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# AUTOMATIC CONTOURING OF GEOPHYSICAL DATA USING BICUBIC SPLINE INTERPOLATION 

(Report and 7 figures)
M. T. Holroyd and B. K. Bhattacharyya


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#### Abstract

A method for automatic contouring of two-dimensional geophysical data is presented in this paper. The method is based on piecewise bicubic spline representation of uniformlygridded data points derived from irregularly-spaced observations. The testing of this method has been carried out with a set of data digitized along flight lines at points of intersection with contour lines on a published aeromagnetic map. The map is conspicuous by the presence of steep gradient as well as broad, smoothly-varying regions. A critical examination of the machine-contoured map with respect to the published hand-drawn map establishes the accuracy and reliability of the automatic contouring program. In this paper a short account of bicubic splines is followed by a description of the contouring program, the input data required by the program, and the results.


# AUTOMATIC CONTOURING OF GEOPHYSICAL DATA USING BICUBIC SPLINE INTERPOLATION 

## INTRODUCTION

Basic geological and geophysical data, when available in twodimensional form, are normally presented as a contoured map. The map compiler relies basically on linear interpolation between data points and manual smoothing. In the process of contouring the compiler is greatly influenced by the trends and characteristic patterns apparent in the data. So the contoured map is essentially a product of his subjective interpretation of the patterns in the data, his ability to perform rapid linear interpolation and his skill in smoothing. The accuracy with which the map is drawn determines to a large degree the nature of application of the map to the problem of delineating geological features. For example, on an aeromagnetic map, the broad characteristics such as the direction and extent of a particular trend and the linear feature corresponding to a major fault stand out prominently and the degree of accuracy of the map can, therefore, be relaxed considerably. However, for detailed geological mapping and for thorough investigation of important anomalies, various quantitative methods of interpretation have to be used. Many such methods employ not only the amplitudes of the magnetic field but also its gradient and curvature. Thus in order that the results of interpretation become reliable, the amplitude, gradient and curvature of the field should be accurately maintained in the map. To achieve this objective, the interpolation scheme used for the production of aeromagnetic maps should provide continuity of the field, its gradient and curvature at all the data points and should introduce the minimum amount of smoothing.

An automatic machine contouring method has some distinct advantages over manual contouring. Firstly, there is a reduction in time between the acquisition of data and its presentation in the form of a map. Secondly, the map is contoured objectively. Thirdly, accurate interpolation schemes can be used. This third factor is the most important of all because the interpolated positions of the contours between the original data points determine essentially the accuracy of the map.

There have been a few papers recently on automatic contouring of twodimensional data. Smith (1968) proposed a method of interpolation in which a circle or an ellipse of specified shape and angle of rotation is used to define a neighbourhood around a grid point. An average of the values of the independent variable in this neighbourhood, each weighted by the reciprocal of distance from the grid point or by the reciprocal of square of that distance or by some mixture of the two, is computed and stored at the grid point. Smith also tried to use a priori information on strike of the causative body to influence contouring.

Pelto et al. (1968) described a method of automatic contouring of irregularly spaced data. The method consists of three steps. Firstly, in

[^0]regions with few or no observations, control values.are interpolated by either linear or nonlinear methods. Secondly, a regional polynomial surface, commonly of eighth or twelfth degree, is fitted by least squares to the original and interpolated control points. Thirdly, a moving, weighted, least squares surface, which is essentially a continuous surface of first or second degree, with smoothly varying parameters, is forced to pass through the deviations between the regional surface and the observations. In this process all data points are weighted according to an inverse power of distance from the grid point. The final product of this method of contouring looks very similar to hand-drawn maps for smoothly-varying data.

It is obvious that the quality of the contoured map depends largely on the method of interpolation used, not only in the reduction of irregularlydistributed data to equispaced form but also in determining the co-ordinates of the points along individual contours. Crain and Bhattacharyya (1967) evaluated various methods of interpolation currently in use at many places, developed some new methods and published the results of application of these methods to actual aeromagnetic data. Of all the methods, only two, the Gram-Schmidt orthogonalization procedure and the quadratic, weighted-average method, are found to be satisfactory for interpolation in regions characterized by smooth and gentle variations of the observed variable. The results obtained with these methods in the presentation of small-scale features and large gradients in an aeromagnetic map are, in general, not of high quality and cannot surpass the performance achieved by manual contouring of the data.

For the above reasons a new technique using spline interpolation (Bhattacharyya, 1969) has been recently developed for computing values of an observed variable at equispaced points along two orthogonal directions with the help of irregularly-distributed data. The interpolation technique applied to aeromagnetic data obtained along nonlinear flight lines shows high resolution by maintaining the separation of neighbouring anomalies and small-scale features. The shapes, peaks and troughs of both large and small amplitude anomalies are faithfully reproduced. The gradients of the magnetic field are not found to undergo any appreciable distortion.

The excellent quality of the results obtained with spline interpolation suggests its use in the possible development of a good automatic method for contouring two-dimensional geophysical data. A method so developed is presented in this paper. The details of treatment of irregular observations for computing field values at regular grid points are given in the paper by Bhattacharyya (1969). In this paper it is therefore assumed that regular gridded data in two orthogonal directions are readily available. A short account of bicubic splines which are extensively used in the contouring method, precedes the description of the contouring program, the data to be contoured and the results. Finally, a listing of the program in FORTRAN language written for IBM 360 system is given.

## BICUBIC SPLINES

Thin elastic splines are used by a draftsman to draw a smooth curve through a number of given points under the constraint of maintaining the continuity of slopes and curvatures along the curve. The bicubic spline interpolation method is a numerical analog of the draftsman's method for plotting curves in two dimensions. The interpolation scheme essentially generates piecewise cubic polynomials representing the field function in every interval between points of observation. For this generation, the scheme utilizes the continuity of the function and its first two successive derivatives at all data points. The potential strength of this method lies in the fact that the function and its slope and curvature are made continuous throughout the area of observations.

Let the values of a function $g(x, y)$ of two variables ( $x, y$ ) be given at mesh-points $i=0,1, \ldots . ., M$ and $j=0,1, \ldots, N$. At the point ( $1, j$ ) the value of the function is denoted by $g_{1 j}$. The plecewise polynomial function, defined in each rectangular cell

$$
\mathrm{x}_{\mathrm{i}-1} \leq \mathrm{x} \leq \mathrm{x}_{\mathrm{i}}, \mathrm{y}_{\mathrm{j}-1} \leq \mathrm{y} \leq \mathrm{y}_{\mathrm{j}}
$$

of the grid, is of the following form

$$
\begin{equation*}
r_{i j}=\sum_{m=0}^{3} \sum_{n=0}^{3} a_{m n}^{i j}\left(x-x_{i-1}\right)^{m} \cdot\left(y-y_{j-1}\right)^{n} \tag{1}
\end{equation*}
$$

In order to determine the coefficients $\mathrm{a}_{\mathrm{mm}}^{\mathrm{ij}}$, it is required to know the derivatives at the boundary points of each cell, $1 . e .$,

$$
\begin{equation*}
p_{i j}=g_{x}\left(x_{i}, y_{j}\right), q_{i j}=g_{y}\left(x_{i}, y_{j}\right), \tag{2}
\end{equation*}
$$

and

$$
s_{i j}=g_{x y}\left(x_{i}, y_{j}\right),
$$

where the variables with respect to which $g(x, y)$ has been differentlated, are indicated in the subscripts of $g$.

Let there be known values of

$$
\begin{aligned}
P_{i j}, i & =0, M ; j=0,1, \ldots, N \\
q_{i j}, i & =0, I, \ldots M ; j=0, N, \\
\text { and } s_{i j}, i & =0, M ; j=0, N .
\end{aligned}
$$

Then it may be shown (DeBoor, 1962) that for the above values and the values of gi.j at the mesh-points, there exists one and only one piecewise bicubic polynomial of the form (1).

If $g_{i j}, P_{i j}, q_{i j}$ and $s_{i j}$ are known at all the mesh-points and if the station-spacing is assumed to be unity, the coefficients am are given by

$$
\begin{aligned}
& a_{00}^{i j}=g_{i-1, j-1}, a_{10}^{i j}=p_{i-1}, j-1 \\
& a_{01}^{i j}=q_{1-1, j-1}, a_{11}^{i j}=s_{i-1, j-1} \\
& a_{02}^{i j}=\left[3\left(g_{i-1, j}-g_{1-1, j-1}\right)-\left(q_{1-1, j}+2 q_{1-1, j-1}\right)\right], \\
& a_{03}^{i j}=\left[\left(q_{1-1, j}+q_{1-1, j-1}\right)-2\left(g_{i-1, j}-g_{1-1, j-1}\right)\right], \\
& a_{12}^{i j}=\left[3\left(p_{i-1, j}-p_{i-1, j-1}\right)-\left(s_{1-1, j}+2 s_{1-1, j-1}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
a_{13}^{i j} & =\left[\left(s_{i-1, j}+s_{1-1, j-1}\right)-2\left(p_{i-1, j}-p_{i-1, j-1}\right)\right], \\
a_{20}^{i j} & =\left[3\left(g_{i, j-1}-g_{i-1, j-1}\right)-\left(p_{i, j-1}+2 p_{i-1, j-1}\right)\right], \\
a_{21}^{i j} & =\left[3\left(q_{i, j-1}-q_{i-1, j-1}\right)-\left(s_{i, j-1}+2 s_{i-1, j-1}\right)\right], \\
a_{22}^{i j} & =9 \gamma_{1}-3 \gamma_{2}-3 \gamma_{3}+\gamma_{4}, \\
a_{23}^{i j} & =-6 \gamma_{1}+2 \gamma_{2}+3 \gamma_{3}-\gamma_{4}, \\
a_{30}^{i j} & =\left[\left(p_{i}, j-1+p_{i-1, j-1}\right)-2\left(g_{1, j-1}-s_{i-1, j-1}\right)\right], \\
a_{31}^{i j} & =\left[\left(s_{i, j-1}+s_{i-1, j-1}\right)-2\left(q_{i, j-1}-q_{i-1, j-1}\right)\right], \\
a_{32}^{i j} & =-6 \gamma_{1}+3 \gamma_{2}+2 \gamma_{3}-\gamma_{4}, \\
\text { and } a_{33}^{i j} & =4 \gamma_{1}-2 \gamma_{2}-2 \gamma_{3}+\gamma_{4},
\end{aligned}
$$

where $\gamma_{1}=\left(g_{i j}+g_{i-1, j-1}-g_{i, j-1}-g_{i-1, j}\right)+\left(p_{i-1, j-1}-p_{i-1, j}+\right.$

$$
\begin{aligned}
& \left.\quad q_{i-1, j-1}-q_{i, j-1}\right)+s_{i-1, j-1} \\
& r_{2}=p_{i-1}, j-1-p_{i, j-1}-p_{i-1, j}+s_{i-1, j-1}-s_{i, j-1}, \\
& \gamma_{3}=q_{i j}+q_{i-1, j-1}-q_{i-1, j}-q_{i, j-1}+s_{i-1, j-1}-s_{i-1, j}, \\
& \text { and } \gamma_{4}=s_{i j}+s_{i-1, j-1}-s_{i, j-1}-s_{i-1, j} .
\end{aligned}
$$

The derivatives in the above set of equations for the coefficients can be evaluated by assuming the continuity of the following derivatives at all the interior mesh-points:

$$
\begin{aligned}
& g_{x x}\left(x_{i}, y_{j}\right), g_{y y}\left(x_{i}, y_{j}\right), \text { and either } \\
& g_{x^{2} y}\left(x_{i}, y_{j}\right) \text { or } g_{x y^{2}}\left(x_{i}, y_{j}\right)
\end{aligned}
$$

The continuity of $g_{x} 2 y\left(x_{i}, y_{j}\right)$ has been used in the calculations presented in this paper. The scheme used for evaluating $P_{i j}, q_{i j}$ and $s_{i j}$ are briefly indicated in the following:
(i) Calculation of $p_{i j}$ :

For each of the lines $j=0,1, \ldots N$, the observed values $g_{i j}$ are represented by piecewise cubic polynomials in each of the intervals ( $x_{i}-1, x_{i}$ ), 1 = 1 , ...., N. Assuming the continuity of second derivatives at all the knots, we have (Bhattacharyya, 1969)

$$
\begin{align*}
p_{i+1, j}+4 p_{i, j}+p_{i-1, j} & =3\left(g_{1+1, j}-g_{i-1, j}\right),  \tag{3}\\
1 & =1, \ldots M-1 .
\end{align*}
$$

The two other equations necessary for solving all the $p_{i}$ 's are provided by the conditions

$$
\begin{equation*}
g_{x x}\left(x_{i}, y_{j}\right)=0, i=0, M ; j=0,1, \ldots N \tag{4}
\end{equation*}
$$

The assumption of the second derivatives being equal to zero at the end points of a given profile keeps the integral-square measure of approximation to the second derivative at a minimum value (Holladay, 1957; Walsh et al., 1962).
(ii) Calculation of $q_{i j}$ :

The equations for $q$ are similar to those of $p$ and are given below:

$$
\begin{align*}
q_{i, j+1}+4 q_{i j}+q_{i, j-1} & =3\left(g_{i, j+1}-g_{i, j-1}\right), \\
j & =1, \ldots, N-1 ; i=0, \ldots, M . \tag{5}
\end{align*}
$$

and

$$
\begin{equation*}
g_{y y}\left(x_{i}, y_{j}\right)=0, i=0,1, \ldots M ; j=0, N \tag{6}
\end{equation*}
$$

(iii) Calculation of $s_{i j}$ :

For computing $s_{i j}$, the values of $p$ and $q$ in (3) and (5) are used:
(a) For $\mathrm{j}=0, \mathrm{~N}$

$$
s_{i+1, j}+4 s_{i j}+s_{i-1, j}=3\left(q_{i+1, j}-q_{i-1, j}\right), i=1, \ldots, M-1
$$

and

$$
\begin{equation*}
g_{x^{2} y}\left(x_{i}, y_{j}\right)=0, i=0, M ; j=0, N \tag{8}
\end{equation*}
$$

(b) For $1=0,1, \ldots, M$

$$
s_{i, j+1}+4 s_{i j}+s_{i, j-1}=3\left(p_{i, j+1}-p_{i, j-1}\right), j=1, \ldots, N-1
$$

The tridiagonal linear equations (3)-(9) are solved by standard methods. With the values of $p, q$, $s$ thus determined it is now very straightforward to evaluate all the bicubic spline coefficients.

## DETAILS OF THE CONTOURING PROGRAM

A. CONTROL CARDS:

These cards control the details of execution of the program by specifying the data grid, strip height, block width, subgrid numbers, scale, contour interval and title of the map. Some of these parameters are self-explanatory and the rest will be explained in the following paragraphs. Control parameters also determine whether the whole map or one detailed section of the map will be drawn. The total number of blocks per strip and strips per map and their physical dimensions are computed in the program. As will be noted later, various other optional features are available in the program.

## B. BICUBIC SPLINE COEFFICIENTS:

The bicubic spline coefficients are calculated in one operation for the basic data covering the whole area under study. There are sixteen coefficients for each rectangular cell defined by four adjacent mesh points. These coefficients are written on a magnetic tape sequentially from left to right for each row of cells beginning with the southernmost row.

Requirements for computer core storage have to be an acceptable maximum and provision has to be made in the program for coping with large and variable amounts of basic data. It is, therefore, not possible in the majority of cases to consider the whole area of the map for contouring in a single operation. Consequently, the given area is normally divided into a number of strips (Fig. 1a). A strip is defined by the whole width of the area and a specified number of cells in height. A strip is broken up into a number of blocks, each block covering the full height of the strip and a specified number of cells in width (Fig. 1b). This partitioning of the area is needed to keep the working storage requirements for contouring within prescribed limits. In order to achieve this objective, the height of the strip must be inversely proportional to the width. This scheme may occasionally give rise to strips of very small height and exceedingly large width. It is then necessary to divide the original rows of rectangular cells into smaller units of selected width and write the bicubic spline coefficients in the output magnetic tape in the fashion described before for each of these units. The perfect matching of the contours at the boundaries of the blocks is guaranteed by the way spline coefficients are calculated for the basic data.

It is now evident that regardless of the number of units into which the area is divided, the spline coefficients for a specific strip are considered alone for the process of contouring. When the contouring for the strip is completed, the spline coefficients for the next strip are obtained from the input tape for computation.

At this stage it should be noted that a control parameter, called subgrid spacing (Fig. 1c), which is a small fraction of the side of a rectangular cell, is so chosen that linear interpolation between two points separated by the subgrid spacing is valid anywhere in the whole area. The smoothness of the contoured map and the computation time depend on this parameter.

Values of the field to be plotted are calculated at every subgrid interval around the periphery of the block and along certain selected lines crossing the block parallel to its lower boundary. To trace the contours across the block, values of the field are calculated with the bicubic spline coefficients only at those subgrid points which lie in the neighbourhood of the passing contours. The locations of points on the contours are determined by linear interpolation between the values at subgrid points.


## C. STEPS OF COMPUTATION IN A BLOCK:

The block at the extreme left of the strip is first considered. The major steps of computation in a block are given below:
(i) Search for relative maxima and minima:

The data values are searched for relative maxima and minima from left to right row by row beginning with the southernmost row (Fig. 1d, e). The value of the field at a data point is compared with the values of the mesh points immediately adjacent to it for establishing a relative maximum or minimum (SUBROUTINE MAXMIN). The gradient of the field in the two orthogonal directions at such a point, calculated with the help of the spline coefficients, are used to locate correctly the cell containing the relative maximum or minimum point. If the cell lies within the block, the exact location of this point is determined by linear interpolation between the field values at. the subgrid points within the cell. The significance of this maximum or minimum point with regard to the process of contouring is then tested and it is rejected if no contours traverse between the point and either the block boundary or the horizontal line through the previous maximum or minimum. The locations and values of all significant points are stored for future reference.
(ii) Location of contour intercepts:

With the help of bicubic spine coefficients the field values are calculated at all points separated by the subgrid interval along a 'search path' (Fig. If). These values are then used to find the absolute maximum and minimum within the given block and thus to ascertain the range of contours within the block. The starting point of the 'search path' is at the southwest corner of the block. This path first traverses anticlockwise along the boundary of the block and then from left to right along the horizontal lines passing through the significant maxima and minima points. The alternate points of intersection of each contour value with this path are determined by linear interpolation of the field values at subgrid points. The locations of these intersections are recorded as linear distances from the starting point to the contour intercepts along the 'search path'. If the range of contour values within the block is too great for the reserved working storage, the range is suitably divided into smaller groups. Each group is treated separately (SUBROUTINE CFIND).
(iii) Contour tracing:

In this step the locations of contour intercepts, as stored above, are converted from a linear co-ordinate to ( $x, y$ ) co-ordinates (SUBROUTINE CSTART). The field values at only those subgrid points which are adjacent to the path of a contour to be traced, are calculated with the appropriate bicubic spline coefficients (Fig. 1g). These values are used to compute the ( $x, y$ ) co-ordinates of the points necessary for drawing a particular contour. It is to be noted that by considering only the subgrid points in the vicinity of desired contours, computation time is made inversely proportional to the subgrid interval rather than to the square of the interval. The field values at the subgrid points are stored until all the contours in the block are traced.

In the case of contours originating on the boundary of the block, the storage of their alternate intersections with the search path precludes the possibility of a contour being traced twice or missed altogether. Because of this particular process of storage, contours originating on one of the internal maximum or minimum lines could not be missed but might be traced more than once. To prevent the possibility of retracing the same contour, any stored contour intercept on a maximum or minimum line, once considered in the process of tracing, is cancelled. Tracing is terminated when either a boundary is encountered or a contour returns to its starting point.

Due to sharp changes in horizontal gradients of the field, it is often not feasible to trace all the contours according to the contour interval specified in the control card. To avoid tracing a host of contours which will never appear on the map, the local contour density is checked every tenth point along selected contours. This density determines the contours to be traced. The method of checking the density will be described later.

For each point on a contour, its ( $x, y$ ) co-ordinates and the difference between the field values at the two adjacent subgrid points are stored (SUBROUTINE CTRACE).
(iv) Contour drawing and labelling:

In the case of geological and geophysical data, particularly aeromagnetic data, a large variation in horizontal gradient is normally found over an area. A procedure is, therefore, incorporated in the program to achieve adequate contouring in regions of low gradient and to avoid crowding of the contours in places of high gradient.

As has been noted in the previous section, the differences between the field values at subgrid points lying on both sides of traced contours are stored. With this array of differences the contour density about each point on the contour is estimated. This density is averaged over a moving window of several points. The average value at a point of the contour is used to determine whether the contour shall be drawn, terminated or omitted. Values of the contour density are averaged in order to smooth out its fluctuations which may cause repeated termination and recomencement of a contour, thus simulating a dashed line. An input parameter allows the size of the moving window to be varied.

If the minimum value of the contour density at a series of points on a contour allow a label to be written without overwriting adjacent contours, a suitable short segment of the contour is replaced by a label. The centres of the numerals which make up the label, lie on a parabola fitted to the ends and midpoint of the omitted contour segment. Due to the necessary simplicity of the tests applied, labels may occasionally overwrite adjacent contours or labels. This problem may be minimized by narrowing the criteria for insertion of labels, thus reducing their total number (SUBROUTINES CDRAW and CLABEL). One of the control parameters read as input to the contouring program specifies the minimum separation between adjacent contours which permits a label to be inserted. This separation is expressed in terms of label numeral height. Five orders of contour are defined within the program. Different types of line are drawn to distinguish between the orders. They are given below:

| Order | Type of line |  |
| :---: | :--- | :--- |

The types of line and corresponding multiples of basic contour interval are specified as control parameters of the program.

## D. PROGRAM OPTIONS:

In the initial stages of development of the contouring program, it was attempted to determine automatically an appropriate subgrid spacing for each individual block. The values of the curvature of the field obtained from the bicubic spline coefficients were tested along both axial directions at each of the data points. The maximum value of these curvatures was used to determine
the suitable subgrid spacing. The resulting contours were remarkably smooth in every block and the computation time was kept to a reasonable magnitude. However, it was noted that contours did not match perfectly at the boundary of two adjacent blocks with different subgrid spacings. Moreover, highly complex data may result in extremely fine or coarse subgrid spacing in some regions. Because of these reasons, the method has not been used in the program presented in this paper. This method is, however, recommended for use with data having not too many big variations or in cases of production of preliminary maps and large scale maps of a single anomaly.

Sometimes after production of a map it is found that for detailed studies different sections of the map have to be replotted in a smaller scale. For this reason it is a good idea to split the program in two stages. In the first stage all possible contours are traced and the arrays defining them are written on a magnetic tape. In the second stage this tape is used for plotting the desired contours according to the required scale and labeling. In this way the whole map or any particular section can be drawn with different combinations of scale, contour interval, type of line and labelling with modest computation time and storage requirement.

## TEST OF THE CONTOURING PROGRAM

DATA:
In Figure 2 a redrafted version of an aeromagnetic map published by the Geological Survey of Canada is presented. The contouring of this map has been done manually. The basic data needed for production of this map has been used for testing the efficiency of the automatic contouring program. It is to be noted that the same set of data played an important role in the studies of different interpolation methods applicable to nonuniformly spaced data (Crain and Bhattacharyya, 1967; Bhattacharyya, 1969). The main reason for using these data in all these studies is the conspicuous presence of very steep gradients as well as broad, smoothly-varying regions of the map.

The flight lines spaced about one-half mile apart are indicated by thin lines in Figure 2. The data in the map were digitized only along flight lines at points of intersection with contour lines with the aid of an electronic coordinatograph. Figure 3 presents the locations of the digitized data points marked by crosses and the best-fitting straight lines passing through them. It is evident from these two figures that the spacings between flight lines are not uniform, varying from one-quarter of a mile to 0.69 mile. However, for a great majority of the lines, the line-spacing is approximately one-half mile.

With the help of the method described in the paper by Bhattacharyya (1969), the nonequispaced data was reduced to regular gridded data. The area containing this regular data is outlined by dashed lines in Figure 2. The data available for the contouring program consists of 137 points by 119 points in the north-south and east-west directions respectively at a uniform spacing of one-eighth of a mile. Because it was found in earlier studies that a spacing of one quarter mile was sufficient for depicting practically all the details of the map, a set of data containing 69 points by 60 points at a uniform interval of one-quarter of a mile is extracted from the regular data for the present study.

ANALYSIS AND RESULTS:
The bicubic spline coefficients for the whole set of data are computed in one operation. Figure 4 presents a continuous profile along a least squares line, as shown in Figure 3, calculated with the help of the spline coefficients. The dots in the figure indicate the original flight line data values


Figure 2. An aeromagnetic map from the Canadian Northwest Territories contoured in units of 100 gammas.
along the same profile. The correspondence between the actual and interpolated values is regarded as extremely good. The discrepancy between the two sets of values around some of the peaks is due to the insufficiency of the chosen grid spacing for proper definition of the peaks. However, to decrease the spacing for a better matching of the two curves will normally increase the total number of data points to an unmanageable level. The amount of smoothing introduced by spline interpolation at the peaks is, therefore, considered tolerable for practically all purposes and a reasonable compromise between conflicting requirements.

Within the automatic contouring program the data is contoured as 42 independent blocks. The complete machine-contoured map is shown in Figure 5. In the original scale of one inch to one mile of the published aeromagnetic map, it is not possible to detect the block boundaries by visual inspection.


Flgure 3. Best-fitting straight lines approximating actual flight lines from Figure 2 with locations of data marked by crosses.

To trace the contours in Figure 5, values of the field are calculated with the bicubic spline coefficients at the subgrid points being immediately adjacent to the passing contours. The subgrid interval has been chosen to be $1 / 32$ inch. The final tracing of the contours is done by linear interpolation between subgrid values. The positions of the contours so determined may differ from those calculated exactly with bicubic spline interpolation by less than the thickness of the drawn line in the worst case and by a small fraction of this thickness in the majority of cases.

The basic contour interval in Figure 5 is 100 gammas. The interval between adjacent contours increases gradually with the horizontal gradient in the map from 100 gammas to a maximum of 5,000 gammas in regions of highest gradient at steps of 200,500 and 1,000 gammas. As is evident in Figure 4, this automatic increase in contour interval up to 50 times the specified basic value does not give rise to any region in the map with excessive or inadequate coverage.

Figure 4. A continuous profile along one of the least squares lines in Figure 3 calculated with the help


Figure 5. Machine-contoured map of the digitized data, the locations of which are given by Figure 3. The basic contour interval is 100 gammas.


Figure 6. A northwesterly-trending group of anomalies with very steep gradients. This diagram shows the effects of the contour termination and labelling.

The very high resolution of the map presented in Figure 5 is borne out by the separation of neighbouring anomalies and good representation of smallscale features. The shapes of the anomalies are well maintained. The peaks and troughs of both large- and small-scale anomalies are faithfully reproduced. The gradients of the magnetic field in Figure 5 appear to be correct and this is very important from the point of quantitative interpretation of data.

The machine-contoured map (Fig. 5) differs in one respect from the hand-drawn map (Fig. 2). There are more closures of contours and so more isolated anomalies in Figure 5 in comparison to those in Figure 2. An examination of the hand-drawn map reveals a tendency on the part of the compiler to maintain trends beyond the supporting ability of the data. This tendency results in linking a series of contour intercepts across several flight lines by a single continuous contour rather than closing off the contour between flight lines in the absence of definite indication from the data. So it is concluded that Figure 5 presents a more objectively contoured version of the available data than does Figure 2.


Figure 7. A region with a relatively small variation of the magnetic field.

Let us now consider two of the major features of the original aeromag netic map. There is a northwesterly-trending group of anomalies starting at an approximate location of $61^{\circ} 38^{\prime} \mathrm{N}$ and $94^{\circ} 30^{\prime} \mathrm{W}$. Figure 6 shows details of these anomalies from the machine-contoured map. This also illustrates the effects of the contour termination and labeling.

An area with a relatively small variation of the magnetic field is centred at $61^{\circ} 32^{\prime} \mathrm{N}$ and $94^{\circ} 51^{\prime \prime} \mathrm{W}$. The machine-contoured map of this area (Fig. 7) seems to be adequate and reasonably good.

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