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MACHINE METHODS AS AIDS IN THE PREPARATION OF GEOPHYSICAL MAPS

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B. K. Bhattacharyya and D. N. Clay

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ABSTRACT

This paper describes two methods that would aid a map compiler in finishing a map based on the computer-processed results of geophysical data. One method that uses the printer is suitable for the preparation of a map for small rectangular areas. The other method, using an x - y plotter, is suitable for large areas in which the convergence of longitude lines and the curvature of latitude lines cannot be ignored. This latter method uses Lambert Conformal Conic Projection for the preparation of maps. These methods considerably shorten the time required in representing processed values in the form of maps.

MACHINE METHODS AS AIDS IN THE PREPARATION OF GEOPHYSICAL MAPS

INTRODUCTION

The advent of modern data processing equipment has opened up possibilities not conceived of before in the analysis and interpretation of geophysical data. New processes and techniques have been evolved to extract information from the data with maximum efficiency. The rapid increase in the development of new and better tools for interpretation has been prompted on the one hand by the increasing realization of the severe limitations of qualitative interpretation and on the other by the various promising possibilities arising from the availability of digital computers. This desirable trend in the quantitative treatment of data has, however, created some major problems.

This paper is concerned only with the problem associated with the representation of results coming from the computer in the form of a two-dimensional map. The problem of representation arises because there are many advantages in having the results of analysis of the data in the form of a map. In order to understand the problem, consideration must be given to the details of the work that a compiler must do to prepare a map with the help of the results obtained from the computer after completion of mathematical analysis of a set of data.

The work of the map compiler begins with the writing down of the computer output on a graphical sheet, maintaining a fixed ratio of spacings along the two axes of the plot. These values are then contoured to provide a map of the computed quantity, a time-consuming process. A standard aeromagnetic sheet at a scale of one inch to one mile and with a grid spacing of one-quarter of a mile contains about 5,500 data points. For this number of points and for a reasonably complicated map, an experienced compiler will require about seven working days to complete the plotting and contouring, a procedure obviously expensive and prohibitively slow.

To keep the overall cost of the whole interpretive process within economically feasible limits, it is necessary to devise ways to reduce the time requirements. Initially, attempts were made to use an automatic plotter for contouring of a map. Unfortunately, however, no suitable contouring program has yet been written for the often complicated aeromagnetic maps and it was necessary to develop machine methods that would aid the compiler in finishing the map accurately and quickly.

Two methods have been developed, one employing the printer and the other a plotter but which procedure is used is dependent upon the size of the area involved. A small area is arbitrarily defined as one with dimensions in the order of several tens of miles or less and a large area as one with dimensions in the order of a hundred miles or more. For a small area, the convergence of longitude lines and the curvature of latitude lines are not noticeable and in this case, the printer can be used to produce a map rapidly and efficiently. It prints out the computed values at the intersections of rectangular coordinate lines thus forming a map. A constant spacing is maintained along both axes and values may be displayed in either a numerical or symbolic form. The time necessary to print out the data for a standard map-sheet is less than five minutes and the possibility of human error in manually transferring the results is eliminated.

However, for large areas appreciable scale and positional errors arise. Since the printer can only be used to advantage on a rectangular grid, these larger areas are represented on a Lambert Conformal Conic Projection produced with an IBM 1627, model 2, plotter. This projection employs a cone intersecting the spheroid along two parallels of latitude designated as standard parallels. Along these, scale errors are zero. Between the standard parallels, the scale is slightly smaller and beyond them, slightly larger. For an area of the size of the Yukon Territory, the maximum scale error is 0.02 per cent in the values of the coordinates and even for the projection covering the continental United States, scale errors need not exceed 1.2 per cent.

USE OF THE PRINTER IN THE PREPARATION OF A RECTANGULAR MAP

A rectangular map is composed of a number of perpendicular intersecting grid lines. The map compiler initially writes down the value of the quantity being mapped beside each intersecting point. This is a time consuming process, particularly when there are thousands of points in an area. If properly programmed, the printer would print out the computed results in a proper way beside points that constitute the final map, maintaining the correct ratio of spacings between successive points along north and east axes. Thus the output of the printer, which may be one of the quantities calculated by the computer, would show the respective values at the relatively correct locations.

The IBM 1443 on-line printer in conjunction with the IBM 1620 II computer was used in the Department of Mines and Technical Surveys, Ottawa, Canada for the above purpose. In this printer for the numerals 0-9 and the alphabetic characters, the character height is normally 0.1 inch, the vertical spacing between the bottom of one character and the top of another

0.066 inch, the width of any character 0.066 inch on the average, and the horizontal spacing between the characters, 0.033 inch. Hence the vertical spacing between the midpoints of two characters is 0.166 inch contrasted with the horizontal spacing of 0.099 inch. Let the axis parallel to the north direction be the x-axis and the axis perpendicular to it and pointing to the east be the y-axis. The spacings of 0.166 inch and 0.099 inch can now be considered as the unit lengths along the x and y axes respectively. This is the basic information necessary for choosing the correct ratio of spacings between successive points along the two axes.

Let the midpoint of each character be regarded as a point. In the case when equal spacings along both the perpendicular axes are desirable two blank spaces are normally provided between two succesive characters along the x-axis, making the spacing equal to 0.498 inch. On the other hand, four blank characters are inserted between two points in the y-axis to make a spacing of 0.495 inch. Hence the spacings along the two axes differ by about 0.6 per cent, which is extremely good. In order to keep a spacing of 1 inch between two points along both axes, the numbers of blank characters interposed between two points will be 5 and 9 for the x and y axes respectively. By varying the number of blank characters it is possible to arrive at any arbitrary ratio of spacings desired along the two axes. It is not difficult to write a program for any of these schemes. The printer is exceedingly fast, and thus the "execution time" for this sort of program is normally in the order of a few minutes.

The diagram on the right hand side of Figure 1 shows a typical output from the printer. The output may be reduced or enlarged photographically to obtain the desired scale of the map. For decreasing the time of contouring the values, the computer is often used to interpolate between grid lines originally chosen for the purpose of collection of the basic data. The spacings between grid lines in the output of the printer will then be less than the original ones. The greater the density of these lines, the less the time taken by the map compiler to do the contouring.

A finished coloured map with a proper colour code may be all that is required and for this purpose it has been found convenient to prepare a symbol map of the processed values with the help of the computer and the printer. Such a symbol map, shown on the left in Figure 1, corresponds to the numerical map on the right. The whole spectrum of numerical values is divided into nineteen intervals. In the case of aeromagnetic data the numerals from 0 to 9 signify increasing intervals of 100 gammas starting from zero, whereas the decreasing intervals from zero are denoted by the "alphabets" A to I. It is apparent from the symbol map, Figure 1, that colouring this type of map is fast and convenient. A wide choice of colour code suited to different conditions is also offered by such a map.

SYMBOLIC MAP

A	A	A	A	A	A	A	A	A	A
A	A	A	A	A	A	A	A	A	A
A	A	A	A	A	A	A	A	A	A
0	A	A	A	A	A	A	A	A	A
0	0	A	A	A	A	A	A	A	B
0	0	0	0	A	A	B	B	B	B
0	0	0	0	0	A	A	B	B	B
0	1	1	1	1	0	A	A	B	B
0	1	1	2	2	2	1	A	B	C
A	0	1	2	3	2	2	0	A	C
A	0	0	1	2	2	2	1	A	B
A	A	A	0	1	1	1	1	0	0
A	A	A	A	A	0	1	1	1	1
A	A	A	A	A	A	0	1	2	2
0	A	A	A	A	A	0	1	1	1

NUMERICAL MAP

-26	-51	-64	-58	-36	-15	-4	-9	-22	-32
-15	-38	-50	-48	-36	-25	-19	-20	-19	-12
-1	-30	-51	-58	-53	-40	-28	-21	-20	-19
14	-19	-49	-66	-67	-55	-41	-36	-45	-60
33	4	-25	-49	-63	-68	-71	-78	-90	-104
53	43	28	4	-28	-66	-101	-126	-134	-125
63	85	98	88	48	-16	-87	-140	-158	-137
55	111	160	182	160	93	-2	-97	-159	-171
27	106	187	249	267	222	119	-16	-141	-218
-10	69	164	253	306	294	207	61	-98	-223
-52	5	89	179	246	263	215	111	-17	-134
-87	-60	-8	57	117	156	164	139	90	28
-94	-95	-81	-50	-6	48	106	157	184	176
-56	-76	-94	-95	-69	-11	70	154	214	226
3	-23	-58	-81	-74	-30	38	111	161	170

Figure 1. Numerical and symbolic map of the magnetic field values of an area produced by the printer associated with the computer.

It is to be noted that the printer is extremely fast and prints out the data for a standard map in less than five minutes. A map obtained in this way eliminates the possibility of human error in transferring manually the results from the output listing of the computer to the standard graph sheets designed for preparing maps. In spite of all these advantages, the printer has its own constraints. Essentially it can only print out maps in rectangular form and is not too suitable for writing down the values in the form required by the Lambert Conformal Conic Projection. However, this type of projection is needed for preparing maps of a large area in which the convergence of longitude lines and the curvature of latitude lines are prominent.

THEORY OF THE LAMBERT CONFORMAL CONIC PROJECTION

A sphere is a non-developable surface, i.e., it cannot be represented on a plane without introducing distortion. Thus a map projection is a compromise in which certain features are retained at the expense of inaccuracies in others. Such features might be the preservation of areas or of angular relationships; establishing lines of longitude as straight lines and parallels of latitude as concentric circles; or, perhaps requiring that the equator be a straight line. In other words, a projection is selected on the basis that it best meets the requirements for a particular job.

In plotting of geophysical data it is desirable to obtain a similarity of elements, such as (a) preservation of angular relationships and (b) for any suitably restricted locality, invariance of the ratio of the length of a linear element on the spheroid and the length of a corresponding element in the map for all azimuths in which the former element may be oriented. This is termed a conformal representation. Minimizing scale errors is also essential. Both conformality and minimal scale errors can be obtained with the Lambert Conformal Conic Projection.

This projection consists of a cone doubly intersecting the spheroid. The position of the standard parallels is illustrated in Figure 2. If these parallels are chosen at one-sixth and five-sixths of the longitudinal extent of the central meridian for the area to be represented, the scale error would be equally distributed and minimal. This can be seen from a consideration of Figure 3 that indicates the maximum scale errors to be expected in a projection that would cover the continental United States.

The first feature to note is that maximum scale errors occur along the northern and southern limits and along the central latitude of the projection. These are equal and hence the scale error is equally distributed. Secondly, if the standard parallels are moved closer together to increase the accuracy along the central latitudinal zone of the projection, the accuracy at

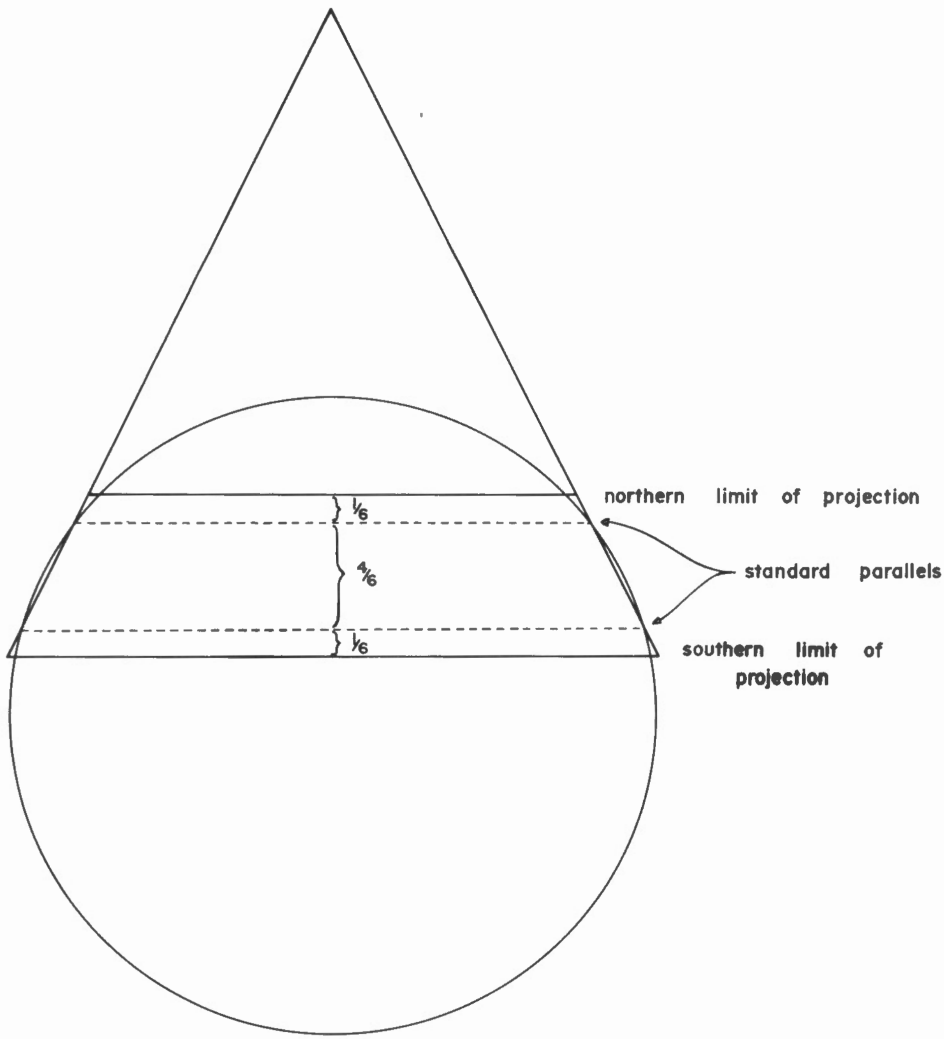


Figure 2. Intersection of a cone and a sphere along two standard parallels.

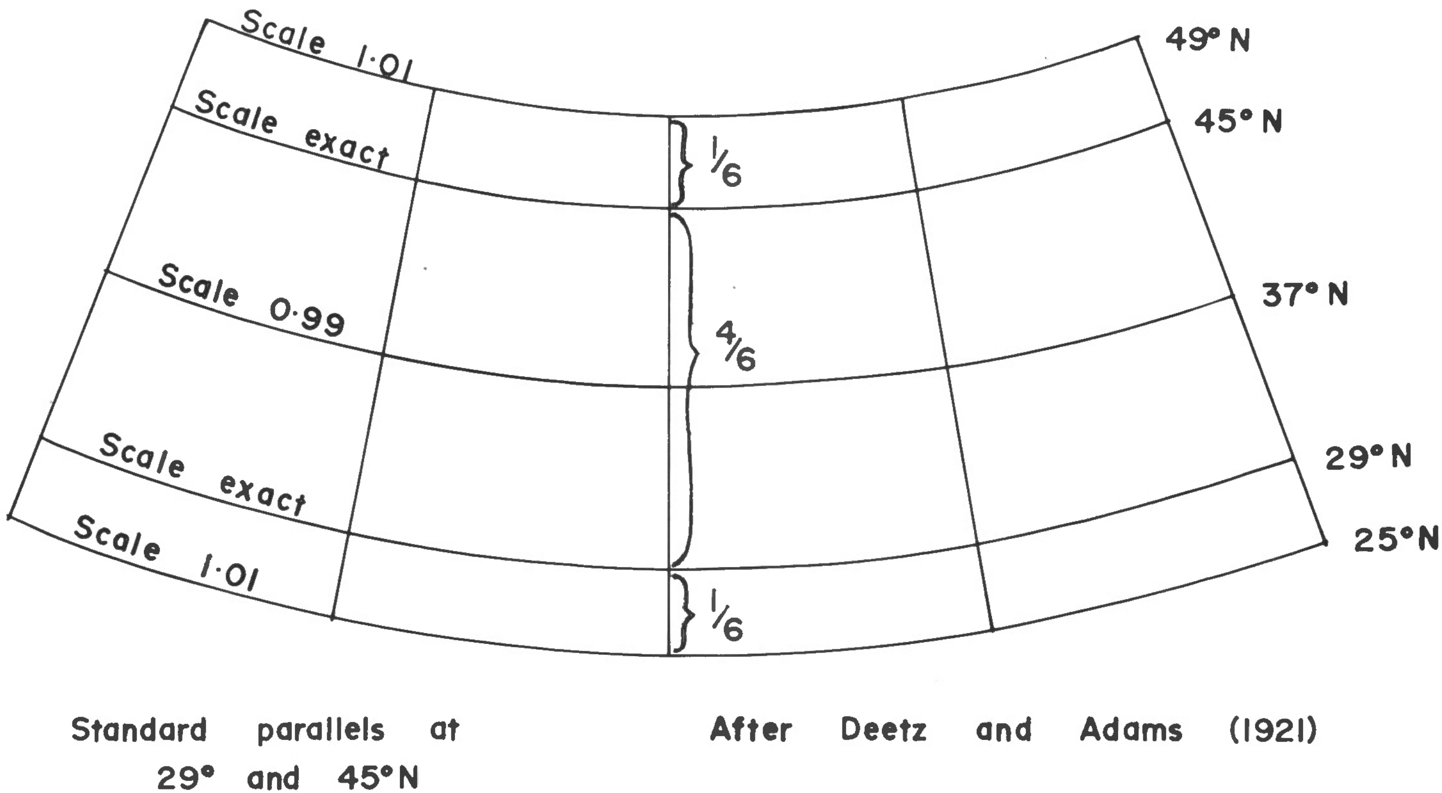


Figure 3. Distortion of the scale with the standard parallels at 25° N and 49° N.

the northern and southern limits decreases and vice versa. Consequently, Figure 3 depicts the spacing of the standard parallels that will give a minimal scale error.

It may also be mentioned that meridians are straight lines intersecting at a common point ouside the map boundary, whereas parallels are concentric circles whose common centre is the point of intersection of the meridians. Thus meridians and parallels intersect at right angles.

Scale errors increase with the latitudinal extent of the projection and this can impose a limitation with regard to the size of the area under consideration. However, most geological or geophysical investigations do not have the areal extent of the example depicted in Figure 3. Therefore, this limitation will seldom be applicable to the type of problems that concern a geologist or a geophysicist.

The following section is a condensation of the mathematics involved in deriving the Lambert Conformal Conic Projection, as presented by Adams (1918). This section provides sufficient details about the projection so that a computer program can be written without too much trouble.

The mathematical development for this projection is reasonably straightforward. The parametric equations for an ellipsoid of revolution may be written as

$$x = a \cos \phi \sin \psi, \quad y = a \sin \phi \sin \psi,$$

$$\text{and } z = b \cos \psi,$$

where a and b are semi-major and semi-minor axes respectively; ϕ is the longitude and ψ, the complement of the reduced latitude (Fig. 4).

An element of length ds on the spheroid is given by

$$ds^2 = dx^2 + dy^2 + dz^2, \qquad \ldots (1)$$

which may be written as

$$ds^2 = a^2 \sin^2 \psi \, d\phi^2 + (a^2 \cos^2 \psi + b^2 \sin^2 \psi) \, d\psi^2 \qquad \ldots (2)$$

The square of the eccentricity of the elipsoid is given

$$\epsilon^2 = (a^2 - b^2) / a^2 \qquad \ldots (3)$$

Now, from Figure 4 we have

$$\tan \lambda = \operatorname{ctn} \rho = \frac{a}{b} \operatorname{ctn} \psi \qquad \ldots (4)$$

A combination of (3) and (4) and some trigonometric manipulations yield

$$\cos^2 \psi = (1 - \epsilon^2) \cdot \cos^2 \rho / (1 - \epsilon^2 \cos^2 \rho), \qquad \ldots (5)$$

and

$$\sin^2 \psi = \sin^2 \rho / (1 - \epsilon^2 \cos^2 \rho) \qquad \ldots (6)$$

With the help of (3), (4) and (5) we have

$$d\psi = \sqrt{(1 - \epsilon^2)} \cdot d\rho / (1 - \epsilon^2 \cos^2 \rho) \qquad \ldots (7)$$

Combining (2) and equations (3)-(7), we obtain

$$ds^2 = a^2 \sin^2 \rho (d\phi^2 + d\theta^2) / (1 - \epsilon^2 \cos^2 \rho), \qquad \ldots (8)$$

where

$$d\theta = (1 - \epsilon^2) \, d\rho / (1 - \epsilon^2 \cos^2 \rho) \sin \rho \qquad \ldots (9)$$

Thus the parameters have been reduced to an orthogonal system.

We now select a conformal representation of the spheroid on the plane using the relation:

$$x + iy = f(\phi - i\theta), \qquad \ldots (10)$$

where

$$f(\phi - i\theta) = K \exp i\ell(\phi - i\theta) \qquad \ldots (11)$$

Equating the real and imaginary parts of (10) and utilizing the expression for θ obtained from (9), we have

$$x = R \cos \ell\phi,$$

and $$y = R \sin \ell\phi \qquad \ldots (12)$$

where R, the radius of a parallel of latitude, is given by

$$R = K \tan^{\ell} z/2, \qquad \text{... (13)}$$

$$\text{and } \tan(z/2) = \tan(\rho/2) \cdot \left(\frac{1 + \epsilon \cos \rho}{1 - \epsilon \cos \rho}\right)^{\epsilon/2} \qquad \text{... (14)}$$

Now (14) can also be written as

$$\log \tan(z/2) = \log \tan(\rho/2) + \epsilon \log \operatorname{ctn}(q/2), \qquad \text{... (15)}$$

where

$$\cos q = \epsilon \cos \rho.$$

K and ℓ have remained as arbitrary constants up to this point in the development. These are now defined in a manner which will ensure the conformality of the projection.

ℓ is selected so that the ratio of the lengths of any two arcs of parallels on the map is equal to the ratio of the lengths of the arcs that they represent. Let N be the radius of curvature perpendicular to the meridian λ or the length of the normal prolonged to the minor axis. Then N is given by

$$N = \frac{a}{\sqrt{(1 - \epsilon^2 \sin^2 \lambda)}} . \qquad \text{... (16)}$$

The ratio of the lengths of two arcs at parallels of latitude λ_1 and λ_2 is

$$\frac{N_1 \cos \lambda_1}{N_2 \cos \lambda_2} . \qquad \text{... (17)}$$

The arc on the map representing the parallel λ_1 has the length

$$\ell R_1 = \ell K \tan^{\ell}\left(\frac{z_1}{2}\right) . \qquad \text{... (18)}$$

Similarly,

$$\ell R_2 = \ell K \tan^{\ell}\left(\frac{z_2}{2}\right) . \qquad \text{... (19)}$$

The ratio of lengths will be preserved if the ratio of the quantities on the right hand sides of (18) and (19) is equal to (17). This is maintained if

$$\ell = \frac{\log \cos \lambda_1 - \log \cos \lambda_2 - \log A_1 + \log A_2}{\log \tan \frac{z_1}{2} - \log \tan \frac{z_2}{2}}, \qquad \ldots (20)$$

where

$$A_1 = 1 \,/\, N_1 \sin 1'',$$

and

$$A_2 = 1 \,/\, N_2 \sin 1''.$$

K is now determined so as to hold the exact length of these parallels, e.g.,

$$\ell K \tan^{\ell}\left(\frac{z_1}{2}\right) = N_1 \cos \lambda_1 .$$

From this equation we have

$$K = \frac{\cos \lambda_1}{A_1 \sin 1'' \, \ell \tan^{\ell} \frac{z_1}{2}}$$

$$= \frac{\cos \lambda_2}{A_2 \sin 1'' \, \ell \tan^{\ell} \frac{z_2}{2}} . \qquad \ldots (21)$$

Selecting the central meridian as the Y-axis and a perpendicular to it as the X axis (Fig. 5), we have

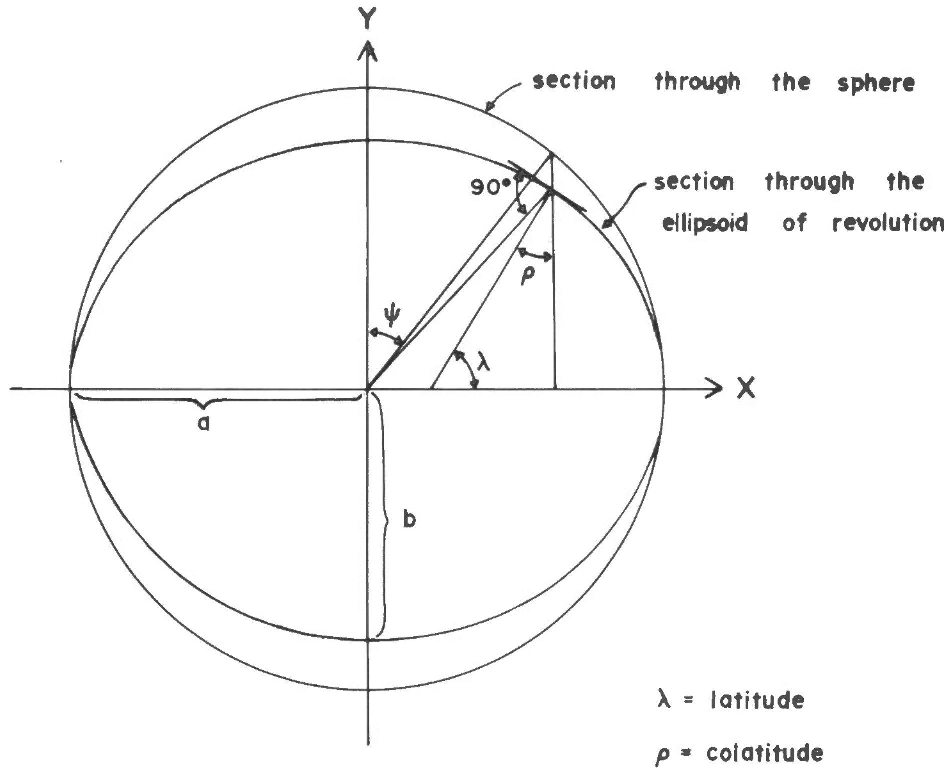
$$x = R \sin \ell \, \phi,$$

$$y = 2 R \sin^2 \left(\ell \, \phi /_2\right), \qquad \ldots (22)$$

and $R = K \tan^{\ell} \left({}^{z}/_{2}\right)$.

USE OF THE PLOTTER FOR PREPARING MAPS

With the help of equations (15), (16), (20), (21) and (22), it is now possible to prepare maps of a large area on the basis of Lambert Conformal Conic Projection. The Clarke spheroid of 1866 is used as the reference spheroid because it has been adopted for geodetic work in North America. It is defined by the following parameters:



Equations of generating ellipse: $X = a \sin \psi$

$Y = b \cos \psi$

Figure 4. Generating ellipse.

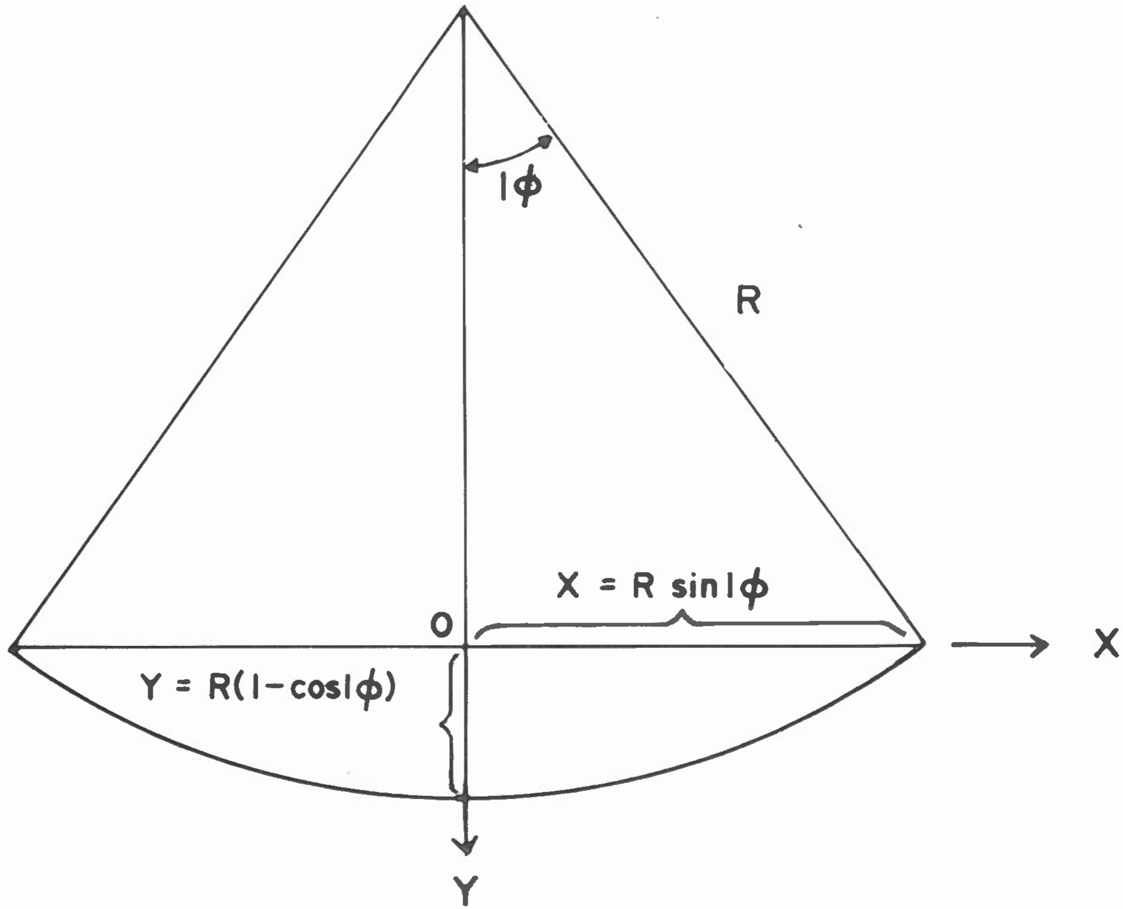


Figure 5. Coordinate system used in the Lambert Conformal Conic Projection.

semi-major axis = a = 6,378,206.4 metres,
semi-minor axis = b = 6,356,583.8 metres

If the above system is used equations (22) will provide (x, y) coordinates of a point in terms of metres with the origin at 0 as shown in Figure 5. It is clear that x values are symmetrical with respect to the central meridian. In order to use the plotter for automatic preparation of numerical or symbol maps, the way in which (x, y) values are plotted by the plotter must be considered.

Discussion will be limited to IBM 1627 plotter, Model 2, available in the Department of Mines and Technical Surveys, Ottawa. Its plotting area is 29 1/2 inches by 120 feet and the incremental step size is 0.01 inch in both the x and y directions. The input for this x - y plotter must be in inches and this is most easily accomplished by introducing into the equations for x and y a factor for converting metres into inches.

Because of the limited width of the plotter, a large area must commonly be broken up into a number of segments, each segment being plotted independently and later on, being tied up with adjoining segments. This must be done in order to obtain a finished map with the reasonable scale of 1 inch to 1 mile or 1 inch to 4 miles. However, this procedure should not conflict with the most efficient use of the computer. As noted above, the latter can be achieved if the symmetric properties of the Lambert Conformal Conic Projection are used. With this in mind, all the calculations for (x, y) coordinates of the points in an area are done initially and these coordinates are stored either in the disk pack if available, or in the scratch tape of the computer. These coordinates are then rearranged on the basis of the position of the segments in the particular area, a process that can be accomplished quickly.

Figure 6 shows how the plotter works. The origin is at the top left hand corner. With respect to Figure 5 this not only means a translation of the origin for each segment of the area, but also a change in the notation of the x and y axes. This can be easily achieved once the coordinates of the points in the original system are available.

The incremental steps in the plotter are in hundredths of an inch, hence, it is necessary to delete significant figures beyond the second decimal place in the values of the coordinates, a procedure that introduces the largest errors into the plot.

An illustration of the technique described above applied to the plotting of aeromagnetic data in the Yukon Territory, Canada is given as a final illustration. The area is bounded by latitudes 60°N and 62°N and by longitudes 128°W and 135°W. For the Yukon data, the spacing along a

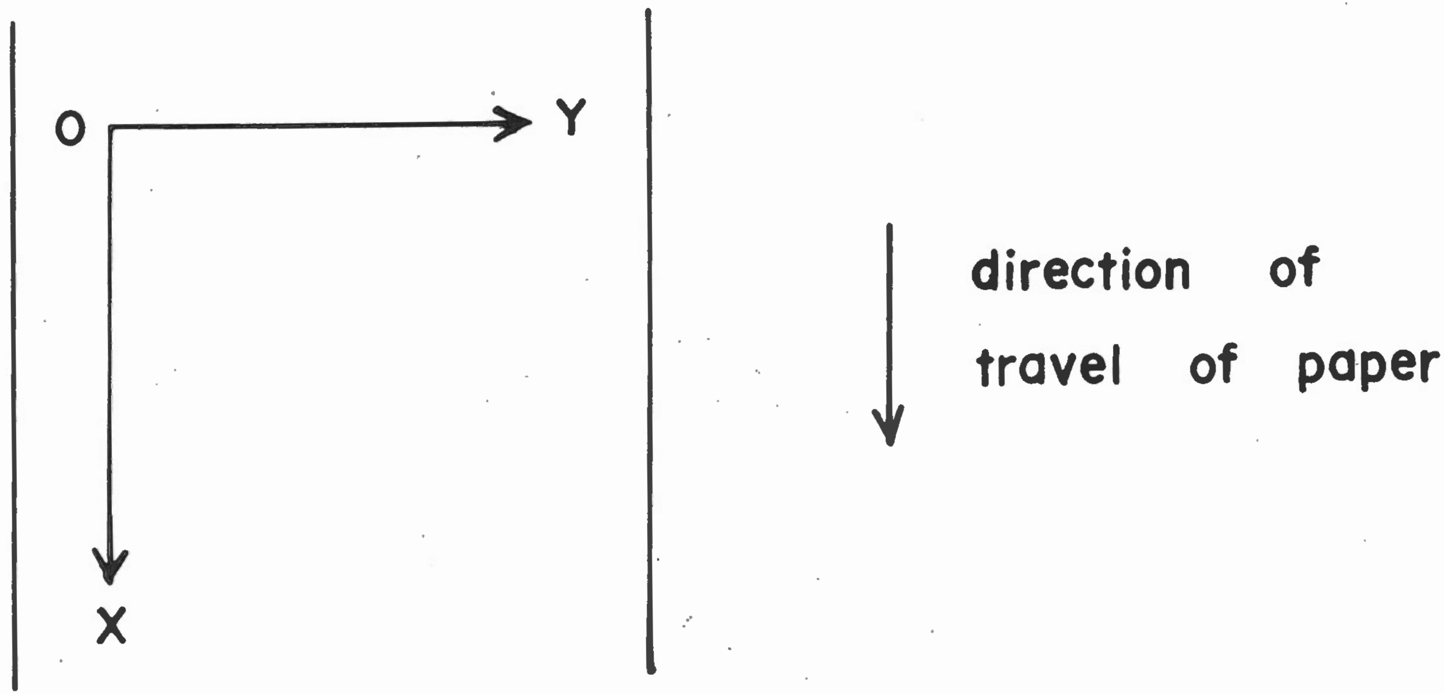


Figure 6. Coordinate axes for the IBM 1627 Plotter, Model 2.

parallel of latitude is 100 seconds and along a meridional line, 150 seconds. Using the procedure of setting the standard parallels at one-sixth and five-sixths of the latitudinal extent for minimal scale errors, these parallels are placed at latitudes 60°20'N and 61°40'N respectively, thus limiting the maximum scale error to only 0.02 per cent. The values of the constants to ℓ and K for the Yukon data will serve to demonstrate the magnitude of the numbers involved:

$$\ell = 0.87463861 \text{ and } K = 11507963.$$

The plot obtained is at the scale of four miles to one inch. The truncation error, mentioned above, introduces the largest errors into the plot. For the Yukon area in the chosen scale, the greatest error arising through the use of the Lambert Conformal Conic Projection is less than one metre. On the other hand, the maximum possible plotter error may be 64 metres which corresponds to 0.01 inch in the scale of four miles to one inch. It can be safely said that the map prepared by the plotter would be much more accurate than the one made by an experienced draftsman.

Figure 7 shows a plot of the magnetic data for the Yukon area. The curvature of the parallels of latitude and the convergence of the lines of longitude are visible in the figure. Magnetic field values have been printed out in the symbolic form. A few important contours drawn by hand are also shown. The total time required by the plotter to print 12,096 symbols at the proper x - y coordinate points, is slightly over five hours.

CONCLUSION

The Lambert Conformal Conic Projection is particularly well-suited to the type of plotting illustrated in this paper. It is extremely accurate and mechanical limitations in the plotter, rather than scale errors in the projection, generally determine the degree of accuracy. Compared to the length of time required to do the same work manually, the procedure is very rapid and several operations can be carried out in one program.

The program outlined in this report handles regularly-spaced data but can be modified to accommodate data with irregular spacings. Many of the restrictions arise from the rather limited capacity of the computing system IBM 1620 II for which this method was developed. A computer with at least double the core storage of the IBM 1620 II computer would be more than adequate to run the entire program as one unit, thereby decreasing the plotting time.

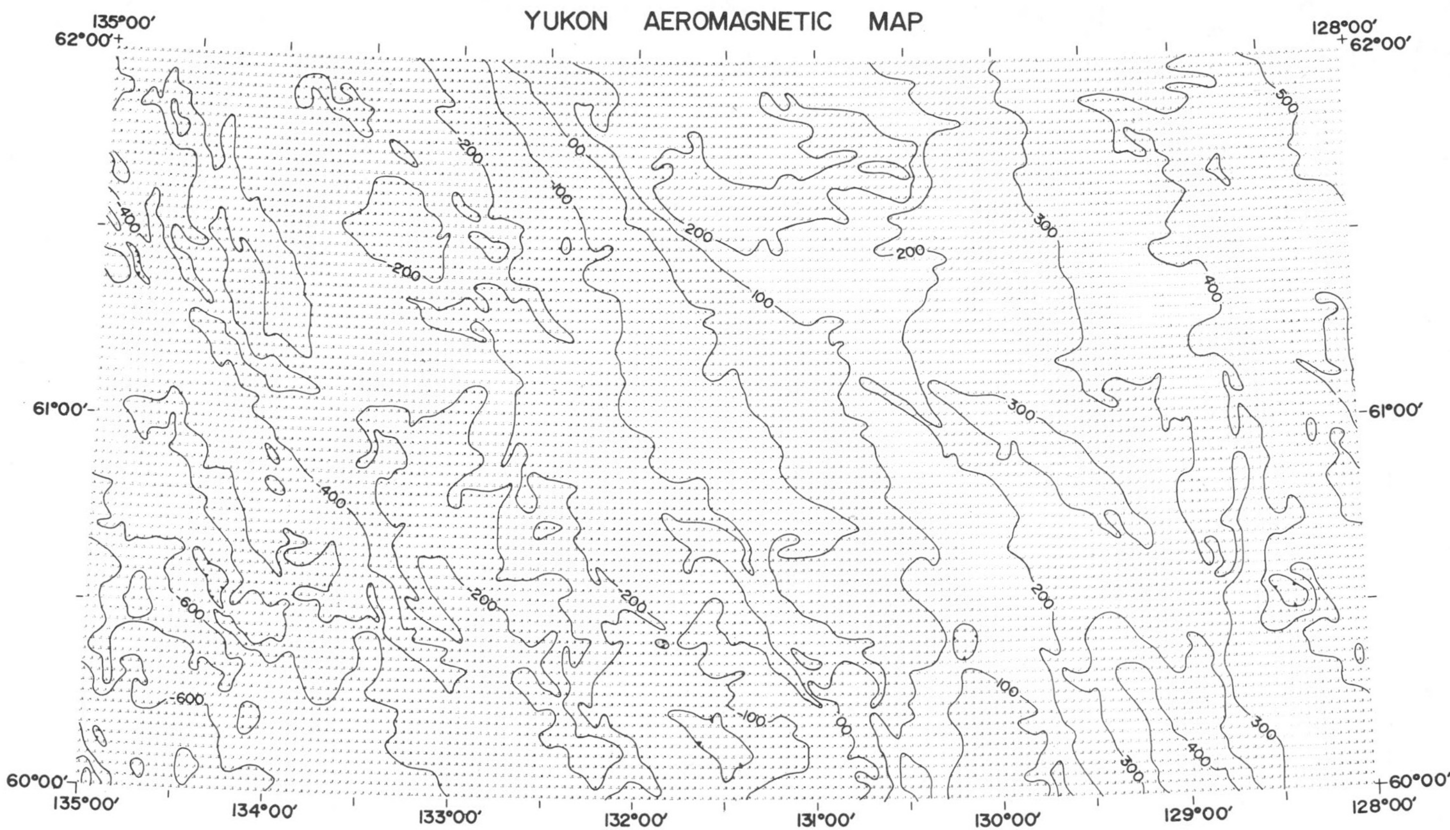


Figure 7. Symbolic map of the aeromagnetic data in the Yukon Territory, Canada, prepared by the plotter.

For optimum efficiency, the plotter should be off-line from the computing system. Otherwise, the computer will of necessity be idle during the period the plotter is operating.

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