GEOLOGICAL SURVEY OF CANADA

DEPARTMENT OF ENERGY, MINES AND RESOURCES, OTTAWA

PAPER 75-21

A COMPUTER PROGRAM FOR POLYNOMIAL TREND-SURFACE ANALYSIS

F.P. AGTERBERG
C.F. CHUNG

1975

Energy, Mines and Resources Canada
Énergie, Mines et Ressources Canada

GEOLOGICAL SURVEY
PAPER 75-21

A COMPUTER PROGRAM FOR POLYNOMIAL TREND-SURFACE ANALYSIS

F.P. AGTERBERG
C.F. CHUNG

Available by mail from *Information Canada*, Ottawa, K1A 0S9

from the Geological Survey of Canada
601 Booth St., Ottawa, K1A 0E8

and

Information Canada bookshops in

HALIFAX — 1683 Barrington Street
MONTREAL — 640 St. Catherine Street W.
OTTAWA — 171 Slater Street
TORONTO — 221 Yonge Street
WINNIPEG — 393 Portage Avenue
VANCOUVER — 800 Granville Street

or through your bookseller

A deposit copy of this publication is also available
for reference in public libraries across Canada

Price - Canada: $3.00
Other Countries: $3.60

Catalogue No. M44-75-21

Price subject to change without notice

Information Canada
Ottawa
1975

Contents

Illustration

Appendices

A COMPUTER PROGRAM FOR POLYNOMIAL TREND-SURFACE ANALYSIS

ABSTRACT

This program performs most calculations commonly expected of a polynomial trend-surface fitting routine and contains two more or less independent built-in numerical precision tests. In addition, it can print half-confidence intervals for the fitted surfaces and evaluate individual residuals for statistical significance.

Throughout the text it is stressed that this method applies exactly only if the observations satisfy the underlying mathematical model. This condition is rarely realized in practice. The user's judgment will be required to evaluate for his own data the validity of approximations resulting from a divergence between mathematical model and geological reality.

RÉSUMÉ

Ce programme sert à réaliser les calculs normalement attendus d'un ajustement de routine algébrique des surfaces représentatives et comporte deux tests de précision numérique, intégrés et plus ou moins indépendants. Il peut également imprimer des demi-intervalles de confiance pour les surfaces ajustées et évaluer l'importance statistique des résidus individuels.

Le texte précise que cette méthode ne peut s'appliquer avec exactitude que si les observations satisfont aux exigences du modèle mathématique sous-jacent. Cette condition est rarement réalisée dans la pratique. L'utilisateur devra faire preuve de jugement pour évaluer, pour ses propres données, la validité des approximations résultant d'une divergence entre le modèle mathématique et la réalité géologique.

Introduction

The technique of trend-surface analysis is well-known to earth scientists. The underlying theory is relatively simple and clearly explained in publications such as Krumbein and Graybill (1965) and Harbaugh and Merriam (1968) to which the reader is referred for more detailed considerations. In most practical applications, large matrices must be manipulated and a digital computer is required for performing the calculations.

Many computer programs for trend-surface analysis are in existence. Published programs, which are documented, include Harbaugh (1963), Whitten (1963), Good (1964), O'Leary *et al.* (1966), Miesch and Connor (1968), and Whitten (1974).

Two sets or systems of computer programs written for geologists and including surface-fitting algorithms have recently been published in book-form by Koch *et al.* (1972) and Davis (1973).

The student or research scientist who wishes to fit trend surfaces has many computer programs to choose from. In fact, he may prefer to prepare his own program, combining selected features from existing programs with subroutines already available on the computer to which he has access.

Our own procedure for surface-fitting in practice has been to maintain one or more FORTRAN programs available for immediate usage as required. This is because the technique is used relatively frequently (on a monthly basis) by ourselves, colleagues within the Geological Survey, and students at the University of Ottawa. The programs have to be periodically revised because of modifications in computer hardware or the replacement of parts of a program by subroutines believed to be more efficient.

During the past year, the program documented in this report has been operational on the Department of Energy, Mines and Resources CYBER 74 machine and the IBM 360/65 of the University of Ottawa. Earlier versions have temporarily been used on IBM 360/85, UNIVAC 1108 and CDC 6400 machines.

In order to avoid the introduction of errors into the program, new versions are run on one or more standard data sets. These consist either of one of the input-output examples appended to published programs or a private data set.

One of our own data sets used for such testing is reproduced at the end of this paper together with the output obtained for it. The input consists of 174 enstatite determinations on orthopyroxene crystals from samples collected and studied by C.H. Smith and I.D. MacGregor for the Mount Albert Peridotite Intrusion, Gaspé, Quebec. (cf. MacGregor and Smith, 1963.) The output is a sequence of low-degree polynomial trend surfaces, their confidence intervals, analysis of variance tables, etc. Much of this material has been discussed at greater length elsewhere (Agterberg, 1964 and 1974).

Original manuscript submitted: January 10, 1975
Final version approved for publication: February 7, 1975

Figure 1 was constructed from output obtained in 1963 by means of an early predecessor of the present program. It shows the shape of the Mount Albert intrusion, approximate distribution of the sampling points, and the cubic trend surface with its 95 per cent half-confidence interval. Today's result as shown in the output listing at the end of this paper, is only slightly more accurate but it can be obtained much more rapidly.

The main reason for publishing the present program is to make more generally available the technique of computing confidence intervals on polynomial trend surfaces. As far as we know, there are no published computer programs in existence with this useful auxiliary tool. It can be helpful for the evaluation of both trend surfaces and residuals. There have been some demands for this technique. For example, Tinkler (1969) has stated for confidence intervals that "these are exceedingly difficult to calculate when n is large and the surface is higher than linear in order. Until computer programs are generally available, this approach cannot be used" (n = number of observations).

This paper presents a general discussion of the capability of the present program followed by a detailed mathematical documentation of some parts of the program mainly where it differs from other published programs. The final part of the report consists of detailed input preparation instructions, the example of input and output, and a complete listing of the computer program.

Usage of confidence intervals

The following word of caution remains in order. Trend-surface analysis is perhaps the most widely used computer-based method in geology and it has been the subject of considerable controversy. Criticisms arise mainly from the fact that the polynomial trend surfaces can be fitted to almost any set of data when the observation points are areally distributed on a map. The resulting contour maps can be partly or entirely meaningless. Analysis of variance is sometimes used as an aid to decide where to cut off the two-dimensional polynomial series.

However, autocorrelation effects would have to be considered during the fitting not only for their influence on the trend surface itself but also with respect to their stronger effect on the analysis of variance results and confidence intervals. When the residuals are autocorrelated a high F-ratio, e. g. for the step of proceeding from a quadratic to a cubic fit, may not be a valid criterion for cut-off. (cf. Agterberg, 1964.)

At present, these effects can only be evaluated by approximate methods, because of a lack of more rigorous statistical tests that are easy to use. The best test that a trend really exists may be simply its reproducibility from subsets of data, or its emergence from polynomial equations of several different degrees.

Trend-surface analysis has been most frequently applied and is probably most useful when there are relatively few data available for statistical analysis and when these data are known to be subject to considerable random variability (noise). If measurements are very abundant, it may be satisfactory to employ a standard contouring program that fits an irregular surface to all or most of the observed variability. If the data remain noisy on a local level, such contouring would not make sense but then a simple moving averaging technique can be useful.

The possible drawbacks of trend surfaces are relatively well-known. Confidence intervals on trend surfaces are even harder to interpret than the trend surfaces themselves. This is because they are much more sensitive to departures from the basic model consisting of a true polynomial trend surface with random residuals that are normally distributed.

For the graphical representation of confidence intervals, we have used the procedures illustrated in Figure 1. Suppose that the values on the trend surface (Fig. 1, left side) are called T_k and values on the 95 per cent half-confidence interval (right side) for corresponding points (k) are called C_k. The 95 per cent – confidence interval for the trend surface T_k then is given by $T_k \pm C_k$. This confidence interval applies to the entire trend surface. Strictly speaking it means that there is a 95 per cent probability that the "true" trend surface (of

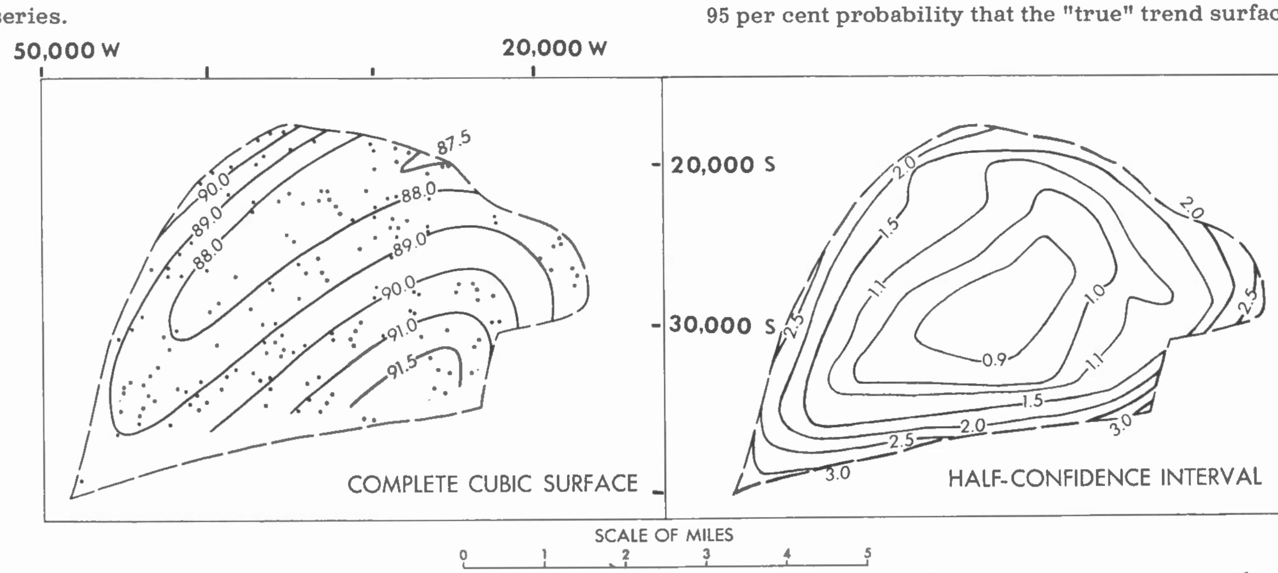


Figure 1. Outline of Mount Albert peridotite intrusion; cubic trend-surface and its 95 per cent half-confidence interval for Mg-content of orthopyroxene. Dots indicate locations of specimens (from Agterberg, 1964).

degree 3 in Fig. 1) does not anywhere intersect the confidence interval $T_k \pm C_k$ placed around the calculated surface T_k.

A confidence interval is readily computed as a companion to any fitted trend surface. However, it is only meaningful if a fairly correct trend has been found and if the other conditions of the basic model (no autocorrelation, normality of residuals) are at least approximately satisfied. In the output example at the end of the paper, the linear trend surface provides a poor fit. Nevertheless, its confidence interval is narrower than those for the quadratic and cubic surfaces which are more meaningful. Of course, this does not mean that the linear surface is more reliable than the higher-degree surfaces.

Trend-surface analysis is based on the ordinary least squares method. Because of the Gauss-Markov theorem, normality of the random variable is not a prerequisite for obtaining the best linear unbiased estimate (BLUE) although the variance of the residuals should theoretically be constant. In order to apply analysis of variance or use confidence intervals, the stronger assumption of normality has to be made. Also, if the residuals are autocorrelated according to a homogeneous autocorrelation function, the trend surface provides a fit that is unbiased although the minimum variance property has been lost. On the other hand, the confidence interval will always be biased (too narrow) when the residuals are autocorrelated.

Another drawback of using confidence intervals is that the underlying statistical theory is not elementary. Excellent coverage of the subject has been given by Miller (1966) in a monograph for advanced students. The subject has never been fully explained in introductory textbooks. Wonnacott and Wonnacott (1970) have succeeded in presenting a clear discussion of the problem but omit mathematical proofs.

The confidence interval for a straight line is a hyperbola the equation of which was originally derived by Working and Hotelling (1929). This case is treated in great detail by Kendall and Stuart (1961, p. 365-370) with the proofs exceeding four pages of text. Scheffé (1953) and Roy and Bose (1953) originally provided the equations of the $\sqrt{F}$ - confidence intervals on which our result is based. Krumbein (1963) was the first to make a practical application of confidence intervals as applied to trend surfaces. The reader interested in pursuing the theory of the subject is referred to Miller (1966) who has provided two kinds of mathematical proofs, one algebraic and the other geometric (Miller, 1966, e.g., p. 63-67).

It seems that the preceding drawbacks have prevented the widespread application of confidence intervals. Howarth (1967) stated: "Confidence limits can be calculated (Krumbein and Graybill, 1965, p. 340-345), but this is a complex procedure and the results might be difficult to use in practice". It seems that the latest newly computed confidence interval depicted in print was by Agterberg *et al.* (1967). We are now providing an algorithm for the automatic computation of confidence intervals, because this constitutes useful auxiliary information provided that the results are interpreted with caution.

In recent publications by Doveton and Parsley (1970) and Davis (1973), it has been emphasized that the contours of polynomial trend surfaces fitted to sets of random data have a distinct tendency to follow the margins of the clusters of observation points. These authors have stressed that in many geologic applications of trend surface analysis, the observation points are confined to a geologic entity such as an igneous body with distinct boundaries. Under such circumstances, even random data would yield contours tending to be parallel to the geological discontinuities and such phenomena might be easily misinterpreted.

Strong edge effects are normally predicted by confidence intervals as illustrated in the example of Figure 1. The evaluation of edge effects may in fact be the main purpose of computing confidence intervals. In places where the values on the plotted half-confidence interval are relatively large, a computed trend surface is bound to be unreliable.

The following problem concerning residuals is also closely related to that of the edge effects. It seems that, so far, it has not been considered in print in the context of polynomial trend surfaces. A good interpretation of the so-called residual variance (mean square for residuals in analysis of variance table) is that it expresses the scatter of the residuals about their mean value which is equal to zero. It can be informative to represent the residuals by means of a histogram or to plot them against the calculated values. They may be subjected to a normality test. However, in absolute value the residuals may decrease sharply at the edges of the cluster of observation points. The theory of the relationship between the width of confidence intervals for residuals and the values assumed by the independent variables is well-known (e.g. Draper and Smith, 1968, p. 93-94).

In most applied statistical regression runs, changes in width of confidence intervals on residuals can be neglected. Some trend-surface applications may provide an exception to this rule, especially if the observation points are irregularly distributed. For example, if a single observation point occurs at some distance from the cluster of observation points, then its residual is relatively close to zero. In particular, a higher-degree trend surface has a distinct tendency to pass exactly through values at isolated observation points. This may make it difficult or impossible to evaluate for statistical significance the value of a residual near the edge of the cluster of observation points. For this reason, we have provided every residual with its own t-test. These values form the last column in the optional printout of the list of residuals.

Discussion of Input-Output; precision of algorithms

The following explanations are in addition to those given in the list of Input Data Description and the example. Care was taken to ensure that the calculations were accurate, especially for higher-degree surfaces, by subjecting results particularly sensitive to distortions due to the propagation of round-off errors to two more or less independent tests for numerical precision (see below).

Our program uses single precision (approx. 14 significant digits) on a CDC – machine with 60 bit word-length. It may be wise to use double-precision when the program is converted for usage on other machines, e. g. IBM 360s where single precision corresponds to approximately 7 significant digits.

In the input one needs to specify the degree of the trend surface to be computed. Unless the dimension statements are changed, 8 is the highest degree permitted in the program. It should be kept in mind that usually several lower-degree surfaces produce more tractable results than a single higher-degree surface. After selecting a maximum degree, most users will wish to estimate simultaneously all surfaces of lower degrees from the linear surface upward.

A well-known problem of polynomial trend-surface analysis is that it can be very difficult to obtain a numerically precise estimate of the inverse matrix for higher-degree surfaces. This is a problem of multicollinearity. In general, there exists one or more approximately linear or even exactly linear relationships between larger groups of variables all of which are functions of powers of only two variables (geographic coordinates). It may be useful to employ orthogonal polynomials or other methods such as consideration of the eigenvalues and eigenvectors to attack the problem of multicollinearity. However, no method is entirely foolproof for all situations and some, rather arbitrary, cut-off levels may have to be set.

It is well-known that some of the problems of an imprecise inverse can be avoided by re-scaling of the independent variables. We have standardized all independent variables so that the matrix to be inverted is always the correlation matrix. An additional advantage of this procedure is that it becomes possible to apply the same multicollinearity test in different situations. The standard Doolittle method is used for matrix inversion but a variable is eliminated when its pivotal element is less than 10^{-8}. This procedure becomes comparable with the one used by Efroymson (1967) when the level of significance for Efroymson's F-test is set equal to zero. The latter author has stated that he obtained good results for multiple regression by choosing a tolerance value between 0. 001 and 0. 00001. We have chosen a lower cut-off value because the problem is especially severe in trend-surface analysis. When two or more variables are eliminated because of this test, a run is completely terminated. In the example this happened after degree 5, although the initial specification was to compute trend surfaces up to and including degree 6.

A second, more or less independent test of the precision of the inverse matrix consists of postmultiplication of the computed inverse by its original matrix. This should yield the identity matrix. A warning will appear in the output of the program when one or more of the elements in this product matrix deviate by more than 0. 001 from corresponding elements in the identity matrix. However, a run is not terminated if this should happen.

A normal output consists of a sequence of results for polynomial surfaces of increasing degrees. The analysis of variance table in the output permits three of the more frequently used tests for statistical evaluation of the "strength" of a trend surface or difference between two successive trend surfaces. These three methods are discussed below and illustrated by the results for the example.

1. The multiple correlation coefficient squared (R^2) increases as follows: 0. 217 (degree 1), 0. 389 (degree 2), 0. 490 (degree 3), 0. 519 (degree 4), 0. 536 (degree 5). After cubic fit R^2 starts levelling off.

2. The residual variance (= mean square due to residuals) shows the following pattern: 3. 13 (degree 0), 2. 47 (degree 1), 1. 97 (degree 2), 1. 68 (degree 3), 1. 63 (degree 4), 1. 64 (degree 5). If one bases the choice of the "best" degree on this test, the quartic trend surface would be selected because it results in the minimum residual variance. However, the values for the cubic and quintic fits are very close to that for the quartic.

3. The best-known statistical test is stepwise analysis of variance. The F-ratios computed for successive steps are: 23. 8 (1), 15. 9 (2), 8. 1 (3), 1. 9 (4), 0. 9 (5). The corresponding values along the probability axes of the (cumulative) F-distributions for appropriate numbers of degrees of freedom are: 100 per cent (degree 1), 100 per cent (degree 2), 100 per cent (degree 3), 91 per cent (degree 4), 50 per cent (degree 5). It seems obvious that the cubic trend surface should be selected according to this test.

A convenient feature of our program is that it automatically converts computed t- and F-values into the corresponding percentage values. The inverse computation is also performed when needed. The equations required for these numerical calculations were taken from Abramowitz and Stegun (1964). The resulting approximations are quite satisfactory for most practical applications. It should be kept in mind, however, that the series expansion underlying the inverse formula of computing F-values from prespecified probabilities does not provide a very good approximation. In order to compute confidence intervals on trend surfaces, an $F_{0.95}$ for the appropriate degrees of freedom has to be known in advance. (The same applies to the t-test for residuals.) The necessary values can be taken from the tables listed at the end of the Input Data Description. The print-out for these values will indicate the quality of the selected value. If no table-value is included in the input, the program will automatically compute a value for any prespecified per cent confidence. However, this calculation is crude because it is based on the inverse formula. This crudeness is evaluated by recomputing the percentage value and the result should be close to the percentage value for confidence selected by the user. For example, the 95 per cent half-confidence interval for the cubic trend surface in the output is actually a 95. 082 per cent half-confidence interval. A slightly more accurate value could be obtained by including the correct table value in the input but this gain in precision is negligibly small. However, the discrepancy becomes greater for F-ratios with lesser numbers of degrees of freedom.

Explanation of main algorithms

Let us consider n observations y_i at points (u_i, v_i) on a two-dimensional geographic rectangular grid. The polynomial trend surface of degree d in two-dimensional space is defined as follows;

$$y_i = f(u_i, v_i; d) + e_i \text{ for } i = 1, \ldots, n \qquad (1)$$

where $f(u, v; d) = \sum_{\lambda=o}^{d} \sum_{\mu=o}^{\lambda} \beta_j u^{\lambda-\mu} v^{\mu}$

is the d^{th} -degree polynomial with two variables u and v, with unknown coefficients β_j,

$j = \mu + \sum_{k=o}^{\lambda} k$ to be estimated for $j = 0, \ldots, p = d + \sum_{k=o}^{d} k$,

and where e_i are independent random variables with $E(e_i) = 0$ and unknown constant variance $Var(e_i) = \sigma^2$ for all $i = 1, \ldots, n$.

For simplification, let $p = d + \sum_{k=o}^{d} k$, $j = \mu + \sum_{k=o}^{\lambda} k$ and $x_{ji} = u_i^{\lambda-\mu} v_i^{\mu}$ for all $\mu = 0, 1, \ldots, \lambda$ and $\lambda = 0, 1, \ldots, d$. Let

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}, \quad X = \begin{bmatrix} x_{01} & x_{11} & \cdots & x_{p1} \\ x_{02} & x_{12} & \cdots & x_{p2} \\ \vdots & \vdots & \ddots & \vdots \\ x_{0n} & x_{1n} & \cdots & x_{pn} \end{bmatrix} \text{ and } \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix}$$

Then, the model (1) for given d is written as

$$Y = X\beta + e \qquad (2)$$

with $E(e) = 0$ and $Var(e) = \sigma^2 I$.

Thus, the least squares (LS) estimator b of β is the best (minimum variance) linear unbiased estimator (BLUE) by the Gauss-Markov theorem and

$$b = (X'X)^{-1} X'Y \qquad (3)$$

provided that $(X'X)^{-1}$ exists.

However, even if $(X'X)^{-1}$ exists, a direct computation of b in (3) for higher degrees and with large quantities of data, can cause entirely invalid results due to rounding errors. In particular, when the numbers in the matrix (X'X) are of different orders of magnitudes, it may be very difficult to obtain significantly valid inverse matrix. Thus, in our program, b is computed by using the correlation matrix R and b is written as;

$$b = \begin{bmatrix} \bar{y} - \sqrt{s_{yy}}\, D' S^{-1} R^{-1} R_y \\ \sqrt{s_{yy}}\, S^{-1} R^{-1} R_y \end{bmatrix}$$

where

$$D = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_p \end{bmatrix}, \quad R_y = \begin{bmatrix} r_{y1} \\ r_{y2} \\ \vdots \\ r_{yp} \end{bmatrix}, \quad S = \begin{bmatrix} s_{11} & & \\ & s_{22} & 0 \\ 0 & & s_{pp} \end{bmatrix}, \quad R = \begin{bmatrix} r_{11} & \cdots & r_{1p} \\ & r_{22} & \\ \vdots & & \vdots \\ r_{p1} & \cdots & r_{pp} \end{bmatrix}$$

$$r_{ij} = \frac{s_{ij}}{(s_{ii}\, s_{jj})^{\frac{1}{2}}}, \quad r_{yi} = \frac{s_{iy}}{(s_{ii}\, s_{yy})^{\frac{1}{2}}}$$

$$s_{ij} = \sum_{k=o}^{n} (x_{ik} - \bar{x}_i)(x_{jk} - \bar{x}_j)$$

continued....

$$s_{iy} = \sum_{k=o}^{n} (x_{ik} - \bar{x}_i) (y_k - \bar{y})$$

$$s_{yy} = \sum_{k=o}^{n} (y_k - \bar{y})^2 \qquad \text{for all } i, j = 1, \ldots, p.$$

When the problem of multicollinearity occurs, that is, when an independent variable is approximately or exactly a linear combination of other independent variables, the correlation matrix is almost singular or singular. Then the computer run will be terminated.

A standard type of analysis of variance table is printed for every degree. It contains the multiple correlation coefficient and the mean square of residuals, s^2 which will be used as an estimator of the unknown constant variance σ^2.

The variance-covariance matrix Var (b) of b is decomposed into

$$\text{Var}(b) = \begin{bmatrix} \text{Var}(b_0) & \text{Cov}(b_0, c') \\ \text{Cov}(b_0, c) & \text{Var}(c) \end{bmatrix}$$

where

$$c' = (b_1, \cdots, b_p) = \sqrt{s_{yy}} S^{-1} R^{-1} R_y$$

$$\text{Var}(c) = \sigma^2 S^{-1} R^{-1} S^{-1}$$

$$\text{Cov}(b_0, c) = -\sigma^2 D' S^{-1} R^{-1} S^{-1}$$

$$\text{Var}(b_0) = \sigma^2 (\frac{1}{n} + D' S^{-1} R^{-1} S^{-1} D)$$

Thus,

$$\text{Var}(b) = \sigma^2 \begin{bmatrix} \frac{1}{n} + D' S^{-1} R^{-1} S^{-1} D & -S^{-1} R^{-1} S^{-1} D \\ -D' S^{-1} R^{-1} S^{-1} & S^{-1} R^{-1} S^{-1} \end{bmatrix}$$

However, we also know that $\text{Var}(b) = \sigma^2 (X'X)^{-1}$ since $b = (X'X)^{-1} X'Y$.
Hence,

$$(X'X)^{-1} = \begin{bmatrix} \frac{1}{n} + D' S^{-1} R^{-1} S^{-1} D & -S^{-1} R^{-1} S^{-1} D \\ -D' S^{-1} R^{-1} S^{-1} & S^{-1} R^{-1} S^{-1} \end{bmatrix}$$

and the estimator of Var (b) is

$$\text{Var}(b) = s^2 (X'X)^{-1}.$$

Let us denote as c_{j+1}^2 the $(j+1)^{th}$ diagonal element of the matrix $s^2 (X'X)^{-1}$; c_{j+1} is the standard deviation of b_j for $j=0, 1, 2, \ldots, p$.

It must be emphasized that the LS estimator b for β should be used with caution. First of all, when the underlying distribution of the observations is known, the LS estimator b is not always to be recommended because the b is not the BLUE in most of the situations. In some cases, other estimators should be utilized, for instance, the maximum likelihood estimator. For a normal population, the LS estimator b is the BLUE and it also coincides with the maximum likelihood estimator. However, when the underlying distribution is not known, the LS estimator b is the BLUE under the two assumptions of independence and constant variance. If the distribution is not known and the observations do not satisfy one or both of these assumptions, that is, if the observations are autocorrelated or do not have a constant variance, then the generalized least squares (GLS) estimator or the weighted least squares (WLS) estimator should be used, respectively. Under those circumstances, the GLS and WLS estimators are BLUEs.

When we make the normality assumption for the underlying distribution of observations together with the independence and constant variance assumptions, we have the following results. Let

$$Y \sim N(X\beta,\ \sigma^2 I)$$

where $N(X\beta,\ \sigma^2 I)$ denotes a normal distribution with $E(Y) = X\beta$, and $Var(Y) = \sigma^2 I$. Then, since $b = (X'X)^{-1}X'Y$, $E(b) = \beta$ and

$$b \sim N(\beta,\ \sigma^2 (X'X)^{-1})$$

It follows that

$$\frac{b_j - \beta_j}{c_{j+1}} \sim t(n-m)$$

where $t(n-m)$ is the t-distribution with $(n-m)$ degrees of freedom and $m = p + 1$. Thus, the confidence interval for β_j is obtained by

$$(\ b_j - s\ c_{j+1}\ t_{\frac{1}{2}\alpha}(n-m),\ \ b_j + s\ c_{j+1}\ t_{\frac{1}{2}\alpha}(n-m)\ )$$

for a given significance level α for all $j = 0, 1, \ldots, p$.

Likewise, since $\hat{e} = (I - X(X'X)^{-1}X')Y$, $E(\hat{e}) = e$, $Var(\hat{e}) = \sigma^2\ (I - X(X'X)^{-1}X')$ and

$$\hat{e} \sim N(e,\ \sigma^2 (I - X(X'X)^{-1}X')\ )$$

Hence

$$\frac{\hat{e}_i - e_i}{s(\ 1 - x_i'\ (X'X)^{-1} x_i)^{\frac{1}{2}}} \sim t(n-m)$$

so that the confidence interval for e_i is given by

$$(\hat{e}_i - s(1 - x_i'\ (X'X)^{-1}x_i)^{\frac{1}{2}}\ t_{\frac{1}{2}\alpha}\ (n-m),\ \hat{e}_i + s(1 - x_i'\ (X'X)^{-1}x_i)^{\frac{1}{2}}\ t_{\frac{1}{2}\alpha}\ (n-m)\ )$$

for a given significance level α for all $i = 1, 2, \ldots, n$.

Furthermore, the quadratic form

$$(\beta - b)'\ (X'X)\ (\beta - b) \sim \chi^2(m) \qquad \text{and also}$$

$$(n-m)\ s^2 \sim \chi^2\ (n-m)$$

where $\chi^2(m)$ is Chi-square distribution with m degrees of freedom, lead to the ratio

$$\frac{(\beta - b)'\ (X'X)\ (\beta - b)}{m\ s^2} \sim F\ (m,\ n-m)$$

where $F\ (m,\ n-m)$ is the F-distribution with m and $(n-m)$ degrees of freedom. The confidence region Q_α of β in the m-dimensional parameter space, for a given α can be obtained from

$$Q_\alpha = \left\{ \mathring{\beta} \in R^m \ \middle|\ (\mathring{\beta} - b)'\ (X'X)\ (\mathring{\beta} - b) \leq m\, s^2\ F_\alpha(m,\ n-m) \right\}$$

Since $(X'X)$ is a positive definite matrix, Q_α forms an m-dimensional hyperellipsoid of which the semi-axes are the square roots of eigen-values of $(X'X)$ in the parameter space. The ellipsoid can be transposed into the original m-dimensional observation (Y, X) space. The translated representation of Q_α in our observation space which consists of the m dimensional hyperplanes generated by the linear combinations of $q \in Q_\alpha$ will be

$$P_\alpha = \left\{ P' = (P_o,\ P_1,\ \ldots,\ P_{m-1}) \in R^m \ \middle|\ P_o \in I_r = (f^-(r),\ f^+(r)\ ) \right\}$$

where $r' = (1,\ P_1,\ P_2,\ \ldots,\ P_{m-1})$, $f^-(r) = b'r - w^{\frac{1}{2}}$ and $f^+(r) = b'r + w^{\frac{1}{2}}$ with $w = m\ s^2\ F_\alpha\ (m,\ n-m)\ r'\ (X'X)^{-1}r$. P_α is called a half-confidence interval of the polynomial trend surface for a given significance level α.

It can be shown that

for $z' = (1, z_1, \ldots, z_{m-1}) \varepsilon R^m$ and $q \varepsilon Q_\alpha$, let $h = (z'q, z_1, \ldots, z_{m-1})$, then $h \varepsilon P_\alpha$. On the other hand, for $h = (h_o, h_1, \ldots, h_{m-1}) \varepsilon P_\alpha$ there exists at least one $q \varepsilon Q_\alpha$ such that $h_o = (1, h_1, \ldots, h_{m-1})q$.

For a polynomial trend-surface any vector $z' = (1, z_1, \ldots, z_{m-1}) \varepsilon R^m$ is uniquely generated by one point (u, v) in two-dimensional space, i.e., $z' = (1, u, v, u^2, uv, v^2, \ldots, v^d)$. Thus the half-confidence interval can be easily shown in two-dimensional space. At each map point, (u,v), $(m\, s^2 F\,(m, n-m) z'\,(X'X)^{-1} z)^{\frac{1}{2}}$ is computed and these values are plotted as a half-confidence interval.

References

Abramowitz, M. and Stegun, A. (Editors)
1964: Handbook of mathematical functions; Applied Math. Ser. 55, National Bureau of Standards, Washington D.C., 1046 pp.

Agterberg, F.P.
1964: Methods of trend surface analysis; Color. Sch. Mines, Q., v. 59 no. 4, p. 111-130.

1974: Geomathematics; Elsevier, Amsterdam, 596 p.

Agterberg, F.P., Hills, L.V. and Trettin, H.P.
1967: Paleocurrent trend analysis of a delta in the Bjorne Formation (Lower Triassic) of northeastern Melville Island, Arctic Archipelago; J. Sediment. Petrol., v. 37, p. 852-862.

Davis, J.C.
1973: Statistics and data analysis in geology; Wiley, New York, N.Y., 550 p.

Doveton, J.H. and Parsley, A.J.
1970: Experimental evaluation of trend surface distortions induced by inadequate data-point distributions; Inst. Min. Metall., Trans., sec. B., v. 79, no. 768, p. B197-207.

Draper, N.R. and Smith, H.
1966: Applied regression analysis; Wiley, New York, N.Y., 407 p.

Efroymson, M.A.
1967: Multiple regression analysis. In: A. Ralston and H.S. Wilf (Editors), Mathematical methods for digital computers, Wiley, New York, N.Y., p. 191-203.

Good, D.I.
1964: FORTRAN II trend-surface program for the IBM 1620; Kansas Geol. Surv., Spec. Dist. Publ. 14: 54 pp.

Harbaugh, J.W.
1963: BALGOL program for trend-surface mapping using an IBM 7090 computer; Kansas Geol. Surv., Spec. Distr. Publ. 3, 54 p.

Harbaugh, J.W. and Merriam, D.F.
1968: Computer applications in stratigraphic analysis; Wiley, New York, N.Y., 282 p.

Howarth, R.J.
1967: Trend-surface fitting to random data – an experimental test; Am. J. Sci., v. 265, p. 619-625.

Kendall, M.G. and Stuart, A.
1961: The advanced theory of statistics, v. 2; Hafner, New York, N.Y., 676 p.

Koch, G.S., Jr., Link, R.F. and Schuenemeyer, J.H.
1972: Computer programs for geology; Artronic Information Systems Inc., New York, N.Y., 142 p.

Krumbein, W.C.
1963: Confidence intervals on low-order polynomial trend surfaces; J. Geophys. Res., v. 68, p. 5869-5878.

Krumbein, W.C. and Graybill, F.A.
1965: An introduction to statistical models in geology; McGraw-Hill, New York, N.Y., 475 p.

MacGregor, I.D. and Smith C.H.
1963: The use of chrome spinels in petrographic studies of ultramafic intrusions; Can. Mineral., v. 7, p. 403-412.

Miesch, A.T. and Connor, J.J.
1968: Stepwise regression and nonpolynomial models in trend analysis; Kansas Geol. Surv., Computer Contr. 27, 40 p.

Miller, R.G., Jr.
1966: Simultaneous statistical inference; McGraw-Hill, New York, N.Y., 272 p.

O'Leary, M., Lippert, R.H. and Spitz, O.T.
1966: FORTRAN IV and MAP program for computation and plotting of trend surfaces for degrees 1 through 6; Kansas Geol. Surv., Computer Contr. 3, 47 p.

Roy, S. N. and Bose, R. C.
1953: Simultaneous confidence interval estimation; Ann. Mathem. Stat., v. 24, p. 513-536.

Scheffé, H.
1953: A method for judging all contrasts in the analysis of variance; Biometrika, v. 40, p. 87-104.

Tinkler, K. J.
1969: Trend surfaces with low "explanations"; the assessment of their significance; Am. J. Sc., v. 267, p. 114-123.

Whitten, E. H. T.
1963: A surface-fitting program suitable for testing geological models which involve areally-distributed data; Off. Naval Res., Geogr. Br., Tech. Rep. 2, ONR Task No. 389-135, Contract No. 1228 (26), 56 p.

Whitten, E. H. T. (cont'd.)

1974: Orthogonal-polynomial contoured trend-surface maps for irregularly-spaced data. Tech. Rep. 2, U. S. Army Research Office – Durham Grant DA-ARO-D-31-124-72-G54, Dep. Geol. Sc., Northwestern Un., Evanston, Ill., 24 p.

Wonnacott, R. J. and Wonnacott, T. H.
1970: Econometrics; Wiley, New York, N. Y., 445 p.

Working, H. and Hotelling, H.
1929: The application of the theory of error to the interpretation of trends; J. Am. Stat. Assoc., v. 24, p. 73-89.

APPENDIX I

Input data description

1. Card 1.

Specifies degree of polynomial and controls input and output.

Columns	Format	Variable	Description
1	I1	NDEG	Degree of polynomial, $1 \leq NDEG \leq 6$. If NDEG = 0, the run will be terminated.
2	I1	ITER	= 1, analysis will be repeated for polynomials of degrees 1, 2, ..., NDEG. = 0, analysis will be performed for polynomial of degree NDEG only.
3	I1	ITABLE	= 1, statistical table will be used to specify ALPF and ALPT. = 0, ALPF and ALPT will specify the level of significance (see Cards 3 and 4 for ALPF and ALPT, respectively).
4	I1	IMAP	= 1, print-out of trend surface. = 0, no action taken.
5	I1	ICONF	= 1, print-out of half-confidence interval for the entire trend surface. = 0, no action taken.
6	I1	IRES	= 1, print-out of observed and calculated values, residuals and confidence intervals for residuals. = 0, no action taken.

2. Card 2.

Specifies indentification of data.

Columns	Format	Variable	Description
1-80	8A10	NAME	Any alphanumeric information used as an output page heading.

3. Card 3.

When the ITABLE = 0 (see Card 1), Card 3 specifies only one value, ALPF(1) = α, the level of significance α for the 100(1-α)% half-confidence interval of the entire trend surface. However, if ITABLE = 1, then the user specifies

Card 4. (cont'd)

respectively. For example, if ITABLE = 1, NDEG = 2, ITER = 1, IRES = 1, n = 100 and α = 0.05 then (cf. table in Appendix 2):

$$ALPT(1) = t_{\frac{1}{2}0.05}(n-m=100-3) = t_{0.025}(97) = 1.984.$$

$$ALPT(2) = t_{\frac{1}{2}0.05}(n-m=100-6) = t_{0.025}(94) = 1.985.$$

Columns	Format	Variable	Description
1-10	F10.0	ALPT(1)	
11-20	F10.0	ALPT(2)	
21-30	F10.0	ALPT(3)	
31-40	F10.0	ALPT(4)	$t_{\frac{1}{2}\alpha}$ (n-m)
41-50	F10.0	ALPT(5)	
51-60	F10.0	ALPT(6)	

5. Card 5.

Specifies values of the parameters for print-outs of the trend surfaces and their half-confidence intervals. This card may be blank if IMAP = 0 and ICONF = 0. Otherwise, it should be specified. The parameters AA and AM should be selected in such a manner that most of printed values of the print-out map will lie more or less uniformly between 0.0 and 20.0. In the print-out map, A=11, B=13, ..., E=19; - = <0, += > 20. We have: Printed value = AA + AM* Calculated value.

Columns	Format	Variable	Description
1-10	F10.0	XA	X-co-ordinate of lower right corner of print-out map
11-20	F10.0	XM	X-co-ordinate of upper left corner
21-30	F10.0	YA	Y-co-ordinate of lower right corner
31-40	F10.0	YM	Y-co-ordinate of upper left corner
41-50	F10.0	XVAL	Interval between the print-out symbols on X-co-ordinate. $XVAL \geq \frac{\lvert XA - XM \rvert}{100.0}$
51-60	F10.0	AA	Additive constant for the calculated values.
61-70	F10.0	AM	Multiplicative factor.
75-80	I5	LINE	Number of lines per inch of line-printer.

Card 3. (cont'd)

$F_{\alpha}(m, n-m)$ from a statistical table of the $100(1-\alpha)$% fractiles of the F-distribution with degrees of freedom m and n-m where n is number of observations and m is number of independent variables (m = 3, 6, 10, 15, 21 and 28 for NDEG = 1, 2, 3, 4, 5 and 6, respectively). If ICONF = 0 on Card 1, this card may be blank. If ICONF = 1 and ITER = 0, ALPF (1) = F(m, n-m) will be specified where m depends on NDEG of Card 1. However, if ICONF = 1 and ITER = 1, then NDEG values, ALPF(1), ALPF(2), ALPF(NDEG) should be specified for all polynomials of degree 1, 2 . . . , NDEG, respectively. For example, if ITABLE = 0, ALPF(1) = 0.05 for 95% half-confidence interval. However, if ITABLE = 1, NDEG = 2, ITER = 1, ICONF = 1, n(number of samples) = 102 and α = 0.05, then

$$ALPF(1) = F_{0.05}(m=3,\ n-m=99) = 2.70$$

$$ALPF(2) = F_{0.05}(m=6,\ n-m=96) = 2.19$$

For tables of the 90, 95 and 99% fractiles of the F-distribution, see Appendix 2.

Columns	Format	Variable	Description
1-10	F10.0	ALPF(1)	
11-20	F10.0	ALPF(2)	
21-30	F10.0	ALPF(3)	F_{α} (m, n-m)
31-40	F10.0	ALPF(4)	
41-50	F10.0	ALPF(5)	
51-60	F10.0	ALPF(6)	

4. Card 4.

When the ITABLE = 0, Card 4 specifies only one value, ALPT(1) = α, the level of significance for the $100(1-\alpha)$% confidence intervals of the residuals. However, if ITABLE = 1, then the user specifies $t_{\frac{1}{2}\alpha}(n-m)$ from a statistical table of the $100(1-\frac{1}{2}\alpha)$% fractiles of the t-distribution with n-m degrees of freedom. If IRES = 0 on Card 1, this card may be blank. If ITABLE = 1, IRES = 1 and ITER = 0, then only one value, ALPT(1) = $t_{\frac{1}{2}\alpha}(n-m)$ need be specified. However, if ITABLE = 1, IRES = 1 and ITER = 1, then NDEG values, ALPT(1), ALPT(2), . . . , ALPT(NDEG) should be specified for all polynomials of degree 1, 2, . . . , NDEG,

6. Card 6.

Specifies format of the data block (user's data) that will follow. The parameter ITAPE in this card allows the use of data stored on tape or disk file. A single record on tape or disk, or a set of cards, corresponds to a single observation with X-co-ordinate (XI), Y-co-ordinate (YI) and observed value (ZI). XI, YI, and ZI can be arranged in any order but this order must be specified by using IORDER. FMT specifies the format of these three variables. ILOG allows a logarithmic transformation of the observed values.

Columns	Format	Variable	Description
1	I1	ITAPE	= 1 if the data block is stored on tape or disk. = 0 is the data block is on punched cards.
2	I1	ILOG	= 0, no transformation = 1, takes $\log_{10}$ (observed value) = 2, takes $\log_e$ (observed value)
3	I1	IORDER	= 1, XI, YI, ZI = 2, XI, ZI, YI = 3, YI, XI, ZI = 4, YI, ZI, XI = 5, ZI, XI, YI = 6, ZI, YI, XI
11-40	3A10	FMT	Format of the data. For example, (10x, F20.5, 2x, 2F10.7)

7. Data Block.

Consists of records on tape or disk file or sets of cards. The parameters of the data block were specified on Card 6. Each record for tape or disk file or set of punched cards should contain X-co-ordinate, Y-co-ordinate and observed value in an order according to IORDER and under the format FMT. The data block is normally ended by an end of file mark for tape or disk file, or the special end card (XI = 999.0, YI = 999.0, and ZI = 999.0) for a deck of punched cards.

8. End card.

Specifies end of job. A single blank card will be sufficient.

APPENDIX II

Statistical tables for input preparation for confidence intervals.

Fractiles of the t-distribution

Probability in per cent / n-m Degrees of freedom	95	97.5	99.5
1	6.314	12.71	63.66
2	2.920	4.303	9.925
3	2.353	3.182	5.841
4	2.132	2.776	4.604
5	2.015	2.571	4.032
6	1.943	2.447	3.707
7	1.895	2.365	3.499
8	1.860	2.306	3.355
9	1.833	2.262	3.250
10	1.812	2.228	3.169
11	1.796	2.201	3.106
12	1.782	2.179	3.055
13	1.771	2.160	3.012
14	1.761	2.145	2.977
15	1.753	2.131	2.947
16	1.746	2.120	2.921
17	1.740	2.110	2.898
18	1.734	2.101	2.878
19	1.729	2.093	2.861
20	1.725	2.086	2.845
21	1.721	2.080	2.831
22	1.717	2.074	2.819
23	1.714	2.069	2.807
24	1.711	2.064	2.797
25	1.708	2.060	2.787
26	1.706	2.056	2.779
27	1.703	2.052	2.771
28	1.701	2.048	2.763
29	1.699	2.045	2.756
30	1.697	2.042	2.750
40	1.684	2.021	2.704
50	1.676	2.009	2.678
60	1.671	2.000	2.660
80	1.664	1.990	2.639
100	1.660	1.984	2.626
200	1.653	1.972	2.601
500	1.648	1.965	2.586
∞	1.645	1.960	2.576

90 per cent fractiles of the F-distribution

Degrees of freedom	Linear equation	Quadratic equation	Cubic equation	Quartic equation	Quintic equation	Sextic equation
n-m	m = 3	m = 6	m = 10	m = 15	m = 21	m = 28
1	53.6	58.2	60.2	61.2	61.7	62.3
2	9.16	9.33	9.39	9.42	9.44	9.46
3	5.39	5.28	5.23	5.20	5.18	5.17
4	4.19	4.01	3.92	3.87	3.84	3.82
5	3.62	3.40	3.30	3.24	3.21	3.17
6	3.29	3.05	2.94	2.87	2.84	2.80
7	3.07	2.83	2.70	2.63	2.59	2.56
8	2.92	2.67	2.54	2.46	2.42	2.38
9	2.81	2.55	2.42	2.34	2.30	2.25
10	2.73	2.46	2.32	2.24	2.20	2.16
11	2.66	2.39	2.25	2.17	2.12	2.08
12	2.61	2.33	2.19	2.10	2.06	2.01
13	2.56	2.28	2.14	2.05	2.01	1.96
14	2.52	2.24	2.10	2.01	1.96	1.91
15	2.49	2.21	2.06	1.97	1.92	1.87
16	2.46	2.18	2.03	1.94	1.89	1.84
17	2.44	2.15	2.00	1.91	1.86	1.81
18	2.42	2.13	1.98	1.89	1.84	1.78
19	2.40	2.11	1.96	1.86	1.81	1.76
20	2.38	2.09	1.94	1.84	1.79	1.74
22	2.35	2.06	1.90	1.81	1.76	1.70
24	2.33	2.04	1.88	1.78	1.73	1.67
26	2.31	2.01	1.86	1.76	1.71	1.65
28	2.29	2.00	1.84	1.74	1.69	1.63
30	2.28	1.98	1.82	1.72	1.67	1.61
40	2.23	1.93	1.76	1.66	1.61	1.54
50	2.20	1.90	1.73	1.63	1.57	1.50
60	2.18	1.87	1.71	1.60	1.54	1.48
80	2.15	1.85	1.68	1.57	1.51	1.44
100	2.14	1.83	1.66	1.56	1.49	1.42
200	2.11	1.80	1.63	1.52	1.46	1.38
500	2.10	1.79	1.61	1.50	1.44	1.38
∞	2.08	1.77	1.60	1.49	1.42	1.34

95 per cent fractiles of the F-distribution

Degrees of freedom	Linear equation	Quadratic equation	Cubic equation	Quartic equation	Quintic equation	Sextic equation
n-m	m = 3	m = 6	m = 10	m = 15	m = 21	m = 28
1	216	234	242	246	248	250
2	19.2	19.3	19.4	19.4	19.4	19.5
3	9.28	8.94	8.79	8.70	8.66	8.62
4	6.59	6.16	5.96	5.86	5.80	5.75
5	5.41	4.95	4.74	4.62	4.55	4.50

95 per cent fractiles of the F-distribution (cont'd)

Degrees of freedom n-m	Linear equation m = 3	Quadratic equation m = 6	Cubic equation m = 10	Quartic equation m = 15	Quintic equation m = 21	Sextic equation m = 28
6	4.76	4.28	4.06	3.94	3.87	3.82
7	4.35	3.87	3.64	3.51	3.44	3.39
8	4.07	3.58	3.35	3.22	3.14	3.09
9	3.86	3.37	3.14	3.01	2.93	2.87
10	3.71	3.22	2.98	2.85	2.76	2.71
11	3.59	3.09	2.85	2.72	2.64	2.58
12	3.49	3.00	2.75	2.62	2.53	2.48
13	3.41	2.92	2.67	2.53	2.45	2.39
14	3.34	2.85	2.60	2.46	2.38	2.32
15	3.29	2.79	2.54	2.40	2.32	2.26
16	3.24	2.74	2.49	2.35	2.27	2.21
17	3.20	2.70	2.45	2.31	2.22	2.16
18	3.16	2.66	2.41	2.27	2.18	2.12
19	3.13	2.63	2.38	2.23	2.15	2.08
20	3.10	2.60	2.35	2.20	2.11	2.05
21	3.07	2.57	2.32	2.18	2.09	2.02
22	3.05	2.55	2.30	2.15	2.06	2.00
23	3.03	2.53	2.27	2.13	2.04	1.97
24	3.01	2.51	2.25	2.11	2.02	1.95
25	2.99	2.49	2.24	2.09	2.00	1.93
26	2.98	2.47	2.22	2.07	1.98	1.91
27	2.96	2.46	2.20	2.06	1.96	1.90
28	2.95	2.45	2.19	2.04	1.95	1.88
29	2.93	2.43	2.18	2.03	1.93	1.87
30	2.92	2.42	2.16	2.01	1.92	1.85
32	2.90	2.40	2.14	1.99	1.89	1.83
34	2.88	2.38	2.12	1.97	1.88	1.80
36	2.87	2.36	2.11	1.95	1.86	1.79
38	2.85	2.35	2.09	1.94	1.84	1.77
40	2.84	2.34	2.08	1.92	1.83	1.76
42	2.83	2.32	2.06	1.91	1.82	1.74
44	2.82	2.31	2.05	1.90	1.80	1.73
46	2.81	2.30	2.04	1.89	1.79	1.72
48	2.80	2.29	2.03	1.88	1.78	1.71
50	2.79	2.29	2.03	1.87	1.77	1.70
55	2.77	2.27	2.01	1.85	1.75	1.68
60	2.76	2.25	1.99	1.84	1.73	1.66
65	2.75	2.24	1.98	1.82	1.72	1.65
70	2.74	2.23	1.97	1.81	1.71	1.64
80	2.72	2.21	1.95	1.79	1.69	1.62
90	2.71	2.20	1.94	1.78	1.67	1.60
100	2.70	2.19	1.93	1.77	1.66	1.59
125	2.68	2.17	1.91	1.75	1.64	1.57
150	2.66	2.16	1.89	1.73	1.63	1.55
200	2.65	2.14	1.88	1.72	1.61	1.53

95 per cent fractiles of the F-distribution (cont'd)

Degrees of freedom n-m	Linear equation m = 3	Quadratic equation m = 6	Cubic equation m = 10	Quartic equation m = 15	Quintic equation m = 21	Sextic equation m = 28
300	2.63	2.13	1.86	1.70	1.60	1.51
500	2.62	2.12	1.85	1.69	1.58	1.50
1000	2.61	2.11	1.84	1.68	1.57	1.49
∞	2.60	2.10	1.83	1.67	1.56	1.48

99 per cent fractiles of the F-distribution

Degrees of freedom n-m	Linear equation m = 3	Quadratic equation m = 6	Cubic equation m = 10	Quartic equation m = 15	Quintic equation m = 21	Sextic equation m = 28
1	540	586	606	616	621.5	625
2	99.2	99.3	99.4	99.4	99.45	99.5
3	29.5	27.9	27.2	26.9	26.65	26.5
4	16.7	15.2	14.5	14.2	14.0	13.9
5	12.1	10.7	10.1	9.72	9.53	9.40
6	9.78	8.47	7.87	7.56	7.37	7.25
7	8.45	7.19	6.62	6.31	6.13	6.02
8	7.59	6.37	5.81	5.52	5.34	5.22
9	6.99	5.80	5.26	4.96	4.79	4.67
10	6.55	5.39	4.85	4.56	4.39	4.27
11	6.22	5.07	4.54	4.25	4.08	3.96
12	5.95	4.82	4.30	4.01	3.84	3.72
13	5.74	4.62	4.10	3.82	3.64	3.53
14	5.56	4.46	3.94	3.66	3.49	3.37
15	5.42	4.32	3.80	3.52	3.85	3.24
16	5.29	4.20	3.69	3.41	3.24	3.12
17	5.18	4.10	3.59	3.31	3.14	3.03
18	5.09	4.01	3.51	3.23	3.06	2.94
19	5.01	3.94	3.43	3.15	2.98	2.87
20	4.94	3.87	3.37	3.09	2.92	2.80
21	4.87	3.81	3.31	3.03	2.86	2.74
22	4.82	3.76	3.26	2.98	2.80	2.69
23	4.76	3.71	3.21	2.93	2.76	2.64
24	4.72	3.67	3.17	2.89	2.72	2.60
25	4.86	3.63	3.13	2.85	2.68	2.56
26	4.64	3.59	3.09	2.82	2.64	2.53
27	4.60	3.56	3.06	2.78	2.61	2.49
28	4.57	3.53	3.03	2.75	2.58	2.46
29	4.54	3.50	3.00	2.73	2.55	2.44
30	4.51	3.47	2.98	2.70	2.53	2.41
32	4.46	3.43	2.93	2.66	2.48	2.36
34	4.42	3.39	2.89	2.62	2.44	2.32
36	4.38	3.35	2.86	2.58	2.40	2.29
38	4.34	3.32	2.83	2.55	2.38	2.26

99 per cent fractiles of the F-distribution (cont'd)

Degrees of freedom n-m	Linear equation m = 3	Quadratic equation m = 6	Cubic equation m = 10	Quartic equation m = 15	Quintic equation m = 21	Sextic equation m = 28
40	4.31	3.29	2.80	2.52	3.35	2.23
42	4.29	3.27	2.78	2.50	2.32	2.20
44	4.26	3.24	2.75	2.47	2.30	2.18
46	4.24	3.22	2.73	2.45	2.28	2.16
48	4.22	3.20	2.72	2.44	2.26	2.14
50	4.20	3.19	2.70	2.42	2.25	2.12
55	4.16	3.15	2.66	2.38	2.20	2.08
60	4.13	3.12	2.63	2.35	2.17	2.05
65	4.10	3.09	2.61	2.33	2.15	2.03
70	4.08	3.07	2.59	2.31	2.13	2.01
80	4.04	3.04	2.55	2.27	2.10	1.97
90	4.01	3.01	2.52	2.24	2.06	1.94
100	3.98	2.99	2.50	2.22	2.05	1.92
125	3.94	2.95	2.47	2.19	2.00	1.88
150	3.92	2.92	2.44	2.16	1.98	1.85
200	3.88	2.89	2.41	2.13	1.95	1.82
300	3.85	2.86	2.38	2.10	1.91	1.79
500	3.82	2.84	2.36	2.07	1.90	1.76
1000	3.80	2.82	2.34	2.06	1.87	1.74
∞	3.78	2.90	2.32	2.04	1.85	1.72

APPENDIX III

Example of input and selected output.

Example of Input

The input consists of 6 control cards (*see* cards 1-6 in Input data description) and Data Block of 174 cards each of which has the orthopyroxene determination value in the 5th field (columns 28-33). Note that the last control card specifies reading of the geographic co-ordinates (in units of 1 000 ft) and the orthopyroxene value only. For an explanation of the other values on the cards, which are not used here *see* Agterberg, 1974, Table III, p. 65.

```
610111
ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE
0.05
0.05
15.0      50.0      40.0      15.0      -0.5      -170.0    2.0                 6
001       (5X,2F8.0,6X,F6.0)
    6 30.320  18.330  88.9  87.5  8.150  2.81
    8 27.980  19.020  91.2  87.5  8.150  2.75
   12 28.740  18.890  90.0  88.3         2.64
   17 30.810  21.570  90.2  87.5  8.170  2.67
   23 32.620  22.200  90.6  87.1         2.76
   28 33.960  18.280  91.0  90.6         2.70
   37 17.720  26.860  88.0  87.6  8.178  3.34
   41 25.070  27.280  91.6  90.3
   44 25.520  28.010  90.5  88.4  8.315  2.56
   52 24.450  32.970  91.2  93.1  8.315  2.61
   56 22.550  29.620  90.9  90.3  8.270  2.68
   57 23.280  29.810        89.5  8.225  2.60
   59 24.060  31.110        91.4  8.273  2.68
   60 24.400  31.270  90.5  92.0
   61 25.710  32.820  90.2  90.3  8.245  2.67
   63 26.340  34.400        88.3  8.215  2.99
  80C 22.920  23.710  90.3  89.9  8.233  3.15
  81A 22.960  23.280        89.1  8.140  3.32
   82 22.850  23.000  89.6  88.3
   91 35.810  17.420  91.0  89.1  8.290  2.67
   95 35.600  21.460        87.6
  103 39.120  19.900  90.3  88.3  8.167  2.88
  104 40.090  20.100        89.9  8.157  2.67
  105 41.200  20.900  90.5  87.6  8.170  2.67
  112 37.850  22.300  90.9  90.3  8.170
 112A 37.850  22.300  90.9  88.3  8.167  2.54
  117 40.120  28.100  89.6  86.6  8.150  2.86
  118 40.700  28.780        87.6  8.155  2.87
  121 41.680  24.480        87.4  8.157  2.70
  124 39.340  21.620  89.7  89.1  8.155  2.82
  126 37.220  19.230  91.2  90.3  8.280  2.68
  127 40.720  26.210  89.2  88.0  8.162  2.80
  131 40.270  30.830  90.0  87.6  8.145
 131A 40.270  30.830  89.6  88.0  8.169  2.80
  135 33.270  30.960  89.6  89.1  8.183
  140 46.130  33.310  89.7  88.3  8.177  2.88
  150 36.750  30.720  90.0  89.5  8.160  2.85
  246 49.050  39.010  91.0  90.3  8.225  2.75
  265 44.450  32.530  90.5  86.1         2.74
  268 41.020  34.320  91.2  88.4  8.135  2.63
  269 41.540  34.820  90.1  89.5         2.73
  270 43.350  35.600        90.0  8.300  2.56
  504 25.900  19.700  90.6  88.4  8.150  2.55
  509 27.040  20.500  89.3  87.8  8.170
  512 28.610  21.820  91.6  86.6  8.166
  522 29.260  25.480  89.2  88.4  8.192
  524 29.100  23.080  89.7  87.8
  528 26.870  21.970  89.4  89.1  8.200  2.72
  542 30.330  27.780  90.0  89.1  8.198
  553 32.910  27.210  90.7  87.7  8.168  2.69
```

Input example (continued)

```
610111
ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE
0.05
0.05
15.0      50.0      40.0      15.0      -0.5      -170.0    2.0                 6
001       (5X,2F8.0,6X,F6.0)
 570  26.430  30.720  90.6  90.3  8.260  2.64
 573  27.340  31.400  89.5  93.2  8.290
 575  27.100  31.900  91.0  91.5  8.255  2.58
 577  26.850  32.200  92.0  93.2  8.285
 579  26.750  34.970  90.6  89.9  8.236
 586  30.790  35.650        91.4  8.265  2.53
 587  30.980  35.730        93.2  8.274  2.54
 593  18.790  24.480  89.0  88.4  8.153  3.24
 598  18.150  26.320  91.2  89.2  8.184
 601A 19.280  26.440  89.5  89.9  8.186  2.93
 607  17.880  28.000  90.9  88.4         3.19
 612  35.810  24.100  89.0  87.5  8.150  2.83
 613  35.380  24.930  87.9  88.1  8.175
 620  35.090  28.430  90.2  86.2  8.180
 621  33.980  28.920  89.5  87.5  8.160  2.78
 623  33.230  30.160  90.0  90.8         2.71
 628  32.460  32.660  91.2  93.0  8.280  2.56
 632  36.300  31.600  89.7  91.5  8.225
 633  36.230  30.230  90.0  92.9  8.282
 638  43.310  26.090  89.2  88.1
 641  44.160  26.050  89.9  89.9  8.271
 642A 43.500  26.480  90.7  89.1
 750  22.950  28.150  90.5  91.8  8.279  2.74
 752  23.900  28.200  90.5  93.2  8.281  2.56
 753  24.010  27.800        89.9  8.263  2.65
 761  27.090  24.920  90.1  86.6  8.167  2.72
 762  26.950  23.500  89.0  86.6  8.163  2.84
 767A 33.430  21.540  90.1  88.3  8.163  2.87
 767B 33.430  21.540  89.6  86.6  8.168  2.85
 768  33.500  22.010  89.3  89.9         2.88
 772A 38.080  28.580  91.3  92.2  8.302  2.67
 775  27.710  28.020  90.0  89.9  8.245  2.63
 776A 27.840  27.810  89.9  89.9  8.229  2.74
 779  31.800  30.200  89.2  91.3  8.258  2.64
 780A 31.090  29.620  89.3  91.1  8.268  2.68
 800  30.200  20.100  91.0  87.6  8.174  2.79
 801  30.250  19.150  90.2  88.1  8.248  2.81
 802  30.170  18.260  90.5  88.1  8.176  2.65
 803  28.800  19.720  90.9  86.6  8.329  2.63
 811  32.800  23.700  90.4  88.2  8.174  2.78
 813  31.220  24.950  89.5  88.3  8.174  2.89
 814  32.410  23.000  91.0  88.2         2.90
 816  31.550  23.220  89.7  88.3  8.165  2.74
 817  32.900  21.450  89.2  86.6  8.168  2.74
 818  42.800  29.750  89.7  89.9  8.177  2.72
 819  43.550  30.120        90.6         2.80
 823  44.000  31.980  87.9  86.6         2.72
 826  44.940  33.050        86.7  8.159  2.78
 827  45.760  32.940        88.9  8.163  2.70
 835  46.700  36.090  89.6  86.8  8.166  2.69
 836  46.390  34.720        89.8         2.62
 837  46.190  33.520        87.8  8.290  2.74
 838  44.620  33.760        88.1         2.73
 839  41.360  33.050        90.8  8.174  2.64
 840  41.710  33.750        89.0  8.232  2.58
 841  42.080  34.410        91.6  8.251  2.65
 842  42.390  33.610        88.9  8.165  2.53
 843  42.890  33.400        89.1  8.215  2.53
 845  44.390  34.100        86.9  8.169  2.77
 846  45.000  34.550        90.8  8.244  2.54
 847  45.280  35.030        89.9  8.199  2.63
 848  45.300  35.540        87.8  8.165  2.72
```

Input example (continued)

```
610111
ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE
0.05
0.05
15.0       50.0      40.0      15.0      -0.5      -170.0    2.0                 6
001        (5X,2F8.0,6X,F6.0)
 850   35.420  18.700           88.7   8.164  2.86
 851   37.510  19.890           89.6          2.69
 861   44.980  29.150           88.3   8.174  2.60
 864   45.660  29.820           87.6          2.68
 865   46.510  30.510           89.9          2.64
 881   34.690  22.090           88.3   8.183  2.67
 885   33.780  25.620           87.8          2.69
 886   34.580  26.320           86.5   8.181  2.76
 888   36.720  27.350           87.6   8.191  2.76
 889   36.940  26.950           87.8          2.74
 891   35.000  25.990           87.8   8.168  2.67
 894   36.210  21.980           86.4   8.166  2.81
 897   42.390  25.250           88.8   8.165  2.63
 899   42.290  26.520           87.8   8.161  2.66
 902   39.000  26.650           88.5   8.165  2.76
 908   31.820  30.880           91.3   8.275  2.68
 909   31.860  30.550           93.2          2.52
 912   30.050  30.450           89.5   8.291  2.65
 916   27.800  29.220           91.6          2.63
 917   28.450  29.100           91.5          2.54
 925   37.590  29.820           87.8   8.174  2.86
 926   36.750  29.690           86.8   8.177  2.82
 927   34.780  30.870           87.9   8.213  2.75
 928   37.480  30.620           88.3   8.188  2.72
 930   37.220  31.380           87.8   8.215  2.72
 931   36.280  31.280           87.8   8.190  2.85
 932   35.420  31.220           88.9   8.229  2.82
 934   27.510  30.400           92.0          2.60
 935   25.900  30.100           93.9          2.66
 936   24.660  28.790           89.9          2.57
 937   24.210  27.280           91.8          2.62
 939   28.760  24.500           88.9   8.235  2.71
 940   28.920  23.880           88.9   8.235  2.73
 941   28.300  23.200   90.8    87.7   8.175  2.82
 944   29.360  22.000   89.6    87.8   8.163  2.65
 945   29.320  23.650           87.8   8.170  2.67
 948   30.250  24.190   89.0    88.4   8.164  2.78
 949   30.250  23.260   87.9    87.8   8.165  2.89
 950   30.250  22.280   89.5    89.6   8.164  2.79
 952   37.020  32.960           89.6   8.191  2.82
 953   37.110  33.650   89.2    91.4   8.172  2.71
 955   37.620  32.090   89.0    88.9   8.204  2.78
 956   38.100  32.590   91.5    90.8   8.221  2.73
 958   39.220  32.400           89.1   8.187  2.60
 959   39.750  32.800   90.5    89.9   8.194  2.76
 967   36.880  17.690   89.0    89.0   8.262  2.73
 968   37.440  18.180   90.9    93.1   8.268  2.69
 989   18.880  25.000   89.5    87.8          3.25
 990   19.400  25.200   89.5    87.8   8.170  3.26
 991   19.790  25.400   90.3    87.1   8.160  3.10
1023   37.450  34.680           90.0   8.238  2.70
1029   35.550  34.950           91.5   8.275  2.67
1035   34.780  33.280           90.7   8.248  2.62
1041   34.120  33.000           90.7   8.232  2.69
1044   34.790  34.300           91.6   8.261  2.73
1046   34.780  35.560           92.0   8.268  2.63
1047   33.960  35.150           90.0   8.251  2.66
1048   33.680  34.750           91.3   8.242  2.71
1049   33.350  33.970           91.1   8.235  2.68
1083   33.650  26.520           89.9   8.199  2.67
1085   31.500  26.440           89.9   8.202  2.62
1087   30.260  27.350           89.9   8.249  2.66
     999.0   999.0           999.0
```

Example of selected Output

In order to save space, a number of sheets have been omitted from the full output for the run specified in the Input. In particular, the listing of residuals was restricted to the first 50 residuals of the cubic surface (degree 3).

```
ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

NUMBER OF OBSERVATIONS    N  =        174

NUMBER OF INDEPENDENT VARIABLES INCLUDING CONSTANT    M  =          28


   ACCORDING TO   INPUT(SEE INPUT DATA DESCRIPTION IN  TEXT)

DEGREES OF POLYNOMIAL   NDEG =     6
ITER   =       1
ITABLE =       0
IMAP   =       1
ICONF  =       1
IRES   =       1

LEVEL OF SIGNIFICANCE FOR T-VALUE =    .05000
LEVEL OF SIGNIFICANCE FOR F-VALUE =    .05000

ITAPE     =        0
IORDER    =        1
ILOG      =        0
FORMAT    =     (5X,2F8.0,6X,F6.0)

XM =        50.0000000000                     YM =        15.0000000000
XA =        15.0000000000                     YA =        40.0000000000
XVAL =        -.5000000000                    LINE =           6
AA =      -170.0000000000                     AM =         2.0000000000


MEAN OF DEPENDENT VARIABLE   =   8.9199425287341E+01

VARIANCE OF DEPENDENT VARIABLE = 3.1261846399032E+00
```

ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

ANALYSIS OF VARIANCE TABLE OF POLYNOMIAL TREND SURFACE OF DEGREE 1

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	CALCULATED F - VALUE	PROBABILITY IN %
TOTAL(UNCORRECTED)	1.384978350E+06	174			
MEAN(B(1))	1.384437520E+06	1			
TOTAL(CORRECTED)	5.408299427E+02	173			
RESIDUAL	4.232202529E+02	171	2.4749722392		
REGRESSION ENTIRE EQN /B(1)	1.176096898E+02	2	58.8048448995	23.7597997941	100.000

MULTIPLE CORRELATION COEFFICIENT SQUARED = .2174615

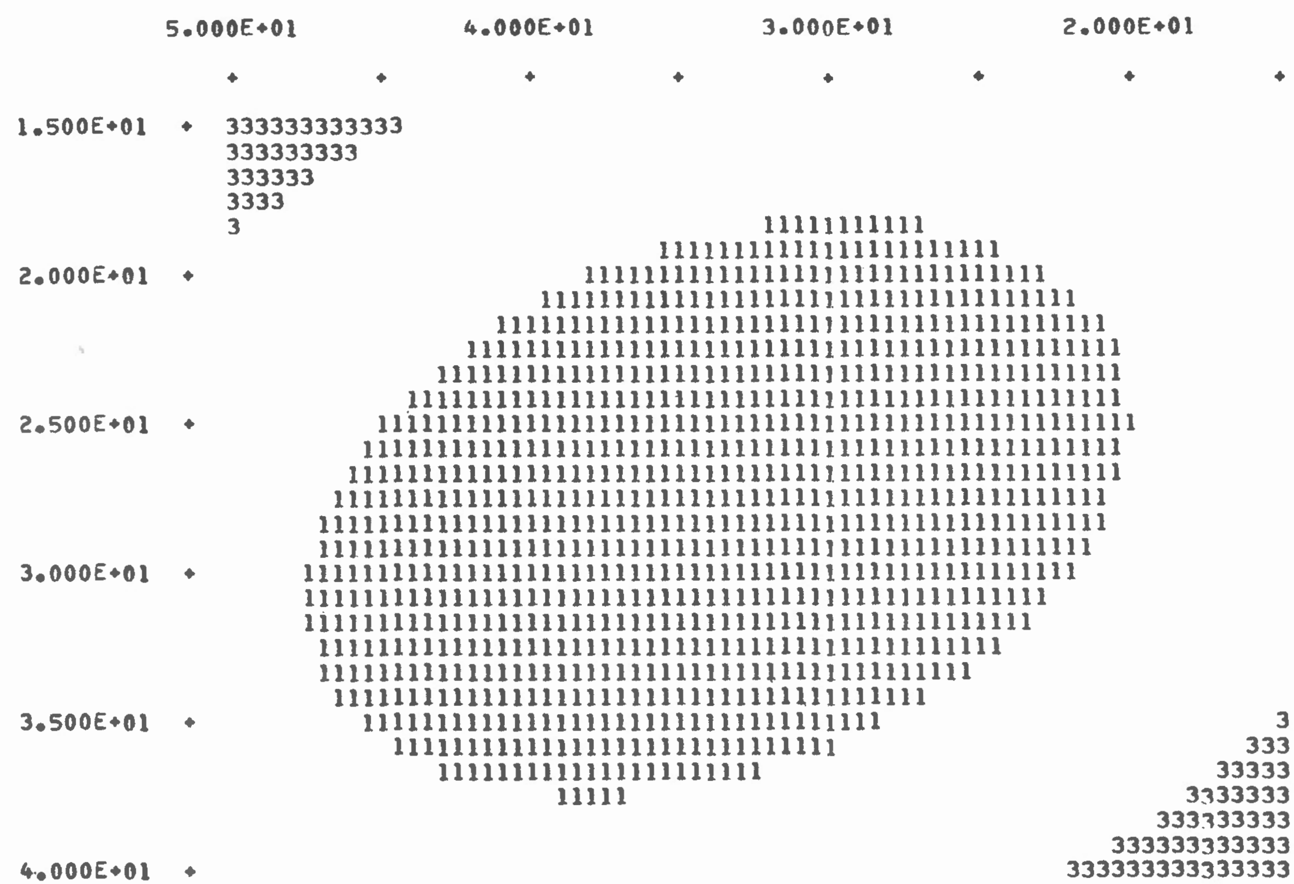
	ESTIMATES OF COEFFICIENTS	STANDARD DEVIATIONS
1	8.7240649264513E+01	7.6431423265612E-01
2	-7.3948277562327E-02	1.7421011623897E-02
3	1.6016631200103E-01	2.4732243679042E-02

```
ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

 XM =        50.0000000000            YM =        15.0000000000

 XA =        15.0000000000            YA =        40.0000000000

 XVAL =         -.5000000000          YVAL =          .8333333333

 X-SIZE =         71         Y-SIZE =         31

ADDITIVE CONSTANT =      -170.0000000000

MULTIPLICATIVE FACTOR =          2.0000000000

CALCULATED VALUE = ( PRINTED VALUE -  -.170E+03 ) /   .200E+01


MAP FOR PREDICTED VALUES

           5.000E+01           4.000E+01           3.000E+01           2.000E+01

               +         +         +         +         +         +         +         +

1.500E+01   +           3333333333333              5555555555555              77777777
                    33333333333333             55555555555555             777777777777
                 3333333333333              5555555555555              7777777777777
               33333333333              5555555555555              77777777777777
               33333333             55555555555555             77777777777777
               3333              5555555555555              7777777777777
2.000E+01   +  3             55555555555555             77777777777777             999
                          5555555555555              7777777777777              999999
                      5555555555555              7777777777777              9999999999
                  55555555555555             77777777777777             99999999999999
               5555555555555              7777777777777              9999999999999
               5555555555             77777777777777             99999999999999
2.500E+01   +  555555              7777777777777              9999999999999
               55              7777777777777              9999999999999              A
                           77777777777777             99999999999999             AAAAA
                        7777777777777              9999999999999              AAAAAAAA
                    77777777777777             99999999999999             AAAAAAAAAAAA
                777777777777777             99999999999999              AAAAAAAAAAAAA
3.000E+01   +  77777777777              9999999999999              AAAAAAAAAAAAA
               77777777             99999999999999             AAAAAAAAAAAAAA
               7777              9999999999999              AAAAAAAAAAAAA
               7             99999999999999             AAAAAAAAAAAAAA             BBB
                         99999999999999             AAAAAAAAAAAAAA              BBBBBB
                      9999999999999              AAAAAAAAAAAAA              BBBBBBBBBB
3.500E+01   +     99999999999999             AAAAAAAAAAAAAA             BBBBBBBBBBBBBB
               9999999999999              AAAAAAAAAAAAA              BBBBBBBBBBBBB
               9999999999             AAAAAAAAAAAAAA             BBBBBBBBBBBBBB
               999999             AAAAAAAAAAAAAA             BBBBBBBBBBBBBB
               99              AAAAAAAAAAAAA              BBBBBBBBBBBBB              C
                           AAAAAAAAAAAAAA             BBBBBBBBBBBBBB             CCCCC
4.000E+01   +           AAAAAAAAAAAAA              BBBBBBBBBBBBB              CCCCCCCC
```

Print-out of
Linear
Trend Surface

```
XM =          50.0000000000             YM =          15.0000000000

XA =          15.0000000000             YA =          40.0000000000

XVAL =           -.5000000000           YVAL =            .8333333333

X-SIZE =           71          Y-SIZE =          31

MULTIPLICATIVE FACTOR =           2.0000000000

CALCULATED VALUE = PRINTED VALUE /   .200E+01
```

MAP FOR HALF CONFIDENCE INTERVAL   95.935 %



ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

ANALYSIS OF VARIANCE TABLE OF POLYNOMIAL TREND SURFACE OF DEGREE 2

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	CALCULATED F - VALUE	PROBABILITY IN %
TOTAL(UNCORRECTED)	1.384978350E+06	174			
MEAN(B(1))	1.384437520E+06	1			
TOTAL(CORRECTED)	5.408299427E+02	173			
RESIDUAL	3.303019676E+02	168	1.9660831404		
REGRESSION ENTIRE EQN /B(1)	2.105279751E+02	5	42.1055950226	21.4159788857	100.000
1 DEGREE LESS/B(1)	1.176096898E+02	2			
PARTIAL EQN	9.291828531E+01	3	30.9727617714	15.7535361220	100.000

MULTIPLE CORRELATION COEFFICIENT SQUARED = .3892683

ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

	ESTIMATES OF COEFFICIENTS	STANDARD DEVIATIONS
1	7.9116112542545E+01	4.4089550708597E+00
2	5.8063561550150E-01	1.3273309899235E-01
3	-1.2454646617880E-01	2.4787227864988E-01
4	-4.9609665949939E-04	2.1244994007048E-03
5	-2.2288959481350E-02	4.1897493428754E-03
6	1.9661192945580E-02	4.6163861054109E-03

```
ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

 XM =        50.0000000000            YM =        15.0000000000

 XA =        15.0000000000            YA =        40.0000000000

 XVAL =         -.5000000000          YVAL =         .8333333333

 X-SIZE =          71          Y-SIZE =          31

ADDITIVE CONSTANT =      -170.0000000000

MULTIPLICATIVE FACTOR =          2.0000000000

CALCULATED VALUE = ( PRINTED VALUE -  -.170E+03 ) /   .200E+01


MAP FOR PREDICTED VALUES

           5.000E+01           4.000E+01           3.000E+01           2.000E+01

            +         +         +         +         +         +         +         +

1.500E+01 + CCCCCC     BBBB     AAAAA     99999    77777    55555    33333    11111
                  BBBBB      AAAAA     99999      77777     55555    33333     1111
            BBBBBB      AAAAAA      999999      77777      55555      33333     111
                   AAAAAAA       9999999      777777      555555      333333      1
             AAAAAAAA        99999999       7777777       5555555       333333
            AAA         999999999         77777777        55555555        3333333
2.000E+01 +       99999999999           777777777          555555555         333333
            99999999999              777777777777           55555555555          33
            99                   7777777777777777              5555555555555
                           77777777777777777777777                   55555555555555
            7777777777777777777777777777777777777777777                          55
            77777777777777777777777777777777777777777777777777777777777
2.500E+01 +    77777777777777777777777777777777777777777777777777777777777777777777
                       77777777777777777777777777777777
                          7777777777777777777                        99999999999999
            55            77777777777777                999999999999999999
            5555          77777777777            999999999999              AAAAAAAA
            55555         777777777         9999999999          AAAAAAAAAAA
3.000E+01 + 55555        7777777        999999999        AAAAAAAAA         BBBBBBBB
            55555       777777       9999999       AAAAAAAA       BBBBBBB         C
            55555     777777      9999999      AAAAAA       BBBBBBB      CCCCCCC
            5555     777777     999999     AAAAAA      BBBBB      CCCCCC       DDDD
            555     77777     99999     AAAAA     BBBBB     CCCCCC     DDDDDD     E
            55     7777     9999     AAAA     BBBBB    CCCCC     DDDDD     EEFEE
3.500E+01 + 5     7777    9999    AAAA     BBBBB    CCCC     DDDD     EEEE     +++++
                7777    9999    AAAA    BBBB    CCCC    DDDD     EEEEE    +++++++++++
               777    999    AAAA   BBBB    CCC    DDDD    EEE    +++++++++++++++++
              777   999    AAA    BBB   CCCC   DDDD   EEEE   ++++++++++++++++++++++
            777   9999   AAA   BBB   CCCC   DDD   EEEE   ++++++++++++++++++++++++++
            77   999   AAA   BBB   CCC   DDD   EEE   ++++++++++++++++++++++++++++++
4.000E+01 +    999   AA   BBB   CCC   DD   EEE   ++++++++++++++++++++++++++++++++++
```

Print-out of
Quadratic
Trend Surface

```
XM =          50.0000000000            YM =          15.0000000000

XA =          15.0000000000            YA =          40.0000000000

XVAL =           -.5000000000          YVAL =           .8333333333

X-SIZE =            71          Y-SIZE =          31

MULTIPLICATIVE FACTOR =            2.0000000000

CALCULATED VALUE = PRINTED VALUE /   .200E+01


MAP FOR HALF CONFIDENCE INTERVAL      95.192 %

         5.000E+01           4.000E+01           3.000E+01           2.000E+01

             +         +         +         +         +         +         +         +

1.500E+01  + AAA   9999   7777      5555555555         555555555555        7777777
             A   999    7777    555555                         555555555       77777
               999   7777    55555        33333333333333            55555555      77
             999   7777    5555      3333333333333333333333333          5555555
             99   777    5555     333333333           3333333333333        555555
                777    5555     333333                     33333333333       555555
2.000E+01  +  7777   5555     33333                            3333333333      55555
             777    555     33333           111111                333333333      555
             77   5555    333333        11111111111111               3333333      55
                 5555    33333         11111111111111111               3333333     5
                555     33333        111111111111111111111              3333333
              5555     33333         111111111111111111111111            333333
2.500E+01  + 5555    333333         111111111111111111111111111           333333
             555    333333         111111111111111111111111111111         333333
             55    333333          1111111111111111111111111111111        33333
             5    333333          11111111111111111111111111111111        33333    5
                 333333         111111111111111111111111111111111        33333     5
                333333        1111111111111111111111111111111111        33333     55
3.000E+01  +   333333        1111111111111111111111111111111111        33333     555
               33333        11111111111111111111111111111111          33333    5555
              33333        111111111111111111111111111111           33333     5555
              33333        111111111111111111111111              333333     5555
             333333        11111111111111111                  3333333     5555    77
             333333                                        3333333      5555    777
3.500E+01  +  333333                                  333333333      55555    7777
               3333333                         333333333333       55555        777 9
                33333333333333333333333333333333333333         55555     7777    999
                   3333333333333333333333333333           5555555     7777    999
             5                                      555555555      7777    9999   AA
             555555                         555555555555       77777    9999    AAA
4.000E+01  +   5555555555555555555555555555555555         777777     9999    AAA   B
```

ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

ANALYSIS OF VARIANCE TABLE OF POLYNOMIAL TREND SURFACE OF DEGREE 3

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	CALCULATED F - VALUE	PROBABILITY IN %
TOTAL(UNCORRECTED)	1.384978350E+06	174			
MEAN(B(1))	1.384437520E+06	1			
TOTAL(CORRECTED)	5.408299427E+02	173			
RESIDUAL	2.756481119E+02	164	1.6807811702		
REGRESSION ENTIRE EQN /B(1)	2.651818308E+02	9	29.4646478664	17.5303295810	100.000
1 DEGREE LESS/B(1)	2.105279751E+02	5			
PARTIAL EQN	5.465385568E+01	4	13.6634639211	8.1292342892	99.999

MULTIPLE CORRELATION COEFFICIENT SQUARED = .4903239

ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION. GASPE, QUE

	ESTIMATES OF COEFFICIENTS	STANDARD DEVIATIONS
1	7.1984425329025E+01	2.2635728620405E+01
2	2.6330498368436E+00	1.0001829201801E+00
3	-1.8014520204859E+00	1.8296807904578E+00
4	-5.6523071887582E-02	2.1597912531613E-02
5	-5.6595806279255E-02	4.4911721088482E-02
6	1.1435370238121E-01	6.1879468890295E-02
7	1.3793332875910E-03	3.0789832244831E-04
8	-2.8847755574474E-03	7.5857324238531E-04
9	4.1378548593926E-03	8.7240058438602E-04
10	-2.8996830598765E-03	8.2567026236469E-04

ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

```
XM =          50.0000000000            YM =        15.0000000000

XA =          15.0000000000            YA =        40.0000000000

XVAL =           -.5000000000          YVAL =          .8333333333

X-SIZE =           71          Y-SIZE =          31

ADDITIVE CONSTANT =      -170.0000000000

MULTIPLICATIVE FACTOR =         2.0000000000

CALCULATED VALUE = ( PRINTED VALUE -  -.170E+03 ) /   .200E+01
```

MAP FOR PREDICTED VALUES

```
               5.000E+01          4.000E+01          3.000E+01          2.000E+01

                  +         +         +         +         +         +         +         +

 1.500E+01   +  +++++++++++++++++++++++++ E D  C BB  AA  99   777   555   333  111-----
                ++++++++++++++++++++++ E D  C BB AA  999   777    5555   3333  111 ----
                ++++++++++++++++++++ EDD C BB AA  99   7777     55555    3333   11 ----
                +++++++++++++++++ E D C BB AA  99   7777      5555555     3333   111 ---
                +++++++++++++++E D C  B AA  99   777       555555555      333   11 ---
                ++++++++++++ E D C B  A  99   777       555555555555555    3333  11 --
 2.000E+01   +  +++++++++++ ED C  B AA 99   777      5555555555555555555555   333  11 -
                ++++++++  D C B  A  99  777     55555555555555      55555555  33  11
                ++++++  D C B AA 99  777     555555555555              5555  33  11
                ++++  D C B AA 99  777     5555555555        777777777    555  33  1
                ++ ED C B AA 99  777     55555555555      777777777777777777   55  33 1
                 E DC B  A 99  777    5555555555     777777             7777  55  3
 2.500E+01   +   D C B A  9  777    555555555     77777     999999999    77  55 33
                 C B AA 99  77    55555555     77777    999999999999999   77  5  3
                C B A  9  777    555555     77777   99999             9999  77 5  3
                B  A 99  777              7777   9999    AAAAAAAAA    99  77 5  3
                 AA 99  777             77777  9999    AAAAAAAAAAAAAAA   99 77 5 3 1
                AA 99  777             7777   9999   AAAAA           AAAA  99 77 5 3 1
 3.000E+01   +  A 99  7777        77777   999   AAAA             AAA  9  7 5 3 1-
                 99   77777    777777   9999  AAAA    BBBBBBBBBB   AAA  9 77 5 31 -
                 99   77777777777777   9999  AAAA    BBBBBBBBBBBBB   AA  9  7 5 3  --
                99   777777777777    999   AAA    BBBBBBBBBBBBBBBB  AA  99 7 5 3  ---
                99    77777777     999   AAAA   BBBBBB      BBBBB   AA 99 7 5 3  ----
                99     77        9999   AAAA   BBBBBB      BBBBB  AA  9  7 5  1------
 3.500E+01   +  99          99999   AAA   BBBBBBB     BBBB   AA 99 7 5 3 1-------
                99         99999   AAAA   BBBBBBBBBBBBBBBB  AA  9  7 5 31 --------
                999         999999   AAAA   BBBBBBBBBBBBBBB  AA  9  7 5 3 1----------
                9999      9999999   AAAAA     BBBBBBBBB   AAA  9  7 5 3 1------------
                999999999999999     AAAAA                AAA  99 77 5 3 1-------------
                999999999999999      AAAAAA          AAAA   99  7  5 3 1---------------
 4.000E+01   +  999999999999999      AAAAAAAAAAAAA    999  7  5 3 1 ----------------
```

Print-out of
Cubic
Trend Surface

```
XM =        50.0000000000            YM =        15.0000000000

XA =        15.0000000000            YA =        40.0000000000

XVAL =         -.5000000000          YVAL =          .8333333333

X-SIZE =          71          Y-SIZE =          31

MULTIPLICATIVE FACTOR =          2.0000000000

CALCULATED VALUE = PRINTED VALUE /  .200E+01


MAP FOR HALF CONFIDENCE INTERVAL     95.082 %

           5.000E+01          4.000E+01          3.000E+01          2.000E+01

               +         +         +         +         +         +         +         +

 1.500E+01  +  ++++++++++++++ E D C  B  AAA     99999       AAAA  BBB  CC DD  E ++++++
               ++++++++++++ ED C B  A  999    7777777       9999  AAA  BB  C  D  E +++
               ++++++++++ ED C B A 99  777                  7777    99  AAA BB  C  D EE
               ++++++++ ED C B A 9  7   5555          555555    777  999  A  BB CC D
               +++++++E DC B A 9 77  55   33333333333       555   777  99   AA  B  C D
               +++++ ED CB A 9 77 55  333333333333333333333    555   777  99  A  BB C
 2.000E+01  +  ++++  DC B A 9 7  55  3333                   333333   555   77  99  AA BB
               +++E  C  A 9 77 55  3333                         3333   555   77  99  AA
               + ED CB A 9 7  55  3333                            33333   555   777  9  A
                ED  B A 9 7  5   3333                              33333    555  777 99
                DC B A 9 7  5   3333                               3333333   555  77  9
               DC B A99 7 55  33333                                   3333333   555 77 9
 2.500E+01  +  C B  99 7 55  3333                                      3333333   55  7
                B  99 7 55  3333                                          33333   55 7
               B A 9 7 55  3333                                             333   5  7 9
                A 9 7  5  3333                                             3333  5  7 9
               A 9 7  5   333                                            3333  5  7 9 A
                9 77 55  333                                           3333  55 77 9 A
 3.000E+01  +  9  7 55  333                                          3333   5  7 9 A B
               9 7  5  3333                                        3333   55 77 9 A BC
                77 55  333                                       3333   55 77 9 A B CD
                7 55  333                                      3333   55 77 9 A B C  E+
               77 55  333                                    33333   55 77 99 A BC DE ++
               7  5  333                              333333333   55   77 9  A B  D  ++++
 3.500E+01  +  7 55  3333         3333333333333333333333      55   7   9 AA B  D E++++++
               7 55   33333333333333333            5555   77   99 A B C D E++++++++
               7  555                 55555555555555     777   99 A  B C D  ++++++++++
               77   55555555555555            77777777    99   AA   B C D E ++++++++++++
                 77777      7777777777        99999     AA  BB   C D E +++++++++++++++
                9999        999999999       AAAAAA   BBB  CC DD E +++++++++++++++++
 4.000E+01  +  AAAAA       AAAAAAAAA       BBBBBB    CC   DD EE ++++++++++++++++++++++
```

ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

OBSERVED	CALCULATED	RESIDUAL	CONFIDENCE INTERVAL 95.005 %
8.75000000E+01	8.79342206E+01	-4.34220570E-01	2.42740862E+00
8.75000000E+01	8.75829115E+01	-8.29115049E-02	2.42763034E+00
8.83000000E+01	8.76434172E+01	6.56582772E-01	2.43801468E+00
8.75000000E+01	8.76778203E+01	-1.77820309E-01	2.52258728E+00
8.71000000E+01	8.76394531E+01	-5.39453114E-01	2.51968597E+00
9.06000000E+01	8.88128280E+01	1.78717204E+00	2.45116030E+00
8.76000000E+01	8.81238850E+01	-5.23885007E-01	2.31337956E+00
9.03000000E+01	9.03257450E+01	-2.57450449E-02	2.51135887E+00
8.84000000E+01	9.05979276E+01	-2.19792761E+00	2.51144090E+00
9.31000000E+01	9.12166573E+01	1.88334272E+00	2.38758763E+00
9.03000000E+01	9.06603671E+01	-3.60367053E-01	2.48417533E+00
8.95000000E+01	9.08753529E+01	-1.37535289E+00	2.49358735E+00
9.14000000E+01	9.11911048E+01	2.08895196E-01	2.47682586E+00
9.20000000E+01	9.12763942E+01	7.23605818E-01	2.48042592E+00
9.03000000E+01	9.15554361E+01	-1.25543615E+00	2.45948398E+00
8.83000000E+01	9.14875636E+01	-3.18756361E+00	2.35911542E+00
8.99000000E+01	8.88003109E+01	1.09968906E+00	2.46814897E+00
8.91000000E+01	8.86248332E+01	4.75166808E-01	2.45682277E+00
8.83000000E+01	8.85033215E+01	-2.03321482E-01	2.44499470E+00
8.91000000E+01	9.03416877E+01	-1.24168767E+00	2.33312250E+00
8.76000000E+01	8.78086098E+01	-2.08609811E-01	2.51226653E+00
8.83000000E+01	8.98410825E+01	-1.54108249E+00	2.43842091E+00
8.99000000E+01	9.02393824E+01	-3.39382367E-01	2.39175449E+00
8.76000000E+01	9.02048683E+01	-2.60486825E+00	2.35840385E+00
9.03000000E+01	8.79546801E+01	2.34531994E+00	2.50543773E+00
8.83000000E+01	8.79546801E+01	3.45319936E-01	2.50543773E+00
8.66000000E+01	8.78523224E+01	-1.25232241E+00	2.52174833E+00
8.76000000E+01	8.79333137E+01	-3.33313690E-01	2.52221028E+00
8.74000000E+01	8.82272711E+01	-8.27271080E-01	2.47597896E+00
8.91000000E+01	8.86998592E+01	4.00140791E-01	2.48465471E+00
9.03000000E+01	8.94240890E+01	8.75911022E-01	2.47011358E+00
8.80000000E+01	8.77175779E+01	2.82422112E-01	2.51051946E+00
8.76000000E+01	8.85030417E+01	-9.03041653E-01	2.52526245E+00
8.80000000E+01	8.85030417E+01	-5.03041653E-01	2.52526245E+00
8.91000000E+01	9.04046355E+01	-1.30463547E+00	2.53058292E+00
8.83000000E+01	8.86180698E+01	-3.18069788E-01	2.47298652E+00
8.95000000E+01	8.93100189E+01	1.89981078E-01	2.53033731E+00
9.03000000E+01	8.93509013E+01	9.49098681E-01	1.62977050E+00
8.61000000E+01	8.84600337E+01	-2.36003370E+00	2.50975976E+00
8.84000000E+01	8.93956172E+01	-9.95617159E-01	2.51259504E+00
8.95000000E+01	8.94108793E+01	8.91206798E-02	2.50423205E+00
9.00000000E+01	8.92413142E+01	7.58685823E-01	2.48629542E+00
8.84000000E+01	8.75431600E+01	8.56839990E-01	2.40111806E+00
8.78000000E+01	8.76803121E+01	1.19687922E-01	2.47566744E+00
8.66000000E+01	8.78747928E+01	-1.27479279E+00	2.51876318E+00
8.84000000E+01	8.90417264E+01	-6.41726397E-01	2.52673688E+00
8.78000000E+01	8.81680660E+01	-3.68065962E-01	2.52581736E+00
8.91000000E+01	8.80498488E+01	1.05015118E+00	2.50353385E+00
8.91000000E+01	8.98201490E+01	-7.20149047E-01	2.52686491E+00
8.77000000E+01	8.89434822E+01	-1.24348219E+00	2.53281659E+00

Partial Listing
of Residuals
from Cubic
Trend Surface

ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

ANALYSIS OF VARIANCE TABLE OF POLYNOMIAL TREND SURFACE OF DEGREE 4

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	CALCULATED F - VALUE	PROBABILITY IN %
TOTAL(UNCORRECTED)	1.384978350E+06	174			
MEAN(B(1))	1.384437520E+06	1			
TOTAL(CORRECTED)	5.408299427E+02	173			
RESIDUAL	2.598986171E+02	159	1.6345824973		
REGRESSION ENTIRE EQN /B(1)	2.809313256E+02	14	20.0665232592	12.2762376889	100.000
1 DEGREE LESS/B(1)	2.651818308E+02	9			
PARTIAL EQN	1.574949483E+01	5	3.1498989662	1.9270357852	90.732

MULTIPLE CORRELATION COEFFICIENT SQUARED = .5194448

	ESTIMATES OF COEFFICIENTS	STANDARD DEVIATIONS
1	9.9929411236585E+01	1.3063245027918E+02
2	5.8589566771821E+00	9.1717889548943E+00
3	-9.8887377913472E+00	1.2522066706757E+01
4	-2.2154906219472E-01	2.5779667113284E-01
5	-3.4085049212193E-03	5.0629232447826E-01
6	5.2100256184166E-01	6.4332033722049E-01
7	2.8596302097349E-03	4.2151372079139E-03
8	5.0309462881526E-03	9.5831921186161E-03
9	-9.0354186808592E-03	1.2059351767563E-02
10	-6.7265174414934E-03	1.6919591618775E-02
11	-5.2875131218405E-05	4.3195643463905E-05
12	1.9734686471495E-04	1.3265405799474E-04
13	-5.0133524500844E-04	2.0089762357766E-04
14	5.7036009904565E-04	1.9520439877785E-04
15	-1.4190296564615E-04	1.8094680558727E-04

```
ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

 XM =         50.0000000000           YM =        15.0000000000

 XA =         15.0000000000           YA =        40.0000000000

 XVAL =          -.5000000000         YVAL =          .8333333333

 X-SIZE =          71          Y-SIZE =          31

ADDITIVE CONSTANT =      -170.0000000000

MULTIPLICATIVE FACTOR =          2.0000000000

CALCULATED VALUE = ( PRINTED VALUE -  -.170E+03 ) /   .200E+01


MAP FOR PREDICTED VALUES

           5.000E+01           4.000E+01           3.000E+01           2.000E+01

              +         +         +         +         +         +         +         +

 1.500E+01  + -----------1 35 77 9  AA    BBBBBBBBBBBB     AAAA  999  77  5  3 1 ----
              ----------1  5  7 99  AAAA            AAAAAA   999   777  55  3   1 -----
              ------  3 5  7  99    AAAAAAAAAAAA        9999    777   555  33   1 ------
              ----1 3 5 77  999               99999      77777   5555  333  11-------
              -1 3 5  77  9999          9999999     77777       5555   333   1 -------
              3 55 77  9999        9999999     777777       555555   333   11 ------
 2.000E+01  + 5 77  9999       9999999     777777      555555555    333   11 -----
                 999         999999     77777      555555555555555    333  11 ----
              99            99999    77777      5555555555555555555555   33  1 ---
                 AAA      99999    77777      55555555555        5555  33 1 --
              AAAAAA     9999    7777       5555555          777777    555 33 1 -
              AAAAAA    999    7777        5555         777777777    77777  55  3 1-
 2.500E+01  + AAAAA   9999   77777                   777777      9999    77  5 3 1-
              AAA    999   7777                   77777  9999999    9999   7 55  1-
              AA   9999  77777                 7777   9999      AAA    999 77 53 1-
                 9999   7777                7777   9999   AAAAAAAAAAAA  99 7 5 3 --
                999   77777              77777  9999   AAAA          AAA  9 7 53  --
              999   77777             77777   999   AAA     BBBBBBB   AA 9 7 5   ---
 3.000E+01  + 9   777777           77777   999   AAA    BBBBBBBBBBB  AA 9 7 53 -----
                 7777777       7777777   9999   AAA   BBBBB      BBB  AA 9 7  31------
               77777777     7777777   999   AAA   BBBB         BBB  A  97 53 --------
              7777777777777777777    9999   AAA   BBBB          BB  AA 9  5  ---------
              77777777777777777   999   AAAA   BBBB        BB  AA 9 75   -----------
              7777777777777   9999   AAA   BBBB        BBB  A  97 53 --------------
 3.500E+01  + 777777777   9999   AAA   BBBBB      BBB  AA 9 7 531-----------------
              777777   9999   AAAA   BBBBBB   BBBBB  A  9 75  1------------------
              777    9999   AAAA   BBBBBBBBBBBBBB  AA 9 7 53 --------------------
                 99999   AAAA   BBBBBBBBBBBBBB  AA 9 7 5  1-----------------------
               9999   AAAA   BBBBBBBBBBBBBB  AA 99 75 31-------------------------
              9   AAAA   BBBBBBB   BBBBB  A  9 7 53  ----------------------------
 4.000E+01  + AAAA    BBBBB          BBBB  AA 9 7 5  1-----------------------------
```

Print-out of
Quartic
Trend Surface

```
XM =          50.0000000000            YM =          15.0000000000

XA =          15.0000000000            YA =          40.0000000000

XVAL =           -.5000000000          YVAL =            .8333333333

X-SIZE =           71         Y-SIZE =          31

MULTIPLICATIVE FACTOR =          2.0000000000

CALCULATED VALUE = PRINTED VALUE /  .200E+01


MAP FOR HALF CONFIDENCE INTERVAL     95.046 %

           5.000E+01           4.000E+01           3.000E+01           2.000E+01

              +         +         +         +         +         +         +         +

 1.500E+01  + +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
              ++++++++++++++++++++++++ E D  CCCCCCCCCCC  DD  E ++++++++++++++++++++++
              +++++++++++++++++++++EDC B AA             AAA  B  C  E ++++++++++++++++++
              ++++++++++++++++++  CB  9 777      777777   99 A B C DE++++++++++++++++
              ++++++++++++++++  CBA9 7 555      55555555   77 9 A B CD  +++++++++++++
              +++++++++++++++  BA9 7 5                    55  7  9 A  C   +++++++++++
 2.000E+01  + +++++++++++++ D   97 5          3333333333   55 77 9 A BC  E+++++++++
              ++++++++++++E BA9 7 5          3333333333333   55 77 9 A BC  E+++++++
              +++++++++++ CBA97 55         333333333333333333  55  7 9 A B  DF+++++
              +++++++++ D   97 55        33333333333333333333   55  7  9 AB CDE+++
              ++++++++E      7 5       33333333333   333333333333   55  77 9 A  CDE+
              +++++++E  A  7 5      3333333333     333333333333    555  7    A
 2.500E+01  + ++++++E BA9 7 5    33333333333     33333333333333      55 7 9    D
              +++++E BA9  55    33333333333     33333333333333        5 7   A
              ++++E BA9 755    333333333333     33333333333333        5 7    CD
              +++E     7 5   3333333333333333           333333333        5 7    D
              ++ D    7 5   3333333333333333            33333333      5 7 9ABC ++
              ++ CBA97 5  333333333333333333            33333333    5  79    E+++
 3.000E+01  + +E     755 3333333333333       333333333333333333333  55 7  ABC  ++++
              + CBA97 5  33333333333       33333333333333333333  5 7 9A  DE++++++
               D    7 5 3333333333       3333333333333333333  5    9     ++++++++
              E  A    5  3333333333     333333333333333333   55 79   B  ++++++++++
               CBA9   5  3333333333333333333333333333333       5  79   BC  +++++++++++
               CBA97 5 333333333333333333333333333         55 7 9      +++++++++++++
 3.500E+01  + D     75   3333                             5555  7 9 AB   ++++++++++++++++
                A97 5     5555                           777 99 AB    +++++++++++++++++
              CB   7 5555  777    9999999999999999999   A  B CD  ++++++++++++++++++++
                  7  77 99   AA    BBBBBBBBBBBBBB   CC   D E+++++++++++++++++++++++++
                 999   A B  CC  DDD        EEEEEE    ++++++++++++++++++++++++++++++++
              C BBB C D E  +++++++++++++++++++++++++++++++++++++++++++++++++++++++++
 4.000E+01  +    E +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
```

ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

ANALYSIS OF VARIANCE TABLE OF POLYNOMIAL TREND SURFACE OF DEGREE 5

SOURCE OF VARIATION	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	CALCULATED F - VALUE	PROBABILITY IN %
TOTAL(UNCORRECTED)	1.384978350E+06	174			
MEAN(B(1))	1.384437520E+06	1			
TOTAL(CORRECTED)	5.408299427E+02	173			
RESIDUAL	2.510793053E+02	153	1.6410412109		
REGRESSION ENTIRE EQN /B(1)	2.897506374E+02	20	14.4875318721	8.8282559732	100.000
1 DEGREE LESS/B(1)	2.809313256E+02	14			
PARTIAL EQN	8.819311812E+00	6	1.4698853021	.8957028576	50.030

MULTIPLE CORRELATION COEFFICIENT SQUARED = .5357518

ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

	ESTIMATES OF COEFFICIENTS	STANDARD DEVIATIONS
1	3.3337886422083E+02	9.8873741901465E+02
2	-8.2773494887719E+01	8.3315995407663E+01
3	6.3083523479854E+01	1.1030905361707E+02
4	2.9755787866044E+00	2.8388914874756E+00
5	4.9503603734169E+00	6.2483839379365E+00
6	-8.2403051772317E+00	6.8496704292239E+00
7	-3.6406553249660E-02	5.8239728248433E-02
8	-2.0093240728164E-01	1.5213376909911E-01
9	-1.5703386372035E-02	1.8437911197767E-01
10	3.2317140670723E-01	2.5508210402275E-01
11	3.5526658631071E-04	8.1286032960967E-04
12	1.1089932214661E-03	1.8195211336110E-03
13	5.1447998118183E-03	3.4515537144593E-03
14	-4.0057653791477E-03	3.4026011786209E-03
15	-4.8178814016888E-03	4.9151642233592E-03
16	-4.7495273104811E-06	6.5322320427319E-06
17	1.3300426124913E-05	2.1273122369098E-05
18	-4.6601279240321E-05	4.5634194823449E-05
19	-1.2401289695834E-05	5.2537739419977E-05
20	5.0792640719214E-05	4.8242558101583E-05
21	2.1536641990405E-05	4.0805840691289E-05

```
ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

 XM =        50.0000000000             YM =        15.0000000000

 XA =        15.0000000000             YA =        40.0000000000

 XVAL =         -.5000000000           YVAL =             .8333333333

 X-SIZE =          71         Y-SIZE =          31

ADDITIVE CONSTANT =      -170.0000000000

MULTIPLICATIVE FACTOR =          2.0000000000

CALCULATED VALUE = ( PRINTED VALUE -  -.170E+03 ) /   .200E+01


MAP FOR PREDICTED VALUES

          5.000E+01           4.000E+01           3.000E+01           2.000E+01

              +         +         +         +         +         +         +         +

 1.500E+01  + ---------------    9  AAAAA  9  7 5  3 11 ---------- 1  5    CD +++++++
              --------------1357 9 AA  AAAA 99 77 55  33     1    3  5  9       ++++++++
              ---------------   79  AAA AAAA  99  7  555    33     55 7 9  BC    +++++++
              -------------135  9  AAAAAAA   99  77   5555555555  77 9 A B  DE+++++++
              -------------  57 99          999   777   555555   77  9  A  C DE++++++
              ------------13  7  999    9999   7777     55       77  99 AA B  D E++++
 2.000E+01  + -----------   5 77  9999999   7777                777  99  A BB C  E +
              ----------  3 5 77          77777          55     7777  999  AA R  C D
              ---------  3 5  7777   7777777        55555       77777    999  AA   B
              ---------1 3 5   7777777777        555555       77777777     9999
              --------13  5   7777777        5555555      7777777777777777777777
              -----1 3 55   777777        5555555      777777777777777777777
 2.500E+01  + ---1 3 5   7777777         55555       777777            7777    555
              -1 3 5  7777777777                   7777       999999999    777    55
                55 77777 7777777                7777    99999         9999    77   55  3
              5  77        777777             7777   9999      AAAAAAA     999   77   55
               77           77777          7777    999    AAAA          AAAA  99   77 55
              7            777777       77777    999    AAA     BBBBBBB    AA  99 77   55
 3.000E+01  + 7           7777777777777777    999    AAA   BBBBBBBBBBBBB   AA 99  7   55
              77        777777777777777     999    AA   BBBB          BBB  AA  99 77 55
                7777777777777777777777    999  AAA   BBB           BBB  AA  9  7   5  33
              55       77777777777777    999   AAA   BBBB         BBB  AA 99 77 5   33  1
                555       777777     9999   AAA   BBBB         BBBB  AA 99 7  5 3   1 ----
               3  555     777     999    AAAA   BBBBB      BBBB  AA  9 77 5 3 1 ---------
 3.500E+01  + 1 33 55   77   999    AAA    BBBBB      BBBB  AA   9 77 5 31 --------------
               1 3 5 77 99   AAA     BBB            BB  AA 99 7 5   1-----------------
              -13 5 7 9 AA  BB  CCCC          CCC  BB A   9 75 31 --------------------
              13  79 A B  C  DD              DD  CC B A 97  31------------------------
              3579A  C D E  +++++++++++++ EE D C   A9   31-------------------------
              79   DE +++++++++++++++++++++++  DC      31---------------------------
 4.000E+01  + B  ++++++++++++++++++++++++++++    A9 3 ------------------------------
```

Print-out of
Quintic
Trend Surface

```
XM =          50.0000000000            YM =          15.0000000000

XA =          15.0000000000            YA =          40.0000000000

XVAL =           -.5000000000          YVAL =            .8333333333

X-SIZE =            71          Y-SIZE =           31

MULTIPLICATIVE FACTOR =           2.0000000000

CALCULATED VALUE = PRINTED VALUE /   .200E+01


MAP FOR HALF CONFIDENCE INTERVAL      95.028 %

          5.000E+01           4.000E+01           3.000E+01           2.000E+01

              +         +         +         +         +         +         +         +

1.500E+01  +  +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
              +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
              ++++++++++++++++++++++++++E DD DDD  E  ++++++++++++++++++++++++++++++++
              +++++++++++++++++++++++D  9999999   AAA BBC DE+++++++++++++++++++++++++
              ++++++++++++++++++++ C              77777  9A    E+++++++++++++++++++++
              ++++++++++++++++++   97 5555     5555555555 7 9AB  ++++++++++++++++++++
2.000E+01  +  +++++++++++++++++DB    55    55555555      55 7 9ABC  +++++++++++++++++
              +++++++++++++++ C 97  55555555555            55 7 9ABCDE+++++++++++++++
              ++++++++++++++E A97  55555555                555   7 9    +++++++++++++
              +++++++++++++  A 7   555555     333333        555    7 9A   E++++++++++
              ++++++++++++DB  7   555555      333333333     55555    77 9A CD++++++++
              +++++++++++DB  7   5555555       333333333     55555       7      +++++
2.500E+01  +  ++++++++++DB  7  55555555         33333333     55555          7  B ++++
              +++++++++DB    555555555           33333        5555         5  7 BD+++
              ++++++++DB   555555555                           5555           79 D+++
              +++++++     55555555                               555    7777779ACE+++
              ++++++E A 755555555       333333333                 55   777 9AB   ++++
              ++++++  97 5555555      333333333333                55  7    AB E++++++
3.000E+01  +  +++++ B   55555555     3333333333333               55  7   AB E++++++++
              ++++ C 97 555555      3333333333333               55 7 9AB E+++++++++++
              ++++     55          3333333333333      555555555555 7 BCE+++++++++++++
              ++++DB 7 5        33333333         555555    5555    9 C ++++++++++++++
              ++++  975       333333           5555            79A  +++++++++++++++++
              ++++  975                      55555          7 9A  +++++++++++++++++++
3.500E+01  +  ++++  97 55555            555555555     7777 9AB E+++++++++++++++++++++
              +++ C 97   7 99    AA     9999       99   A BCD +++++++++++++++++++++++
              +++ B 77 9A  C  DD      DDD   CCCCC  DD E++++++++++++++++++++++++++++++
              ++    9 CD ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
              ++CAACE++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
              +   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4.000E+01  +  +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
```

```
ENSTATITE CONTENT(IN PERCENT) OF ORTHOPYROXENE, MT. ALBERT INTRUSION, GASPE, QUE

POLYNOMIAL TREND SURFACE OF DEGREE          6

THE INPUT MATRIX HAS A RANK           20

VARIABLE NUMBER        22           IS ELIMINATED

VARIABLE NUMBER        23           IS ELIMINATED

VARIABLE NUMBER        24           IS ELIMINATED

VARIABLE NUMBER        25           IS ELIMINATED

VARIABLE NUMBER        26           IS ELIMINATED

VARIABLE NUMBER        27           IS ELIMINATED

VARIABLE NUMBER        28           IS ELIMINATED

THE RUN IS TERMINATED DUE TO MULTICOLLINEARITY
```

APPENDIX IV

Listing of Program

```
      PROGRAM TREND(INPUT,OUTPUT,TAPE99,TAPE10)
C        *         *         *         *         *         *         *
C
C                  T R E N D     S U R F A C E     A N A L Y S I S
C
C     C. F. CHUNG  AND F. P. AGTERBERG, G. S. C., E. M. R.
C
C        *         *         *         *         *         *         *
      DIMENSION A(45,45),BB(45,45),B(45),D(45),F(45),X(45),NS(45)
      DIMENSION NAME(8),FMT(3),ZZ(45,45),ATT(8),AFF(8)
C    *      *     *     *
C     READ IN DATA
C    *      *     *     *
  100 READ 1,NDEG,ITER,ITABLE,IMAP,ICONF,IRES
    1 FORMAT(10I1)
      IF(NDEG.EQ.0)GO TO 999
      READ 2,NAME
    2 FORMAT(8A10)
      READ 3,AFF
      READ 3,ATT
    3 FORMAT(8F10.0)
      READ 5,XA,XM,YA,YM,XVAL,AA,AM,LINE
      READ 4,ITAPE,ILOG,IORDER,FMT
    4 FORMAT(3I1,7X,3A10)
    5 FORMAT( 7F10.0,I10)
      ALPM=AM
      IN=1
      REWIND 10
      IF(ITAPE.EQ.1)GO TO 230
      N=0
  110 GO TO(120,130,140,150,160,170),IORDER
  120 READ FMT,XI,YI,ZI
      GO TO 180
  130 READ FMT,XI,ZI,YI
      GO TO 180
  140 READ FMT,YI,XI,ZI
      GO TO 180
  150 READ FMT,YI,ZI,XI
      GO TO 180
  160 READ FMT,ZI,XI,YI
      GO TO 180
  170 READ FMT,ZI,YI,XI
  180 IF((XI.EQ.999.0).AND.(YI.EQ.999.0).AND.(ZI.EQ.999.0))GO TO 330
  190 CALL POLA(NDEG,IN,XI,YI,IOUT,X)
      IF(ILOG-1)200,210,220
  200 X(IOUT)=ZI
      GO TO 225
  210 IF(ZI.LE.0.0)GO TO 350
      X(IOUT)=ALOG10(ZI)
      GO TO 225
  220 IF(ZI.LE.0.0)GO TO 350
      X(IOUT)=ALOG(ZI)
  225 N=N+1
      WRITE(10)N,(X(K),K=1,IOUT)
      IF(ITAPE.EQ.1)GO TO 240
      GO TO 110
  230 REWIND 99
      N=0
  240 GO TO(250,260,270,280,290,300),IORDER
  250 READ(99,FMT)XI,YI,ZI
      GO TO 310
  260 READ(99,FMT)XI,ZI,YI
      GO TO 310
  270 READ(99,FMT)YI,XI,ZI
```

```
      GO TO 310
  280 READ(99,FMT)YI,ZI,XI
      GO TO 310
  290 READ(99,FMT)ZI,XI,YI
      GO TO 310
  300 READ(99,FMT)ZI,YI,XI
  310 IF(EOF(99))330,320
  320 GO TO 180
  330 M=IOUT-1
      REWIND 10
      IF(ITAPE.EQ.1)REWIND 99
C     *     *     *     *
C     PRINT ALL INFORMATION FOR INPUT
C     *     *     *     *
  340 PRINT 6,NAME
    6 FORMAT(1H1,8A10)
      PRINT 7,N
    7 FORMAT(1H0,"NUMBER OF OBSERVATIONS    N  = ",I10)
      PRINT 8,IOUT
    8 FORMAT(1H0,"NUMBER OF INDEPENDENT VARIABLES INCLUDING CONSTANT
     1M  =  ",I10)
      PRINT 9
    9 FORMAT(1H0)
      PRINT 10
   10 FORMAT(1H0,"    ACCORDING TO   INPUT(SEE INPUT DATA DESCRIPTION IN
     1 TEXT)")
      PRINT 11,NDEG,ITER,ITABLE,IMAP,ICONF,IRES
   11 FORMAT(1H0,"DEGREES OF POLYNOMIAL   NDEG = ",I5,/1X,"ITER   =  ",
     1I5,/1X,"ITABLE =  ",I5,/,1X,                "IMAP   =  ",I5,/,
     21X,"ICONF  =  ",I5,/1X,"IRES   =  ",I5)
      IF(ITABLE.EQ.0)GO TO 880
      PRINT 12,ATT,AFF
   12 FORMAT(1H0,"T - VALUE FROM TABLE   =  ",8F10.5,/1X,"F - VALUE FROM
     1 TABLE   =  ",8F10.5)
      GO TO 870
  880 PRINT 13,ATT(1),AFF(1)
   13 FORMAT(1H0,"LEVEL OF SIGNIFICANCE FOR T-VALUE =",F10.5,/,1X,
     1  "LEVEL OF SIGNIFICANCE FOR F-VALUE =",F10.5)
  870 PRINT 14,ITAPE,IORDER,ILOG,FMT
   14 FORMAT(1H0,"ITAPE    =     ",I5,/1X,"IORDER    =     ",I5,/1X,
     1"ILOG     =     ",I5,/1X,"FORMAT    =     ",3A10)
      PRINT 15,XM,YM,XA,YA,XVAL,LINE,AA,AM
   15 FORMAT(1H0,"XM = ",F20.10,20X,"YM = ",F20.10,/1X,"XA = ",F20.10,
     120X,"YA = ",F20.10,/1X,"XVAL = ",F18.10,20X,"LINE = ",I10,/1X,
     2"AA = ",F20.10,20X,"AM = ",F20.10)
      IF(ITABLE.EQ.1)GO TO 830
      ALPT=ATT(1)
      ALPF=AFF(1)
  830 PRINT 9
      L=IOUT
      DO 500 I=1,L
      NS(I)=-1
      B(I)=0.0
      D(I)=0.0
      F(I)=0.0
      DO 500 J=1,L
      A(I,J)=0.0
      BB(I,J)=0.0
      ZZ(I,J)=0.0
  500 CONTINUE
      DO 515 K=1,N
      READ(10)NN,(X(KK),KK=1,L)
      DO 510 I=1,L
      D(I)=D(I)+X(I)
      F(I)=F(I)+X(I)*X(I)
      DO 510 J=1,L
      A(I,J)=A(I,J)+X(I)*X(J)
  510 CONTINUE
  515 CONTINUE
      SSN=0.0
      YSS=F(L)
```

```
      XNN=FLOAT(N)
      DO 520 I=1,L
      D(I)=D(I)/XNN
      F(I)=F(I)-XNN*D(I)*D(I)
  520 CONTINUE
      DO 525 I=1,L
      DO 525 J=1,L
      XXX=A(I,J)-XNN*D(I)*D(J)
      YYY=F(I)*F(J)
      A(I,J)=XXX/SQRT(YYY)
      BB(I,J)=A(I,J)
  525 CONTINUE
      VAR=F(L)/(XNN-1.0)
      REWIND 10
      PRINT 51,D(L),VAR
   51 FORMAT(1H0,"MEAN OF DEPENDENT VARIABLE    =  ",1PE20.13,//1X,
     1             "VARIANCE OF DEPENDENT VARIABLE =",1PE20.13)
      FFFF=F(L)
      ND=NDEG
      IF(ITER.EQ.0)GO TO 712
      ND=0
      M=0
  710 ND=ND+1
      M=M+ND+1
      IF(ND.GT.NDEG)GO TO 360
  712 PRINT 6,NAME
      PRINT 81,ND
   81 FORMAT(1H0,"POLYNOMIAL TREND SURFACE OF DEGREE  ",I10)
      CALL STEPINV(L,M,A,BB,NS)
      BBBB=A(L,L)
      KOLA=0
      DO 715 I=1,M
      IF(NS(I).EQ.1)GO TO 715
      J=I+1
      KOLA=KOLA+1
      PRINT 82,J
   82 FORMAT(1H0,"VARIABLE NUMBER",I10,10X,"IS ELIMINATED")
  715 CONTINUE
      IF(KOLA.GE.2)PRINT 83
   83 FORMAT(1H0,"THE RUN IS TERMINATED DUE TO MULTICOLLINEARITY")
      IF(KOLA.GE.2)GO TO 999
      PRINT 6,NAME
      CALL ANOVA(ND,N,M,ITER,SSN,YSS,FFFF,BBBB,SS)
      ZZ(1,1)=0.0
      DO 730 I=1,M
      J=I+1
      ZZ(1,J)=0.0
      IF(NS(I).EQ.-1)GO TO 720
      FFF=F(L)/F(I)
      B(J)=A(I,L)*SQRT(FFF)
      GO TO 730
  720 B(J)=0.0
  730 CONTINUE
      CC=D(L)
      DO 740 I=1,M
      J=I+1
      CC=CC-D(I)*B(J)
  740 CONTINUE
      B(1)=CC
      DO 750 I=1,M
      IF(NS(I).EQ.-1)GO TO 750
      KI=I+1
      DO 760 J=1,M
      IF(NS(J).EQ.-1)GO TO 760
      KJ=J+1
      FFF=F(I)*F(J)
      ZZ(KI,KJ)=SS*A(I,J)/SQRT(FFF)
  760 CONTINUE
  750 CONTINUE
      DO 780 I=1,M
      IF(NS(I).EQ.-1)GO TO 780
```

```
      KI=I+1                                                            TREN 207
      DO 770 J=1,M                                                      TREN 208
      IF(NS(J).EQ.-1)GO TO 770                                          TREN 209
      KJ=J+1                                                            TREN 210
      ZZ(1,1)=ZZ(1,1)+ZZ(KI,KJ)*D(I)*D(J)                               TREN 211
      ZZ(1,KI)=ZZ(1,KI)-ZZ(KI,KJ)*D(J)                                  TREN 212
  770 CONTINUE                                                          TREN 213
      ZZ(KI,1)=ZZ(1,KI)                                                 TREN 214
  780 CONTINUE                                                          TREN 215
      ZZ(1,1)=ZZ(1,1)+ SS/XNN                                           TREN 216
      S=SQRT(SS)                                                        TREN 217
      PRINT 6,NAME                                                      TREN 218
      PRINT 53                                                          TREN 219
   53 FORMAT(1H0," ESTIMATES OF COEFFICIENTS",20X,"STANDARD DEVIATIONS")TREN 220
      NGM=M+1                                                           TREN 221
      DO 535 K=1,NGM                                                    TREN 222
      FFF=SQRT(ZZ(K,K))                                                 TREN 223
      PRINT 61,K,B(K),FFF                                               TREN 224
   61 FORMAT(1H0,I5,5X,1PE20.13,20X,1PE20.13)                           TREN 225
  535 CONTINUE                                                          TREN 226
  570 IF(IMAP.EQ.0)GO TO 580                                            TREN 227
C      *     *     *                                                    TREN 228
C     CALL SUBROUTINE MAP                                               TREN 229
C      *     *     *                                                    TREN 230
      PRINT 6,NAME                                                      TREN 231
      CALL MAP(XA,XM,YA,YM,XVAL,LINE,NGM,ND,AA,AM,B)                    TREN 232
  580 IF(ICONF.EQ.0)GO TO 590                                           TREN 233
C      *     *     *                                                    TREN 234
C     CALL SUBROUTINE CONF                                              TREN 235
C      *     *     *                                                    TREN 236
      PRINT 6,NAME                                                      TREN 237
      NGL=N-M-1                                                         TREN 238
      NGM=M+1                                                           TREN 239
      IF(ITABLE.EQ.1)GO TO 810                                          TREN 240
      CALL INVFDN(ALPF,NGM,NGL,AAAA)                                    TREN 241
      GO TO 850                                                         TREN 242
  810 AAAA=AFF(ND)                                                      TREN 243
  850 CALL PROB(AAAA,NGM,NGL,Q)                                         TREN 245
      CALL CONF(XA,XM,YA,YM,XVAL,LINE,Q,AAAA,ALPM,NGM,ND,ZZ)            TREN 246
  590 IF(IRES.EQ.0)GO TO 630                                            TREN 247
      PRINT 6,NAME                                                      TREN 248
      IF(ITABLE.EQ.1)GO TO 820                                          TREN 249
      ALPP=ALPT/2.0                                                     TREN 250
      CALL INVTDN(ALPP,NGL,AKK)                                         TREN 251
      GO TO 840                                                         TREN 252
  820 AKK=ATT(ND)                                                       TREN 253
  840 AKM=AKK*AKK                                                       TREN 254
      NGM=1                                                             TREN 255
      CALL PROB(AKM,NGM,NGL,Q)                                          TREN 256
      PRINT 56,Q                                                        TREN 257
      DO 620 K=1,N                                                      TREN 258
      READ(10)NN,(X(KK),KK=1,L)                                         TREN 259
      SUM=ZZ(1,1)                                                       TREN 260
      NGM=M+1                                                           TREN 261
      DO 601 I=2,NGM                                                    TREN 262
      J=I-1                                                             TREN 263
      SUM=SUM+2.0*ZZ(1,I)*X(J)                                          TREN 264
  601 CONTINUE                                                          TREN 265
      DO 600 I=1,M                                                      TREN 266
      KI=I+1                                                            TREN 267
      DO 600 J=1,M                                                      TREN 268
      KJ=J+1                                                            TREN 269
      SUM=SUM+ZZ(KI,KJ)*X(I)*X(J)                                       TREN 270
  600 CONTINUE                                                          TREN 271
      SUM=S*S-SUM                                                       TREN 272
      SUM=SQRT(SUM)*AKK                                                 TREN 273
      ZZZ=B(1)                                                          TREN 274
      DO 610 I=1,M                                                      TREN 275
      J=I+1                                                             TREN 276
      ZZZ=ZZZ+B(J)*X(I)                                                 TREN 277
  610 CONTINUE                                                          TREN 278
```

```
      EE=X(L)-ZZZ                                                       TREN 279
      PRINT 57,X(L),ZZZ,EE,SUM                                          TREN 280
   57 FORMAT(1H ,3(1PE15.8, 5X),6X,1PE15.8)                             TREN 281
  620 CONTINUE                                                          TREN 282
      REWIND 10                                                         TREN 283
   56 FORMAT(1H0,"   OBSERVED             CALCULATED            RESIDUAL  TREN 284
     1           CONFIDENCE INTERVAL",F10.3," %")                       TREN 285
  630 CONTINUE                                                          TREN 286
      IF(ITER.EQ.0)GO TO 360                                            TREN 287
      GO TO 710                                                         TREN 288
  350 NFG=N+1                                                           TREN 289
      PRINT 16,NFG                                                      TREN 290
   16 FORMAT(1H0,"ERROR IN",I5,"  TH CARD",/1X,"ZI=0,HENCE LOG(ZI)=UNDEFTREN 291
     1INE",1X,"IGNORING THIS SAMPLE AND PROCEEDING")                    TREN 292
      IF(ITAPE.EQ.1)GO TO 240                                           TREN 293
      GO TO 110                                                         TREN 294
  360 GO TO 100                                                         TREN 295
  999 STOP                                                              TREN 296
      END                                                               TREN 297
      SUBROUTINE ANOVA(NDEG,N,M,IR,SSN,YSS,F,A,SR)                      ANOV 001
      NNN=1                                                             ANOV 002
      PRINT 1,NDEG                                                      ANOV 003
    1 FORMAT(1H0," ANALYSIS  OF  VARIANCE  TABLE   OF   POLYNOMIAL ",   ANOV 004
     1   "TREND SURFACE OF DEGREE",I5)                                  ANOV 005
      PRINT 2                                                           ANOV 006
    2 FORMAT(1H0,"    SOURCE  OF              SUM  OF        DEGREES OF ANOV 007
     1    MEAN            CALCULATED         PROBABILITY")              ANOV 008
      PRINT 3                                                           ANOV 009
    3 FORMAT(1H ,"    VARIATION               SQUARES        FREEDOM    ANOV 010
     1    SQUARE            F - VALUE           IN  %")                 ANOV 011
    4 FORMAT(1H0,"TOTAL(UNCORRECTED)",3X,1PE17.9,3X,I7)                 ANOV 012
    5 FORMAT(1H0,"MEAN(B(1))        ",3X,1PE17.9,3X,I7)                 ANOV 013
    6 FORMAT(1H0,"TOTAL(CORRECTED)  ",3X,1PE17.9,3X,I7)                 ANOV 014
    7 FORMAT(1H0,"RESIDUAL          ",3X,1PE17.9,3X,I7,3X,0PF17.10)     ANOV 015
    8 FORMAT(1H0,"    REGRESSION    ")                                  ANOV 016
    9 FORMAT(1H ,"ENTIRE EQN /B(1)  ",3X,1PE17.9,3X,I7,3X,0PF17.10,3X,  ANOV 017
     1   F17.10,3X,F17.3)                                               ANOV 018
   10 FORMAT(1H0,"1 DEGREE LESS/B(1)",3X,1PE17.9,3X,I7)                 ANOV 019
   11 FORMAT(1H0,"PARTIAL EQN       ",3X,1PE17.9,3X,I7,3X,0PF17.10,3X,  ANOV 020
     1   F17.10,3X,F17.3)                                               ANOV 021
   12 FORMAT(1H0)                                                       ANOV 022
   13 FORMAT(1H0,"MULTIPLE CORRELATION COEFFICIENT SQUARED = ",F15.7)   ANOV 023
      PRINT 4,YSS,N                                                     ANOV 024
      SV=YSS-F                                                          ANOV 025
      PRINT 5,SV,NNN                                                    ANOV 026
      LLL=N-1                                                           ANOV 027
      PRINT 6,F,LLL                                                     ANOV 028
      LL=N-M-1                                                          ANOV 029
      SSR=F*A                                                           ANOV 030
      SSE=F-SSR                                                         ANOV 031
      SR=SSR/FLOAT(LL)                                                  ANOV 032
      PRINT 7,SSR,LL,SR                                                 ANOV 033
      LLL=M                                                             ANOV 034
      PRINT 8                                                           ANOV 035
      SE=SSE/FLOAT(LLL)                                                 ANOV 036
      FE=SE/SR                                                          ANOV 037
      CALL PROB(FE,LLL,LL,QE)                                           ANOV 038
      PRINT 9,SSE,LLL,SE,FE,QE                                          ANOV 039
      IF((IR.EQ.0).OR.(NDEG.EQ.1))GO TO 14                              ANOV 040
      LLL=M-NDEG-1                                                      ANOV 041
      PRINT 10,SSN,LLL                                                  ANOV 042
      LLL=NDEG+1                                                        ANOV 043
      SSM=SSE-SSN                                                       ANOV 044
      SM=SSM/FLOAT(LLL)                                                 ANOV 045
      FM=SM/SR                                                          ANOV 046
      CALL PROB(FM,LLL,LL,QM)                                           ANOV 047
      PRINT 11,SSM,LLL,SM,FM,QM                                         ANOV 048
   14 RR=SSE/F                                                          ANOV 049
      SSN=SSE                                                           ANOV 00
      SSN=SSE                                                           ANOV 050
      PRINT 12                                                          ANOV 051
```

```
      PRINT 13,RR
      RETURN
      END
      SUBROUTINE MAP(XA,XI,YA,YI,XVAL,LINE,M,NDEG,AA,AM,C)
      DIMENSION L(23),LL(130),C(45),X(45),AP(6)
      DATA L/1H ,1H1,1H ,1H3,1H ,1H5,1H ,1H7,1H ,1H9,1H ,1HA,1H ,1HB,
     1       1H ,1HC,1H ,1HD,1H ,1HE,1H ,1H+,1H-/
      PRINT 1
    1 FORMAT(1H0)
      PRINT 2,XI,YI,XA,YA
    2 FORMAT(1H0," XM = ",F20.10,10X,"   YM = ",F20.10,//1X,
     1          " XA = ",F20.10,10X,"   YA = ",F20.10)
      XVAL=ABS(XVAL)
      XLINE=FLOAT(LINE)
      YVAL=10.0*XVAL/XLINE
      IF(XI.GE.XA)XVAL=-XVAL
      IF(YI.GE.YA)YVAL=-YVAL
      PRINT 3,XVAL,YVAL
    3 FORMAT(1H0," XVAL = ",F20.10,10X," YVAL = ",F20.10)
      T=(XA-XI)/XVAL
      T=T+1.5
      NP=INT(T)
      T=(YA-YI)/YVAL
      T=T+1.5
      NQ=INT(T)
      IF(NP.GT.110)NP=110
      PRINT 4,NP,NQ
    4 FORMAT(1H0," X-SIZE = ",I10,10X,"Y-SIZE = ",I10)
      PRINT 7,AA,AM
    7 FORMAT(1H0,"ADDITIVE CONSTANT = ",F20.10,//1X,
     1    "MULTIPLICATIVE FACTOR = ",F20.10)
      PRINT 11,AA,AM
   11 FORMAT(1H0,"CALCULATED VALUE = ( PRINTED VALUE - ",E10.3," ) / ",
     1    E10.3)
      PRINT 1
      PRINT 5
    5 FORMAT(1H0,"MAP FOR PREDICTED VALUES")
      DO 145 I=1,NP,20
      J=(I+19)/20
      AP(J)=FLOAT(I-1)*XVAL+XI
  145 CONTINUE
      PRINT 8,(AP(K),K=1,J)
    8 FORMAT(1H0,10X,6(1PE10.3,10X))
      PRINT 9,(L(22),K=1,NP,10)
    9 FORMAT(1H0,15X,11(A1,9X))
      PRINT 21
   21 FORMAT(1H )
      JJ=-1
      IN=2
      X(1)=1.0
      DO 170 J=1,NQ
      JJ=JJ+1
      UJ=FLOAT(J-1)
      DO 160 I=1,NP
      UI=FLOAT(I-1)
      XX=XI+UI*XVAL
      YY=YI+UJ*YVAL
      CALL POLA(NDEG,IN,XX,YY,IOUT,X)
      XL=0.0
      DO 150 K=1,M
      XL=XL+X(K)*C(K)
  150 CONTINUE
      XL=AM*XL+AA
      IF(XL)210,220,220
  210 LL(I)=L(23)
      GO TO 160
  220 XL=XL+1.5
      ID=INT(XL)
      IF(ID.GE.22) ID=22
      LL(I)=L(ID)
  160 CONTINUE
```

```
      IF((JJ.EQ.0).OR.(JJ.EQ.LINE))GO TO 165
      PRINT 6,(LL(K),K=1,NP)
    6 FORMAT(1H ,15X,110A1)
      GO TO 170
  165 PRINT 10,YY, L(22),(LL(K),K=1,NP)
   10 FORMAT(1H ,1PE10.3,2X,A1,2X,110A1)
      JJ=0
  170 CONTINUE
      RETURN
      END
      SUBROUTINE CONF(XA,XI,YA,YI,XVAL,LINE,Q2,A2,AM2,M,NDEG,A)
      DIMENSION L(22),X(45),A(45,45),LL(110),AP(6)
      DATA L/1H ,1H1,1H ,1H3,1H ,1H5,1H ,1H7,1H ,1H9,1H ,1HA,1H ,1HB,
     1       1H ,1HC,1H ,1HD,1H ,1HE,1H ,1H+/
      PRINT 1
    1 FORMAT(1H0)
      PRINT 2,XI,YI,XA,YA
    2 FORMAT(1H0," XM = ",F20.10,10X,"   YM = ",F20.10,//1X,
     1            " XA = ",F20.10,10X,"   YA = ",F20.10)
      XVAL=ABS(XVAL)
      XLINE=FLOAT(LINE)
      YVAL=10.0*XVAL/XLINE
      IF(XI.GT.XA)XVAL=-XVAL
      IF(YI.GT.YA)YVAL=-YVAL
      PRINT 3,XVAL,YVAL
    3 FORMAT(1H0," XVAL = ",F20.10,10X," YVAL = ",F20.10)
      T=(XA-XI)/XVAL
      T=T+1.5
      NP=INT(T)
      T=(YA-YI)/YVAL
      T=T+1.5
      NQ=INT(T)
      IF(NP.GT.110)NP=110
      PRINT 4,NP,NQ
    4 FORMAT(1H0," X-SIZE = ",I10,10X,"Y-SIZE = ",I10)
      PRINT 7,AM2
    7 FORMAT(1H0,"MULTIPLICATIVE FACTOR = ",F20.10)
      PRINT 11,AM2
   11 FORMAT(1H0,"CALCULATED VALUE = PRINTED VALUE /",E10.3)
      PRINT 1
      PRINT 5,Q2
    5 FORMAT(1H0,"MAP FOR HALF CONFIDENCE INTERVAL ",F10.3," *")
      DO 145 I=1,NP,20
      J=(I+19)/20
      AP(J)=FLOAT(I-1)*XVAL+XI
  145 CONTINUE
      PRINT 8,(AP(K),K=1,J)
    8 FORMAT(1H0,10X,6(1PE10.3,10X))
      PRINT 9,(L(22),K=1,NP,10)
    9 FORMAT(1H0,15X,11(A1,9X))
      PRINT 21
   21 FORMAT(1H )
      JJ=-1
      IN=2
      X(1)=1.0
      DO 170 J=1,NQ
      JJ=JJ+1
      UJ=FLOAT(J-1)
      DO 160 I=1,NP
      UI=FLOAT(I-1)
      XX=XI+UI*XVAL
      YY=YI+UJ*YVAL
      CALL POLA(NDEG,IN,XX,YY,IOUT,X)
      XL=0.0
      DO 150  K1=1,M
      DO 150  K2=1,M
      XL=XL+X(K1)*A(K1,K2)*X(K2)
  150 CONTINUE
      XL=A2*XL*FLOAT(M)
      XL=SQRT(XL)
      XL=XL*AM2+1.5
```

```
      ID=INT(XL)                                                        CONF 062
      IF(ID.GE.22)ID=22                                                 CONF 063
      LL(I)=L(ID)                                                       CONF 064
  160 CONTINUE                                                          CONF 065
      IF((JJ.EQ.0).OR.(JJ.EQ.LINE))GO TO 165                            CONF 066
      PRINT 6,(LL(K),K=1,NP)                                            CONF 067
    6 FORMAT(1H ,15X,110A1)                                             CONF 068
      GO TO 170                                                         CONF 069
  165 PRINT 10,YY,L(22),(LL(K),K=1,NP)                                  CONF 070
   10 FORMAT(1H ,1PE10.3,2X,A1,2X,110A1)                                CONF 071
      JJ=0                                                              CONF 072
  170 CONTINUE                                                          CONF 073
      RETURN                                                            CONF 074
      END                                                               CONF 075
      SUBROUTINE POLA(NDEG,IN,X,Y,IOUT,Z)                               POLA 001
      DIMENSION Z(45)                                                   POLA 002
      M=IN                                                              POLA 003
      IF(NDEG.EQ.0)GO TO 3                                              POLA 004
      DO 2 K=1,NDEG                                                     POLA 005
      IX=K                                                              POLA 006
      IY=0                                                              POLA 007
      KK=K+1                                                            POLA 008
      DO 1 I=1,KK                                                       POLA 009
      Z(M)=(X**IX)*(Y**IY)                                              POLA 010
      IY=IY+1                                                           POLA 011
      IX=IX-1                                                           POLA 012
      M=M+1                                                             POLA 013
    1 CONTINUE                                                          POLA 014
    2 CONTINUE                                                          POLA 015
    3 IOUT=M                                                            POLA 016
      RETURN                                                            POLA 017
      END                                                               POLA 018
      SUBROUTINE PROB(F,M1,M2,Q)                                        PROB 001
C     *     *     *                                                     PROB 002
C     F-DISTRIBUTION                                                    PROB 003
C     COMPUTE APPROXIMATE PROBABILITY FROM F-VALUE                      PROB 004
C     COMPUTE  P FROM  Q(F,M1,M2)=P WHERE  F - F(M1,M2)                 PROB 005
C     SECTION 26. 6. 8. ON PAGE  946,  ABRAMOWITZ AND STEGUN (1964)     PROB 006
C     *     *     *                                                     PROB 007
      IF((F.LE.0.0).OR.(M1.LE.0).OR.(M2.LE.0))GO TO 150                 PROB 008
      A1=FLOAT(M1)                                                      PROB 009
      A2=FLOAT(M2)                                                      PROB 010
      IF((MOD(M1,2).EQ.1).AND.(MOD(M2,2).EQ.1))GO TO 60                 PROB 011
      X=A1*F+A2                                                         PROB 012
      X=A2/X                                                            PROB 013
      IF (MOD(M1,2).EQ.1)GO TO 30                                       PROB 014
C     *     *     *                                                     PROB 015
C     M2 IS ODD OR EVEN AND M1 IS EVEN                                  PROB 016
C     *     *     *                                                     PROB 017
      S=1.0                                                             PROB 018
      M=M1/2                                                            PROB 019
      IF(M.EQ.1)GO TO 20                                                PROB 020
      DD=1.0                                                            PROB 021
      Y=1.0-X                                                           PROB 022
      MM=M-1                                                            PROB 023
      DO 10 I=1,MM                                                      PROB 024
      XM=2.0*FLOAT(I)                                                   PROB 025
      YM=A2+XM-2.0                                                      PROB 026
      DD=DD*YM/XM                                                       PROB 027
      DD=DD*Y                                                           PROB 028
      S=S+DD                                                            PROB 029
   10 CONTINUE                                                          PROB 030
   20 PP=A2/2.0                                                         PROB 031
      Q=S*X**PP                                                         PROB 032
      GO TO 150                                                         PROB 033
C     *     *     *                                                     PROB 034
C    M1 IS ODD OR EVEN AND M2 IS EVEN                                   PROB 035
C     *     *     *                                                     PROB 036
   30 S=1.0                                                             PROB 037
      M=M2/2                                                            PROB 038
      IF(M.EQ.1)GO TO 50                                                PROB 039
```

```
      DD=1.0
      MM=M-1
      DO 40 I=1,MM
      XM=2.0*FLOAT(I)
      YM=A1+XM-2.0
      DD=DD*YM/XM
      DD=DD*X
      S=S+DD
   40 CONTINUE
   50 PP=A1/2.0
      PP=(1.0-X)**PP
      Q=1.0-PP*S
      GO TO 150
C     *     *     *
C     M1 AND M2  ARE ODD
C     *     *     *
   60 PH=3.14159265359
      THETA=SQRT(A1*F/A2)
      THETA=ATAN(THETA)
      SN=SIN(THETA)
      CN=COS(THETA)
      IF(M1.EQ.1)GO TO 100
      S=1.0
      M=M1-2
      IF(M.EQ.1)GO TO 80
      DD=1.0
      DO 70 I=3,M,2
      XM=FLOAT(I)
      YM=A2+XM-2.0
      DD=DD*YM/XM
      DD=DD*SN*SN
      S=S+DD
   70 CONTINUE
   80 N1=M2-1
      N2=N1/2
      Z=1.0
      DO 90 I=1,N2
      XM=FLOAT(N2+1-I)
      YM=XM-0.5
      Z=Z*XM/YM
   90 CONTINUE
      Z=Z*2.0/PH
      BB=Z*S
      BB=BB*SN*CN**M2
      GO TO 110
  100 BB=0.0
  110 IF(M2.EQ.1)GO TO 130
      M=M2-2
      DD=1.0/CN
      S=0.0
      DO 120 I=1,M,2
      J=I-1
      IF(J.EQ.0)J=1
      XM=FLOAT(J)
      YM=FLOAT(I)
      DD=DD*XM/YM
      DD=DD*CN*CN
      S=S+DD
  120 CONTINUE
      PP=SN*S+THETA
      AA=2.0*PP/PH
      GO TO 140
  130 AA=2.0*THETA/PH
  140 Q=1-AA+BB
  150 Q=(1.0-Q)*100.0
      RETURN
      END
      SUBROUTINE STEPINV(M,MM,A,B,NS)
      DIMENSION A(45,45),B(45,45),C(45),NS(45)
    3 DO 4 I=1,M
      C(I)=0.0
```

```
    4 CONTINUE
      DO 5 I=1,MM
      XX= ABS(A(I,I))
      IF((XX.LE.1.0E-8).OR.(NS(I).EQ.1))GO TO 5
      K=I
      GO TO 6
    5 CONTINUE
      GO TO 11
    6 XX=A(K,K)
      NS(K)=1
      C(K)=1.0/XX
      DO 7 J=1,M
      A(K,J)=A(K,J)/XX
    7 CONTINUE
      DO 9 I=1,M
      IF(I.EQ.K)GO TO 9
      XX=A(I,K)
      C(I)=-XX*C(K)
      DO 8 J=1,M
      A(I,J)=A(I,J)-A(K,J)*XX
    8 CONTINUE
    9 CONTINUE
      DO 10 I=1,M
      A(I,K)=C(I)
   10 CONTINUE
      GO TO 3
   11 K=0
      DO 12 I=1,MM
      IF(NS(I).EQ.-1)GO TO 12
      K=K+1
   12 CONTINUE
      IF(K.NE.MM)GO TO 14
      PRINT 13,MM
   13 FORMAT(1H0,"THE INPUT MATRIX HAS FULL RANK",I10)
      GO TO 16
   14 PRINT 15,K
   15 FORMAT(1H0,"THE INPUT MATRIX HAS A RANK",I13)
   16 DO 22 K=1,MM
      IF(NS(K).EQ.-1)GO TO 22
      DO 18 I=1,MM
      CC=0.0
      IF(NS(I).EQ.-1)GO TO 18
      DO 17 J=1,MM
      IF(NS(J).EQ.-1)GO TO 17
      CC=CC+A(K,J)*B(J,I)
   17 CONTINUE
      C(I)=CC
   18 CONTINUE
      DO 19 L=1,MM
      IF(L.EQ.K)GO TO 19
      CC=ABS(C(L))
      IF(CC.LE.1.0E-3)GO TO 19
      GO TO 20
   19 CONTINUE
   22 CONTINUE
      GO TO 23
   20 PRINT 21
   21 FORMAT(1H0,"WARNING - - - NOT SIGNIFICANT INVERSE MATRIX")
   23 RETURN
      END
      SUBROUTINE INVFDN(P,N1,N2,F)
C    *    *    *
C     INVERSE F-DISTRIBUTION
C     COMPUTE APPROXIMATE F FROM PROBABILITY
C     COMPUTE F FROM Q(F,N1,N2)=P WHERE F - F(N1,N2)
C     SECTION 26. 5.22. ON PAGE 945 AND SECTION 26. 6.16. ON PAGE 947
C                 ABRAMOWITZ AND STEGUN (1964)
C    *    *    *
      CALL INVNOR(P,Y)
      R1=1.0/(FLOAT(N1)-1.0)
      R2=1.0/(FLOAT(N2)-1.0)
```

```
      H=2.0/(R1+R2)                                                     INFD 012
      T=(Y*Y-3.0)/6.0                                                   INFD 013
      W1=(Y*SQRT(H+T))/H                                                INFD 014
      W2=(R1-R2)*(T+5.0/6.0-2.0/(3.0*H))                                INFD 015
      W=2.0*(W1-W2)                                                     INFD 016
      F=EXP(W)                                                          INFD 017
      RETURN                                                            INFD 018
      END                                                               INFD 019
      SUBROUTINE INVNOR(P,XP)                                           INNO 001
C     *     *     *                                                     INNO 002
C     INVERSE NORMAL DISTRIBUTION                                       INNO 003
C     COMPUTE APPROXIMATE XP FROM PROBABILITY                           INNO 004
C     COMPUTE XP FROM Q(XP)=P WHERE XP - N(0,1)                         INNO 005
C     SECTION 26. 2.23. ON PAGE 933 ,  ABRAMOWITZ AND STEGUN (1964)     INNO 006
C     *     *     *                                                     INNO 007
      Q=P                                                               INNO 008
      IF(P.GT.0.5)Q=1.0-P                                               INNO 009
      CI=2.515517                                                       INNO 010
      CJ=0.802853                                                       INNO 011
      CK=0.010328                                                       INNO 012
      DI=1.432788                                                       INNO 013
      DJ=0.189269                                                       INNO 014
      DK=0.001308                                                       INNO 015
      TT=1.0/(Q*Q)                                                      INNO 016
      TT=ALOG(TT)                                                       INNO 017
      T=SQRT(TT)                                                        INNO 018
      UP=CI+CJ*T+CK*T*T                                                 INNO 019
      DN=1.0+DI*T+DJ*T*T+DK*T*T*T                                       INNO 020
      XP=T-UP/DN                                                        INNO 021
      IF(P.GT.0.5)XP=-XP                                                INNO 022
      RETURN                                                            INNO 023
      END                                                               INNO 024
      SUBROUTINE INVTDN(P,N,T)                                          INTD 001
C     *     *     *                                                     INTD 002
C     INVERSE T-DISTRIBUTION                                            INTD 003
C     COMPUTE APPROXIMATE T FROM PROBABILITY                            INTD 004
C     COMPUTE T FROM Q(T,N)=P WHERE T - T(N)                            INTD 005
C     SECTION 26. 7. 5. ON PAGE 949 ,  ABRAMOWITZ AND STEGUN (1964)     INTD 006
C     *     *     *                                                     INTD 007
      R=FLOAT(N)                                                        INTD 008
      CALL INVNOR(P,X)                                                  INTD 009
      D1=4.0                                                            INTD 010
      D2=96.0*R                                                         INTD 011
      D3=384.0*R*R                                                      INTD 012
      D4=92160.0*R*R*R                                                  INTD 013
      U1=1.0+X*X                                                        INTD 014
      U2=3.0+16.0*X*X+5.0*(X**4.0)                                      INTD 015
      U3=-15.0+17.0*X*X+19.0*(X**4.0)+3.0*(X**6.0)                      INTD 016
      U4=-945.0-1920.0*X*X+1482.0*(X**4.0)+776.0*(X**6.0)+ 79.0*(X**8.0)INTD 017
      TT=U1/D1+U2/D2+U3/D3+U4/D4                                        INTD 018
      T=X+(TT*X)/R                                                      INTD 019
      RETURN                                                            INTD 020
      END                                                               INTD 021
```