CANADA
DEPARTMENT OF MINES
HON. CHARLES STEWART, MINISTER; CHARLES CAMSELL, DEPUTY MINISTER

GEOLOGICAL SURVEY
W. H. COLLINS, DIRECTOR

MEMOIR 164

The Niagara Falls Survey of 1927

BY

W. H. Boyd

OTTAWA
F. A. ACLAND
PRINTER TO THE KING'S MOST EXCELLENT MAJESTY
1930

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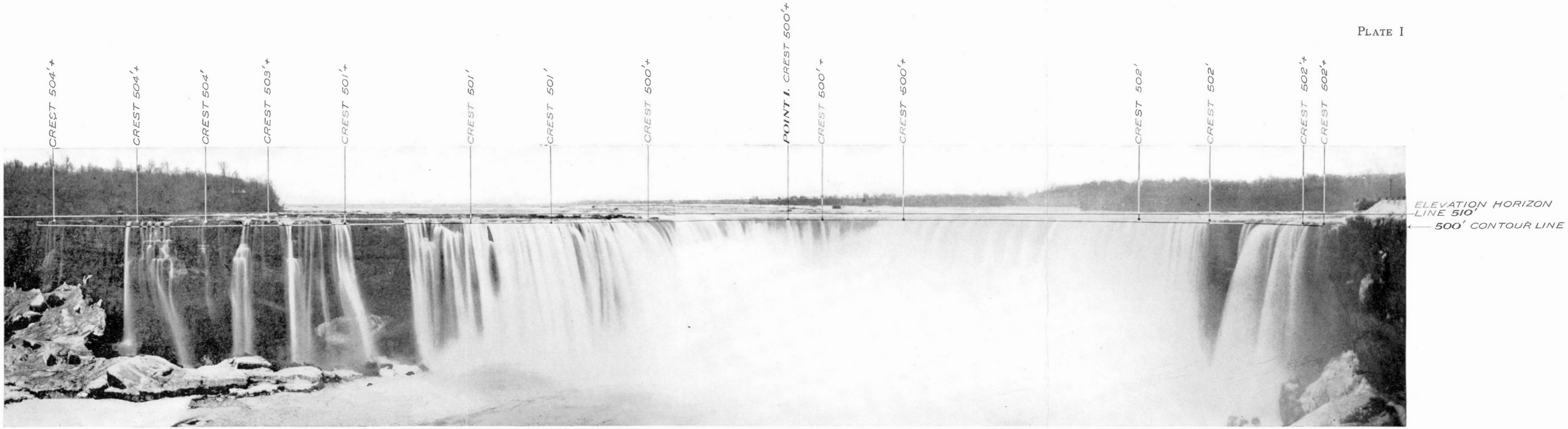
PLATE I



Horseshoe fall from station M. (Page 1.)

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The Niagara Falls Survey of 1927

INTRODUCTION

The survey of the Horseshoe and American falls, which together form Niagara falls, was carried out by the writer[1] in April, 1927, at the request of Dr. Charles Camsell, Deputy Minister of Mines, who was one of the Canadian representatives on the enlarged Niagara Board. Messrs. K. G. Chipman and A. C. T. Sheppard, topographical engineers of the Geological Survey, assisted throughout the work.

The writer wishes to acknowledge his indebtedness to Mr. E. R. Waldenberger, Superintendent of the State Reservation at Niagara, and to Mr. J. H. Jackson, General Manager of the Niagara Parks Commission, for many courtesies, and to officials of the Ontario Power Company and Dominion Water Power and Reclamation Service, whose generous co-operation greatly aided the work.

GROUND SURVEY METHODS

The regular Canadian phototopographical methods were used for the survey of the American fall. The fall is comparatively easy to survey by this method, as nearly all of the rock crest is clearly seen owing to the thinness of the sheet of water that goes over the edge.

For the survey of the Horseshoe fall, however, the regular methods of phototopography are not so well suited owing to the central part of the fall being completely covered with heavy falling water with, seemingly, no well-marked points of identification. In order to overcome the great difficulty of identifying points over the central part of the fall from three camera stations occupied in turn, and in order to obtain correct direction lines to all points identified, a new method of carrying out the phototopographical survey was used. This new method consisted in using three surveying cameras, one at each of three stations, and taking photographs of the same portion of the falls, from all camera stations simultaneously. By this method, the appearance of the water along the crest line is recorded at the same instant at each of the three stations. All points along the crest line will then appear the same on each of the three photographs. In this way, it becomes an easy matter to identify the same point from each of the three stations. The direction line to the points are automatically recorded in each picture. For a fuller account of the reasons for adopting this new method and for the procedure followed in carrying it out, the reader is referred to a paper presented by the writer before the Royal Society of Canada[2].

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[1]Chief Topographical Engineer, Geological Survey, Canada.

[2]"A New Method of Determining the Rate of Recession of Niagara Falls"; Trans., Roy. Soc., Canada, sect. IV, vol. XXII (1928).

Triangulation, using a Berger transit reading to 10 seconds of arc, was carried out for the control of the surveys. This triangulation was extended from the triangle "Transformer", "Terrapin", and "Canal" established by the United States Lake Survey in 1906. Connexion was made to boundary monuments 19, 20, and 21, to station T.P. 6 of the 1842 survey; to stations M and N of the 1890 survey; to stations "Cliff" and "Bluff" of the 1906 survey. The geographic positions of these stations, as well as those of the 1927 survey that were permanently marked, are given in table I. All camera stations were stations on the main network of triangulation.

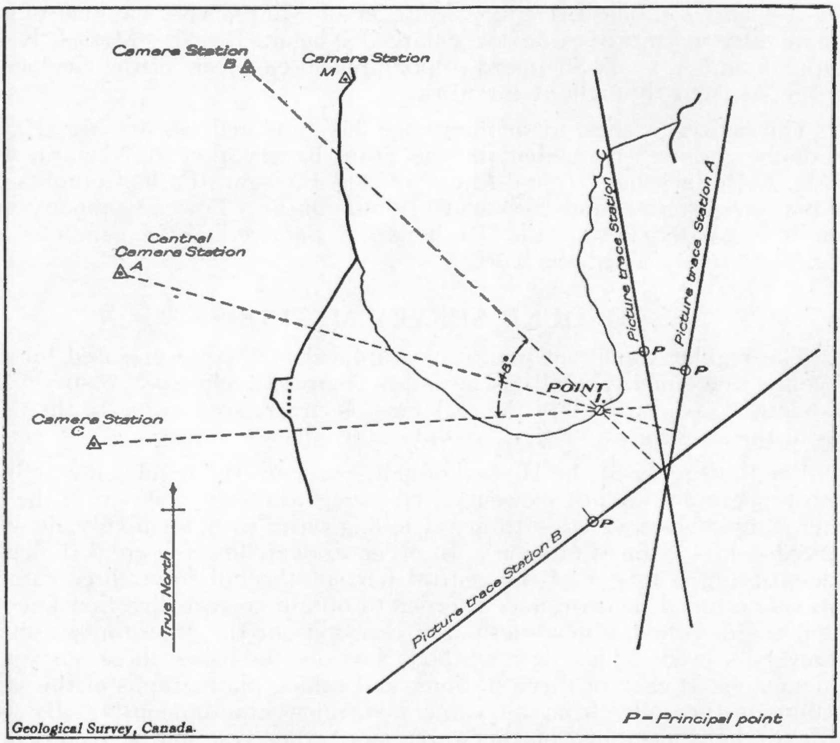

FIGURE I. Showing a set of camera stations in relation to Horseshoe fall.

For the survey of the Horseshoe fall, three camera stations A, B, and C, comprising one set of stations for synchronized photographs, were located on top of the bank extending along the west side of the fall. The relation of these stations to the crest line of the fall is shown on Figure 1. The set of synchronized photographs taken from these stations is shown in Plate II. Another set of three stations, D, E, and F (*See* Figure 2) were located on Goat island. These two sets formed the main stations for mapping the crest line. Other camera stations, Ea, M, and N, were used to supplement the information obtained from the main stations and to obtain other data relating to the crest line. Station M was selected to obtain photographs of the fall from a point at as near the height of the crest line as possible. The actual station site used was an iron post placed by some unknown previous survey and now constituting a permanently marked point of the 1927 survey.

The survey work was carried out for a map scale of 1:2,000. There was no hesitation in using phototopographical methods for the survey of the falls. The experience of the Geological Survey has proved that the method is suitable for large scale surveys provided that all field and office work is planned and carried out to suit the ground conditions and the map scale requirements. All points selected along the crest line were located by at least three rays, and in many cases by four or five rays. All rays intersected clearly and sharply in a definite point. This was only brought about by exercising the greatest care and precision in all phases of the work.

Points selected along the central part of the Horseshoe fall are shown in Plate III, and points along the crest of the American fall in Plate IV.

Elevations along the crest of the Horseshoe fall were obtained for the first time. These are shown in Plate I. All elevations were based on lines of levels run by the Dominion Water Power and Reclamation Service, connexion being made to the camera stations. In Plate I, the horizon line is 510 feet above sea-level. The 500-foot contour line was computed from several points along the crest and is shown plotted on the picture in relation to these points. The interval between the two lines, at any point on the crest, represents 10 feet at that point. By means of this scale, the height of any projecting ledge of rock, or thickness of any stratum of rock shown in the photograph can be measured. The height of the fall at any point can also be obtained.

Figure 2 shows the resulting map of the Horseshoe fall and Figure 3 that of the American fall.

The geographical co-ordinates of all points established on the crest lines of the Horseshoe and American falls are given in tables II and III. These co-ordinates will enable the crest lines to be replotted at any future date.

AERIAL PHOTOGRAPHIC METHODS

HORSESHOE FALL

As the ground survey was made with as much accuracy as possible and was plotted on the scale of 1:2,000, one hundredth of an inch on this map scale being equivalent to 1·66 feet on the ground, it was necessary, for purposes of comparison, to obtain, if possible, a commensurate degree of accuracy in the work of the graphical plotting of the data given on the aerial photograph. The aerial photos that were available were contact prints, on heavy paper, from films. It was realized, at the outset, that, for work of the nature required, these prints were hardly the proper medium to work from; however, the work was carried forward with as much care as possible and was checked in every possible way. The results that were obtained seem to justify the use of the prints.

The scale of the photos being about 1 inch to 203·33 feet and that of the plot being 1 inch to 166·66 feet, it will readily be seen that, owing to the data from the photo being enlarged during the process of plotting, it was very difficult to take the information from the photo without appreciable errors creeping into the work. Throughout the work beam compasses were used for laying off distances and for making locations.

The method, hereinafter described, used to plot the crest line of the Horseshoe fall from the aerial photo, was based on the projection of the photo plane on to the map plane, obliquely in the direction of tilt (principal distance line projection). By this method, what are believed to be the best obtainable results were secured in a very simple way. The results compare very favourably with those obtained by the ground survey.

Plate V represents the aerial photograph, taken by the Royal Canadian Air Force, of the Horseshoe fall. This photo, which was taken at the time the ground survey was made, was used throughout for the graphical constructions. The images of the three main control points *a*, *b*; and *c*, which were used throughout, are shown, as well as other control points which were used as check points during the progress of the constructions.

The points selected for delineating the crest line are shown encircled. The points *s* and *d* are the two used throughout in the graphical constructions to illustrate the methods of plotting. They correspond to the points *1* and *36* of the ground survey plot.

The portions of the crest line outlined in black are those portions where the actual crest is obscured by the water. Points along these sections were selected with the aid of a stereoscope, after careful study.

The focal length of the camera used is 11·68 inches. To carry out the work on the scale used, it was necessary to take into account appreciable differences of elevation and refer all to one common datum of reference. The elevation of the top of the water along the apex zone, at the time the photo was taken, was about 505 feet above sea-level. The elevation of the easterly portion was about 504 feet and that of the westerly portion about 502 feet (*See* Plate I).

Since it was found that the displacement on the photo of a point on the crest line lying farthest away from the plumb point, and having an elevation of 502 feet, was an amount under one hundredth of an inch on the map, when referred to a datum of 505 feet, the reference datum for all of the work was taken at 505 feet, and the crest line of the Horseshoe fall was considered to be horizontal, for all practical purposes, and to lie in this plane.

The elevations of the three main control points A, B, and C, are 505, 515, and 508 feet, respectively. The plane of reference then passes through station A. The difference in elevation between A and B, 10 feet, is an amount sufficiently large to make an appreciable difference in the graphical constructions on the map scale used.

The constructions throughout, to find the tilt, ground plumb point, height of air station, etc., are based on Finsterwalder's method and its application as described in the war office publication[1].

Figure 4 is a diagram illustrating the principles of Finsterwalder's method of plotting any point on a photo where the ground is horizontal, given the positions of three points on the ground, the images of which appear on the photo. The focal length of the camera and the position of the principal point must also be known.

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[1]**"Graphical Methods of Plotting from Air Photographs", 1925, War Office, London.**

Aerial photograph of Horseshoe fall. (Page 4.)

In the figures S is the air station, So the principal distance line (focal length), o the principal point, a, b, and c the photo images of the three ground control points the map positions of which are given by A, B, and C, respectively; abc is the photo triangle and ABC the map triangle; d is a point on the photo, the position of which it is required to plot on the map.

$Sbca$ is the photo pyramid and $SBCA$ the map pyramid. Draw the lines ad, bd, and cd and extend them to meet the photo triangle sides in w, t, and v, respectively; next draw the lines Sw, St, and Sv and extend them to meet the map triangle sides in W, T, and V, respectively. The points W, T, and V are the homologues on the map plane of the points w, t, and v on the photo plane; therefore, by drawing the lines AW, BT, and CV, their intersection D will give the homologue of the point d, or in other words the map position of the point d.

The flattened photo and map pyramids, shown in the upper left hand corner of Figure 4, illustrate the method of carrying out the graphical solution of the problem. For details of this method the reader is referred to the publication mentioned.

Figure 5 shows the graphical construction to find the ground plumb point, tilt, height of air station, principal line, etc., using the hill plane method. As stated before, the plane of reference is the plane through Station A, elevation 505 feet. The elevation of B is 515 feet and that of C 508 feet. The focal length of the camera is 11·68 inches.

The ground triangle ABC is the triangle on the plane of reference. The hill triangle is the triangle which contains the actual sites of the stations. As the point B is 10 feet higher than A, and point C is 3 feet higher than A, a point D on AB, to have the same height as C must fall at a point $\frac{3}{10}$ of the length of AB measured from A. CD will then be the direction of a horizontal line on the hill plane. Lines parallel to CD are called "strike lines" and lines perpendicular to it are called "dip lines". The main strike line is the line shown drawn through A and parallel to CD.

The hill triangle side corresponding to AB is found by erecting BZ perpendicular to AB at B and with a length equal to the difference in height between A and B, in this case 10 feet. With A as centre and AZ as radius describe an arc; where this arc meets the dip line through B is the point of the hill triangle corresponding to B. It is obvious that this point of the hill triangle coincides with the point B on the plot, owing to the small difference in height between A and B. Similarly the point of the hill triangle corresponding to C coincides with C on the plot. The hill triangle, therefore, coincides on the plot with the ground triangle ABC. It would require a difference in height between the stations of at least 50 feet before any appreciable difference would be made in the plot between the lengths of the sides of the hill and ground triangles.

ABC is taken as the hill triangle. The flattened hill pyramid $SC''BCAC'$ and flattened photo pyramid $Sc''bcac'$ are shown adjusted in their correct relative positions.

The lines SN, SL', and SM' are the perpendiculars from S to the sides AB, BC'', and AC', respectively. BL equal to BL' is set off on BC, and AM equal to AM' is set off on AC. Perpendiculars to the sides of the hill triangle at L, M, and N, meet in the point P_1, which is the foot of the perpendicular from S on the hill plane.

P_1T drawn perpendicular to AT, the main strike line, is the main dip line.

P_1R perpendicular to P_1N is drawn, and with N as centre and NS as radius an arc is described intersecting the perpendicular in R. The distance P_1R is the length of the perpendicular from the air position S on the hill plane.

P_1S' perpendicular to the main dip line P_1T and equal to P_1R is drawn. At S' the angle $P_1S'Q$ equal to I, the inclination of the hill plane to the horizontal, is laid off. The point Q on the main dip line is the "hill plane plumb point".

TP' perpendicular to $S'Q$ is drawn, and on TP_1, the main dip line, TP is set off equal to TP'. P is the "ground plumb point". Owing to the small angle of inclination of the hill plane to the horizontal, P coincides on the plot with Q.

$S'P'$ is the height of the air position above the reference plane through A; it scales 2,375 feet.

The plate plumb point p is found as follows: Join AP and produce the line to meet BC in H. On BC'' lay off BH' equal to BH. Join SH'; this ray cuts bc'' in h'. Lay off bh on bc equal to bh'. Join ah. The line ah contains the required point. Again: join BP and produce the line to meet AC in J. On AC' lay off AJ' equal to AJ. Join SJ'; this ray cuts ac' in j'. Lay off aj on ac equal to aj'. Join bj. The line bj also contains the required point. Again: join CP and produce the line to meet AB in K. Join SK; this ray cuts ab in k. Join ck. The line ck also contains the required point. The intersection p of the three lines ah, bj, and ck, is the plate plumb point.

The homologue O_1, on the hill plane, of the principal point o, is found by a process the reverse of the above. O_1 on the hill plane will coincide on the plot with O on the ground plane owing to the small angle of inclination of the hill plane to the horizontal.

To find the tilt. On ae lay off op' equal to op. At o erect a perpendicular oS'' to ao. With centre a and radius aS describe an arc cutting the perpendicular in S''. The distance oS'' should be equal to the focal length. Join $S''p'$. The angle $oS''p'$ is the angle of tilt θ. In this case it equals $1°10'$. The principal line will pass through op, and the trace of the principal plane through OP.

In the foregoing construction, the triangle ABC was taken as the hill triangle. If ABC were taken as the ground triangle on the plane of reference, correction for image displacement, due to difference in height of stations, would have to be made. The results would be the same as in the hill plane construction.

Figure 6 shows the construction when ABC is taken as the ground triangle on the plane of reference through A, corrections for image displacement having been duly made.

The flattened ground pyramid $SC''BCAC'$ and flattened photo pyramid $Sc''bcac'$ are shown adjusted in their correct relative positions. As in the hill plane method, the lines SN, SL', and SM' are the perpendiculars from S on the sides AB, BC'', and AC' respectively. The points N, L', and M' are transferred as before to the ground triangle sides. Perpendiculars to the respective sides of the ground triangle at N, L, and M meet in a common point P, which, in this case, is the "ground plumb point".

PS' perpendicular to PA is drawn, and with centre A and radius AS an arc is described meeting the perpendicular in S'. PS' is then the height of the air position above the plane of reference. It scales, in this case, 2,372 feet.

The plate plumb point p, the homologue O, on the reference plane, of the principal point o, and the angle of tilt θ, are found in the same manner as described for the hill plane construction. The angle of tilt in this case equals 1°08′.

Any number of points on the photo can be plotted by Finsterwalder's method, or by the four point method, but the process in either case is a very tedious and slow one. Equally good results, in this particular case, can be obtained in a much simpler way. This is by projecting the photo plane into the map plane either vertically (plumb line projection) or obliquely in the direction of the tilt (principal distance line projection).

PROJECTION VERTICALLY ON TO MAP PLANE. PLUMB LINE PROJECTION

Figure 7 is a diagram showing the conditions existing when the photo plane is projected vertically on to the map plane.

S is the air station, abc the photo triangle, ABC the map triangle, p the plate plumb point, and P the map plumb point; d is a point on the photo, its map position D being shown as plotted by Finsterwalder's method.

As the projection of the photo plane is vertical, it is obvious that p, when thus projected on to the map plane, will coincide with P.

a_1, b_1, c_1, the projections of a, b, and c, will lie on the radials from P to A, B, and C respectively. $a_1b_1c_1$ is the photo triangle projected on to the map plane. d_1 the projection of d, and r_1 the projection of r, will lie on the radial PDR. v_1, w_1, and t_1, the projections of v, w, and t will lie on the radials PV, PW, and PT respectively. It will be seen that all photo points, projected vertically on to the map plane, will lie on the radials drawn from the map plumb point P.

In the projected photo triangle the map position of d_1 can be plotted as follows: Draw the radial Pd_1. Join a_1d_1 and produce it to meet b_1c_1 in w_1. Draw the radial Pw_1 and produce it to meet the side BC in W. Join AW. The required point lies along this line. Again, join b_1d_1 and produce the line to meet the side a_1c_1 in t_1. Draw the radial Pt_1 and produce it to meet the side AC in T. Join BT. The required point lies along this line. Similarly join c_1d_1 and produce it to meet the side a_1b_1 in v_1. Draw the radial Pv_1 and produce it to meet the side AB in V. Join CV. The required point lies along this line. The intersection of the three lines AW, BT, and CV is the required point D and should fall on the radial Pd_1. The final fixing of D will be seen to be identical with that by Finsterwalder's method.

The proof of the relations existing in this method is shown in the graphical solution of the problem Figure 8.

The flattened photo pyramid $Sc''bcac'$ and the flattened map pyramid $SC''BCAC'$ correspond to those shown on Figure 6.

P is the map plumb point and p the plate plumb point, s and d, two points on the crest line of the Horseshoe fall (*See* Plate V), are the ones used throughout in the constructions. Points s and d correspond to points *1* and *36* of the ground survey respectively. The map positions of these two points, S and D, are shown plotted by Finsterwalder's method in order to show the relations between the two methods.

The map plumb point is found as in Figure 6. SL' is the perpendicular from S on BC''; SN is the perpendicular from S on AB, and SM' is the perpendicular from S on AC'. Again, NP is perpendicular to AB; LP is perpendicular to BC; and MP is perpendicular to AC.

Join PA, PB, and PC. The projections vertically of a, b, and c will lie on these radials respectively.

To obtain the projection vertically of a. From a draw a perpendicular to AB (or parallel to SN or NP) meeting AB in a_1'. Produce aa_1' to meet PA in a_1. a_1 is the projection vertically of a on the map plane. This position is checked as follows: from a draw a perpendicular to AC' (or parallel to SM') meeting AC' in a_1''. On AC lay off the distance Aa''_1. At a_1'', on AC, erect a perpendicular to AC (or a line parallel to MP) cutting PA in a_1. If the construction is correct the two positions of a_1 will coincide.

b_1 and c_1, the projections of b and c, are found in a similar manner. Join a_1, b_1, and c_1. $a_1b_1c_1$ is then the photo triangle abc projected vertically on to the map plane and is shown in its correct relation to A, B, C, and P.

Consider the point w where the line joining a and d meets the photo triangle side bc. As in Finsterwalder's method, bw' is set off on bc'' equal to bw. The line Sw' is drawn and produced to meet BC'' in W'. BW on BC is laid off equal to BW' and the line AW is drawn. The map position of d lies along this line.

Now from w' on bc'' draw a perpendicular to BC'' meeting BC'' in w_1'. On BC lay off the distance Bw_1'. At w_1' on BC draw a perpendicular to BC meeting the projected photo triangle side b_1c_1 in w_1. w_1 is the point w projected vertically on to the map plane. Join a_1w_1, this line is then the line aw projected vertically on to the map plane and contains d_1, the projection of d. Draw the radial Pw_1, it will meet the side BC in W as shown.

Similarly, the points t_1 and v_1 are the projections vertically of t and v respectively, and the radials Pt_1 and Pv_1 meet the map triangle sides in T and V respectively.

It will be seen then, that with the photo triangle projected vertically, the points W, T, and V can be obtained in a very simple way, viz.: join a_1d_1 and produce to meet b_1c_1 in w_1. Draw the radial Pw_1 and produce it to meet BC in W. Join AW. The map position of d lies on AW as before. The points T and V are obtained in a like manner and also the lines BT and CV. The intersection of these lines gives D; a check on the position is had by drawing the radial Pd_1 which should pass through D.

Point s lying outside the photo triangle is shown plotted in S on the map plane. A little careful study of the construction will bring out the relations clearly.

PROJECTION OBLIQUELY OF PHOTO PLANE ON TO MAP PLANE

Figure 9. This diagram illustrates the conditions existing when the photo plane is projected obliquely, in the direction of tilt (principal distance line projection) on to the map plane.

abc is the photo triangle; *ABC* the map triangle; *S* the air station; *o* the principal point and *O* its homologue on the map plane; *So* the principal distance line (focal length of camera).

d a point on the photo is shown as plotted by Finsterwalder's method.

As the photo plane is projected obliquely on to the map plane in the direction of tilt, or in other words, parallel to the principal distance line *So*; the principal point *o*, when thus projected, will coincide with its homologue *O* on the map plane. a_2, b_2, c_2 the projections obliquely of *a*, *b*, and *c* will lie on the radials from *O*, to *A*, *B*, and *C* respectively. $a_2b_2c_2$ is the photo triangle projected obliquely on to the map plane.

d_2 the projection of *d*, and u_2 the projection of *u*, will lie on the radial *ODU*. v_2, w_2, and t_2 the projections of *v*, *w*, and *t*, will lie on the radials *OV*, *OW*, and *OT* respectively.

Without proceeding farther, it is apparent that the relations existing in this case are analogous to those existing in the plumb line projection method. In this case, however, all photo points will lie on the radials from the principal point.

The proof of the relations existing in this method is shown in the graphical solution Figure 10.

The flattened photo pyramid *Sc''bcac'* and the flattened map pyramid *SC''BCAC'* correspond to those shown in Figures 6 and 8.

o is the principal point and *O* its homologue on the map plane. *d* and *s* are the same points shown in Figure 8 and are again shown plotted by Finsterwalder's method for purposes of comparing the relation between the methods.

As has been stated, *o*, when projected obliquely in the manner described, will coincide with *O* on the map plane.

The photo triangle is projected as follows: from *o* draw the perpendiculars *on*, *ol*, and *om* to the three sides of the photo triangle *ba*, *bc*, and *ac* respectively. On *bc''* lay off the distance *bl'* equal to *bl*. Join *Sl'*, if the construction is correct *Sl'* will be perpendicular to *bc''*. Produce *Sl'* to meet *BC''* in l_2'. On *BC* lay off the distance Bl_2'. Join Ol_2'.

Again, on *ac'* lay off *am'* equal to *am*. Join *Sm'*. *Sm'* will be perpendicular to *ac'*. Produce *Sm'* to meet *AC'* in m_2'.

On *AC* lay off the distance Am_2'. Join Om_2'.

Again, join *Sn* and produce *Sn* to meet *AB* in n'_2. *Sn* will be perpendicular to *ab*, and it is obvious that *Sn* produced will pass through *o*. Join On'_2.

The lines Ol'_2, Om_2', and On'_2 will be the perpendiculars from *O* to the obliquely projected photo triangle sides b_2c_2, a_2c_2, and a_2b_2 respectively.

Join *OA*, *OB*, and *OC*.

The lines $Snon'_2$, $Sl'l_2'$ and $Sm'm'_2$ will be lines parallel to the direction of the principal distance line on the three pyramid faces *Sab*, *Sbc''*, and *Sac'* respectively.

From *a* draw aa_2' parallel to $Snon_2'$, meeting *AB* in a_2'. From a_2' draw the line $a_2'a_2$ parallel to On_2' and cutting *OA* in a_2.

a_2 will be the projection of *a*. As a check on this position, from *a* draw aa_2'' parallel to $Sm'm_2'$ meeting *AC'* in a_2''. On *AC* lay off the distance Aa_2''. At a_2'' on *AC* draw the line $a_2''a_2$ parallel to Om_2'. If the construction is correct, this line should cut *OA* in a_2.

Similarly, b_2 and c_2 are found. They will lie on the radials OB and OC respectively. $a_2b_2c_2$ is then the obliquely projected photo triangle in its correct relation to A, B, C, and O.

As stated, On_2' will be the perpendicular from O on the projected triangle side a_2b_2. Consequently, n_2, where On_2' cuts a_2b_2, will be the obliquely projected point n.

Similarly, m_2 and l_2 are the obliquely projected points m and l.

Take the point w, where the line ad produced meets the side bc. As in Finsterwalder's method bw' is set off on bc'' equal to bw. The line Sw' is drawn and produced to meet BC'' in W'. On BC, BW is laid off equal to BW'. The line AW is drawn. The map position of d lies along this line.

Now from w' on bc'', draw the line $w'w'_2$ parallel to $Sl'l_2'$ and meeting BC'' in w_2'. On BC lay off the distance Bw_2'. At w_2', on BC, draw a line parallel to Ol_2' meeting the projected triangle side b_2c_2 in w_2. w_2 is the obliquely projected point w.

Join a_2w_2, this line is the obliquely projected line aw on to the map plane and contains d_2 the projection of d.

Draw the radial Ow_2; it will meet the side BC in W as shown.

Similarly the points t_2 and v_2 are the projections obliquely of t and v respectively, and the radials Ot_2 and Ov_2 meet the map triangle sides in T and V respectively.

Again it will be seen that with the photo triangle thus projected obliquely the points W, T, and V can be obtained in the same simple way, viz.—join a_2d_2 and produce to meet b_2c_2 in w_2. Draw the radial Ow_2 and produce it to meet BC in W. Join AW. The map position D lies on AW. The points T and V are obtained in a manner similar.

The method is analogous to the method of projecting vertically into the map plane. The same results are obtained in a shorter and simpler way than by Finsterwalder's method or by the Four Point Method.

Figure 11 shows the relation, on the map plane, between the vertically projected and obliquely projected photo triangles. It will be seen that the vertical and oblique projections of any point lie along a line which is parallel to the trace of the principal plane.

In both the vertical and oblique methods of projection as shown, correction for image displacement was made. It was found that, for the scale used for the constructions, small tilts made no appreciable difference in the amount of image displacement, nor in the position of the corrected image point, when calculated and plotted from the principal point instead of the plumb point; therefore, for ordinary map scale work, and for the usual small tilts in vertical aerial photos, the image displacement, if it is necessary to correct for it, can be calculated and plotted from the principal point of the photo.

The method naturally has its limitations for practical use, but for problems such as plotting many points in a vertical aerial photograph, particularly when the principal point is over water, where as much accuracy as possible is required, it gives results in a simple and quick manner.

The simplicity of the method lies in the fact that the photo plane is projected obliquely on to the map plane by an operation which is similar to the tracing paper solution of the three point problem, thus: given three, four, or five control points on the ground, and a vertical aerial photo, in the general sense of the term, which contains the images of the control points, also the principal point on the photo.

On a piece of tracing linen, or tracing paper of good quality, fixed over the photo, mark the principal point and the image points of the ground control points (correction for displacement being made if conditions require it) and draw the radials from the principal point to each image point.

Lay this tracing over the map plot and move it about until the radials to the image points cut precisely through the map positions of the corresponding ground control points. When this is accomplished, the photo plane is projected obliquely in the direction of tilt on to the map plane.

Figure 12 shows the crest line of the Horseshoe fall plotted by the method of projecting vertically and obliquely on to the map plane.

Four control points were used for this work. These are the three points *A*, *B*, *C*, used in the graphical constructions throughout, and an additional one "TERRAPIN".

Construction lines used for plotting the points *s* and *d* are shown. These lines with their lettering are the same as shown in Figures 8 and 10.

As stated elsewhere in this paper, the points *S* and *D* correspond to points *1* and *36* of the ground survey.

Figure 13. This shows the oblique projection method used for plotting a lake shore and islands on a map scale of 1 inch to $\frac{1}{2}$ mile.

The scale of the aerial photo is about 1 inch to 1,100 feet.

A, *B*, *C*, *D*, and *E* are the map positions of the control points. Their respective images on the photograph are shown in *a*, *b*, *c*, *d*, and *e*.

o is the principal point on the photo.

The dotted shorelines are the photographed shorelines and the solid shorelines are the same plotted to map scale.

To project the photo obliquely on to the map plane, on a piece of good transparent tracing paper, or linen, mark the map positions of the control points *A*, *B*, *C*, *D*, and *E*.

On the photograph carefully draw the radials *Oa*, *Ob*, *Oc*, *Od*, and *Oe*, in a fine white line.

The tracing paper with the control points marked thereon is then laid over the photo and carefully oriented until the points *A*, *B*, *C*, *D*, and *E* fall precisely on their respective radials *Oa*, *Ob*, *Oc*, *Od*, and *Oe*. When this is achieved the photo is projected obliquely on to the map plane.

Join *AB*, *BC*, *CD*, *DE*, and *EA*, also *ab*, *bc*, *cd*, *de*, and *ea*.

To plot the point *m* shown on the photo.

Draw the radial *Om*, the map position of the point lies along this line.

Join *am*, cutting the side *de* in *w*. Join *Ow*, this radial cuts the side *DE* in *W*.

Join *AW* and produce it to meet the radial *Om* in *M*. M is the map position of *m*.

This position is checked by joining *cm*, which cuts *de* in *v*. Draw the radial *Ov*, which cuts *DE* in *V*. Join *CV* and produce to meet the radial *Om*. If the construction is correct, this line will cut the radial *Om* in *M*.

The radials are shown to other points plotted in this way. The shoreline between the plotted points (shown encircled) was put in by radials and parallels from the nearest point.

TABLE I

Geographical Co-ordinates of Triangulation Stations

Station	Latitude Longitude	Azimuth	Distance metres Log distance	Back azimuth	To station	Latitude Longitude
	° ′ ″	° ′ ″		° ′ ″		° ′ ″
Terrapin	43-04-48.90 79-04-28.06	231-33-19	496.60 2.6960067	51-33-31	Canal	43-04-38.89 79-04-45.25
		288-42-19	420.33 2.6235904	108-42-31	Transformer	43-04-53.27 79-04-45.66
		256-14-29	585.40 2.7674536	76-14-46	A	43-04-44.39 79-04-53.20
		281-37-07	436.70 2.6401832	101-37-20	B	43-04-51.75 79-04-46.97
		241-16-45	680.71 2.8329621	61-17-03	C	43-04-38.30 79-04-54.45
		283-05-28	324.08 2.5106534	103-05-38	M(1927)	43-04-51.28 79-04-42.02
		233-00-46	823.12 2.9154653	53-01-06	Loretto	43-04-32.85 79-04-57.13
		35-50-48	753.53 2.8770984	215-50-35	Boundary Mon. 19	43-05-08.69 79-04-08.55
Canal	43-04-38.89 79-04-45.25	358-48-33	443.63 2.6470209	178-48-33	Transformer	43-04-53.27 79-04-45.66
		313-20-13	247.09 2.3928504	133-20-18	A	43-04-44.39 79-04-53.20
		354-24-37	398.62 2.6005544	174-24-38	B	43-04-51.75 79-04-46.97
		264-58-32	208.84 2.3198137	84-58-38	C	43-04-38.30 79-04-54.45
Transformer	43-04-53.27 79-04-45.66	203-17-39	502.77 2.7013694	23-17-45	C	43-04-38.30 79-04-54-45
		60-26-51	964.96 2.9845088	240-26-26	Boundary Mon. 19	43-05-08.69 79-04-08.55
Boundary Mon. 19	43-05-08.69 79-04-08.55	211-15-48	342.36 2.5344888	31-15-53	Bluff (1906)	43-04-59.21 79-04-16.41
N (1890)	43-05-19.17 79-04-27.99					
M (1890)	43-04-50.03 79-04-24.57					
T.P.6 (1842)	43-04-45.67 79-04-21.66					
Boundary Mon. 21	43-04-45.40 79-04-20.39					
Cliff (1906)	43-05-09.69 79-04-32.65					
Boundary Mon. 20	43-04-44.20 79-04-42.81					

TABLE II

Geographical Co-ordinates of Points Along Crest Line of Horseshoe Fall

Point	Latitude		Longitude		Point	Latitude		Longitude	
1	43°-04′-	39″.47	79°-04′-	29″.96	55	43°-04′-	40.04	79°-04′-	28.82
2	" "	39.30	" "	30.06	56	" "	40.21	" "	28.75
3	" "	39.06	" "	30.50	57	" "	40.47	" "	28.82
4	" "	39.00	" "	30.59	58	" "	40.63	" "	28.79
5	" "	38.91	" "	31.13	59	" "	41.02	" "	29.01
6	" "	38.80	" "	31.33	60	" "	41.24	" "	29.10
7	" "	38.64	" "	31.85	61	" "	41.49	" "	29.16
8	" "	38.62	" "	32.11	62	" "	41.65	" "	29.14
9	" "	38.63	" "	32.17	63	" "	41.84	" "	29.25
10	" "	38.55	" "	32.55	64	" "	42.01	" "	29.48
11	" "	38.53	" "	32.85	65	" "	42.18	" "	29.62
12	" "	38.50	" "	33.07	66	" "	42.37	" "	29.68
13	" "	38.57	" "	33.69	67	" "	42.52	" "	29.76
14	" "	38.64	" "	34.19	68	" "	42.68	" "	29.92
15	" "	38.84	" "	34.70	69	" "	42.78	" "	30.00
16	" "	38.87	" "	35.13	70	" "	42.87	" "	30.01
17	" "	38.97	" "	35.41	71	" "	42.99	" "	30.12
18	" "	39.32	" "	36.24	72	" "	43.09	" "	30.18
19	" "	39.48	" "	36.50	73	" "	43.19	" "	30.18
20	" "	39.76	" "	36.90	74	" "	43.35	" "	30.27
21	" "	39.87	" "	37.09	75	" "	43.59	" "	30.45
22	" "	40.13	" "	37.39	76	" "	43.83	" "	30.50
23	" "	40.20	" "	37.60	77	" "	44.02	" "	30.58
24	" "	40.35	" "	37.94	78	" "	44.28	" "	30.69
25	" "	40.45	" "	38.08	79	" "	44.46	" "	30.90
26	" "	40.74	" "	38.80	80	" "	44.72	" "	30.89
27	" "	40.85	" "	38.93	81	" "	44.76	" "	30.90
28	" "	41.10	" "	39.40	82	" "	45.10	" "	30.77
29	" "	41.23	" "	39.57	83	" "	45.27	" "	30.72
30	" "	41.48	" "	39.84	84	" "	45.46	" "	30.67
31	" "	41.66	" "	40.03	85	" "	45.71	" "	30.48
32	" "	41.77	" "	40.13	86	" "	45.76	" "	30.38
33	" "	41.80	" "	40.23	87	" "	45.86	" "	30.21
34	" "	41.88	" "	40.24	88	" "	46.14	" "	30.32
35	" "	41.91	" "	40.40	89	" "	46.29	" "	30.31
36	" "	42.03	" "	40.40	90	" "	46.42	" "	30.30
37	" "	42.14	" "	40.69	91	" "	46.59	" "	30.28
38	" "	42.33	" "	40.68	92	" "	46.84	" "	30.26
39	" "	42.46	" "	40.75	93	" "	46.92	" "	30.33
40	" "	42.52	" "	40.87	94	" "	47.02	" "	30.33
41	" "	42.78	" "	40.87	95	" "	47.07	" "	30.29
42	" "	42.85	" "	41.00	96	" "	47.21	" "	30.25
43	" "	43.00	" "	41.15	97	" "	47.30	" "	30.16
44	" "	43.61	" "	41.34	98	" "	47.48	" "	30.00
45	" "	43.97	" "	41.47	99	" "	47.60	" "	30.10
46	" "	44.26	" "	41.40	100	" "	47.70	" "	30.12
47	" "	44.34	" "	41.33	101	" "	47.76	" "	30.02
48	" "	44.49	" "	41.30	102	" "	47.77	" "	29.93
49	" "	44.64	" "	41.47	103	" "	47.84	" "	29.74
50	" "	44.84	" "	41.58	104	" "	47.91	" "	29.68
51	" "	44.99	" "	41.64	105	" "	48.08	" "	29.80
52	" "	39.57	" "	29.75	106	" "	48.20	" "	29.80
53	" "	39.69	" "	29.33	107	" "	48.24	" "	29.73
54	" "	39.88	" "	29.02	108	" "	48.31	" "	29.68

TABLE II—(*Continued*)

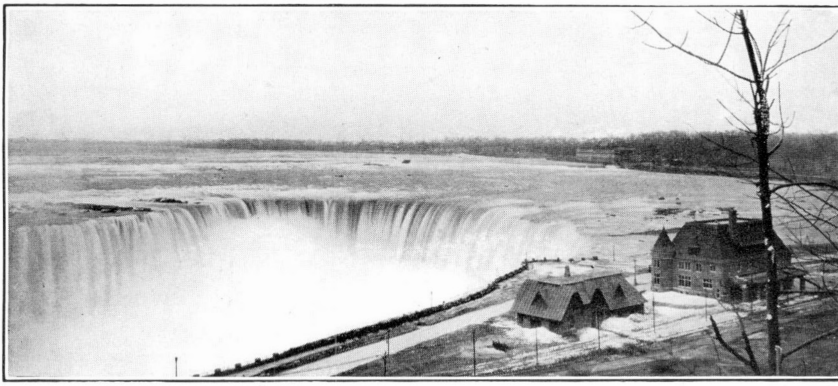
Geographical Co-ordinates of Points Along Crest Line of Horseshoe Fall (Continued)

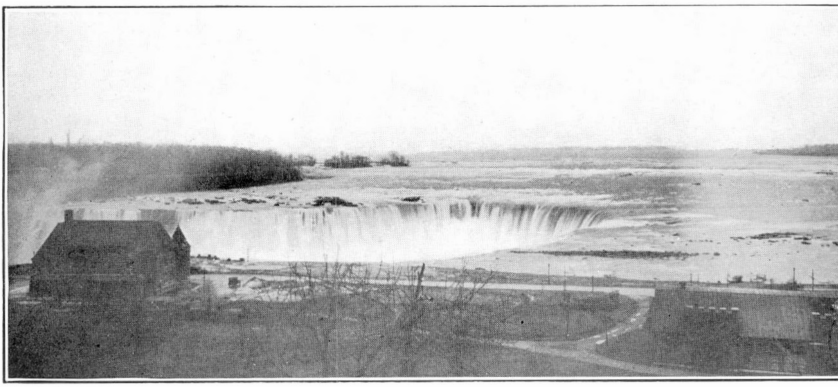
Point	Latitude		Longitude		Point	Latitude		Longitude	
109	43°-04′-	48.36	79°-04′-	29.60	121	43°-04′-	49.10	79°-04′-	28.04
110	" "	48.37	" "	29.53	122	" "	49.17	" "	27.78
111	" "	48.48	" "	29.49	123	" "	49.22	" "	27.60
112	" "	48.52	" "	29.35	124	" "	49.30	" "	27.43
113	" "	48.63	" "	29.33	125	" "	49.34	" "	27.22
114	" "	48.70	" "	29.26	126	" "	49.41	" "	26.96
115	" "	48.80	" "	29.08	127	" "	49.45	" "	26.74
116	" "	48.89	" "	28.98	128	" "	49.50	" "	26.66
117	" "	48.93	" "	28.84	129	" "	49.56	" "	26.53
118	" "	48.94	" "	28.68	130	" "	49.63	" "	26.30
119	" "	48.99	" "	28.55	131	" "	49.67	" "	26.00
120	" "	49.02	" "	28.30					

TABLE III

Geographical Co-ordinates of Points Along Crest Line of American Fall

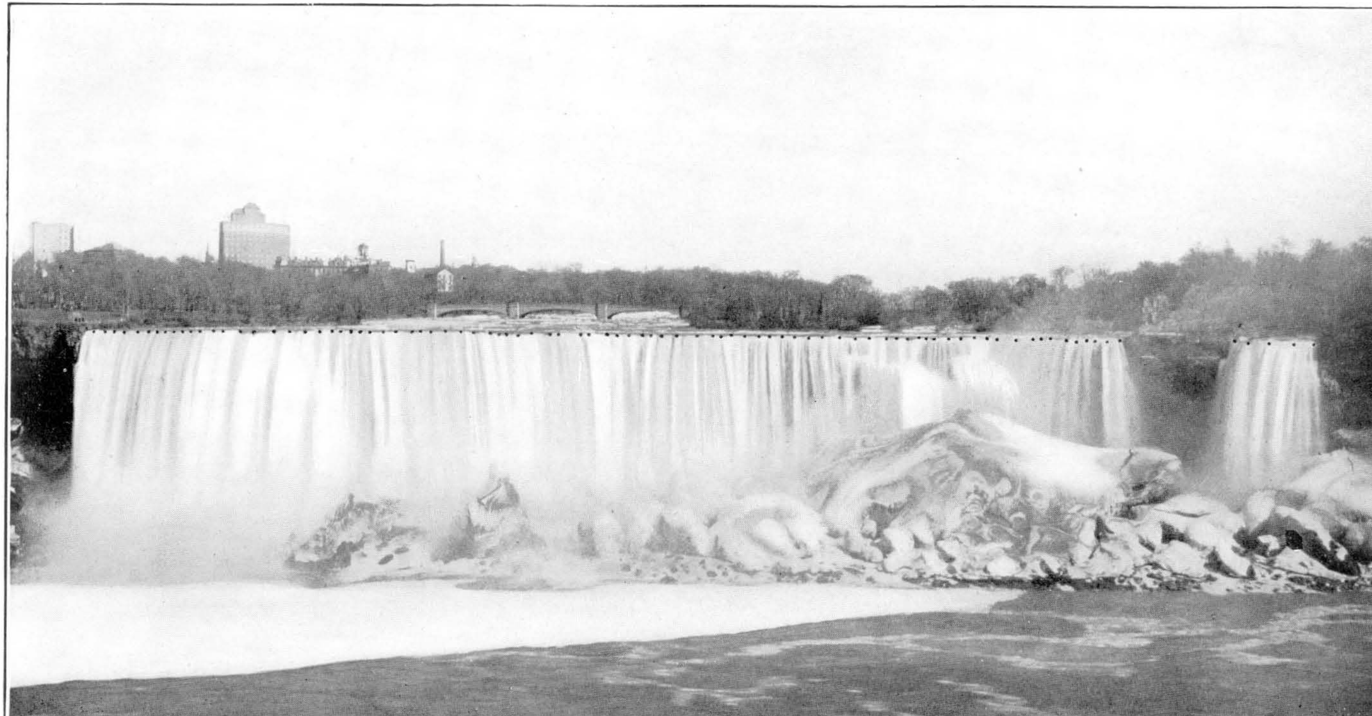
Point	Latitude	Longitude	Point	Latitude	Longitude
1	43°-05′- 00″.31	79°-04′- 16″.06	47	43°-05′- 04.56	79°-04′- 11.38
2	" " 00.47	" " 15.96	48	" " 04.64	" " 11.31
3	" " 00.60	" " 15.86	49	" " 04.75	" " 11.25
4	" " 00.71	" " 15.66	50	" " 04.83	" " 11.22
5	" " 00.75	" " 15.50	51	" " 04.98	" " 11.18
6	" " 01.46	" " 14.84	52	" " 05.02	" " 11.13
7	" " 01.57	" " 14.80	53	" " 05.22	" " 11.14
8	" " 01.62	" " 14.70	54	" " 05.29	" " 11.09
9	" " 01.67	" " 14.62	55	" " 05.45	" " 11.03
10	" " 01.70	" " 14.49	56	" " 05.58	" " 11.07
11	" " 01.73	" " 14.36	57	" " 05.68	" " 11.12
12	" " 01.77	" " 14.19	58	" " 05.74	" " 11.07
13	" " 01.78	" " 14.09	59	" " 05.82	" " 11.06
14	" " 01.82	" " 18.98	60	" " 05.93	" " 11.05
15	" " 01.84	" " 13.90	61	" " 06.06	" " 11.08
16	" " 01.88	" " 13.65	62	" " 06.21	" " 11.06
17	" " 01.95	" " 13.30	63	" " 06.32	" " 11.00
18	" " 02.03	" " 13.26	64	" " 06.40	" " 10.98
19	" " 02.17	" " 13.34	65	" " 06.47	" " 10.93
20	" " 02.30	" " 13.40	66	" " 06.61	" " 10.99
21	" " 02.45	" " 13.47	67	" " 06.70	" " 11.01
22	" " 02.53	" " 13.34	68	" " 06.83	" " 10.94
23	" " 02.55	" " 13.23	69	" " 06.90	" " 10.77
24	" " 02.57	" " 13.18	70	" " 06.95	" " 10.64
25	" " 02.61	" " 13.13	71	" " 07.04	" " 10.57
26	" " 02.63	" " 13.00	72	" " 07.17	" " 10.45
27	" " 02.75	" " 13.02	73	" " 07.30	" " 10.47
28	" " 02.80	" " 12.93	74	" " 07.43	" " 10.41
29	" " 02.94	" " 12.89	75	" " 07.54	" " 10.35
30	" " 03.03	" " 12.76	76	" " 07.73	" " 10.45
31	" " 03.14	" " 12.68	77	" " 07.82	" " 10.31
32	" " 03.27	" " 12.60	78	" " 07.96	" " 10.44
33	" " 03.37	" " 12.49	79	" " 08.02	" " 10.40
34	" " 03.46	" " 12.40	80	" " 08.13	" " 10.39
35	" " 03.54	" " 12.33	81	" " 08.20	" " 10.36
36	" " 03.64	" " 12.26	82	" " 08.28	" " 10.30
37	" " 03.71	" " 12.16	83	" " 08.33	" " 10.17
38	" " 03.80	" " 12.02	84	" " 08.41	" " 10.11
39	" " 03.90	" " 11.95	85	" " 08.47	" " 10.00
40	" " 03.97	" " 11.92	86	" " 08.52	" " 09.90
41	" " 04.03	" " 11.91	87	" " 08.60	" " 09.81
42	" " 04.18	" " 11.90	88	" " 08.71	" " 09.70
43	" " 04.29	" " 11.79	89	" " 08.79	" " 09.61
44	" " 04.34	" " 11.74	90	" " 08.86	" " 09.54
45	" " 04.44	" " 11.62	91	" " 08.95	" " 09.51
46	" " 04.50	" " 11.45	92	" " 09.00	" " 09.40





Simultaneous photographs of Horseshoe fall taken from three stations: upper view, from station B; middle, from station A; lower, from station C. (Page 2.)

Showing intersected points along upper curve of Horseshoe fall. (Page 3.)



Showing intersected points along crest of American fall. (Page 3.)