

CANADA  
DEPARTMENT OF ENERGY, MINES AND RESOURCES  
Geological Survey of Canada



USER'S GUIDE FOR SIMGRA 1:  
AN INTERACTIVE GRAPHIC PROGRAM FOR SIMULATING THE  
DISTRIBUTION OF TRANSFORMATIONS OF SEVERAL  
INDEPENDENT RANDOM VARIABLES

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## TABLE OF CONTENTS

	Page
Introduction .....	1
Operational Procedure .....	1
(1) Initialization of the terminal .....	1
(2) Definition of distribution of the initial random variables and entry of their parameters .....	2
(i) NORMAL .....	2
(ii) LOGNORMAL .....	2
(iii) CAUCHY .....	2
(iv) GAMMA .....	2
(v) BETA .....	2
(vi) TRIANGULAR .....	2
(vii) UNIFORM .....	3
(3) Definition of a transformation to be simulated of the random variables defined in (2) .....	3
(4) Computation of simulation, plotting of the results, and modification of the initial variables .....	3
(4A) Simulation .....	3
(4B) Storage .....	3
(4C) Redefinition of distributions and parameters of initial random variables .....	3
(4D) Replotting of initial distributions and stored simulated distributions .....	3
Control Commands for CYBER 74 .....	4
Concluding Remarks .....	4
APPENDIX A .....	5
APPENDIX B .....	7



## INTRODUCTION

SIMGRA 1 is an interactive graphic computer program for simulating the density distribution and the cumulative distribution of any transformation, in a FORTRAN-type functional form, of several initial random variables defined by the user. The program has been presented at the Tenth Annual Symposium on Interface: Computer Science and Statistics, National Bureau of Standards, U.S.A., April 13-15, 1977. A paper on this contribution is being published in the proceedings of the Symposium. Program SIMGRA 1 is written in FORTRAN for a CYBER 74 computer and utilizes the IMS Subroutine Library and the Terminal Control System Library for the 4014/4015 series TEKTRONIX graphic terminals.

The distributions of initial random variables and their parameters are entered via the terminal keyboard interactively. At present the following seven types of continuous distributions can be entered as distributions of initial variables: normal, lognormal, Cauchy, gamma, beta, uniform and triangular (see figure 3). The maximum number of initial random variables which can be entered during one session is eight.

Random numbers from each initial distribution are generated first, by either calling one of the IMS routines or calling one of the programmed subroutines of SIMGRA 1; then the value of the function defined by the user for the transformation is computed. This process is repeated from a minimum of 50,000 times to a maximum of 500,000 depending on the option selected by the user. Additionally, the program allows the user to modify the initial random variables and their parameters interactively. This interactive capability can be useful for studying the robustness of the distribution of the transformation defined.

The flowchart in Figure 1 describes the operational procedure of SIMGRA 1. The procedure of the program can be outlined in steps as follows: (1) initialization of the terminal; (2) definition of distributions of the initial random variables and entry of their parameters; (3) definition of a transformation, in functional form, of initial random variables;

(4) computation of simulation, plotting of the results and modification of the initial variables; options in (4) are as follows: (4A) simulation, (4B) storage of simulation plots, (4C) redefinition of random variables, (4D) replotting of initial distributions and of stored simulated distributions; (5) entering of new random variables; and finally (6) termination of the program.

When displaying the data, the screen is divided into four areas (see figure 2): an Input area on the left, a Menu of Options area in the center, a Simulation area for density distribution and cumulative distribution plots on the upper right, and a Storage area on the lower right of the screen.

The density distribution and cumulative distribution for each of the initial random variables are plotted separately in the Input area. Simulated density distributions and cumulative distributions are overlaid in both Simulation area and Storage area. This overlapping feature is particularly useful for the comparison of several different simulations.

## OPERATIONAL PROCEDURE

### (1) Initialization of the terminal

Upon logging into the system, attaching the necessary files, and providing other necessary commands (see section: Control Commands) the program is called. Beginning execution, the program title is displayed on the screen accompanied by the prompting message.

**ENTER NO. OF CHARACTERS PER SECONDS  
(IE. 960, 480, 120, 30)**

The user responds by entering the operating speed of his TEKTRONIX terminal. For example, if the baud rate is 1200 enter 120, etc. The number of characters per second is equal to the baud rate at which the terminal is operating divided by ten. Upon entering the characters per second, the display screen will be erased, the cross-hair will appear on the screen and a bell will prompt for data input.

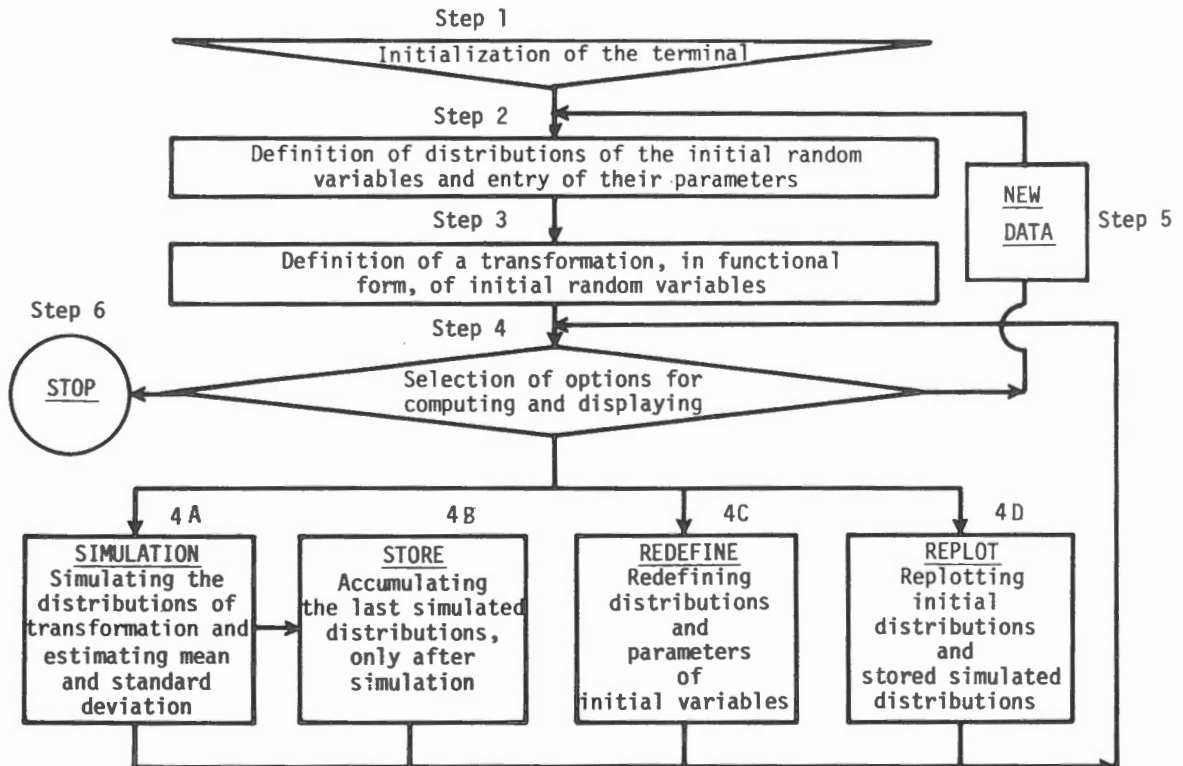


Figure 1. SIMGRA 1, flow chart.

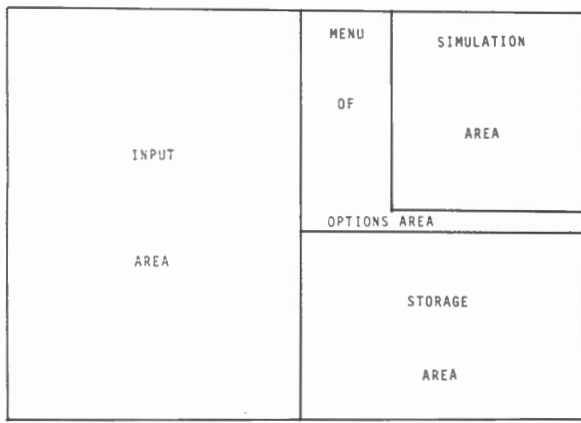


Figure 2. Subdivision of screen into four areas as designed for SIMGRA 1.

(2) Definition of distribution of the initial random variables and entry of their parameters

Upon entering any alphanumeric character and pressing the RETURN key, step (2) is initiated. The menu of options will appear on the screen with the cross-hair, and the bell will sound. The Menu of Options area contains the user selectable options. The user selects the options he desires by means of the screen cursor, cross-hair, activated by the thumb wheels. Entries in this step are made as follows:

An option for the  $x$ -values specified distributions in the menu is selected by placing the cross-hair intersection inside the proper square, by entering an alphanumeric character, and by pressing the RETURN key.

As can be seen in figure 3, a sequence number from 1 to 8 will appear in the appropriate space in the Input area to the left. The name of the option will be written beside the sequence number. A bell will prompt for input parameters which will be printed just below the initial distribution name. In figure 3, some examples of entries of different initial distributions are given. Further comments on figure 3 are given in Appendix A.

The initial distribution types and their parameters can be described as follows:

(i) NORMAL ( $\mu, \sigma$ )

The density function is:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left\{ -1/2 \left( \frac{x-\mu}{\sigma} \right)^2 \right\} \quad -\infty < x < \infty$$

The first parameter to be entered is  $\mu$ ; the second parameter is  $\sigma$ . The range of  $x$ -values is from  $\mu - 2.5\sigma$  to  $\mu + 2.5\sigma$ . The cumulative distribution will be drawn as a solid line, while the density distribution curve will be drawn as a dashed line. Note that the scale value which will appear on the right side of the density distribution is the standardized value,  $\frac{1}{\sqrt{2\pi}} = 0.4$ . Thus, in order to obtain the proper scale of the density function, the user has to multiply  $\frac{1}{\sigma}$  by 0.4.

(ii) LOGNORMAL ( $\alpha, \beta$ ) or ( $\mu, \sigma$ ).

The density function is:

$$f(x) = \begin{cases} 0 & x \leq 0 \\ \frac{1}{\sqrt{2\pi}\sigma} \exp \left\{ -1/2 \left( \frac{\ln x - \mu}{\sigma} \right)^2 \right\} & x > 0 \end{cases}$$

The mean  $\alpha = e^{\mu+1/2\sigma^2}$  and the variance  $\beta^2 = \alpha^2 \eta^2$

where  $\eta^2 = e^{\sigma^2} - 1$ . Also we have that the logarithmic mean  $\mu = 1/2 \log_e \left( \frac{\alpha^4}{\alpha^2 + \beta^2} \right)$ , and the logarithmic variance  $\sigma^2 = \log_e \left( \frac{\alpha^2 + \beta^2}{\alpha^2} \right)$ .

The program allows both types of input: ( $\mu, \sigma$ ) or ( $\alpha, \beta$ ) to be selected by the user. For entering ( $\alpha, \beta$ ), any alphanumeric key will be depressed before the RETURN key. For entering ( $\mu, \sigma$ ), the "L" key will have to be depressed when the cross-hair is located inside the LOGNORMAL square; then the RETURN key will have to be depressed. When entering ( $\alpha, \beta$ ) the first parameter is  $\alpha$ , the second  $\beta$ . The range for  $x$ -values is from 0.0 to  $\exp(2.5\sigma + \mu)$ . As in the case of NORMAL, the scale value of the density distribution is the standardized value, 1.25.

(iii) CAUCHY ( $\mu, \sigma$ ).

The density function is:

$$f(x) = \frac{\sigma}{\pi} \frac{1}{\sigma^2 + (x - \mu)^2} \quad -\infty < x < \infty$$

The first parameter to be entered is  $\mu$ , the second is  $\sigma$ . The range of  $x$  will be from  $\mu - 3.0\sigma$  to  $\mu + 3.0\sigma$ .

(iv) GAMMA ( $\alpha, \beta$ )

The density function is:

$$f(x) = \begin{cases} 0 & x \leq 0 \\ \frac{1}{\Gamma(\alpha) \beta^\alpha} x^{\alpha-1} \exp \left( -\frac{x}{\beta} \right) & x > 0 \end{cases}$$

The first parameter to be entered is  $\alpha$ , the second is  $\beta$ . The range of the  $x$ -values plotted is from 0.0 to 8.0.

(v) BETA ( $\alpha, \beta$ ).

The density function is:

$$f(x) = \begin{cases} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^\beta & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

The first parameter to be entered is  $\alpha$ , the second is  $\beta$ .

(vi) TRIANGULAR ( $p, q, r$ ).

The density function is:

$$f(x) = \begin{cases} \frac{2(x-p)}{(r-p)(q-p)} & p \leq x \leq q \\ \frac{-2(x-r)}{(r-p)(r-q)} & q < x \leq r \\ 0 & \text{otherwise} \end{cases}$$

$p$  is the lower bound,  $q$  is the apex and  $r$  is the upper bound. The values for the parameters will be entered in the order  $p, q, r$ .

(vii) UNIFORM (p, q).

The density function is:

$$f(x) = \begin{cases} \frac{1}{(q-p)} & p < x < q \\ 0 & \text{otherwise} \end{cases}$$

The first parameter to be entered is p, the second is q.

As soon as the user completes to enter the necessary parameters, the program responds immediately by plotting the cumulative distribution (solid line) and density distribution (dashed line) of the random variable selected by the user.

At the completion of the first plot, the cross-hair will reappear with the bell sound for the reselection of the next cycle of entries. When the last cycle of entries desired has been terminated, the function step begins. Figure 3 shows the plots generated by seven sets of different entries.

(3) Definition of a transformation, in functional form and to be simulated, of the initial variables defined in (2)

In the step, the cross-hair is placed inside the FUNCTION square, a character is entered and the RETURN key is depressed. A bell will prompt for the input function (\*\*) to be entered beside the word FUNCTION, for example,  $VV = V(1) + V(2) + V(3) + V(4) + V(5) + V(6) + V(7) + V(8)$  (as in figure 3). By pressing the RETURN key after having entered the function, the cross-hair will reappear with the bell sound. Now, the program is ready for simulation.

(4) Computation of simulation, plotting of the results, and modification of the initial variables

(4A) Simulation: "SIMULATION"

In this step, the density distribution and the cumulative distribution of the function entered in step (3) are simulated. The cross-hair should be set within the square beside the word SIMULATION.

Several options are available for simulation, of random numbers for each initial distribution, by depressing one of the following keys:

any alphabetic character	50,000 random numbers
the character "1"	100,000 random numbers
the character "2"	200,000 random numbers
the character "3"	300,000 random numbers
the character "4"	400,000 random numbers
the character "5"	500,000 random numbers

Note: computer time will increase proportionally to the number of random numbers generated, and also will depend on the type of input distribution chosen.

By depressing one of the above-mentioned optional keys and depressing the RETURN key, a density distribution plot and a separate cumulative distribution plot will appear in the Simulation area, with the expectation  $E(x)$  and the standard deviation  $\sigma(x) = \sqrt{E(x^2) - E^2(x)}$ .

(\*\*) at present, the use of the FUNCTION is limited to addition. If other types of transformations are required, the user should supply corresponding subroutines as is done in figure 4 of example 2. Routines for general functions are being developed and will be released soon.

(4B) Storage "STORE"

If so desired, the simulated density distribution and cumulative distribution plots can be stored on disk and displayed in the Storage area. This is obtained by placing the cross-hair in the square beside the word STORE, entering any alphanumeric character and depressing the RETURN key. If this option is chosen, the stored plots of the Simulation area will be reproduced in the Storage area below with an expanded x-axis scale which further enables the user to notice the smoothness or irregularity of the plots. Several simulation plots can be stored and displayed in the Storage area. This option enables the user to visually compare the density distributions and the cumulative distributions of the functions based on several sets of initial random variables simulated consecutively.

(4C) Redefinition of initial random variables: "REDEFINE"

After simulating the function defined, the user could redefine any or all the initial random variables, one at a time. For this option the cross-hair is placed inside the square beside the word REDEFINE and any alphanumeric character is entered. A bell will prompt for a one-digit number from 1 to 8 to be entered just to the right of the word REDEFINE. This number indicates which of the random variables corresponding to the initial random variable 1 to 8 is to be redefined with a new entry of type of distribution and/or its parameters, and then replotted in the Input area. By depressing the RETURN key, the entire display area will be cleared, the initial distribution plots before the one to be redefined will be redrawn, the menu of options will also be redrawn and the cross-hair will reappear for the selection of a new entries prompted by the bell sound. A new option for initial distribution can be selected at this point and new parameters entered. Upon completion of the new entry, the corresponding density distribution and cumulative distribution plots will be drawn in the area which was occupied by the previous variable. Then the remainder of the initial distribution plots will reappear on the screen. All the stored simulated plots will reappear as well. This procedure can be repeated, if the user desires to redefine any of the input variables. No plot will be redrawn in the Simulation area.

(4D) Replotting of input distributions and of stored simulated distributions: "REPLOTT"

The option is selected by placing the cross-hair inside the square beside the word REPLOTT, by entering any alphanumeric character and then depressing the RETURN key. First the entire area of the screen will be cleared, and then all the initial distribution plots will be redrawn in the display area. The menu of options and the stored distribution plots will also reappear. The Simulation area to the upper right of the screen will have no distribution curve displayed. The REPLOTT option can be used either to clear the Simulation area for new simulated distributions, or simply to eliminate overprinting of corrections which might have been entered during step (2).

(5) Entering of new random variables: "NEW DATA"

The option is selected by placing the cross-hair inside the square beside the word NEW DATA, by entering any alphanumeric character and by depressing the RETURN key. This will reinitiate step (2) of the program by clearing the entire screen and the disk area where plots have been stored. The menu of options will reappear on the screen with the cross-hairs and the bell sound prompting for new initial distributions.

(6) Termination of the program: "STOP"

This option is selected by placing the cross-hairs inside the square beside the word STOP by entering any alphanumeric character, and by depressing the RETURN key. The program will terminate execution and this will bring back to COMMAND mode. Key in LOGOUT to terminate the session.

CONTROL COMMANDS FOR CYBER 74 OF THE  
COMPUTER SCIENCE CENTER OF THE DEPARTMENT OF  
ENERGY, MINES AND RESOURCES

After logging into the system with a valid User name/Password, from the user's TEKTRONIX terminal, the following Control Commands are required to execute the program.

- (1) ATTACH (SIMGRA1, GEOMATHLIB, CY=4, ID=MATH, MR=1)
- (2) ATTACH (LIB1, TEKAGLIB, MR=1)
- (3) ATTACH (LIB2, IMSLIB, MR=1)
- (4) LIBRARY (LIB1, LIB2)
- (5) CONNECT (INPUT, OUTPUT)
- (6) ETL (500)
- (7) DAYFILE (OFF)
- (8) SIMGRA1.

NOTES

- (1) the program which generates the plot is attached. MR=1 allows other users to access the program at the same time.
- (2) the G.S.C. User Library TEKAGLIB, which contains all the TEKTRONIX routines, is attached.
- (3) the G.S.C. User Library IMSLIB, which contains mathematical and statistical routines, is attached.
- (4) the LIBRARY command declares that TEKAGLIB and IMSLIB are global libraries.
- (5) the files INPUT and OUTPUT must be connected for interaction between the user and the system.
- (6) the ETL (500) command is used to set the user's program on command time limit. The default time limit is 50 (in octal seconds).
- (7) this command will eliminate unnecessary system messages from the display screen.
- (8) the SIMGRA1 command loads and executes the program.

CONCLUDING REMARKS

This program has been written for a general purpose, however, it is part of developments for statistical models for resource evaluation. For this specific purpose, the program has been modified. The modified version also allows the entry of a risk factor, a measure of uncertainty which is frequently used for resource appraisals. A documentation of the modified version is in preparation.

APPENDIX A

Example 1.

Let  $V_1, V_2, \dots, V_8$  be independent random variables with

- $V_1 \sim \text{lognormal}(1.0, 0.2)$
- $V_2 \sim \text{normal}(2.5, 1.0)$
- $V_3 \sim \text{gamma}(1.2, 2.0)$
- $V_4 \sim \text{beta}(0.4, 0.6)$
- $V_5 \sim \text{triangular}(0.0, 0.3, 1.2)$
- $V_6 \sim \text{uniform}(0.1, 1.3)$
- $V_7 \sim \text{lognormal}(1.5, 0.15)$
- $V_8 \sim \text{normal}(3.0, 1.0)$

and let  $VV = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7 + V_8$ . Then  $VV$  is also continuous random variable and it is extremely difficult to obtain the exact functional form of the density distribution function  $f_{VV}$  of the random variable  $VV$ . The density distribution and cumulative distribution can be approximated by simulation. Simulated density distribution and cumulative distribution of the  $VV$  have been obtained by the program and shown in the Simulation area in Figure 3. Estimated expected value  $E(VV)$  and standard deviation  $\sigma = \sqrt{E(VV^2) - E^2(VV)}$  of  $VV$  are 15.94 and 15.69, respectively. They are also shown in figure 3. The simulated distributions have been stored, and displayed in the Storage area for comparisons with the simulations to follow.

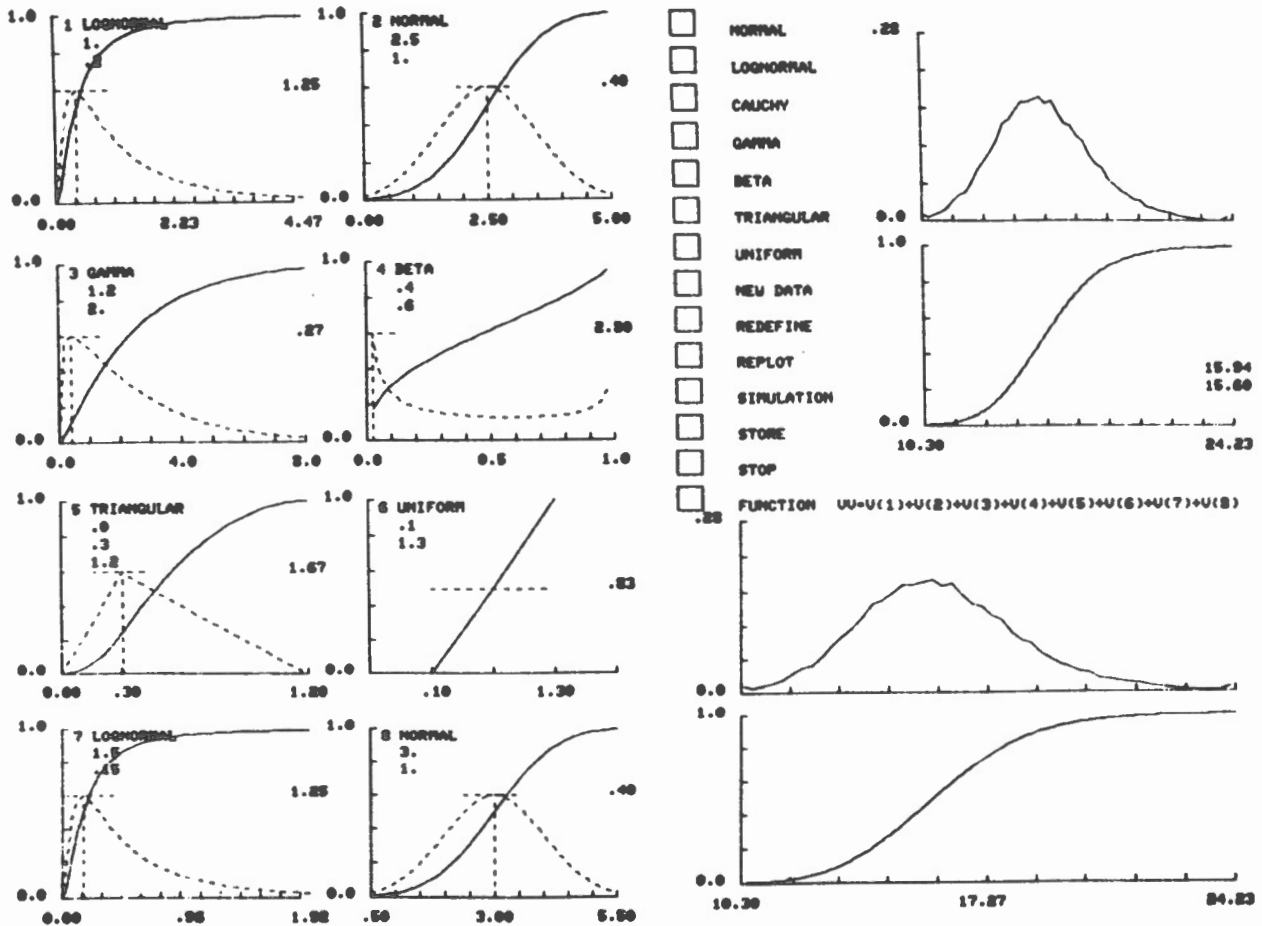


Figure 3. Entry of eight different initial distributions, 50000 random number simulation of their sum, and storage of simulated plots. Explanation of details is in text.



Example 2.

Let  $V_1, V_2, \dots, V_6$  be independent random variables with

$$V_1 \sim \text{lognormal}(1.0, 0.2)$$

$$V_2 \sim \text{normal}(2.5, 1.0)$$

$$V_3 \sim \text{uniform}(1.0, 1.5)$$

$$V_4 \sim \text{beta}(0.4, 0.6)$$

$$V_5 \sim \text{triangular}(0.0, 0.3, 1.2)$$

$$V_6 \sim \text{gamma}(1.2, 2.0)$$

and let  $VV = \frac{(V_1 - V_2)}{V_3} + e^{V_4} - V_5 + \log_e V_6$ . The density distribution and cumulative distribution of the random variable  $VV$  have been simulated and shown in Figure 4. Estimated expected value  $E(VV) = 1.26$  and standard deviation  $\sigma = 1.7$ , are also shown.

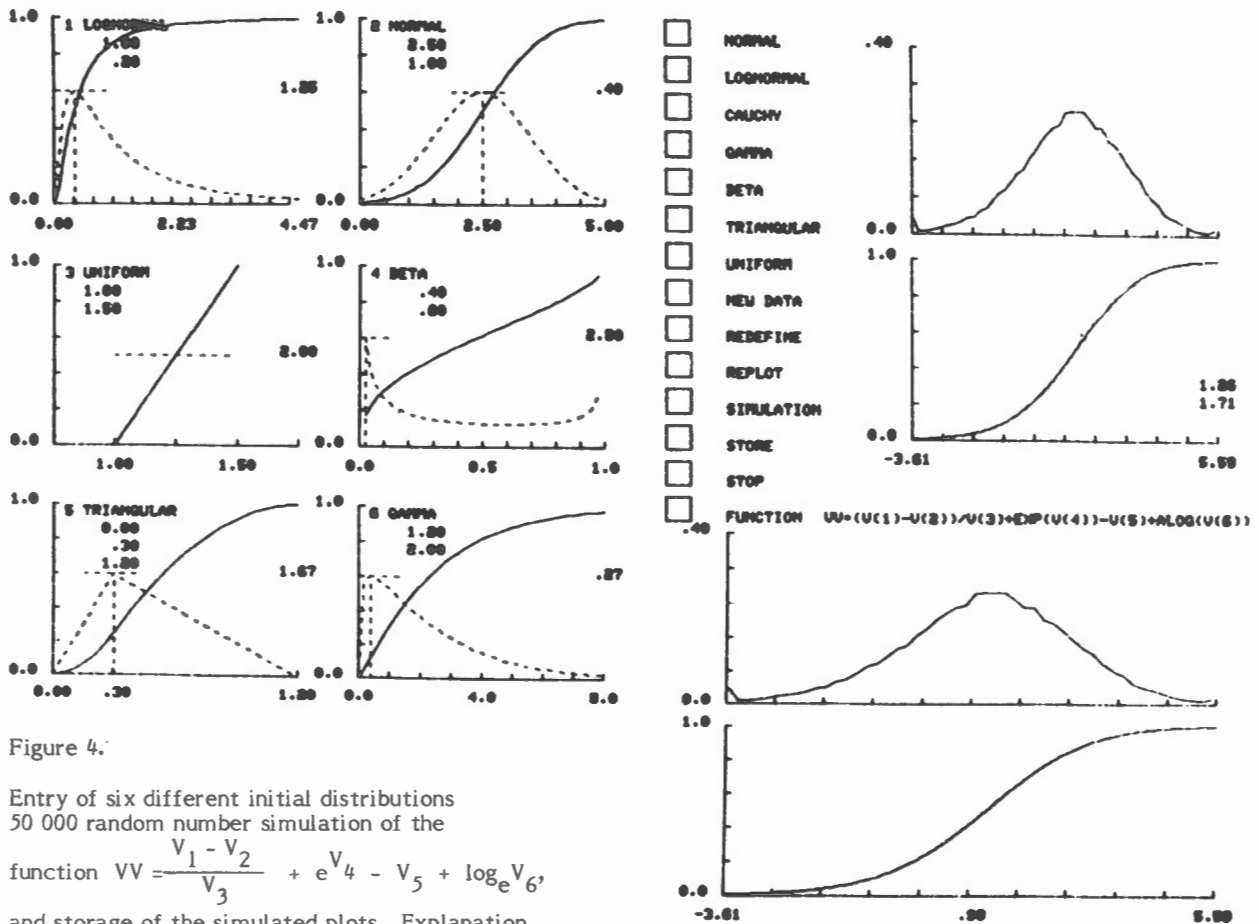


Figure 4.

Entry of six different initial distributions  
 50 000 random number simulation of the  
 function  $VV = \frac{V_1 - V_2}{V_3} + e^{V_4} - V_5 + \log_e V_6$ ,  
 and storage of the simulated plots. Explanation  
 of details is in text.

APPENDIX B

Examples of SIMGRA 1 simulations step by step.

Figures 5 to 7 show displays of 3 sets of entries, simulations and manipulations during a session of SIMGRA 1. Encircled numbers 1 to 50 correspond to individual entries (subscript i) and to displays (subscript o) in sequence in one single session.

Figure 5:

- 1<sub>i</sub> Selection of UNIFORM initial random variable.
- 2<sub>i</sub> and 3<sub>i</sub> Entry of parameters for UNIFORM initial random variable.
- 4<sub>o</sub> Plot of UNIFORM density distribution and cumulative distribution for initial random variable.
- 5<sub>i</sub> Entry of parameters for second UNIFORM initial random variable.
- 6<sub>i</sub> and 7<sub>i</sub> Entry of parameters for second UNIFORM initial variable.
- 8<sub>o</sub> Plot of second UNIFORM density distribution and cumulative distribution.
- 9<sub>i</sub> and 10<sub>i</sub> Entry of FUNCTION  $VV=V(1)+V(2)$ .
- 11<sub>i</sub> Simulation of the FUNCTION, with 50,000 random numbers generated.
- 12<sub>o</sub> and 13<sub>o</sub> Plots of simulated distributions, density distribution and cumulative distribution respectively. Note that the theoretical density distribution of  $VV$  is triangular with the lower bound 0, apex 1 and upper bound 2; i.e.  $VV \sim \text{triangular}(0, 1, 2)$ . As can be seen, the simulated distribution is nearly identical to triangular (0, 1, 2).
- 14<sub>i</sub>, 15<sub>o</sub> & 16<sub>o</sub> Storage and display of stored simulated plots.
- 17<sub>i</sub> & 18<sub>i</sub> Redefinition of random variable 2.

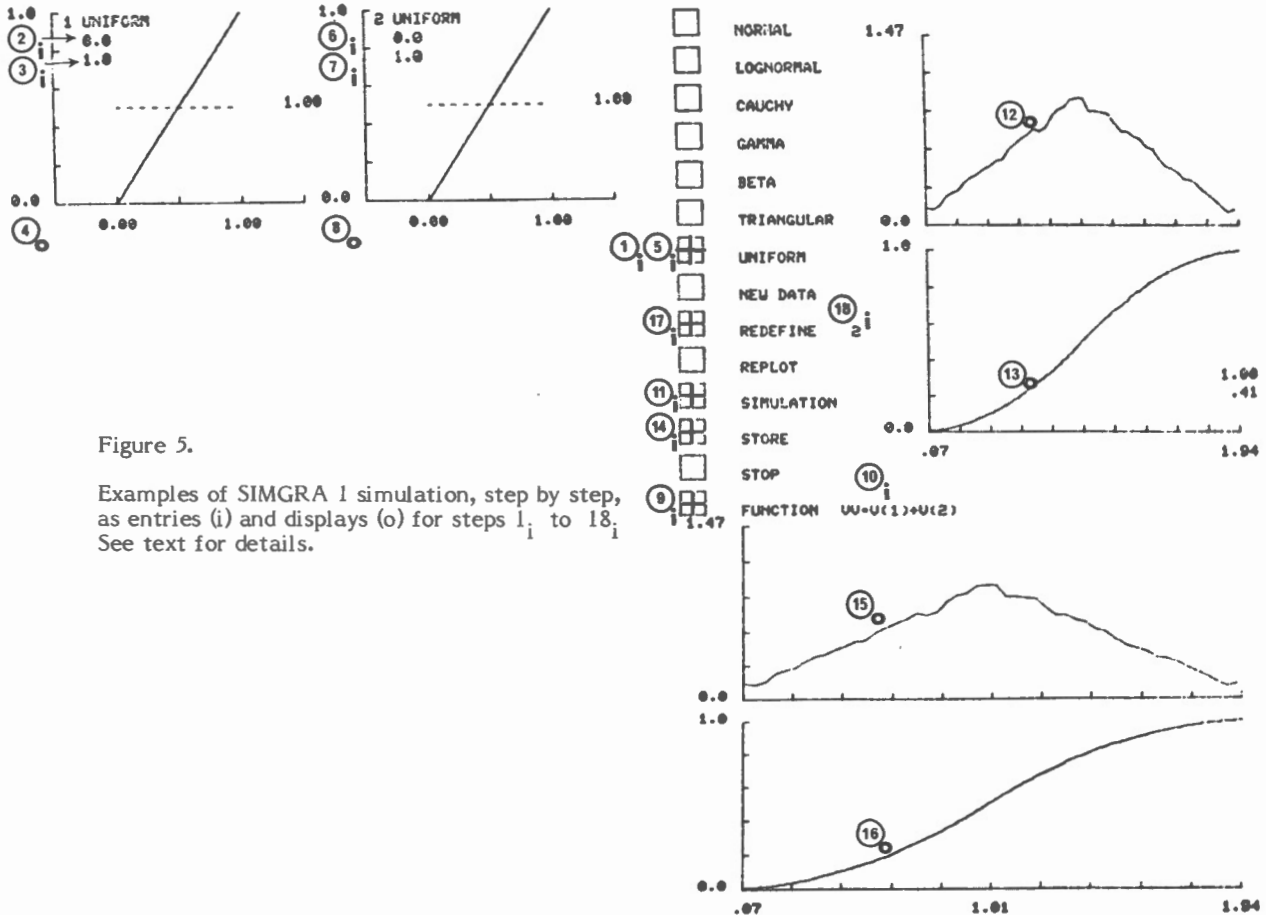


Figure 5.

Examples of SIMGRA 1 simulation, step by step, as entries (i) and displays (o) for steps 1<sub>i</sub> to 18<sub>i</sub>. See text for details.

Figure 6:

- 19<sub>o</sub>            Redisplay of plot for initial random variable 1.
- 20<sub>i</sub>            Selection of TRIANGULAR random variable type for redefined variable 2.
- 21<sub>i</sub>, 22<sub>i</sub> & 23<sub>i</sub>    Entry of parameters for variable 2.
- 24<sub>o</sub>            Plot of TRIANGULAR density distribution and cumulative distribution for initial variable 2.
- 25<sub>i</sub>            New simulation with 50000 random numbers generated.
- 26<sub>o</sub> & 27<sub>o</sub>      Plots of density distribution and cumulative distribution for the new simulated function.
- 28<sub>i</sub>            Storage of second simulated plots.
- 29<sub>o</sub> & 30<sub>o</sub>      Display of previously stored simulated plots.
- 31<sub>o</sub> & 32<sub>o</sub>      Display fo second stored simulated plots.
- 33<sub>i</sub> & 34<sub>i</sub>      Second redefinition of initial random variable 2.

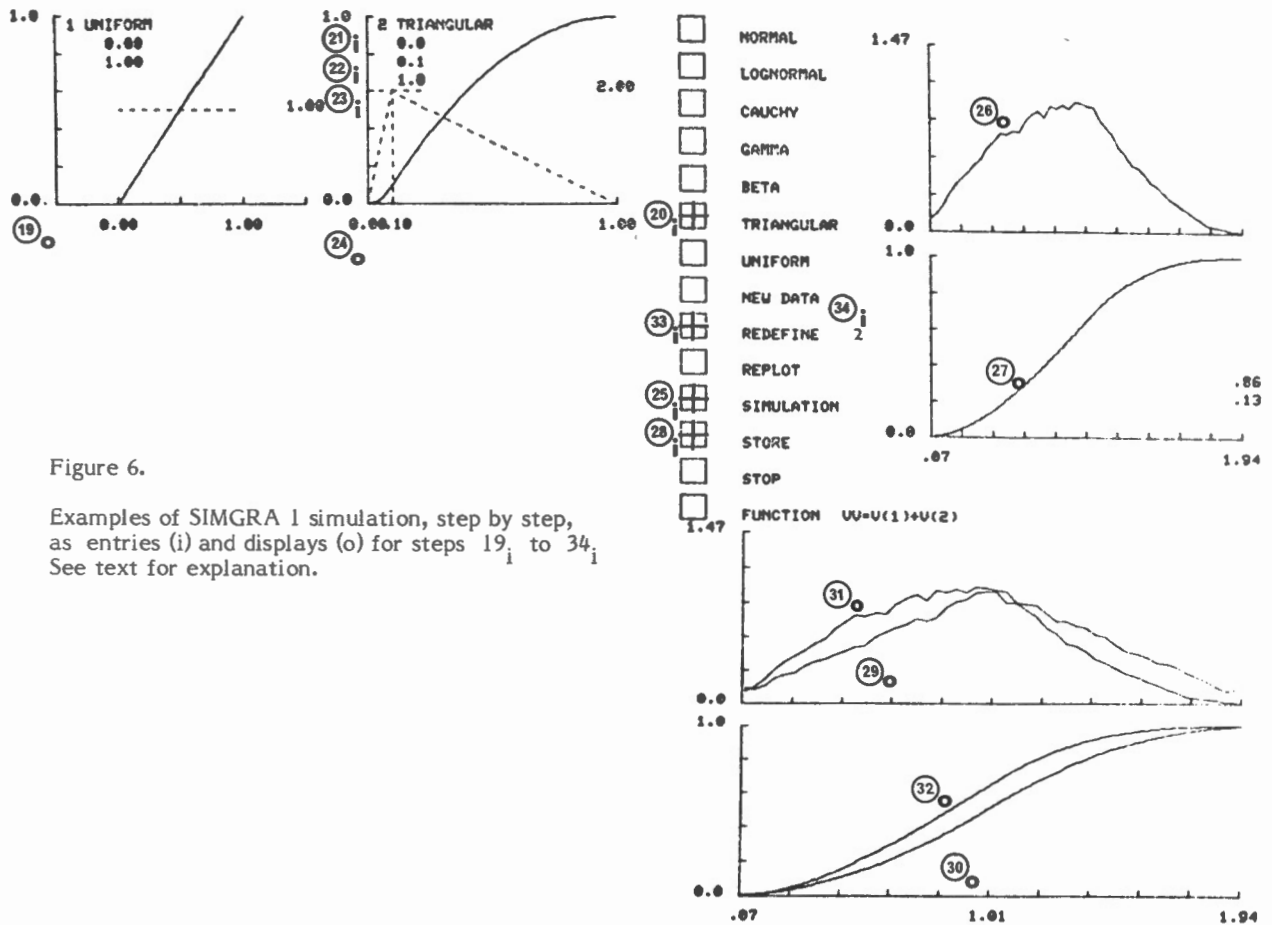


Figure 6.

Examples of SIMGRA 1 simulation, step by step, as entries (i) and displays (o) for steps 19<sub>i</sub> to 34<sub>i</sub>. See text for explanation.

Figure 7:

- 35<sub>o</sub>                   Redisplay of random variable 1 plot.
- 36<sub>i</sub>                   Selection of BETA random variable for initial distribution 2.
- 37<sub>i</sub> & 38<sub>i</sub>           Entry of parameters for BETA random variable.
- 39<sub>o</sub>                   Plot of second redefined BETA random variable.
- 40<sub>i</sub>                   Simulation of the function of a new third pair of random variables.
- 41<sub>o</sub> & 42<sub>o</sub>         New simulated function plot.
- 43<sub>i</sub>                   Storage of new plots.
- 44<sub>o</sub>-45<sub>o</sub>
- 46<sub>o</sub>-47<sub>o</sub> &
- 48<sub>o</sub>-49<sub>o</sub>           Plots for comparasion of all three simulated functions numbered according to sequence of simulation.
- 50<sub>i</sub>                   Termination of session.

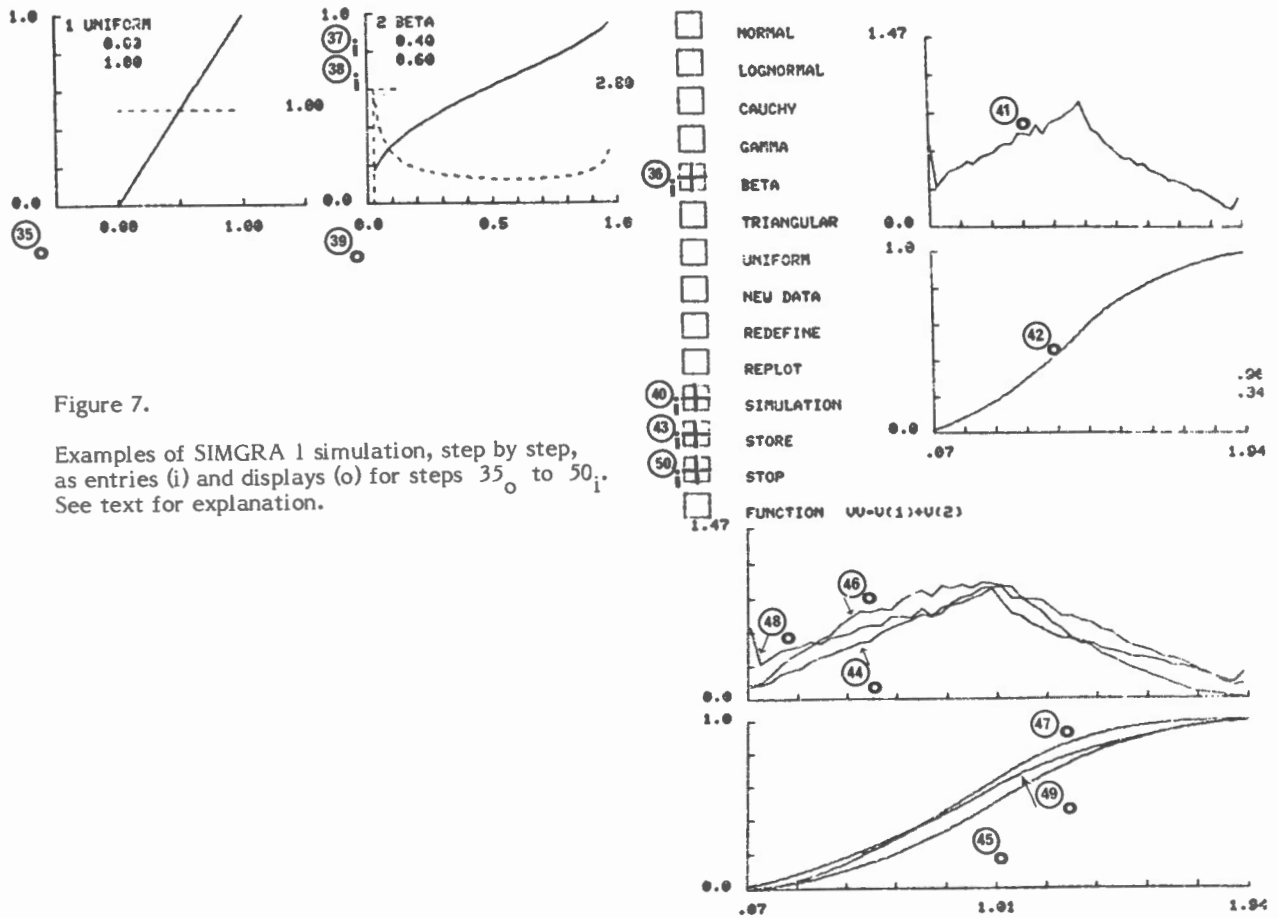


Figure 7.

Examples of SIMGRA 1 simulation, step by step, as entries (i) and displays (o) for steps 35<sub>o</sub> to 50<sub>i</sub>. See text for explanation.