les travaux de cartographie régionale et d'exploration minière sont habituellement peu nombreuses. On a donc besoin d'outils qui permettent à l'interprète de contrôler parfaitement les données graphiques, tout en l'obligeant à utiliser des données de contrôle précises tirées d'observations de terrain. Nous présentons les premiers résultats d'un projet de conception de logiciel et de programmation destiné à la visualisation de structures géologiques complexes d'échelle régionale en utilisant des graphiques de type Bézier. En outre, nous présentons le concept d'un ruban structural, représenté par un tracé sur la carte, ainsi que des méthodes permettant d'optimiser la reconstitution des structures de surface à l'aide de *canevas* graphiques.

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INTRODUCTION

The history of geological mapping methodology is filled with a host of graphics techniques that attempt to enhance the interpretation of complex structures. A range of visualization techniques in support of this process has evolved from the early manual to more recent digital methods. For example, various projections of 3D orientation data on lower hemisphere plots (Phillips, 1971; Smith and Gardoll, 1997; Knox-Robinson and Gardoll, 1998; Stesky, 1998), 3D block diagrams (Lobeck, 1958; Ragan, 1985; Dueholm et al., 1993; Hatch, 1994), stacked cross-sections in perspective views (Ragan, 1985; Schetselaar, 1995), structural contouring techniques (Marshak and Mitra, 1988; Ragan, 1985), and, more recently, integrated Geographic Information Systems (GIS) products such as structurally symbolized image maps (Harris et al., 1994; Nash et al., 1996; Schetselaar, 1996). The recent proliferation of 3D programs designed to provide visualization and interpretive capabilities to the geologist wishing to explore 3D data, or extend 2D data into the subsurface, is a reflection of a new mind-set in the mineral exploration industry. This attitude is dominated by a desire to efficiently create geological scenarios that can be tested, and which at a minimum, are constrained by real field observations, however

limited they may be (McGaughey and Vallée, 1997). The focus of this project is to further develop 3D interpretation tools so that the more classical-artistic approach to 3D visualization, so exemplified by early alpine workers (e.g. 3D fold-nappes mapped by Emile Argand (1922)), can benefit from modern 3D graphics and a more rigorous geospatial approach. The end goal is to shorten the time it takes to create 3D geological models and to increase the quality by putting these tools closer to the geological expert.

INTERPOLATION AND EXTENSION IN STRUCTURAL GEOLOGY

Structural information is rarely sufficiently dense to use in fully automated approaches to geological modelling. However, many structural geologists are intuitively comfortable with extension, curve fitting, and interpreting the subsurface geometry from limited field data (Ragan, 1985). The graphics tools presented here are designed to keep this interactive and interpretive capacity in the hands of the geologist, while relegating the more onerous task of rendering and perspective view generation to the computer-graphics engine. We believe that these utilities are needed so that a range of speculative

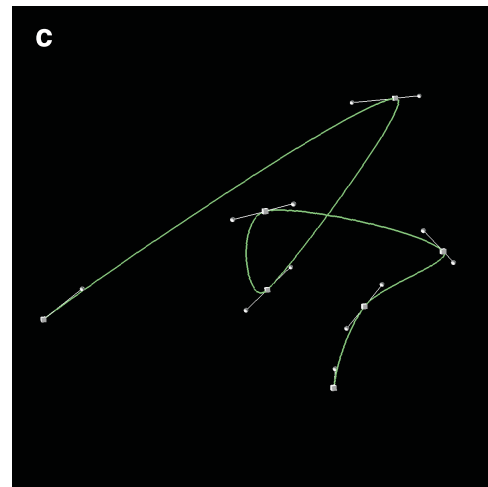
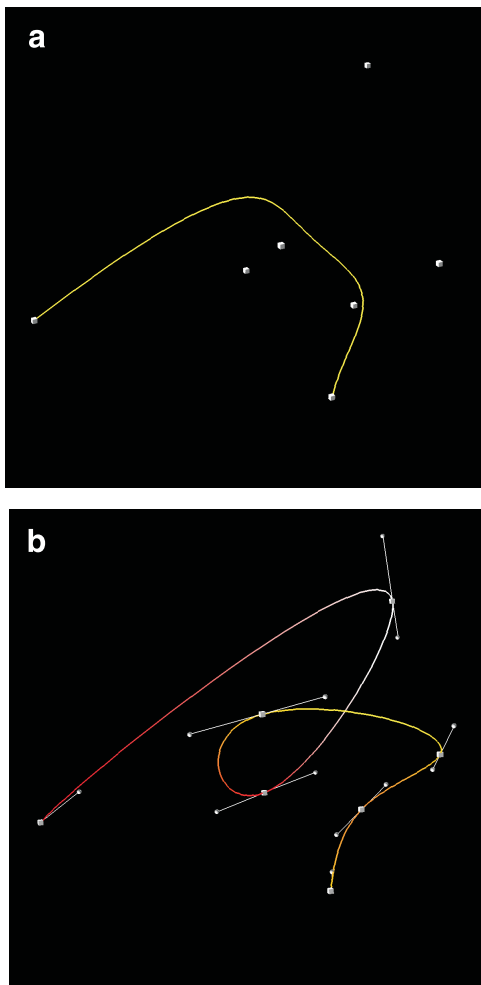


Figure 1a): The Bézier curve of a set of control nodes (cubes). **b)** A C^1 interpolating B-spline and its intermediate nodes (spheres). **c)** A C^2 interpolating B-spline and its intermediate nodes (spheres).

mental solutions to complex geological geometries can be generated quickly, and presented to others for discussion, testing, and revision.

An assortment of design tools have been developed and are being utilized from the field of Computer Aided Geometric Design (CAGD), referred to collectively as parametric design tools. Bézier curves belong to this family of tools. The mathematical formulation and extension from existing physical spline tools emerged in the late 1950s and early 1960s. Bézier-based functions are now used extensively in automotive design, ship building, aerospace manufacturing, and a range of other engineering and design applications. In all of these fields, aesthetics must be combined with strict geometrically constrained product specifications (Farin, 1990; De Paor, 1996). These tools produce an infinite number of solutions for fitting a curve to a limited control set (Fig. 1). Bézier-based tools mathematically produce smooth continuous curves with only a few control points that can be easily manipulated to represent complex geological structures. When these tools are combined with trajectory information from down-plunge projections of ground control points (de Kemp, 2000a, b), a series of speculative geological construction lines can be used to densify a model to a level where it can be interpolated directly by other fully automated interpolations such as Discrete Smooth Interpolation (DSI) (Mallet, 1989).

BÉZIER CURVES AND B-SPLINES FOR THE PEDESTRIAN

Bézier and B-spline curves, in combination, can be used to create a very powerful set of design tools suitable for the interpretative modelling of geological structures. Together, they form the foundation of the curve-generating technology employed in this work.

The idea behind a Bézier curve is simple: given a set of ‘control nodes’, construct a polynomial curve that interpolates the end nodes and whose overall shape is determined by the distribution of the intermediate nodes (Fig. 1a). For a data set with n control nodes, the degree of the Bézier polynomial curve is $n - 1$. Points on a Bézier curve are computed as *barycentric combinations* of the control nodes: that is, as weighted sums of points where the weights sum to unity (cf. *centre of mass* coordinates, see Farin, 1990). The generated curve is always within the *convex hull* of the control nodes (i.e. inside the point set representing the minimal convex enclosure of the control polygon (Rockwood and Chambers, 1996)).

In practice, the magnitude of the binomial coefficients that appear in the barycentric combination of control nodes precludes the use of Bézier curves for data sets with overly numerous nodes. However, low-polynomial-order localized Bézier curves may be smoothly (and conveniently) linked together to manufacture curves connecting any number of data points. This leads us quite naturally to *interpolating B-splines*.

The idea behind interpolating B-splines is also very simple: given a set of control nodes, construct a smooth curve that passes through each point. Such B-splines may be formulated in terms of Bézier curves and, in particular, cubic B-splines are found to be especially convenient, wherein a cubic Bézier curve is formed between each pair of adjacent nodes. The adjacent nodes become the end nodes of the Bézier curve, and two ‘phantom’ intermediate nodes are somehow generated. Together, the four nodes act as input into a Bézier-curve-generating algorithm. The positions of the intermediate nodes are routinely derived using constraints imposed by the overall level of continuity desired (see Fig. 1b, c). Generally speaking, the higher the imposed level of continuity, the less the control over the shape of the curve imparted to the user.

There is a measurable degree of variability associated with algorithms that generate B-spline curves — a reflection of the non-unique character of solutions to the problem of interpolation. Geologists should be quite familiar with the generation of non-unique graphics solutions when interpreting 2D geological polygons from limited survey information. The freedom to select one of many possible B-spline interpolation solutions can be regulated through the use of *shape parameters*, which may be imagined as handles controlling the various degrees of freedom. The number and nature of the shape parameters will vary depending on the flavour of the B-spline algorithm chosen. As their name implies, altering the shape parameters of a curve will change the curve’s appearance, some locally, and others globally. In general, curve shape may be affected by such things as curve parameterization, continuity constraints, node weights (*tension*), curve end-node tangents (for open curves), and intermediate-node positioning parameters. Closed-curve (versus open-curve) B-spline solutions may also change the shape considerably (even for ‘nearly’ closed curves), as can the choice of methodology (i.e. iterative versus non-iterative methods).

To describe the position of a curve in space, we use a *curve parameter*. Any given value of the curve parameter corresponds to a unique point on the parameterized curve. The level of continuity of a curve is commonly measured by the continuity of this curve parameterization, typically C^1 (first derivative smooth) or C^2 (first and second derivative smooth) for interpolating B-splines. The derivatives are with respect to the curve parameterization variable. This ‘functional’ parameterization can lead to pitfalls though, since a smooth parameterization can lead to a not-so-smooth-looking curve. However, adjusting the type of curve parameterization and/or balancing the shape parameters often alleviates unsightly effects. Another approach (which we intend to pursue) to characterizing smoothness is the so-called G^2 , or *geometrically continuous* curve, which is a curve with a C^1 parameterization, but one that can be reparameterized so that the new parameterization is C^2 (Farin, 1990). This is likely a sensible middle ground between the flexibility of C^1 curves and smoother-but-more-constrained C^2 curves.

To achieve a higher level of control over the shape of C^2 cubic B-splines, one may assign weights to the control nodes of the curve to produce cubic *rational B-splines* (such weights may also be assigned to Bézier curves to produce

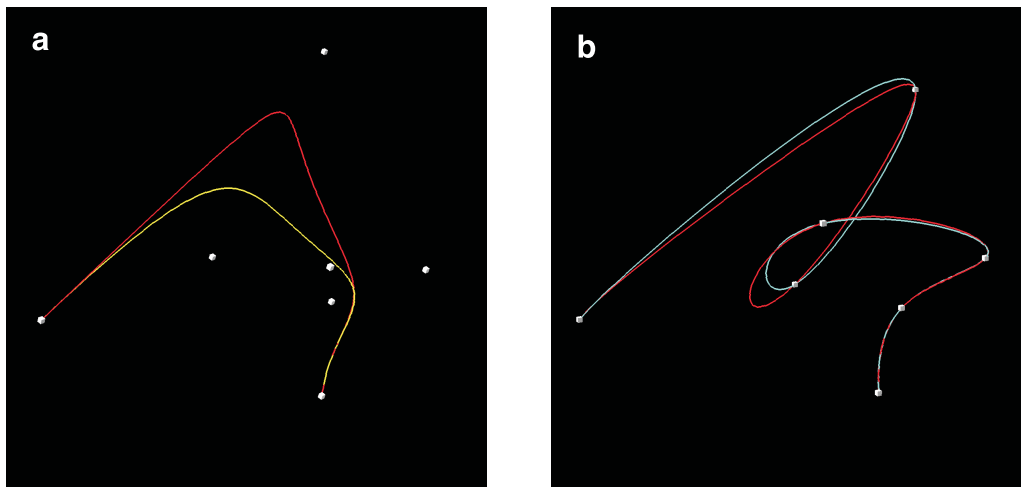


Figure 2a): A non-rational (yellow) and a rational (red) Bezier curve. **b):** A non-rational (cyan) and a rational (red) B-spline.

rational Bézier curves (see Fig. 2a)). Weights have the effect of altering the *tension* of the curve about selected points (Fig. 2b). Higher relative weighting of a node tightens the curve about that node. Rational B-spline curves are generated by multiplying each entry of a control node by its assigned weight, adding an extra coordinate (the weight coefficient, ‘ w ’) to each control node, and then running the new expanded nodes through the same routines used to generate non-rational B-splines. The rational B-spline curve is then found from the higher dimensional curve by projecting it through the origin into the $w=1$ hyperplane (with weights centered about unity). Such curves are also referred to as NURBS (Non-Uniform Rational B-splines) (Farin, 1990; Rockwood and Chambers, 1996). It is worth noting the danger associated with choosing a set of weights that is too unbalanced. This can lead to divergences characterized by zero roots in a *weight polynomial* that ultimately appears as a denominator (Farin, 1990).

WHY NOT JUST USE DSI?

Discrete smooth interpolation (DSI) is a popular and essential component of at least one leading-edge geological modelling software package (*gOcad*®). DSI performs well most of the time. The DSI method proposes to measure the global ‘roughness’ of various interpolating solutions with a specialized quadratic criterion and then selects the solution exhibiting minimal roughness (for details, see Mallet, 1989). In cases where data is sparse, however, a more design-oriented approach to filling in the gaps may be appropriate. Data sets that have been enhanced using Bézier-based curve- and surface-generating routines may then act as interpretively constrained input into DSI algorithms for further smoothing (i.e. see ‘Structural Preparation for Ribbons’, below).

STRUCTURAL PREPARATION FOR RIBBONS

A map trace is a geological interpretation delineating the intersection of a geological boundary as it breaks through the surface of the earth, or as it intersects a given elevation plane. The core idea of a ribbon is that it is an extended 3D map trace. It is a symbolic representation of a localized surface that is derived by extension of a map trace. The structural ribbon is useful, just as map traces are useful, in interpreting more-regional structures. The benefit here, however, is that we are beginning to actually see a part of the surface along a constrained zone that respects the geological mapping and the fold interpretation in 2D. We have developed several methods for assigning slope properties to the 2D map trace before creating the ribbon. These methods are too specific to report on here (Fig. 3). This vital step requires geological decisions that link the proximal field observations to the actual surface being modelled, or they attempt to best calculate these slope values within an appropriate 3D distance. In either case, the geologist must be involved in determining the slope constraints before a ribbon is created.

When spatial line-data is combined with geological strike and dip information, the resulting data set can be used to construct structural *ribbons* that better visualize geological features. Designing a smooth mesh for a structural ribbon requires dense data. Strike and dip measurements, however, are often sparse, and so designing a surface mesh for the ribbon requires a method of interpolating strike and dip measurements along a line. Using the spatial co-ordinates to define a *knot sequence* (a parameterization of the curve), B-spline routines may be employed to smoothly propagate strike and dip measurements to nodes lacking orientation data. This is accomplished by first converting strike and dip measurements to vector cosines, and then interpolating these *orientation vectors* (de Kemp, 1998, 1999).

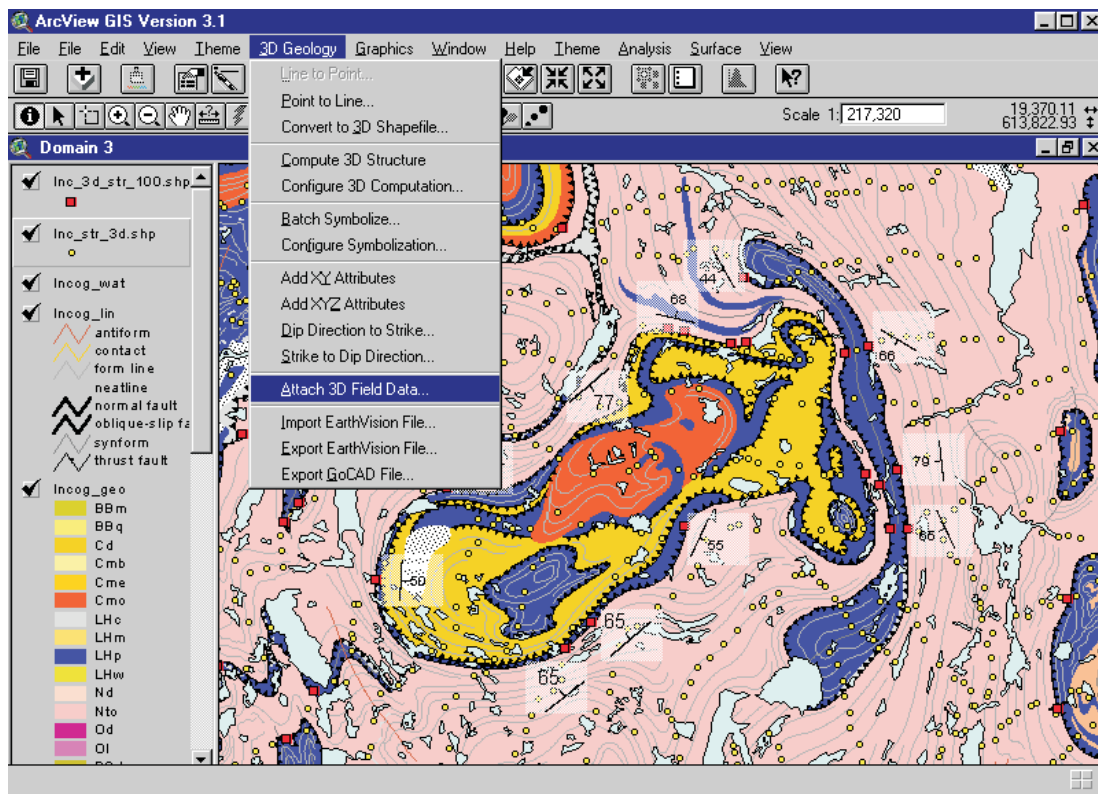


Figure 3. Interoperability between 2D GIS and 3D modelling software, such as *gOcad*[®], is currently underway. Structural observations and thrust traces from previous mapping (St-Onge et al., 1998) are elevation-corrected with topography using the 3DGEOL.AVX extension menu in Arcview. Spatial buffering of field data proximal to the thrust can be used to select and tie field data directly to the thrust. An Arcview Shape file query and import tool, working from within *gOcad*[®], is current being developed.

Once we have constructed a line with dense orientation vector information, a triangulated mesh can be used to generate a ribbon of any width. If desired, the ribbon may then be subjected to further smoothing using DSI (see Fig. 4 for a visual overview of the process).

The ribbons may twist in such a way that inside-outside topology is not maintained (i.e. the ribbon ‘flips’, e.g. Möbius strip), and/or neighbouring ribbons may not have consistent ‘dip-polarities’ (i.e. inside-outside topological mixing near the boundaries). Ribbon-generating schemes should thus provide mechanisms through which the ribbon topology can be forced into consistency with a geological interpretation of the fold structure. Rather than autogenerate such features, it is desirable to allow the practitioner to define and manipulate them in a design-based environment.

The ribbon (Fig. 4c) now represents a more intuitive version of the continuous surface as a geologist would imagine it near the earth’s surface. This example was taken from actual field data collected in 2000 from the Central Baffin Island mapping project (see Corrigan et al., (2001) and de Kemp et al. (2001)).

SKELETON GENERATION

Another related application of Bézier and B-spline curves to geological data sets involves the construction of 3D skeletonized versions of (practically 2D) surface data. Skeletons represent the geologist’s best guess of the 3D form at discrete locations of a geological surface. They are no different than interpretation lines from cross-sections, except that they are truly 3-D and need not be on vertical planes. They can be used to constrain a more automated surfacing or ‘skinning’ interpolation. The idea is to create a dense enough skeleton that maintains the form and topological character of the object.

Spatial surface data, when combined with orientation vector data and *ribbons* (as described above), can be used to estimate the depth-convergence of massive geological structures. This structural ‘depth-line’ can then act as a depth-reference to construct 3-point Bézier or B-spline ‘ribs’. The control nodes for the ‘rib’ curves can be chosen, for example, as the intersection points of surface and depth lines with ‘slicing’ planes along a preferred principal axis or curve (Fig. 5).

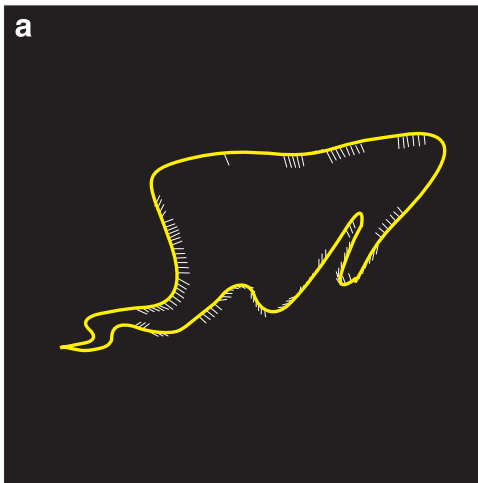


Figure 4a): Surface line with some orientation vectors. **b):** Structural ribbon corrected to eliminate 'flipping'. **c):** Final ribbon after DSI smoothing in gOcad®.

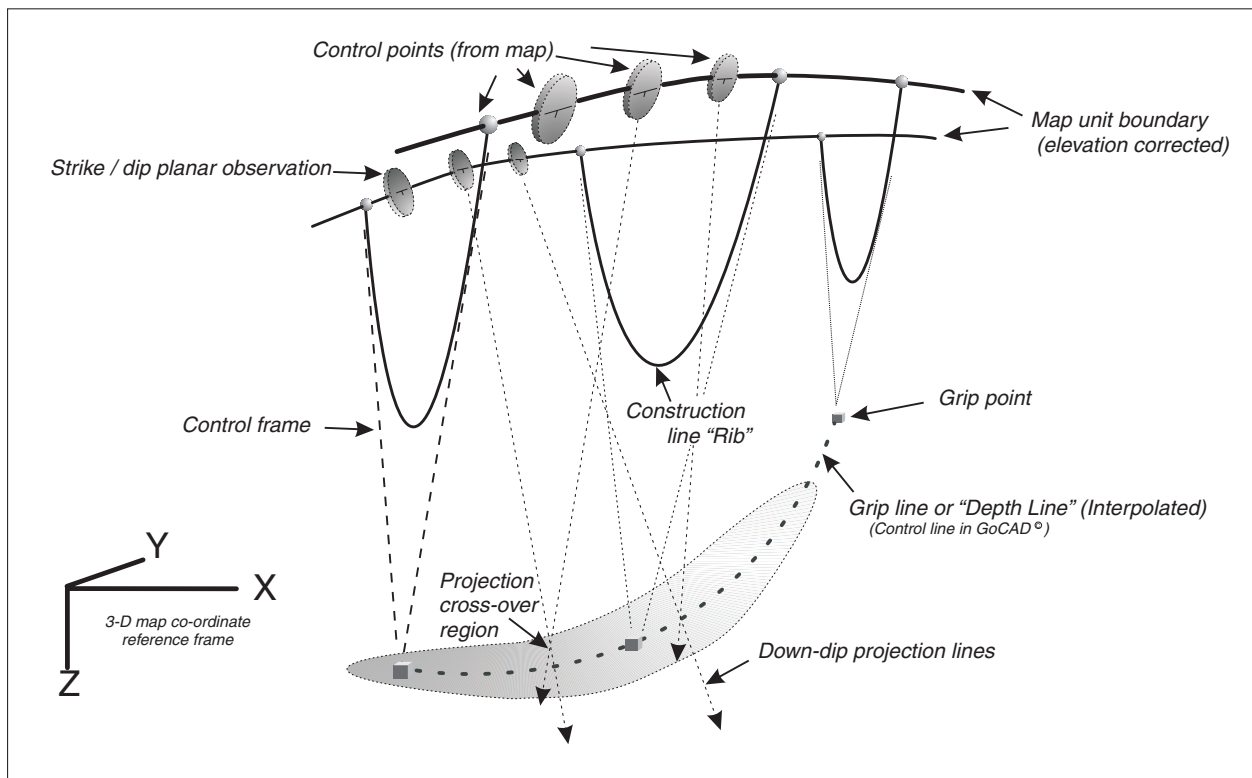
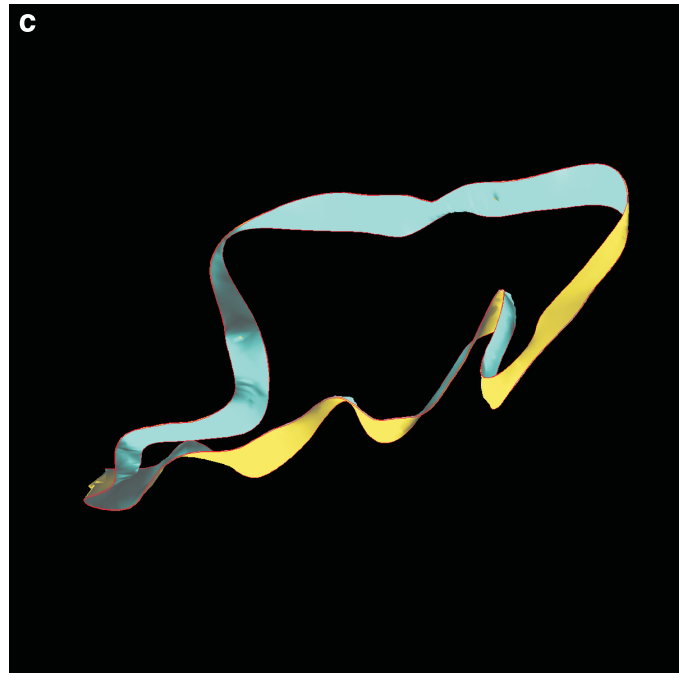
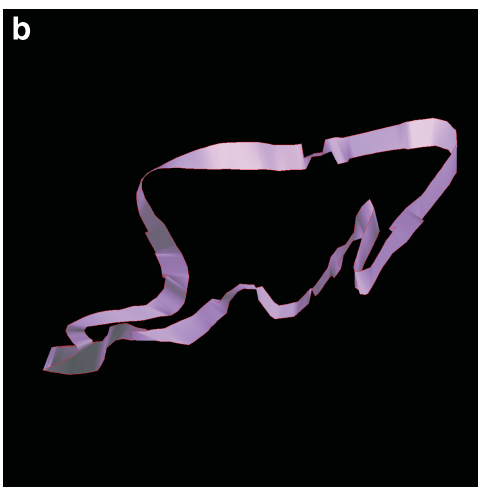


Figure 5. Idealized summary of Bézier-based 3D graphic editing terms used for constructing skeleton graphics.

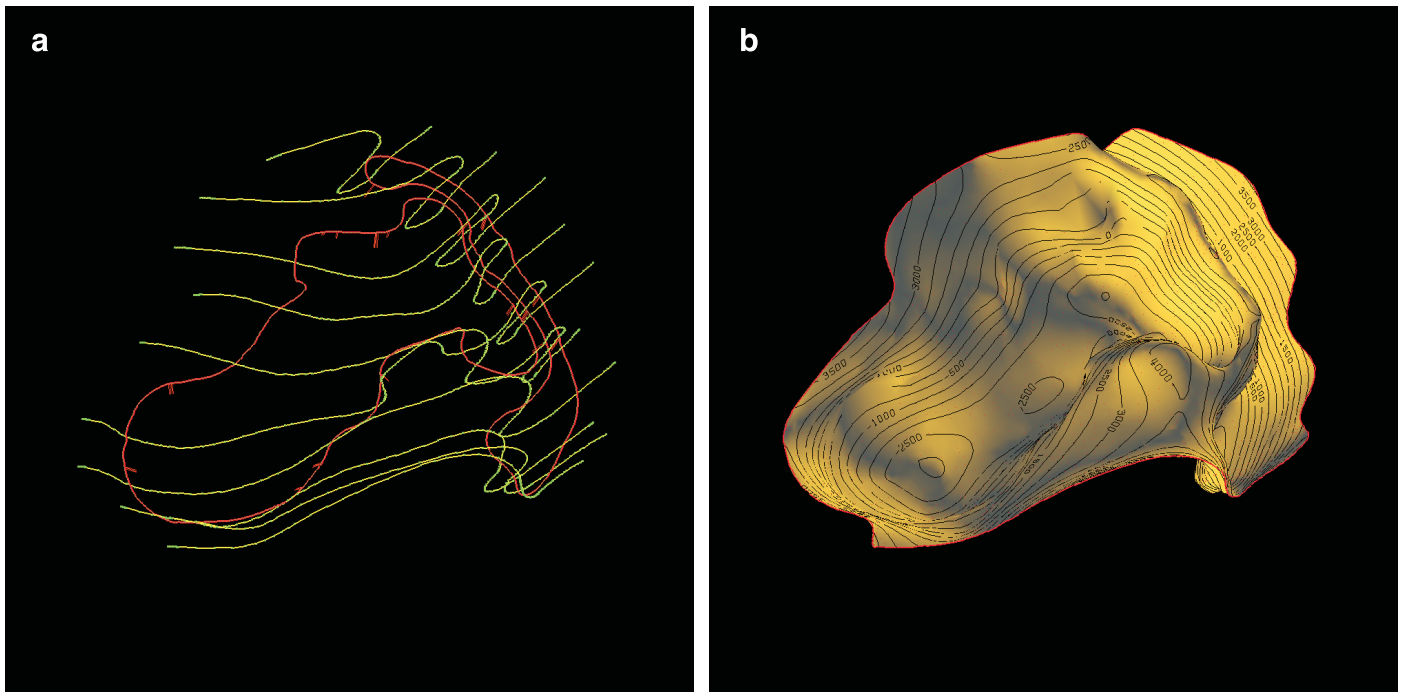


Figure 6a): Skeletonized surface. **b):** Final smoothed surface.

At this stage, the geologist attempts to create a series of raw graphic interpretations which act as construction lines for the final object. These lines act as the ribs of a structural skeleton. A skeleton should be a collection of constraint lines that conform to the field data, and provide a dense enough spatial description for automated surfacing through DSI. The ribs are interactively digitized in a 3D editor, allowing for the dynamic rotation of the field of view, so that lines can be drawn to respect the local dip of a ribbon. Attention is also paid to maintaining correspondence of fold topology as symbolized by the ribbon and the interpreted line. The final surface should reflect this topology so that ‘way-up’ directions of the modelled surface match those of the ribbon (Fig. 6).

CURRENT STATUS

At present, we have developed functional C++ routines for generating rational and non-rational Bézier curves, cubic C^1 and C^2 exact-interpolating B-splines, and C^2 rational exact-interpolating B-splines for *gOcad*[®]-friendly data sets. Both closed- and open- curve solutions have been implemented in all cases, and for the latter, *Bessel Tangents* are employed to generate end-node tangents (Farin, 1990; Ackland, 1915). We also have routines for interpolating strike and dip measurements using a uniform or spatially based knot sequence (parameterization). For the C^1 cubic B-spline case, an assortment of shape-varying parameters has been coded into the framework. The following curve parameterizations are also supported: uniform, chord-length, centripetal, and Foley (for details, *see* Farin (1990)).

We have also developed routines for generating simple structural ribbons as described in the section ‘Structural Preparation for Ribbons’, above.

SOFTWARE AND HARDWARE

Software developed for this project will eventually be embedded as user-friendly menu structures that reflect common geological modelling processes. These menu structures, called ‘wizards’, will call core functions of the 3D modelling package *gOcad*[®] (for details *see*: <http://www.ensg.unancy.fr/GOCAD/>). In addition, new functionality is being added in the form of compiled C++ programs that extend *gOcad*[®] functionality. We are developing within the *gOcad*[®] international academic-government-industry consortium, which allows us to share software code between research groups. The present project is part of ongoing development by the consortium to develop a comprehensive structural toolkit for both petroleum exploration and mining exploration/mapping applications. Our aim is to contribute to tools for sparse data interpretation, and the construction of complex structural objects. Most software in the consortium is presently being developed for Windows NT[®], UNIX-Solaris[™], and SGI[™] platforms. Hardware requirements are for advanced 3D accelerated and frame-buffered graphics cards and large (>10 gigabytes) hard drives. This specific project is being developed for Windows NT, and will likely be ported to other platforms. It is our hope that this development work at the GSC and in the consortium will stimulate the geological software industry in Canada and abroad to develop more interpretive graphics functions for geologists.

FUTURE RESEARCH

We plan to extend the cubic interpolating B-spline options to include G^2 -continuity (as described in the section ‘Bézier Curves and B-splines for the Pedestrian’, above), and furthermore to adapt B-spline and Bézier surface-patch technology to 3D geological model building (Farin, 1990; Rockwood and Chambers, 1996). Methodology for the generation of structural ribbons and skeletons will be defined and implemented into C++ routines, in addition to solidifying structural ribbons to better visualize unusual topological properties. The propagation of various geological properties will also be assessed within the framework of Bézier-based curve- and surface-generating techniques. Concurrently, we plan to develop an easy-to-use interface and *Wizards* within the software package *gOcad*[®] featuring these routines to further catalyze the rapid construction of complex 3D geological models. Alternative methods of interpolation will be investigated, assessed, and/or developed.

CONCLUSIONS

It is clear that there is no shortage of applications for the utilization of Bézier-based curves and surface patches in the construction of 3D spatial geological models, especially for sparse data. Structural ribbons and skeletons, as discussed above, are just two (preliminary) adaptations of this technology to spatial geological data sets. In addition to extending the spatial aspects of a data set, it has been demonstrated that similar techniques can be used effectively to extend certain properties associated with the spatial data (i.e. orientation vectors, above). Once ‘rough’ 3D surface models are constructed, DSI algorithms (as found in *gOcad*[®]) can be used for further surface smoothing.

It must be stressed that the tools introduced in this work are *design* tools, and should not be regarded as anything but an interpretation of the data, rather than a physically constrained or statistical prediction of data values in unmapped territory. It is to be regarded as an extension of the imagination and visualization capabilities of the structural geologist, who must routinely make sensible conclusions about subsurface structures through extrapolation from incomplete data. As more data is made available, the geological models constructed using Bézier-based curve- and surface-generating routines can be updated and ‘redesigned’, eventually converging to the true geological nature of the region.

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